1.6 FORCES ON PLANES

Total Pressure and Centre of Pressure

Total Pressure: It is defined as the force exerted by static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always at right angle (or normal) to the surface.

Centre of Pressure: It is defined as the point of application of the total pressure on the surface. Now we shall discuss the total pressure exerted by a liquid on the immersed surface. The immersed surfaces may be:

1. Horizontal plane surface
2. Vertical plane surface
3. Inclined plane surface
4. Curved surface

Derivation of total pressure

In order to determine the total pressure, we will consider the object in terms of small strips as displayed here in following figure. We will determine the force acting on small strip and then we will integrate the forces on small strips for calculating the total pressure or hydrostatic force on object.

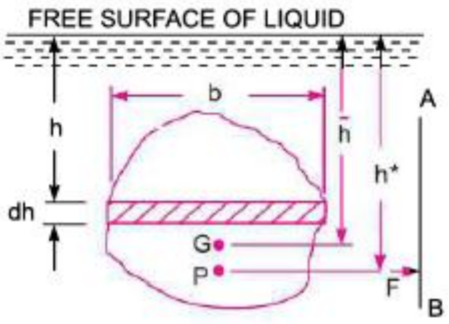


Figure 1.6.1 Vertical plane Immersed surface

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 70]

Let us consider the small strip of thickness dh, width b and at a depth of h from free surface of liquid as displayed here in above figure.

Intensity of pressure on small strip, dp = ρgh

Area of strip, dA = b x dh

Total pressure force on small strip, dF = dP x dA

Total pressure force on small strip, dF = ρgh x b x dh

Total pressure force on whole surface, F = Integration of dF

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $$\int b \times h \times dh = \int h \times dA$$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$$= A \times \bar{h}$$

$$\therefore \quad F = \rho g A \bar{h}$$

Where,

ρ = Density of liquid (Kg/m^3)

g = Acceleration due to gravity (m/s^2)
A = Area of surface (m^2)
ħ = Height of C.G from free surface of liquid (m)

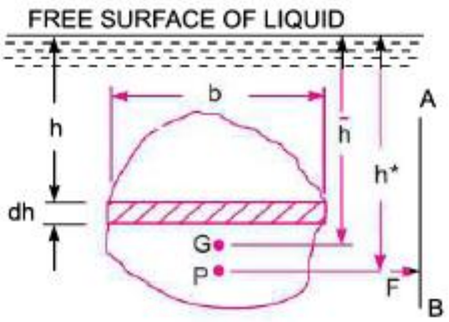
Unit of total pressure

As total pressure is basically a hydrostatic force and therefore total pressure will be measured in terms of N or KN.

Centre of pressure

Centre of pressure is basically defined as a single point through which or at which total pressure or total hydrostatic force will act.

Let us consider that we have one tank filled with liquid e.g. water. Let us consider that there is one object of arbitrary shape immersed inside the water as displayed here in following figure.



Let us consider G is the centre of gravity and P is the centre of pressure. ħ is the height of C.G from free surface of liquid and h* is the height of centre of pressure from free surface of liquid.

Derivation of Centre of Pressure

In order to determine the centre of pressure, we will consider the object in terms of small strips as displayed here in above figure.We will use the concept of "principle of moments" to determine the centre of pressure.

According to the principle of moments, moment of the resultant force about an axis will be equal to the sum of the moments of components about the same axis.

As we have shown above in figure, total hydrostatic force F is applied at centre of pressure P which is at height of h* from the free surface of liquid.

Therefore, let us determine the moment of resultant force F about the free surface of liquid and it will be determined as F x h*.

As we have considered here the object in terms of small strips as displayed here in above figure and hence we will determine the moment of force dF acting on small strip about the free surface of liquid.

$$\text{Moment of force } dF = dF \times h$$
$$\text{Moment of force } dF = \rho\, g\, h \times b\, dh \times h$$

Let us sum of all moments of such small forces about the free surface of liquid and it will be written as mentioned here.

$$= \int \rho g h \times b \times dh \times h = \rho g \int b \times h \times h dh$$

$$= \rho g \int b h^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \quad bdh = dA)$$

But $\int h^2 dA = \int b h^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$= \rho g I_0$

$$F \times h^* = \rho g I_0$$

But $F = \rho g A \bar{h}$

$\therefore$ $\rho g A \bar{h} \times h^* = \rho g I_0$

or $h^* = \dfrac{\rho g I_0}{\rho g A \bar{h}} = \dfrac{I_0}{A \bar{h}}$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \overline{h^2}$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

$$h^* = \frac{I_G + A\overline{h^2}}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Total Pressure and Centre of Pressure for Inclined Plane Surface Immersed in a Liquid

Centre of pressure for inclined plane surface submerged in liquid will be given by

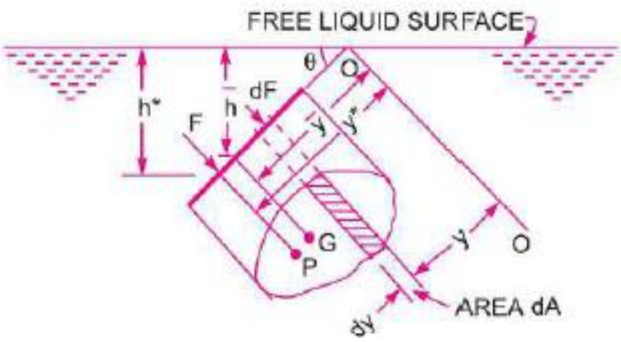


Figure 1.6.2 Inclined Immersed surface

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 86]

Let us consider that we have following data from above figure.
A = Total area of inclined surface
ħ = Height of centre of gravity of inclined area from free surface
h* = Distance of centre of pressure from free surface of the liquid
θ = Angle made by the surface of inclined plane with free surface of the liquid

Total pressure which is basically defined as the hydrostatic force applied by a static fluid on a plane or curved surface when fluid will come in contact with the surfaces.

Total pressure for inclined plane surface submerged in liquid will be given by following formula as mentioned here.

Total pressure = ρ g A ħ

Centre of pressure is basically defined as a single point through which or at which total pressure or total hydrostatic force will act.

Centre of pressure for inclined plane surface submerged in liquid will be given by following formula as mentioned here.

$$\text{Centre of pressure, } h^* = \frac{I_G \sin^2\theta}{A\hbar} + \hbar$$

For a vertical plane submerged surface, θ = 90

Total Hydrostatic Force on Curved Surfaces

Let us consider a curved surface AB sub-merged in a static liquid as displayed here in following figure.

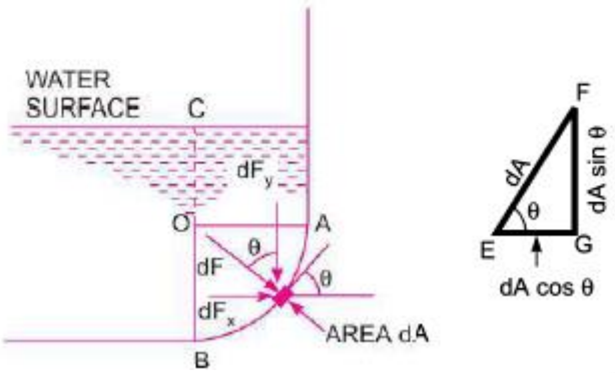


Figure 1.6.3 Curved surface sub-merged in a static liquid
[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 98]

Let us consider one small strip area dA at a depth of h from free surface of liquid. We have following data from above figure.

A = Total area of curved surface
ρ = Density of the liquid
g = Acceleration due to gravity

Pressure intensity on small area dA = ρ g h

Hydrostatic force on small area dA will be given by following formula as mentioned here.

$$dF = \rho\ g\ h \times dA$$

Direction of this hydrostatic force will be normal to the curved surface and will vary from point to point. Therefore, in order to secure the value of total hydrostatic force we will not integrate the above equation.

We will secure the value or expression for total hydrostatic force on curved surface by resolving the force dF in its two components or we can say that dF force will be resolved in X direction i.e. dFx and in Y direction i.e. dFy.

$$dFx = dF \sin\theta = \rho\ g\ h \times dA \sin\theta$$
$$dFy = dF \cos\theta = \rho\ g\ h \times dA \cos\theta$$

Total force in X- direction and in Y- direction will be given as mentioned here.

$$F_x = \int dF_x = \int \rho g h dA \sin\theta = \rho g \int h dA \sin\theta$$

$$F_y = \int dF_y = \int \rho g h dA \cos\theta = \rho g \int h dA \cos\theta$$

Let us analyze the above equation

FG will be dA Sin θ or vertical projection of area dA. Therefore, the expression for Fx will be total pressure force on the projected area of the curved surface on the vertical plane.

Fx = Total pressure force on the projected area of the curved surface on the vertical plane

EG will be dA Cos θ or horizontal projection of dA. Therefore, the expression for Fy will be the weight of the liquid contained between the curved surface extended up to free surface of liquid.

Fy = Weight of the liquid contained between the curved surface extended up to free surface of liquid

Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2}$$

where F_x = Horizontal force on curved surface and is equal to total pressure force on the projected area of the curved surface on the vertical plane,

$= \rho g A \bar{h}$

and F_y = Vertical force on submerged curved surface and is equal to the weight of liquid actually or imaginary supported by the curved surface.