UNIT – I

CONDUCTION

1.1 Heat Energy and Heat Transfer

Heat is a form of energy in transition and it flows from one system to another, without transfer of mass, whenever there is a temperature difference between the systems. The process of heat transfer means the exchange in internal energy between the systems and in almost every phase of scientific and engineering work processes, we encounter the flow of heat energy.

1.2 Importance of Heat Transfer

Heat transfer processes involve the transfer and conversion of energy and therefore, it is essential to determine the specified rate of heat transfer at a specified temperature difference. The design of equipments like boilers, refrigerators and other heat exchangers require a detailed analysis of transferring a given amount of heat energy within a specified time. Components like gas/steam turbine blades, combustion chamber walls, electrical machines, electronic gadgets, transformers, bearings, etc require continuous removal of heat energy at a rapid rate in order to avoid their overheating. Thus, a thorough understanding of the physical mechanism of heat flow and the governing laws of heat transfer are a must.

1.3 Modes of Heat Transfer

The heat transfer processes have been categorized into three basic modes: Conduction, Convection and Radiation.

Conduction - It is the energy transfer from the more energetic to the less energetic particles of a substance due to interaction between them, a microscopic activity.

Convection - It is the energy transfer due to random molecular motion a long with the macroscopic motion of the fluid particles.

Radiation - It is the energy emitted by matter which is at finite temperature. All forms of matter emit radiation attributed to changes m the electron configuration of the constituent atoms or molecules The transfer of energy by conduction and convection requires the presence of a material medium whereas radiation does not. In fact radiation transfer is most efficient in vacuum.

All practical problems of importance encountered in our daily life Involve at least two, and sometimes all the three modes occuring simultaneously When the rate of heat flow is constant, i.e., does not vary with time, the process is called a steady state heat transfer process. When the temperature at any point in a system changes with time, the process is called unsteady or transient process. The internal energy of the system changes in such a process when the temperature variation of an unsteady process describes a particular cycle (heating or cooling of a budding wall during a 24 hour cycle), the process is called a periodic or quasi-steady heat transfer process.

Heat transfer may take place when there is a difference In the concentration of the mixture components (the diffusion thermoeffect). Many heat transfer processes are accompanied by a transfer of mass on a macroscopic scale. We know that when water evaporates, the heal transfer is accompanied by the transport of the vapour formed through an air-vapour mixture. The transport of heat energy to steam generally occurs both through molecular interaction and convection. The combined molecular and convective transport of mass is called convection mass transfer and with this mass transfer, the process of heat transfer becomes more complicated.

1.4 Thermodynamics and Heat Transfer-Basic Difference

Thermodynamics is mainly concerned with the conversion of heat energy into other useful forms of energy and IS based on (i) the concept of thermal equilibrium (Zeroth Law), (ii) the First Law (the principle of conservation of energy) and (iii) the Second Law (the direction in which a particular process can take place). Thermodynamics is silent about the heat energy exchange mechanism. The transfer of heat energy between systems can only take place whenever there is a temperature gradient and thus. Heat transfer is basically a non-equilibrium phenomenon. The Science of heat transfer tells us the rate at which the heat energy can be transferred when there IS a thermal non-equilibrium. That IS, the science of heat transfer seeks to do what thermodynamics is inherently unable to do.

However, the subjects of heat transfer and thermodynamics are highly complimentary. Many heat transfer problems can be solved by applying the principles of conservation of energy (the First Law)

1.5 Dimension and Unit

Dimensions and units are essential tools of engineering. Dimension is a set of basic entities expressing the magnitude of our observations of certain quantities. The state of a system is identified by its observable properties, such as mass, density, temperature, etc. Further, the motion of an object will be affected by the observable properties of that medium in which the object is moving. Thus a number of observable properties are to be measured to identify the state of the system.

A unit is a definite standard by which a dimension can be described. The difference between a dimension and the unit is that a dimension is a measurable property of the system and the unit is the standard element in terms of which a dimension can be explicitly described with specific numerical values.

Every major country of the world has decided to use SI units. In the study of heat transfer the dimensions are: L for length, M for mass, e for temperature, T for time and the corresponding units are: metre for length, kilogram for mass, degree Celsius (^oC) or Kelvin (K) for temperature and second (s) for time. The parameters important In the study of heat transfer are tabulated in Table 1.1 with their basic dimensions and units of measurement.

Table 1.1 Dimensions and units of various parameters

Parameter	Dimension	Unit
Mass	M	Kilogram, kg
Length	L	metre, m
Time	T	seconds, s
Temperature	□	Kelvin, K, Celcius ^oC
Velocity	L/T	metre/second,m/s
Density	ML^{-3}	kg/m^3
Force	$ML^{-1}T^{-2}$	Newton, N = 1 kg m/s^2
Pressure	ML^2T^{-2}	N/m^2, Pascal, Pa
Energy, Work	ML^2T^{-3}	N-m, = Joule, J
Power	ML^2T^{-3}	J/s, Watt, W
Absolute Viscosity	$ML^{-1}T^{-1}$	$N\text{-}s/m^2$, Pa-s
Kinematic Viscosity	L^2T^{-1}	m^2/s
Thermal Conductivity	$MLT^{-3}\,\square^{-1}$	W/mK, W/m^oC
Heat Transfer Coefficient	$MT^{-3}\,\square^{-1}$	W/m^2K, $W/m^{2o}C$
Specific Heat	$L^2\,T^{-2}\,\square^{-1}$	J/kg K, J/kg^oC
Heat Flux	MT^{-3}	W/m^2

1.6 Mechanism of Heat Transfer by Conduction

The transfer of heat energy by conduction takes place within the boundaries of a system, or a cross the boundary of t he system into another system placed in direct physical contact with the first, without any appreciable displacement of matter comprising the system, or by the exchange of kinetic energy of motion of the molecules by direct communication, or by drift of electrons in the case of heat conduction in metals. The rate equation which describes this mechanism is given by Fourier Law

$$\dot{Q} = -\mathbf{kAd}T / dx$$

where $\dot{Q}$ = rate of heat flow in X-direction by conduction in J/S or W,

k = thermal conductivity of the material. It quantitatively measures the heat conducting ability and is a physical property of t he material that depends upon the composition of the material, W/mK,

A = cross-sectional area normal to the direction of heat flow, m^2,

dT/dx = temperature gradient at the section, as shown in Fig. 1 I The neganve sign IS Included to make the heat transfer rate Q positive in the direction of heat flow (heat flows in the direction of decreasing temperature gradient).

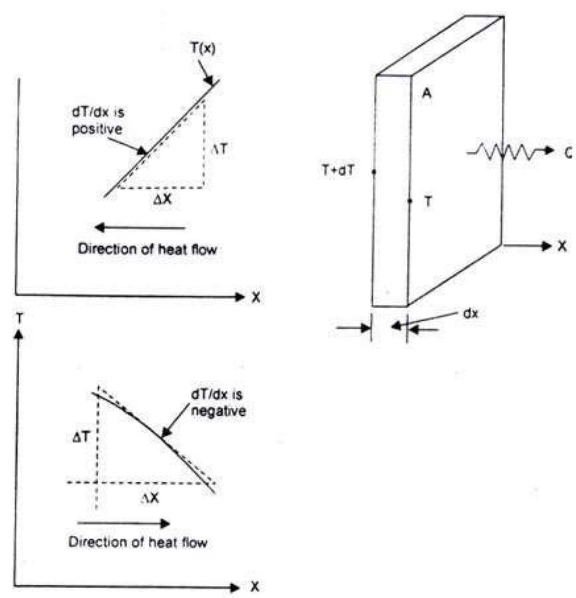


Fig 1.1: Heat flow by conduction

1.7 Thermal Conductivity of Materials

Thermal conductivity is a physical property of a substance and In general, It depends upon the temperature, pressure and nature of the substance. Thermal conductivity of materials are usually determined experimentally and a number of methods for this purpose are well known.

Thermal Conductivity of Gases: According to the kinetic theory of gases, the heat transfer by conduction in gases at ordinary pressures and temperatures take place through the transport of the kinetic energy arising from the collision of the gas molecules. Thermal conductivity of gases depends on pressure when very low «2660 Pal or very high ($> 2 \times 10^9$ Pa). Since the specific heat of gases Increases with temperature, the thermal conductivity Increases with temperature and with decreasing molecular weight.

Thermal Conductivity of Liquids: The molecules of a liquid are more closely spaced and molecular force fields exert a strong influence on the energy exchange In the collision process. The mechanism of heat propagation in liquids can be conceived as transport of energy by way of unstable elastic oscillations. Since the density of liquids decreases with increasing temperature, the thermal conductivity of non-metallic liquids generally decreases with increasing temperature, except for liquids like water and alcohol because their thermal conductivity first Increases with increasing temperature and then decreases.

Thermal Conductivity of Solids (i) Metals and Alloys: The heat transfer in metals arise due to a drift of free electrons (electron gas). This motion of electrons brings about the equalization in temperature at all points of t he metals. Since electrons carry both heat and electrical energy. The thermal conductivity of metals is proportional to its electrical conductivity and both the thermal and electrical conductivity decrease with increasing temperature. In contrast to pure metals, the thermal conductivity of alloys increases with increasing temperature. Heat transfer In metals is also possible through vibration of lattice structure or by elastic sound waves but this mode of heat transfer mechanism is insignificant in comparison with the transport of energy by electron gas. (ii) Nonmetals: Materials having a high volumetric density have a high thermal conductivity but that will depend upon the structure of the material, its porosity and moisture content High volumetric density means less amount of air filling the pores of the materials. The thermal conductivity of damp materials considerably higher than the thermal conductivity of dry material because water has a higher thermal conductivity than air. The thermal conductivity of granular material increases with temperature. (Table 1.2 gives the thermal conductivities of various materials at 0ºC.)

Table 1.2 Thermal conductivity of various materials at 0ºC.

Material	Thermal conductivity (W/m K)	Material	Thermal conductivity (W/m K)
Gases		Solids: Metals	
Hydrogen .	0175	Sliver, pure	410
Helium	0141	Copper, pure	385
A"	0024	AlumllllUm, pure	202
Water vapour (saturated)	00206	Nickel, pure	93
Carbon dioxide	00146	Iron, pure	73
(thermal conductivity of helium and hydrogen are much higher than other gases. because then molecules have small mass and higher mean travel velocity)		Carbon steel, I %C Lead, pure Chrome-nickel-steel (18% Cr, 8% Ni) Non-metals	43 35 16.3
Liquids		Quartz, parallel to axis	41.6
Mercury	821	Magnesite	4.15
Water*	0.556	Marble	2.08 to 2.94
Ammonia	0.54	Sandstone	1.83
Lubricating 011		Glass, window	0.78
SAE 40	0.147	Maple or Oak	0.17
Freon 12	0.073	Saw dust	0.059
		Glass wool	0.038

* water has its maximum thermal conductivity (k = 068 W/mK) at about 150ºC

2. STEADY STATE CONDUCTION ONE DIMENSION

2.1 The General Heat Conduction Equation for an Isotropic Solid with Constant Thermal Conductivity

Any physical phenomenon is generally accompanied by a change in space and time of its physical properties. The heat transfer by conduction in solids can only take place when there is a variation of temperature, in both space and time. Let us consider a small volume of a solid element as shown in Fig. 1.2. The dimensions are: Δx, Δy, Δz along the X-, Y-, and Z-coordinates.

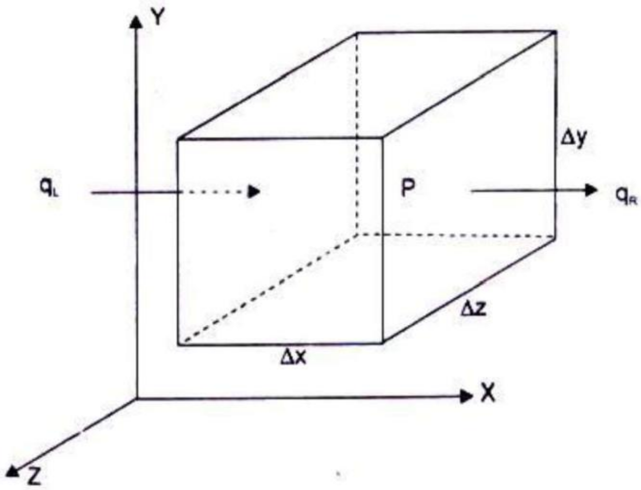


Fig 1.2 Elemental volume in Cartesian coordinates

First we consider heat conduction the X-direction. Let T denote the temperature at the point P (x, y, z) located at the geometric centre of the element. The temperature gradient at the left hand face (x - ~x12) and at the right hand face (x + Δx/2), using the Taylor's series, can be written as:

$$\partial T/\partial x|_L = \partial T/\partial x - \partial^2 T/\partial x^2 . \Delta x/2 \text{ + higher order terms.}$$

$$\partial T/\partial x|_R = \partial T/\partial x + \partial^2 T/\partial x^2 . \Delta x/2 + \text{ higher order terms.}$$

The net rate at which heat is conducted out of the element 10 X-direction assuming k as constant and neglecting the higher order terms,

$$\text{we get } -k\Delta y\Delta z\left[\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2}\frac{\Delta x}{2}-\frac{\partial T}{\partial x}+\frac{\partial^2 T}{\partial x^2}\frac{\Delta x}{2}\right] = -k\Delta y\Delta z\Delta x\left(\frac{\partial^2 T}{\partial x^2}\right)$$

Similarly for Y- and Z-direction,

We have $-k\Delta x\Delta y\Delta z\ \partial^2 T/\Delta y^2$ and $-k\Delta x\Delta y\Delta z\ \partial^2 T/\Delta z^2$.

If there is heat generation within the element as Q, per unit volume and the internal energy of the element changes with time, by making an energy balance, we write

Heat generated within the element — Heat conducted away from the element — Rate of change of internal energy within with the element

or, $\dot{Q}_v(\Delta x \Delta y \Delta z) + k(\Delta x \Delta y \Delta z)\left(\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2\right)$

$$= \rho c(\Delta x \Delta y \Delta z)\, \partial T/\partial t$$

Upon simplification, $\partial^2 T/\partial x^2 + \partial^2 T/\partial y^2 + \partial^2 T/\partial z^2 + \dot{Q}_v/k = \dfrac{\rho c}{k}\partial T/\partial t$

or, $\nabla^2 T + \dot{Q}_v/k = 1/\alpha(\partial T/\partial t)$

where $\alpha = k/\rho c$, is called the thermal diffusivity and is seen to be a physical property of the material of which the solid is composed.

The Eq. (2.la) is the general heat conduction equation for an isotropic solid with a constant thermal conductivity. The equation in cylindrical (radius r, axis Z and longitude □) coordinates is written as: Fig. 2.I(b),

$$\partial^2 T/\partial r^2 + (1/r)\partial T/\partial r + (1/r^2)\partial^2 T/\partial \theta^2 + \partial^2 T/\partial z^2 + \dot{Q}_v/k = 1/\alpha\, \partial T/\partial t \qquad (2.1b)$$

And, in spherical polar coordinates Fig. 2.1(c) (radius, □ longitude, □ colatitudes) is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2} + \frac{\dot{Q}_v}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (2.1c)$$

Under steady state or stationary condition, the temperature of a body does not vary with time, i.e. $\partial T/\partial t = 0$. And, with no internal generation, the equation (2.1) reduces to

$$\nabla^2 T = 0$$

It should be noted that Fourier law can always be used to compute the rate of heat transfer by conduction from the knowledge of temperature distribution even for unsteady condition and with internal heat generation.

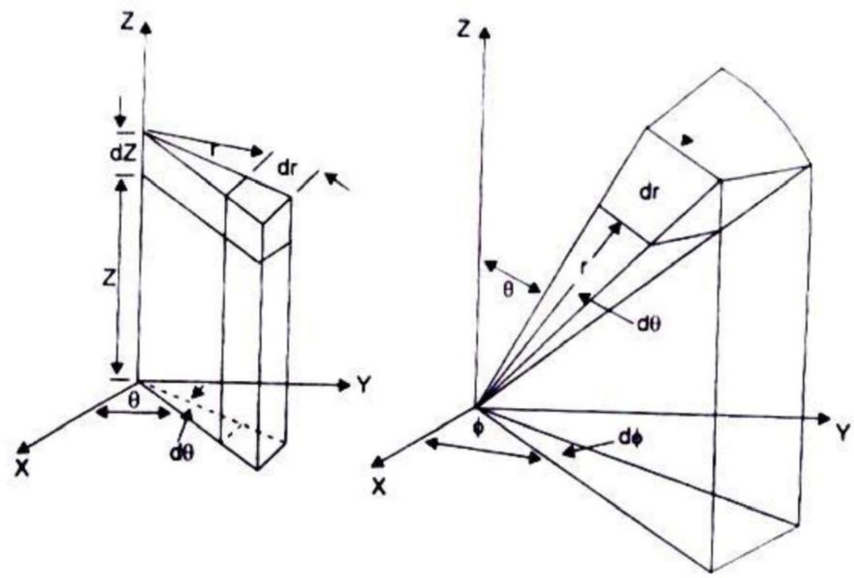


Fig1.3: Elemental volume in cylindrical coordinates (c):spherical coordinates

. **One-Dimensional Heat Flow**

The term 'one-dimensional' is applied to heat conduction problem when:

(i) Only one space coordinate is required to describe the temperature distribution within a heat conducting body;

(ii) Edge effects are neglected;

(iii) The flow of heat energy takes place along the coordinate measured normal to the surface.

3. Thermal Diffusivity and its Significance

Thermal diffusivity is a physical property of the material, and is the ratio of the material's ability to transport energy to its capacity to store energy. It is an essential parameter for transient processes of heat flow and defines the rate of change in temperature. In general, metallic solids have higher value, while non metallics, like paraffin, have a lower value. Materials having large thermal diffusivity respond quickly to changes in their thermal environment, while materials having lower a respond very slowly, take a longer time to reach a new equilibrium condition.

4. TEMPERATURE DISTRIBUTION IN I-D SYSTEMS

4.1 A Plane Wall

A plane wall is considered to be made out of a constant thermal conductivity material and extends to infinity in the Y- and Z-direction. The wall is assumed to be homogeneous and isotropic, heat flow is one-dimensional, under steady state conditions and losing negligible energy through the edges of the wall under the above mentioned assumptions the Eq. (2.2) reduces to

$d^2T / dx^2 = 0$; the boundary conditions are: at $x = 0$, $T = T_1$

Integrating the above equation, $x = L$, $T = T_2$

$T = C_1x + C_2$, where C_1 and C_2 are two constants.

Substituting the boundary conditions, we get $C_2 = T_1$ and $C_1 = (T_2 - T_1)/L$ The temperature distribution in the plane wall is given by

$$T = T_1 - (T_1 - T_2)\, x/L \qquad (2.3)$$

which is linear and is independent of the material.

Further, the heat flow rate, $\dot{Q}/A = k - dT/dx = (T_1 - T_2)k/L$, and therefore the temperature distribution can also be written as

$$T - T_1 = \left(\dot{Q}/A\right)\left(x/k\right) \qquad (2.4)$$

i.e., "the temperature drop within the wall will increase with greater heat flow rate or when k is small for the same heat flow rate,"

4.2 A Cylindrical Shell-Expression for Temperature Distribution

In the cylindrical system, when the temperature is a function of radial distance only and is independent of azimuth angle or axial distance, the differential equation (2.2) would be, (Fig. 1.4)

$d^2T / dr^2 + (1/r)\, dT/dr = 0$

with boundary conditions: at $r = r_l$, $T = T_1$ and at $r = r_2$, $T = T_2$.

The differential equation can be written as:

$$\frac{1}{r}\frac{d}{dr}\left(r\,dT/dr\right)=0, \text{ or, } \quad \frac{d}{dr}\left(r\,dT/dr\right)=0$$

upon integration, $T = C_1 \ln(r) + C_2$, where C_1 and C_2 are the arbitrary constants.

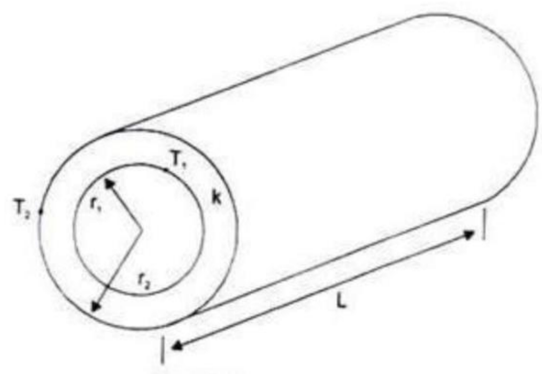


Fig 1.4: A Cylindrical shell

By applying the boundary conditions,

$$C_1=\left(T_2-T_1\right)/\ln\left(r_2/r_1\right)$$

and $$C_2 = T_1-\ln\left(r_1\right).\left(T_2-T_1\right)/\ln\left(r_2/r_1\right)$$

The temperature distribution is given by

$$T = T_1+\left(T_2-T_1\right).\ln\left(r/r_1\right)/\ln\left(r_2/r_1\right) \text{ and}$$

$$\dot{Q}/L=-kA\ dT/dr \quad = 2\pi k\left(T_1-T_2\right)/\ln\left(r_2/r_1\right) \qquad (2.5)$$

From Eq (2.5) It can be seen that the temperature varies 10gantJunically through the cylinder wall In contrast with the linear variation in the plane wall .

If we write Eq. (2.5) as $\dot{Q}=kA_m\left(T_1-T_2\right)/\left(r_2-r_1\right)$, where

$$A_m = 2\pi\left(r_2-r_1\right)L/\ln\left(r_2/r_1\right) \quad =\left(A_2-A_1\right)/\ln\left(A_2/A_1\right)$$

where A_2 and A_1 are the outside and inside surface areas respectively. The term A_m is called ‘Logarithmic Mean Area' and the expression for the heat flow through a cylindrical wall has the same form as that for a plane wall.

4.3 Spherical and Parallelopiped Shells--Expression for Temperature Distribution

Conduction through a spherical shell is also a one-dimensional steady state problem if the interior and exterior surface temperatures are uniform and constant. The Eq. (2.2) in one-dimensional spherical coordinates can be written as

$$\left(1/r^2\right)\frac{d}{dT}\left(r^2 dT/dr\right) = 0$$, with boundary conditions,

at $r = r_1,\ T = T_1;\ \text{at } r = r_2,\ T = T_2$

or, $$\frac{d}{dr}\left(r^2 dT/dr\right) = 0$$

and upon integration, $T = -C_1/r + C_2$, where c_1 and c_2 are constants. substituting the boundary conditions,

$$C_1 = \left(T_1 - T_2\right) r_1 r_2 / \left(r_1 - r_2\right), \text{ and } C_2 = T_1 + \left(T_1 - T_2\right) r_1 r_2 / r_1 \left(r_1 - r_2\right)$$

The temperature distribution m the spherical shell is given by

$$T = T_1 - \left\{\frac{\left(T_1 - T_2\right) r_1 r_2}{\left(r_2 - r_1\right)}\right\} \times \left\{\frac{\left(r - r_1\right)}{r\, r_1}\right\} \qquad (2.6)$$

and the temperature distribution associated with radial conduction through a sphere is represented by a hyperbola. The rate of heat conduction is given by

$$\dot{Q} = 4\pi k \left(T_1 - T_2\right) r_1 r_2 / \left(r_2 - r_1\right) = k\left(A_1 A_2\right)^{1/2} \left(T_1 - T_2\right) / \left(r_2 - r_1\right) \qquad (2.7)$$

where $A_1 = 4\pi r_1^2$ and $A_2 = 4\pi r_2^2$

If A_1 is approximately equal to A_2 i.e., when the shell is very thin,

$$\dot{Q} = kA\left(T_1 - T_2\right)/\left(r_2 - r_1\right) \text{ ; and } \dot{Q}/A = \left(T_1 - T_2\right)/\Delta r/k$$

which is an expression for a flat slab.

The above equation (2.7) can also be used as an approximation for parallelopiped shells which have a smaller inner cavity surrounded by a thick wall, such as a small furnace surrounded by a large thickness of insulating material, although the h eat flow especially in the corners, cannot be strictly considered one-dimensional. It has been suggested that for $(A_2/A_1) > 2$, the rate of heat flow can be approximated by the above equation by multiplying the geometric mean area $A_m = (A_1 A_2)^{½}$ by a correction factor 0.725.]

4.4 Composite Surfaces

There are many practical situations where different materials are placed m layers to form composite surfaces, such as the wall of a building, cylindrical pipes or spherical shells having different layers of insulation. Composite surfaces may involve any number of series and parallel thermal circuits.

4.5 Heat Transfer Rate through a Composite Wall

Let us consider a general case of a composite wall as shown m Fig. 1.5 There are 'n' layers of different materials of thicknesses L_1, L_2, etc and having thermal conductivities k_l, k_2, etc. On one side of the composite wall, there is a fluid A at temperature T_A and on the other side of the wall there is a fluid B at temperature T_B. The convective heat transfer coefficients on the two sides of the wall are h_A and h_B respectively. The system is analogous to a series of resistances as shown in the figure.

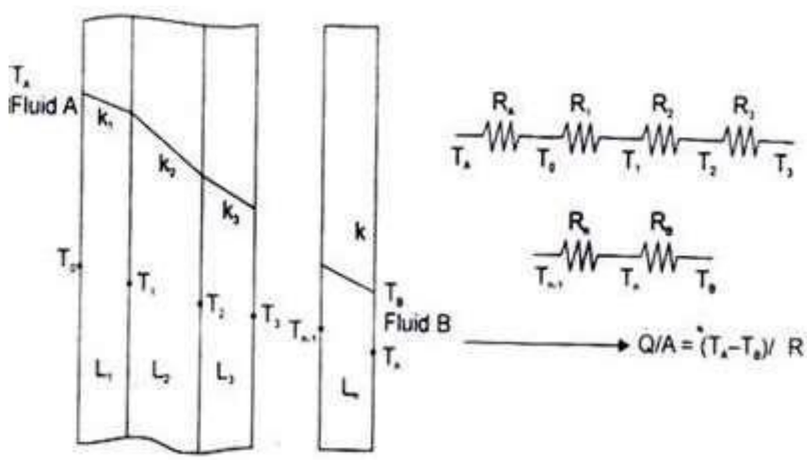


Fig 1.5 Heat transfer through a composite wall

4.6 The Equivalent Thermal Conductivity

The process of heat transfer through compos lie and plane walls can be more conveniently compared by introducing the concept of 'equivalent thermal conductivity', k_{eq}. It is defined as:

$$k_{eq} = \left(\sum_{i=1}^{n} L_i\right) \bigg/ \sum_{i=1}^{n} \left(L_i / k_i\right) \qquad (2.8)$$

$$= \frac{\text{Total thickeness of the composite wall}}{\text{Total thermal resistance of the composite wall}}$$

And, its value depends on the thermal and physical properties and the thickness of each constituent of the composite structure.

Example 1.2 A furnace wall consists of 150 mm thick refractory brick (k = 1.6 W/mK) and 150 mm thick insulating fire brick (k = 0.3 W/mK) separated by an au gap (resistance 0 16 K/W). The outside walls covered with a 10 mm thick plaster (k = 0.14 W/mK). The temperature of hot gases is 1250°C and the room temperature is 25°C. The convective heat transfer coefficient for gas side and air side is 45 W/m2K and 20 W/m^2K. Calculate (i) the rate of heat flow per unit area of the wall surface (ii) the temperature at the outside and Inside surface of the wall and (iii) the rate of heat flow when the air gap is not there.

Solution: Using the nomenclature of Fig. 2.3, we have per m2 of the area, h_A = 45, and $R_A = 1/h_A$ = 1/45 = 0.0222; h_B = 20, and R_B = 1120 = 0.05

Resistance of the refractory brick, $R_1 = L_1/k_1$ = 0.15/1.6 = 0.0937

Resistance of the insulating brick, $R_3 = L_3/k_3$ = 0.15/0.30 = 0.50

The resistance of the air gap, R_2 = 0.16

Resistance of the plaster, R_4 = 0.01/0.14 = 0.0714

Total resistance = 0.8973, m^2K/W

Heat flow rate = $\Delta T/\Sigma R$ = (1250-25)/0.8973= 13662 W/m^2

Temperature at the inner surface of the wall

$$= T_A - 1366.2 \times 0.0222 = 1222.25$$

Temperature at the outer surface of the wall

$$= T_B + 1366.2 \times 0.05 = 93.31\ °C$$

When the air gap is not there, the total resistance would be

$0.8973 - 0.16 = 0.7373$

and the heat flow rate = (1250 – 25)/0/7373 = 1661.46 W/m^2

The temperature at the inner surface of the wall

$= 1250 – 1660.46 \times 0.0222 = 1213.12°C$

i.e., when the au gap is not there, the heat flow rate increases but the temperature at the inner surface of the wall decreases.

The overall heat transfer coefficient U with and without the air gap is

$$U = \left(\dot{Q}/A\right) / \Delta T$$

$= 13662 / (1250 – 25) = 1.115$ Wm^2 °C

and 1661.46/l225 = 1356 W/m^{2o}C

The equivalent thermal conductivity of the system without the air gap

$k_{eq} = (0.15 + 0.15 + 0.01)/(0.0937 + 0.50 + 0.0714) = 0.466$ W/mK.

Example 1.2 A brick wall (10 cm thick, k = 0.7 W/m°C) has plaster on one side of the wall (thickness 4 cm, k = 0.48 W/m°C). What thickness of an insulating material (k = 0.065 W m^{o}C) should be added on the other side of the wall such that the heat loss through the wall IS reduced by 80 percent.

Solution: When the insulating material is not there, the resistances are:

$R_1 = L_1/k_1 = 0.1/0.7 = 0.143$

and $R_2 = 0.04/0.48 = 0.0833$

Total resistance = 0.2263

Let the thickness of the insulating material is L_3. The resistance would then be

$L_3/0.065 = 15.385\ L_3$

Since the heat loss is reduced by 80% after the insulation is added.

$$\frac{\dot{Q}\ \text{with insulation}}{\dot{Q}\ \textbf{without}\ \text{insulation}} = 0.2 = \frac{R\ \text{without insulation}}{R\ \text{with insulation}}$$

or, the resistance with insulation = 0.2263/0.2 = 01.1315

and, 15385 L_3 = I 1315 – 0.2263 = 0.9052

L_3 = 0.0588 m = 58.8 mm

Example 1.3 An ice chest IS constructed of styrofoam (k = 0.033 W/mK) having inside dimensions 25 by 40 by 100 cm. The wall thickness is 4 cm. The outside surface of the chest is exposed to air at 25°C with h = 10 W/m^{2}K. If the chest is completely filled with ice, calculate the time for ice to melt completely. The heat of fusion for water is 330 kJ/kg.

Solution: If the heat loss through the comers and edges are Ignored, we have three walls of walls through which conduction heat transfer Will occur.

(a) 2 walls each having dimensions 25 cm × 40 cm × 4 cm

(b) 2 walls each having dimensions 25 cm × 100 cm × 4 cm

(c) 2 walls each having dimensions 40 cm × 100 cm × 4 cm

The surface area for convection heat transfer (based on outside dimensions)

2(33 × 48 + 33 × 108 + 48 × 108) × 10^{-4} = 2.0664 m^2.

Resistance due to conduction and convection can be written as

$$2\left(\frac{0.04}{0.033\times0.25\times0.4}+\frac{0.04}{0.033\times0.25\times1}+\frac{0.04}{0.033\times0.4\times1}\right)+\frac{1}{10\times2.0664}$$

= 40 + 0.0484 = 40.0484 K/W

$\dot{Q} = \Delta T / \Sigma R$ = (25 – 0.0) / 40.0484 = 0.624 W

Inside volume of the container - 0.25 × 04 × 1 = 0.1 m^3

Mass of Ice stored = 800 × 0.1 = 80 kg; taking the density of Ice as 800 kg/m^3. The time required to melt 80 kg of ice is

$$t = \frac{80\times330\times1000}{0.624\times3600\times24} = 490 \text{ days}$$

Example1.4 A composite furnace wall is to be constructed with two layers of materials (k_1 =

2.5 W/m^{o}C and k_2 = 0.25 W/m^{o}C). The convective heat transfer coefficient at the inside and outside surfaces are expected to be 250 W/m^{2o}C and 50 W/m^{2o}C respectively. The temperature of gases and air are 1000 K and 300 K. If the interface temperature is 650 K, Calculate (i) the thickness of the two materials when the total thickness does not exceed 65 cm and (ii) the rate of heat flow. Neglect radiation.

Solution: Let the thickness of one material (k = 2.5 W / mK) is xm, then the thickness of the other material (k = 0.25 W/mK) will be (0.65 − x)m.

For steady state condition, we can write

$$\frac{\dot{Q}}{A} - \frac{\mathbf{1000} - 650}{\frac{1}{250} + \frac{x}{2.5}} = \frac{1000 - 300}{\frac{1}{250} + \frac{x}{2.5} + \frac{(0.65 - x)}{0.25} + \frac{1}{50}}$$

$$\therefore \quad 700(0.004 + 0.4x) = 350\{0.004 + 0.4x + 4(0.65 - x) + 0.02\}$$

(i) 6x = 3.29 and x= 0.548 m.

and the thickness of the other material = 0.102 m.

(ii) $\dot{Q}/A$ = (350) / (0.004 + 0.4 × 0.548) = 1.568 kW/m^2

Example 1.5 A composite wall consists of three layers of thicknesses 300 rum, 200 mm and 100 mm with thermal conductivities 1.5, 3.5 and is W/mK respectively. The inside surface is exposed to gases at 1200°C with convection heat transfer coefficient as 30W/m^{2}K. The temperature of air on the other side of the wall is 30°C with convective heat transfer coefficient 10 Wm^{2}K. If the temperature at the outside surface of the wall is 180°C, calculate the temperature at other surface of the wall, the rate of heat transfer and the overall heat transfer coefficient.

Solution: The composite wall and its equivalent thermal circuits is shown in the figure.

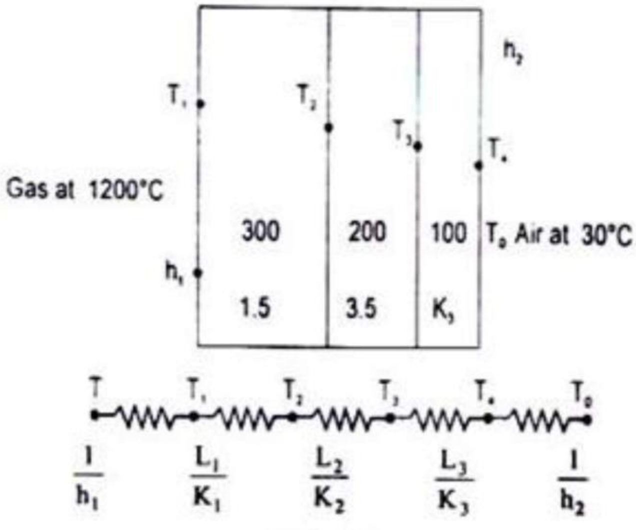


Fig 1.6

The heat energy will flow from hot gases to the cold air through the wall.

From the electric Circuit, we have

$\dot{Q}/A = h_2(T_4 - T_0) = 10 \times (180 - 30) = 1500\ W/m^2$

also, $\dot{Q}/A = h_1(1200 - T_1)$

$T_1 = 1200 - 1500/30 = 1150^o C$

$\dot{Q}/A = (T_1 - T_2)/L_1/k_1$

$T_2 = T_1 - 1500 \times 0.3/1.5 = 850$

Similarly, $\dot{Q}/A = (T_2 - T_3)/(L_2/k_2)$

$T_3 = T_2 - 1500 \times 0.2/3.5 = 764.3^o C$

and $\dot{Q}/A = (T_3 - T_4)/(L_3/k_3)$

$L_3/k_3 = (764.3 - 180)/1500$ and $k_3 = 0.256$ W/mK

Check:

$\dot{Q}/A = (1200 - 30)/\Sigma R;$

where $\Sigma R = 1/h_1 + L_1/k_1 + L_2/k_2 + L_3/k_3 + 1/h_2$

$\Sigma R = 1/30 + 0.3/1.5 + 0.2/3.5 + 0.1/0.256 + 1/10 = 0.75$

and $\dot{Q}/A = 1170/0.78 = 1500 \ W/m^2$

The overall heat transfer coefficient, $U = 1/\Sigma R = 1/0.78 = 1.282 \ W/m^2K$

Since the gas temperature is very high, we should consider the effects of radiation also. Assuming the heat transfer coefficient due to radiation = 3.0 W/m^{2}K the electric circuit would be:

The combined resistance due to convection and radiation would be

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\frac{1}{h_c}} + \frac{1}{\frac{1}{h_r}} = h_c + h_r = 60 W/m^{2o}C$$

$$\therefore \ \dot{Q}/A = 1500 = 60(T - T_1) = 60(1200 - T_1)$$

$$\therefore \ T_1 = 1200 - \frac{1500}{60} = 1175^o C$$

again, $\therefore \ \dot{Q}/A = (T_1 - T_2)/L_1/k_1 \Rightarrow T_2 = T_1 - 1500 \times 0.3/1.5 = 875^o C$

and $\quad T_3 = T_2 - 1500 \times 0.2/3.5 = 789.3^o$ C

$L_3/k_3 = (789.3 - 180)/1500; \ \therefore k_3 = 0.246 \ W/mK$

$$\Sigma R = \frac{1}{60} + \frac{0.3}{1.5} + \frac{0.2}{1.5} + \frac{0.2}{3.5} + \frac{0.1}{0.246} + \frac{1}{10} = 0.78$$

and $\quad U = 1/\Sigma R = 1.282 \ W/m^2K$

Example 1.6 A flat roof (12 m x 20 m) of a building has a composite structure It consists of a 15 cm lime-khoa plaster covering (k = 0 17 W/m°C) over a 10 cm cement concrete (k = 0.92 W/m°C). The ambient temperature is 42°C. The outside and inside heat transfer coefficients are 30 W/m2°C and 10 W/m2 0C. The top surface of the roof absorbs 750 W/m2 of solar radiant energy. The temperature of the space may be assumed to be 260 K. Calculate the temperature of the top surface of the roof and the amount of water to be sprinkled uniformly over the roof surface such that the inside temperature is maintained at 18°C.

Solution: The physical system is shown in Fig. 1.7 and it is assumed we have one-dimensional flow, properties are constant and steady state conditions prevail.

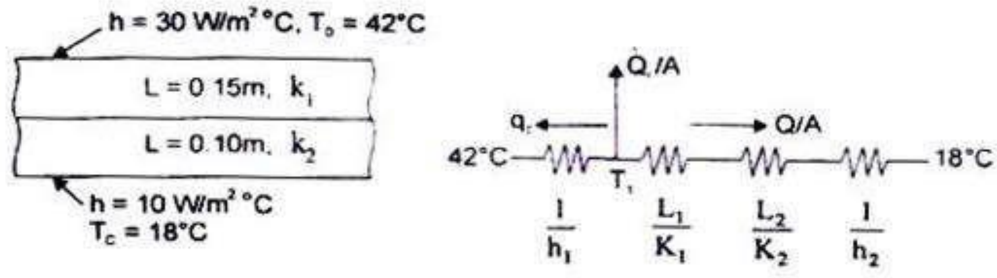


Fig 1.7

Let the temperature of the top surface be T_1°C.

Heat lost by thee top surface by convection to the surroundings is

$$\dot{Q}_c / A = h(\Delta T) = 30 \times (T_1 - 42) = (30T_1 - 1260)$$

Heat energy conducted inside through the roof = $(\Delta T / \Sigma R)$

or, $$\frac{\dot{Q}}{A} = \frac{T_1 - 18}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2}} = (T_1 - 18) / \left(\frac{0.15}{0.17} + \frac{0.1}{0.92} + \frac{1}{10}\right) = 0.918\ (T_1 - 18)$$

Assuming that the top surface of the roof behaves like a black body, energy lost by radiation.

$$\dot{Q}_r / A = \sigma\left[(T_1 + 273)^4 - 260^4\right] = 5.67 \times 10^{-8}\ (T_1 + 273)^4 - 259.1$$

By making an energy balance on the top surface of the roof,

Energy coming in = Energy going out

$750 = (30T, -1260) + 0.918\ (T_1 - 18) + 5.67 \times 10^{-8}\ (T_1 + 273)^4 - 259.1$

or, $2285.624 = 30.918\ T_1 + 5.67 \times 10^{-8}\ (T_1 + 273)^4$

Solving by trial and error, T_1 = 53.4°C, and the total energy conducted through the roof per hour is

0.918 (53.4 – 18) × (12 × 20) × 3600 = 28077.58 kJ/hr

Assuming the latent heat of vaporization of water as 2430 kJ/kg, the quantity of water to be sprinkled over the surface such that it evaporates and consumes 28077.58 kJ/hr, is

$\dot{M}_w$ = 28077.58/2430 = 11.55 kg/hr.

Example 1.7 An electric hot plate is maintained at a temperature of 350°C and is used to keep a solution boiling at 95°C. The solution is contained in a cast iron vessel (wall thickness 25 mm, k = 50 W/mK) which is enamelled inside (thickness 0.8 mm, k = 1.05 WmK) The heat transfer coefficient for the boiling solution is 5.5 kW/m1K. Calculate (i) the overall heat transfer coefficient and (ii) heat transfer rate.

If the base of the cast iron vessel is not perfectly flat and the resistance of the resulting air film is 35 m2K1kW, calculated the rate of heat transfer per unit area. (Gate'93)

Solution: The physical system is shown in the figure below.

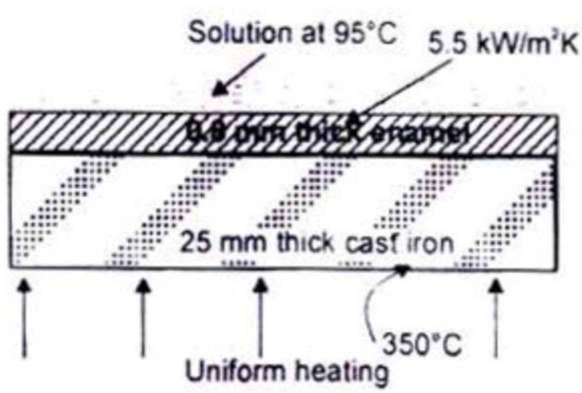


Fig 1.8

Under steady state conditions,

$$\dot{Q}/A = U(\Delta T) = \frac{(\Delta T)}{1/U}$$, where U is the overall heat transfer coefficient.

$$= \frac{(\Delta T)}{R} = \frac{(\Delta T)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h}}$$

Therefore,

$$1/U = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h} = \left(\frac{0.025}{50} + \frac{0.0008}{1.05} + \frac{1}{5500}\right) = 0.00144$$

$$U = 692.65 \text{ W/m}^2\text{K}$$

$$\dot{Q}/A = U(\Delta T) = 692.65 \times (350 - 95) = 176.65 \text{ kW/m}^2.$$

With the presence of air film at the base, the total resistance to heat flow would be:

$$0.00144 + 0.035 = 0.03644 \text{ m}^2\text{K/W}$$

and the rate of heat transfer, $\dot{Q}/A$ = 255/0.03644 = 7 kW/m^2.

(Fig. 1.9 shows a combination of thermal resistance placed in series and parallel for a composite wall having one-dimensional steady state heat transfer. By drawing analogous electric circuits, we can solve such complex problems having both parallel and series thermal resistances.)

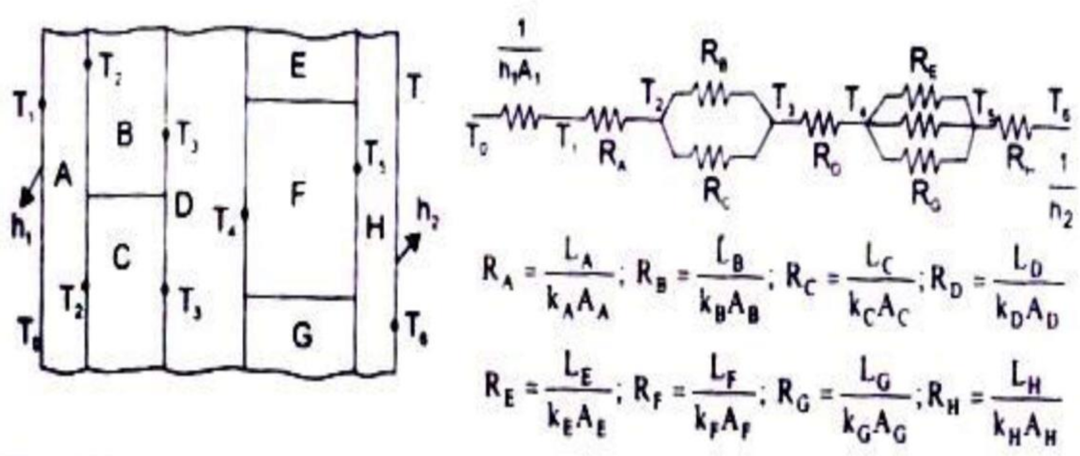


Fig. 1.9 Series and parallel one-dimensional heat transfer through a composite wall with convective heat transfer and its electrical analogous circuit

Example 1.8 A door (2 m x I m) is to be fabricated with 4 cm thick card board (k = 0.2 W /mK) placed between two sheets of fibre glass board (each having a thickness of 40 mm and k = 0.04 W/mK). The fibre glass boards are fastened with 50 steel studs (25 mm diameter, k = 40 W/mK). Estimate the percentage of heat transfer flow rate through the studs.

Solution: The thermal circuit with steel studs can be drawn as in Fig. 1.10.

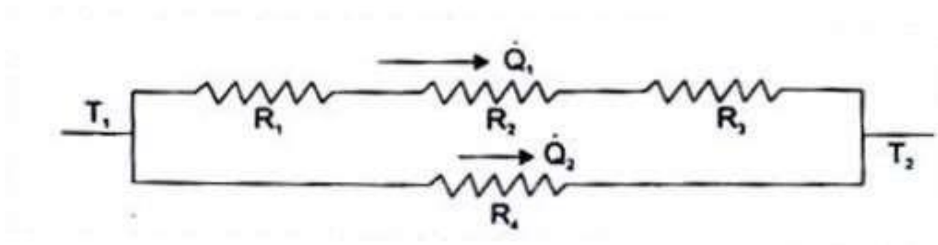


Fig 1.10

The cross-sectional area or the surface area of the door for the heat transfer is $2 m^2$. The cross-sectional area of the steel studs is:

$$50 \times \square/4\ (0.025)^2 = 0.02455\ m^2$$

and the area of the door – area of the steel studs = 2.0 – 0.02455 = 1.97545

R_1, the resistance due to fibre glass board on the outside

$$= L/kA = 0.04/(0.04 \times 1.97545) = 0.506.$$

R_2, the resistance due to card board = 0.101

R_3, the resistance due to fibre glass board on the inside = 0.506

R_4, the resistance due to steel studs = L/kA = 0.121 (40 × 0.2455) = 0 1222

With reference to Fig 2.9, $\dot{Q}_1 = (T_1 - T_2)/\Sigma R = (T_1 - T_2)/1.113$

and $\quad \dot{Q}_2 = (T_1 - T_2)/0.1222$

Therefore, $\quad \dot{Q}_2/(\dot{Q}_1 + \dot{Q}_2) = 8.1833/9.0818 = 0.9$

ie, 90 percent of the heat transfer will take place through the studs.

Example 1.9 Find the heat transfer rate per unit depth through the composite wall sketched. Assume one dimensional heat flow.

Solution: The analogous electric circuit has been drawn in the figure.

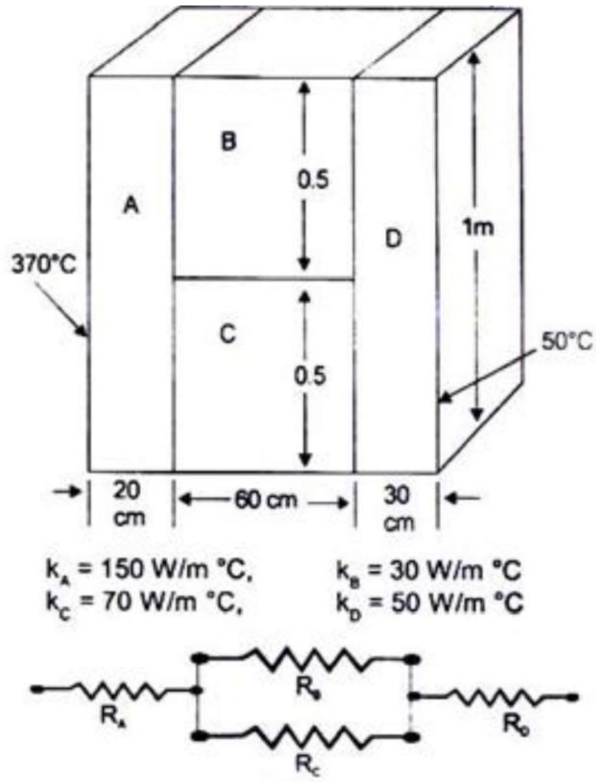


Fig 1.11

$R_A = 0.2/150 = 0.00133$

$R_B = 0.6/(30 \times 0.5) = 0.04$

$R_C = 0.6/(70 \times 0.5) = 0.017$

$R_D = 0.3/50 = 0.006$

$1/R_B + 1/R_C = 1/R_{BC} = 83.82$

Therefore, $R_{BC} = 1/83.82 = 0.0119$

Total resistance to heat flow = 0.00133 + 0.0119 + 0006 = 0.01923

Rate of heat transfer per unit depth = (370‒50)/ 0.01923 = 16.64 kW m.

The Significance of Biot Number

Let us consider steady state conduction through a slab of thickness L and thermal conductivity k. The left hand face of the wall is maintained at T constant temperature T_1 and the right hand face is exposed to ambient air at T_o, with convective heat transfer coefficient h. The analogous electric circuit will have two thermal resistances: $R_1 = L/k$ and $R_2 = l/h$. The drop in temperature across the wall and the air film will be proportional to their resistances, that is, $(L/k)/(1/h) = hL/k$.

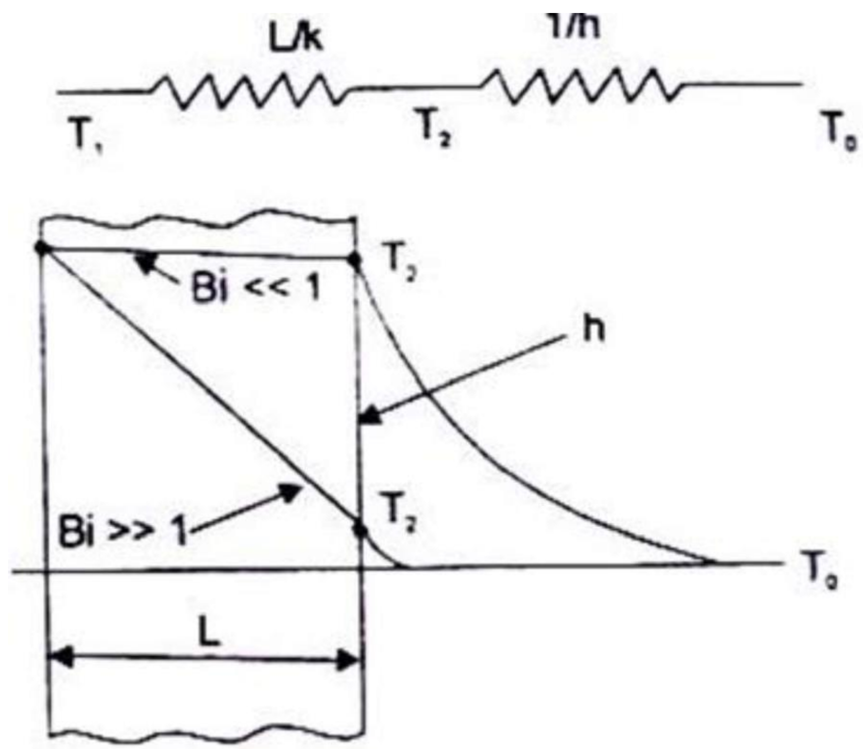


Fig 1.12: Effect of Biot number on temperature profile

This dimensionless number is called 'Biot Number' or,

$$B_i = \frac{\text{Conduction resistance}}{\text{Convection resistance}}$$

When Bi >> 1, the temperature drop across the air film would be negligible and the temperature at the right hand face of the wall will be approximately equal to the ambient temperature. Similarly, when Bi «I, the temperature drop across the wall is negligible and the transfer of heat will be controlled by the air film resistance.

5. The Concept of Thermal Contact Resistance

Heat flow rate through composite walls are usually analysed on the assumptions that - (i) there is a perfect contact between adjacent layers, and (ii) the temperature at the interface of the two plane surfaces is the same. However, in real situations, this is not true. No surface, even a so-called 'mirror-finish surface', is perfectly smooth ill a microscopic sense. As such, when two surfaces are placed together, there is not a single plane of contact. The surfaces touch only at limited number of spots, the aggregate of which is only a small fraction of the area of the surface or 'contact area'. The remainder of the space between the surfaces may be filled with air or other fluid. In effect, this introduces a resistance to heat flow at the interface. This resistance IS called 'thermal contact resistance' and causes a temperature drop between the materials at the interfaces as shown In Fig. 2.12. (That is why, Eskimos make their houses having double ice walls separated by a thin layer of air, and in winter, two thin woolen blankets are more comfortable than one woolen blanket having double thickness.)

Fig. 2.12 Temperature profile with and without contact resistance when two solid surfaces are joined together

Example 1.10 A furnace wall consists of an inner layer of fire brick 25 cm thick k = 0.4 W/mK and a layer of ceramic blanket insulation, 10 cm thick k = 0.2 W/mK. The thermal contact resistance between the two walls at the interface is 0.01 m^2K/w. Calculate the temperature drop at the interface if the temperature difference across the wall is 1200K.

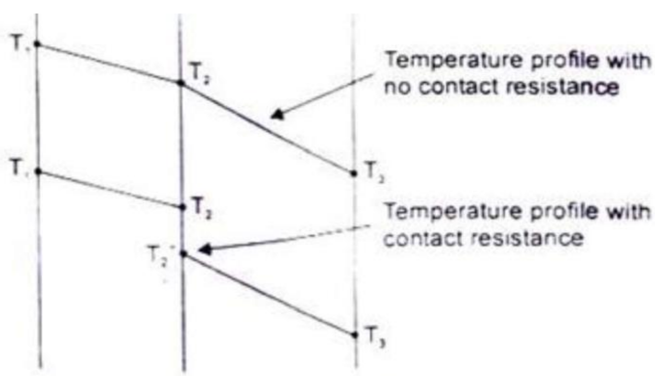


Fig 1.13: temperature profile with and without contact resistance when two solid surfaces are joined together

Solution: The resistance due to inner fire brick = L/k = 0.25/0.4 = 0.625.

The resistance of the ceramic insulation = 0.1/0.2 = 0.5

Total thermal resistance = 0.625 + 0.01 + 0.5 = 1 135

Rate of heat flow, $\dot{Q}/A$ = □t /□□ =1200/1 135 =1057.27 W/m^2

Temperature drop at the interface,

$$\Delta T = \left(\dot{Q}/A\right) \times R = 1057.27 \times 0.01 = 10.57 \text{ K}$$

Example 1.11 A 20 cm thick slab of aluminium (k = 230 W/mK) is placed in contact with a 15 cm thick stainless steel plate (k = 15 W/mK). Due to roughness, 40 percent of the area is in direct contact and the gap (0.0002 m) is filled with air (k = 0.032 W/mK). The difference in temperature between the two outside surfaces of the plate is 200°C Estimate (i) the heat flow rate, (ii) the contact resistance, and (iii) the drop in temperature at the interface.

Solution: Let us assume that out of 40% area m direct contact, half the surface area is occupied by steel and half is occupied by aluminium.

The physical system and its analogous electric circuits is shown in Fig. 2.13.

$$R_1 = \frac{0.2}{230\times1} = 0.00087, \qquad R_2 = \frac{0.0002}{230\times0.2} = 4.348\times10^{-6}$$

$$R_3 = \frac{0.0002}{0.032\times0.6} = 1.04\times10^{-2}, \qquad R_4 = \frac{0.0002}{15\times0.2} = 6.667\times10^{-5}$$

and $R_5 = \dfrac{0.15}{(15\times1)} = 0.01$

Again $1/R_{2,3,4} = 1/R_2 + 1/R_3 + 1/R_4$

$$= 2.3\times10^5 + 96.15 + 1.5\times10^4 = 24.5\times10^4$$

Therefore, $R_{2,3,4} = 4.08\times10^{-6}$

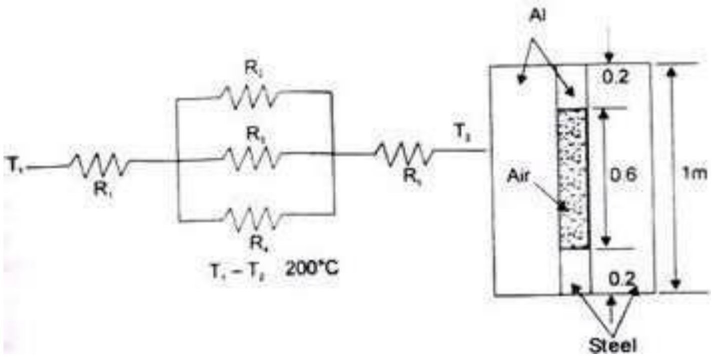


Fig 1.14

Total resistance, $\Sigma R = R_1 + R_{2,3,4} + R_5$

$$= 870\times10^{-6} + 4.08\times10^{-6} + 1000\times10^{-6} = 1.0874\times10^{-2}$$

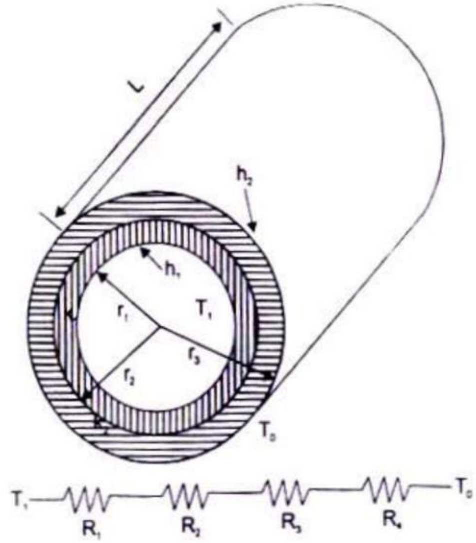
Heat flow rate, $\dot{Q}$ = 200/1.087 × 10^{-2} = 18.392 kW per unit depth of the plate.

Contact resistance, $R\,R_{2,3,4} = 4.08\times10^{-6}$ mK / W

Drop in temperature at the interface, $\Delta T = 4.08 \times 10^{-6} \times 18392 = 0.075^{o}C$

6. An Expression for the Heat Transfer Rate through a Composite Cylindrical System

Let us consider a composite cylindrical system consisting of two coaxial cylinders, radii r_1, r_2 and r_2 and r_3, thermal conductivities k_l and k_2 the convective heat transfer coefficients at the inside and outside surfaces h_1 and h_2 as shown in the figure. Assuming radial conduction under



steady state conditions we have:

Fig 1.15

$$R_1 = 1/h_1A_1 = 1/2\pi r_1 Lh_1$$

$$R_2 = \ln(r_2/r_1)\,2\pi Lk_1$$

$$R_3 = \ln(r_3/r_2)\,2\pi Lk_2$$

$$R_4 = 1/h_2A_2 = 1/2\pi r_3 h_2 L$$

And $\dot{Q}/2\pi L = (T_1 - T_0)/\Sigma R$

$$= (T_1 - T_0)/\left[\left(1/h_1r_1 + \ln(r_2/r_1)/k_1 + \ln(r_3+r_2)/k_2 + 1/h_2r_3\right)\right]$$

Example 1.12 A steel pipe. Inside diameter 100 mm, outside diameter 120 mm (k 50 W/mK) IS Insulated with a 40 mm thick high temperature Insulation (k = 0.09 W/mK) and another Insulation 60 mm thick (k = 0.07 W/mK). The ambient temperature IS 25°C. The heat transfer coefficient for the inside and outside surfaces are 550 and 15 W/m^{2}K respectively. The pipe carries steam at 300^{o}C. Calculate (1) the rate of heat loss by steam per unit length of the pipe (11) the temperature of the outside surface

Solution: I he cross-section of the pipe with two layers of insulation is shown 111 Fig. 1.16. with its analogous electrical circuit.

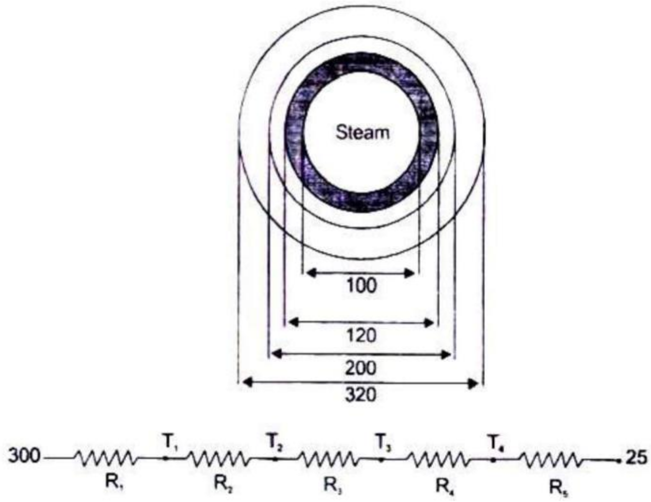


Fig1.16 Cross-section through an insulated cylinder, thermal resistances in series.

For L = 1.0 m. we have

R_1, the resistance of steam film = 1/hA = $1/(500 \times 2 \times 3.14 \times 50 \times 10^{-3}) = 0.00579$

R_2, the resistance of steel pipe = $\ln(r_2/r_1) / 2\,\pi\, k$

$= \ln(60/50)/2\,\pi \times 50 = 0.00058$

R_3, resistance of high temperature Insulation

$\ln(r_3/r_2) / 2\,\pi\, k = \ln(100/60) / 2\,\pi \times 0.09 = 0.903$

$R_4 = 1n(r_4/r_3)/2\,\pi\, k = \ln(160/100)/2\,\pi \times 0.07 = 1.068$

R_5 = resistance of the air film = $1/(15 \times 2\,\pi \times 160 \times 10^{-3}) = 0.0663$

The total resistance = 2.04367

and $\dot{Q}$ $\Delta T / \Sigma R$ = (300 – 25) / 204367 = 134.56 W per metre length of pipe.

Temperature at the outside surface. $T_4 = 25 + R_5$,

$\dot{Q} = 25 + 134.56 \times 0.0663 = 33.92^{o}$ C

When the better insulating material (k = 0.07, thickness 60 mm) is placed first on the steel pipe, the new value of R_3 would be

$R_3 = \ln(120/60) / 2\,\pi \times 0.07 = 1.576$; and the new value of R_4 will be

$R_4 = \ln(160/120)\ 2\,\pi \times 0.09 = 0.5087$

The total resistance = 2.15737 and Q = 275/2.15737 = 127.47 W per m length (Thus the better insulating material be applied first to reduce the heat loss.) The overall heat transfer coefficient, U, is obtained as U = $\dot{Q}/A\,\Delta T$

The outer surface area = π × 320 × 10^{-3} × 1 = 1.0054

and U = 134.56/(275 × 1.0054) = 0.487 W/m^2 K.

Example 1.13 A steam pipe 120 mm outside diameter and 10m long carries steam at a pressure of 30 bar and 099 dry. Calculate the thickness of a lagging material (k = 0.99 W/mK) provided on the steam pipe such that the temperature at the outside surface of the insulated pipe does not exceed 32°C when the steam flow rate is 1 kg/s and the dryness fraction of steam at the exit is 0.975 and there is no pressure drop.

Solution: The latent heat of vaporization of steam at 30 bar = 1794 kJ/kg.

The loss of heat energy due to condensation of steam = 1794(0.99 – 0.975)

= 26.91 kJ/kg.

Since the steam flow rate is 1 kg/s, the loss of energy = 26.91 kW.

The saturation temperature of steam at 30 bar IS 233.84°C and assuming that the pipe material offers negligible resistance to heat flow, the temperature at the outside surface of the uninsulated steam pipe or at the inner surface of the lagging material is 233.84°C. Assuming one-dimensional radial heat flow through the lagging material, we have

$\dot{Q} = (T_1 - T_2)/[\ln(r_2/r_l)]\, 2\pi Lk$

or, 26.91 × 1000 (W) = (233.84 – 32) × 2 π × 10 × 0.99/1n(r/60)

ln (r/60) = 0.4666

r_2/60 = exp (0.4666) = 1.5946

r_2 = 95.68 mm and the thickness = 35.68 mm

Example 1.14 A Wire, diameter 0.5 mm length 30 cm, is laid coaxially in a tube (inside diameter 1 cm, outside diameter 1.5 cm, k = 20 W/mK). The space between the wire and the inside wall of the tube behaves like a hollow tube and is filled with a gas. Calculate the thermal conductivity of the gas if the current flowing through the wire is 5 amps and voltage across the two ends is 4.5 V, temperature of the wire is 160°C, convective heat transfer coefficient at the outer surface of the tube is 12 W/m^{2}K and the ambient temperature is 300K.

Solution: Assuming steady state and one-dimensional radial heat flow, we can draw the thermal circuit as shown In Fig. 1 17.

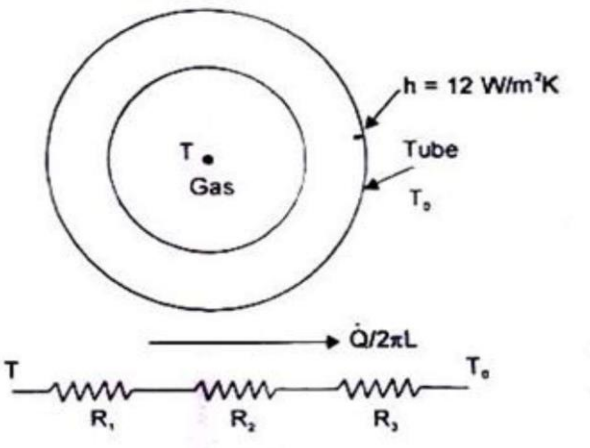


Fig 1.17

The rate of heat transfer through the system,

$\dot{Q}/2\,\pi\,L = VI/2\,\pi\,L = (4.5 \times 5)/(2 \times 3.142 \times 0.3) = 11.935$ (W/m)

R_1, the resistance due to gas = $\ln(r_2/r_l)$, k = ln(0.01/0.0005)/k = 2.996/k.

R_2, resistance offered by the metallic tube = $\ln(r_3 / r_2)$ k

= ln(1.5 /1.0) / 20 = 0.02

R_3, resistance due to fluid film at the outer surface

$l/hr_3 = 1/(l2\times1.5\times I0^{-2}) = 5.556$

and $\dot{Q}$ / 2 π L = □L/□□R = [(273 + 160) – 300]/□R

Therefore, □R = 133/11.935 = 11.l437, and

R_1 = 2.9996/k = 11.1437 – 0.02 – 5.556 = 5.568

or, k = 2.996/5.568 = 0.538 W/mK.

Example 1.15 A steam pipe (inner diameter 16 cm, outer diameter 20 cm, k = 50 W/mK) is covered with a 4 cm thick insulating material (k = 0.09 W/mK). In order to reduce the heat loss, the thickness of the insulation is Increased to 8mm. Calculate the percentage reduction in heat transfer assuming that the convective heat transfer coefficient at the Inside and outside surfaces are 1150 and 10 W/m^2K and their values remain the same.

Solution: Assuming one-dimensional radial conduction under steady state,

$$\dot{Q} / 2\pi L = \Delta T/\Delta R$$

R_1, resistance due to steam film = 1/hr = 1/(1150 × 0.08) = 0.011

R_2, resistance due to pipe material = ln (r_2/r_1)/k = ln (10/8)/50 = 0.00446

R_3, resistance due to 4 cm thick insulation

$= \ln(r_3/r_2)/k = \ln(14/10)/0.09 = 3.738$

R_4, resistance due to air film = 1/hr = 1/(10 × 0.14) = 0.714.

Therefore, $\dot{Q}/2\pi L = \Delta T / (0.011 + 0.00446 + 3.738 + 0.714) = 0.2386\Delta T$

When the thickness of the insulation is increased to 8 cm, the values of R_3 and R_4 will change.

$R_3 = \ln(r_3/r_2)/k = \ln(18/10)/0.09 = 6.53$; and

$R_4 = 1/hr = 1/(10 \times 0.18) = 0.556$

Therefore, $\dot{Q}/2\pi L = \Delta T / (0.011 + 0.00446 + 6.53 + 0.556)$

$= \Delta T / 7.1 = 0.14084 \Delta T$

Percentage reduction in heat transfer = $\dfrac{(0.22386 - 0.14084)}{0.22386} = 0.37 = 37\%$

Example 1.16 A small hemispherical oven is built of an inner layer of insulating fire brick 125 mm thick (k = 0.31 W/mK) and an outer covering of 85% magnesia 40 mm thick (k = 0.05 W/mK). The inner surface of the oven is at 1073 K and the heat transfer coefficient for the outer surface is 10 W/m^2K, the room temperature is 20ºC.

Calculate the rate of heat loss through the hemisphere if the inside radius is 0.6 m.

Solution: The resistance of the fire brick

$$= (r_2 - r_1)/2\pi k r_1 r_2 = \frac{0.725 - 0.6}{2\pi \times 0.31 \times 0.6 \times 0.725} = 0.1478$$

The resistance of 85% magnesia

$$= (r_3 - r_2)/2\pi k r_2 r_3 = \frac{0.765 - 0.725}{2\pi \times 0.05 \times 0.725 \times 0.765} = 0.2295$$

The resistance due to fluid film at the outer surface = 1/hA

$$= \frac{1}{10 \times 2\pi \times (0.765 \times 0.765)} = 0.2295$$

The resistance due to fluid film at the outer surface = 1/hA

$$= \frac{1}{10 \times 2\pi \times (0.765 \times 0.765)} = 0.0272$$

Rate of heat flow, $\dot{Q} = \Delta T / \Sigma R = \dfrac{800 - 20}{0.1478 + 0.2295 + 0.272} = 1930W$

Example 1.17 A cylindrical tank with hemispherical ends is used to store liquid oxygen at $-180^{o}C$. The diameter of the tank is 1.5 m and the total length is 8 m. The tank is covered with a 10 cm thick layer of insulation. Determine the thermal conductivity of the insulating material so that the boil off rate does not exceed 10 kg/hr. The latent heat of vapourization of liquid oxygen is 214 kJ/kg. Assume that the outer surface of insulation is at $27^{o}C$ and the thermal resistance of the wall of the tank is negligible. (ES-94)

Solution: The maximum amount of heat energy that flows by conduction from outside to inside = Mass of liquid oxygen × Latent heat of vapourisation.

= 10 × 214 = 2140 kJ/hr = 2140 × 1000/3600 = 594.44 W

Length of the cylindrical part of the tank = 8 – 2r = 8 – 1.5 = 6.5m

since the thermal resistance of the wall does not offer any resistance to heat flow, the temperature at the inside surface of the insulation can be assumed as - 183°C whereas the temperature at the outside surface of the insulation is 27°C.

Heat energy coming in through the cylindrical part, $\dot{Q}_1 = \dfrac{\Delta T}{\dfrac{\ln(r_2/r_1)}{2\pi Lk}}$

or, $\dot{Q}_1 = \dfrac{(27+183)\times 2\pi\times 6.5k}{\ln(8.5/7.5)} = 68531.84\ k$

Heat energy coming in through the two hemispherical ends,

$\dot{Q}_2 = 2\times(\Delta T\times 2\pi k\ r_2 r_1)/(r_2 - r_1) = \dfrac{2\times 210\times 2\pi k\times 0.85\times 0.75}{0.10}$ = 16825.4 k

Therefore, 594.44 = (68531.84 + 16825.4) k; or, k = 6.96 × 10^{-3} W/mK.

Example 1.18 A spherical vessel, made out of2.5 em thick steel plate IS used to store 10m3 of a liquid at 200°C for a thermal storage system. To reduce the heat loss to the surroundings, a 10 cm thick layer of insulation (k = 0.07 W/rnK) is used. If the convective heat transfer coefficient at the outer surface is W/m^2K and the ambient temperature is 25°C, calculate the rate of heat loss neglecting the thermal resistance of the steel plate.

If the spherical vessel is replaced by a 2 m diameter cylindrical vessel with flat ends, calculate the thickness of insulation required for the same heat loss.

Solution: Volume of the spherical vessel = $10m^3 = \dfrac{4\pi r^3}{3} \quad \therefore\ r = 1.336$ m

Outer radius of the spherical vessle, $r_2 = 1.3364 + 0.025 = 1.361$ m

Outermost radius of the spherical vessel after the insulation = 1.461 m.

Since the thermal resistance of the steel plate is negligible, the temperature at the inside surface of the insulation is 200°C.

Thermal resistance of the insulating material = $(r_3 - r_2)/4\pi\ k\ r_3 r_2$

$$= \frac{0.1}{4\pi\times 0.07\times 1.461\times 1.361} = 0.057$$

Thermal resistance of the fluid film at the outermost surface = 1/hA

$$=1/\left[10\times4\pi\times(1.461)^2\right]=0.00373$$

Rate of heat flow = $\Delta T/\Sigma R=(200-25)/(0.057+0.00373)=2873.8\ \text{W}$

Volume of the insulating material used = $(4/3)\pi\left(r_3^3-r_2^3\right)=2.5\ \text{m}^3$

Volume of the cylindrical vessel $=10\ \text{m}^3=\dfrac{\pi}{4}(d)^2\,L;\ \therefore\ L=10/\pi=3.183\,\text{m}$

Outer radius of cylinder without insulation = 1.0 + 0.025 = 1.025 m.

Outermost radius of the cylinder (with insulation) = r_3.

Therefore, the thickness of insulation = r_3 – 1.025 = □

Resistance, the heat flow by the cylindrical element

$$=\frac{\ln(r_3/1.025)}{2\pi Lk}+1/hA=\frac{\ln(r_3/1.025)}{2\pi\times3.183\times0.07}+\frac{1}{10\times2\pi\times r_3\times3.183}$$

= 0.714 ln (r_3 / 1.025) + 0.005/r_3

Resistance to heat flow through sides of the cylinder

$$=2\delta/kA+1/hA=\frac{2(r_3-1.025)}{0.07\times\pi\times1}+\frac{1}{10\times2\times\pi}$$

$$=9.09(r_3-1.025)+0.0159$$

For the same heat loss, $\Delta T/\Sigma R$ would be equal in both cases, therefore,

$$\frac{1}{0.06073}=\frac{1}{0.714\ \ln(r_3/1.025)+0.005/r_3}+\frac{1}{9.09(r_3-1.025)+0.0159}$$

Solving by trial and error, (r – 1.025j) = □ = 9.2 cm.

and the volume of the insulating material required = 2.692 m^3.

7. Unsteady State Conduction Heat Transfer

7.1 . Transient State Systems-Defined

The process of heat transfer by conduction where the temperature varies with time and with space coordinates, is called 'unsteady or transient'. All transient state systems may be broadly classified into two categories:

(a) Non-periodic Heat Flow System - the temperature at any point within the system changes as a non-linear function of time.

(b) Periodic Heat Flow System - the temperature within the system undergoes periodic changes which may be regular or irregular but definitely cyclic.

There are numerous problems where changes in conditions result in transient temperature distributions and they are quite significant. Such conditions are encountered in - manufacture of ceramics, bricks, glass and heat flow to boiler tubes, metal forming, heat treatment, etc.

7.2. Biot and Fourier Modulus-Definition and Significance

Let us consider an initially heated long cylinder (L >> R) placed in a moving stream of fluid at $T_\infty < T_s$, as shown In Fig. 3.1(a). The convective heat transfer coefficient at the surface is h, where,

$Q = hA\,(T_s - T_\infty)$

This energy must be conducted to the surface, and therefore,

$Q = -kA(dT / dr)_{r = R}$

or, $h(T_s - T_\infty) = -k(dT/dr)_{r=R} \approx -k(T_c - T_s)/R$

where T_c is the temperature at the axis of the cylinder

By rearranging,$(T_s - T_c) / (T_s - T_\infty)$ h/Rk (3.1)

The term, hR/k, IS called the 'BlOT MODULUS'. It is a dimensionless number and is the ratio of internal heat flow resistance to external heat flow resistance and plays a fundamental role in transient conduction problems involving surface convection effects. I t provides a measure 0 f the temperature drop in the solid relative to the temperature difference between the surface and the fluid.

For Bi << 1, it is reasonable to assume a uniform temperature distribution across a solid at any time during a transient process.

Founer Modulus - It is also a dimensionless number and is defind as

$$Fo = \alpha t/L^2 \quad (3.2)$$

where L is the characteristic length of the body, a is the thermal diffusivity, and t is the time

The Fourier modulus measures the magnitude of the rate of conduction relative to the change in temperature, i.e., the unsteady effect. If Fo << 1, the change in temperature will be experienced by a region very close to the surface.

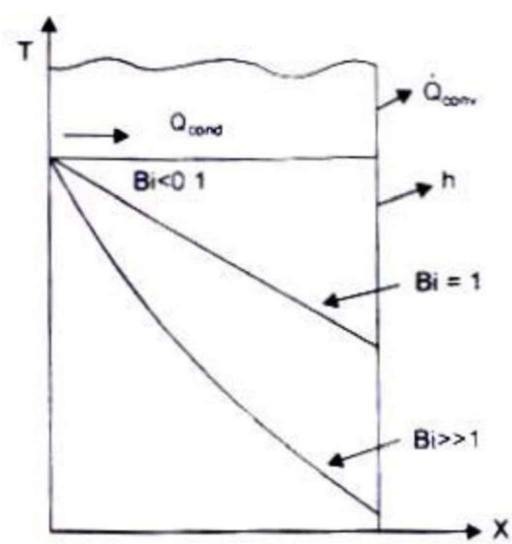


Fig. 1.18 Effect of Biot Modulus on steady state temperature distribution in a plane wall with surface convection.

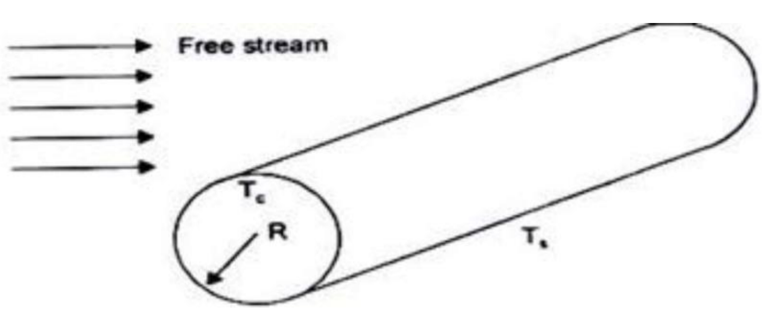


Fig. 1.18 (a) Nomenclature for Biot Modulus

7.3. Lumped Capacity System-Necessary Physical Assumptions

We know that a temperature gradient must exist in a material if heat energy is to be conducted into or out of the body. When Bi < 0.1, it is assumed that the internal thermal resistance of the body is very small in comparison with the external resistance and the transfer of heat energy is primarily controlled by the convective heat transfer at the surface. That is, the temperature within the body is approximately uniform. This idealised assumption is possible, if

(a) the physical size of the body is very small,

(b) the thermal conductivity of the material is very large, and

(c) the convective heat transfer coefficient at the surface is very small and there is a large temperature difference across the fluid layer at the interface.

7.4. An Expression for Evaluating the Temperature Variation in a Solid Using Lumped Capacity Analysis

Let us consider a small metallic object which has been suddenly immersed in a fluid during a heat treatment operation. By applying the first law of

Heat flowing out of the body = Decrease in the internal thermal energy of

during a time dt the body during that time dt

or, $hA_s(T - T_\infty)dt = - pCVdT$

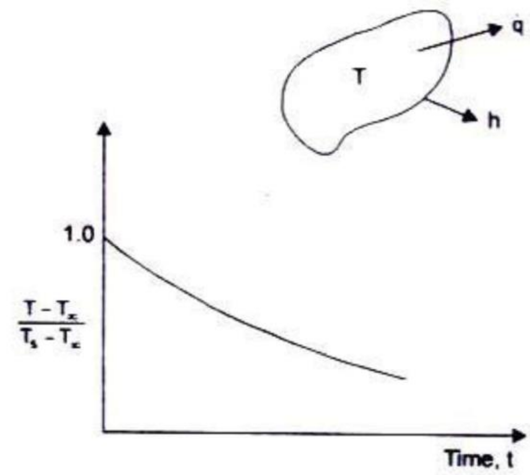
where A_s is the surface area of the body, V is the volume of the body and C is the specific heat capacity.

or, $(hA/\rho CV)dt = - dT/(T - T_\infty)$

with the initial condition being: at t = 0, $T = T_s$

The solution is : $(T - T_\infty)/(T_s - T_\infty) = \exp(-hA/\rho CV)t$ (3.3)

Fig. 3.2 depicts the cooling of a body (temperature distribution □ time) using lumped thermal capacity system. The temperature history is seen to be an exponential decay.



We can express

Bi × Fo = (hL/k)×(αt/L^2) = (hL/k)(k/ ρC)(t/L^2) = (hA/ ρCV)t,

where V / A is the characteristic length L.

And, the solution describing the temperature variation of the object with respect to time is given by

$(T - T_\infty)/(T_s - T_\infty)$ = exp(-Bi· Fo) (3.4)

Example 1.19 Steel balls 10 mm in diameter (k = 48 W/mK), (C = 600 J/kgK) are cooled in air at temperature 35°C from an initial temperature of 750°C. Calculate the time required for the temperature to drop to 150°C when h = 25 W/m2K and density p = 7800 kg/m3.

Solution: Characteristic length, L = VIA = 4/3πr^3/4 π^2 = r/3 = 5 × 10^{-3}/3m

Bi = hL/k = 25 × 5 × 10-^{3}/ (3 × 48) = 8.68 × 10^{-4}<< 0.1,

Since the internal resistance is negligible, we make use of lumped capacity analysis: Eq. (3.4),

$(T - T_\infty) / (T_s - T_\infty)$=exp(-Bi Fo) ; (150 35) / (750 35) = 0.16084

∴ Bi × Fo = 1827; Fo = 1.827/ (8. 68 × 10^{-4}) 2.1× 10^3

or, αt/ L^2 = k/ (ρCL^2)t = 2100 and t = 568 = 0.158 hour

We can also compute the change in the internal energy of the object as:

$$U_0 - U_t = -\int_0^1 \rho CVdT = \int_0^1 \rho CV (T_s - T_\infty)(-hA/\rho CV)\,\text{expt}\,(-hAt/\rho CV)\,dt$$

$$= -\rho CV(T_s - T_\infty)\left[\exp(-hAt/\rho CV) - 1\right] \qquad (3.5)$$

$= -7800 \times 600 \times (4/3)\ \pi (5 \times 10^{-3})^3$ (750-35) (0.16084 - 1)

$= 1.47 \times 10^3$ J = 1.47 kJ.

If we allow the time 't' to go to infinity, we would have a situation that corresponds to steady state in the new environment. The change in internal energy will be $U_0 - U_\infty$ = $[\rho CV(T_s - T_\infty)\exp(-\infty) - 1] = [\rho CV(T_s - T_\infty]$.

We can also compute the instantaneous heal transfer rate at any time.

or. $Q = -\rho VCdT/dt = -\rho VCd/dt[T_\infty + (T_s - T_\infty)\exp(-hAt/\rho CV)]$

$= hA(T_s - T_\infty)[\exp(-hAt/\ \rho CV))$ and for t = 60s,

$Q = 25 \times 4 \times 3.142\ (5 \times 10^{-3})^2 (750\ 35)\ [\exp(-25 \times 3 \times 60/5 \times 10^{-3} \times 7800 \times 600)]$

= 4.63 W.

Example 1.20 A cylindrical steel ingot (diameter 10 cm. length 30 cm, k = 40 W mK. ρ = 7600 kg/m^3, C = 600 J/kgK) is to be heated in a furnace from 50°C to 850°C. The temperature inside the furnace is 1300ºC and the surface heat transfer coefficient is 100 W/m^{2}K. Calculate the time required.

Solution: Characteristic length. $L = V/A = \pi r^2 L/2\ \pi r(r + L) = rL/2(r + L)$

$= 5 \times 10^{-2} \times 30 \times 10^{-2}/2\ (2\ (5 + 30) \times 10^{-2})$

$= 2.143 \times 10^{-2}$ m.

$Bi = hL/k = 100 \times 2.143 \times 10^{-2}/40 = 0.0536 << 0.1$

$Fo = \alpha t/L^2 = (k/\ \rho C) \times (t/L^2)$

$= 40 \times t/(7600 \times 600 \times [2.143 \times 1\ 0^{-2})^2] = 191 \times 10^{-2}\ t$

and $(T - T_\infty)/(T_s - T_\infty) = \exp(-Bi\ Fo)$

or, (850 - 1300) /(50 - 1300) = 0.36 = exp (- Bi Fo)

$\therefore$ Bi Fo= 102

and Fo = 19.06 and t = 19.06/(1.91 × 1 0 $^{-2}$) = 16.63 min

(The length of the ingot is 30 cm and it must be removed from the furnace after a period of 16.63 min. therefore, the speed of the ingot would be 0.3/16.63 = 1.8 × 10^{-2} m/min.)

Example 1.21 A block of aluminium (2cm × 3cm × 4cm, k = 180 W/mK, α = 10 $^{-4}$$m^2$/s) inllially at 300^{o}C is cooled in air at 30^{o}C. Calculate the temperature of the block after 3 min. Take h = 50W/ m^2K.

Solution: Characteristic length, L= [2 × 3 × 4 /2(2 × 3 + 2 × 4 + 3 × 4)] × 10^{-2}

= 4.6 × 10^{-3}m

Bi = hL/k = 50 × 4.6 × 10^{-3}/180 = 1.278 × 10^{-3} << 0.1

Fo = $\alpha t/L^2$ =10^{-4} × 180 / (4.6×10 $^{-3}$)^2 = 850

exp(-Bi Fo) = exp(-1.278 × 10^{-3}×'850) = 0.337

(T - T_∞) (T_s - T_∞) - (T - 30)/(300 - 30) = 0.337

$\therefore$ T= 121.1°C.

Example 1.22 A copper wire 1 mm in diameter initially at 150°C is suddenly dipped into water at 35°C. Calculate the time required to cool to a temperature of 90°C if h = 100 W/ m^2K. What would be the time required if h = 40 W/m^2K. (for copper; k = 370 W/mK, ρ = 8800 kg/m^3. C = 381 J/kgK.

Solution: The characteristic length for a long cylindrical object can be approximated as r/2. As such,

Bi = hL/k = 100 × 0.5 × 10 $^{-3}$/ (2 × 370) = 6.76 × 10^{-5}<< 0.1

Fo = $\alpha t/L^2$ = (k/ ρC) × (t/L^2)

= [370t/(8800 × 381 × (0.25 × 10^{-3})^2] = 1760t

exp(-Bi Fo) = (T - T_∞)(T_s -T_∞)

= (90 - 35)/(150 - 35) = 0.478

$Bi\ Fo = 0.738 = 6.76 \times 10^{-5} \times 1760\ t;$ $\quad \therefore\ t = 6.2s$

when $h = 40\ W/m^2K$, $Bi = 2.7 \times 10^{-5}$ and $2.7 \times 10^{-5} \times 1760\ t = 0.738;$

or, $t = 15.53s$.

Example 1.23 A metallic rod (mass 0.1 kg, C = 350 J/kgK, diameter 12.5 mm, surface area 40cm^2) is initially at 100°C. It is cooled in air at 25°C. If the temperature drops to 40°C in 100 seconds, estimate the surface heat transfer coefficient.

Solution: $hA/\rho CV = hA/mC = h \times 40 \times 10^{-4}/(0.1 \times 350) = 1.143 \times 10^{-4}h$

and, $hAt/\rho CV = 1.143 \times 10^{-4}\ h \times 100 = 1.143 \times 10^{-2}h$

$(T - T_\infty)/(T_s - T_\infty) = (40 - 25)/(100 - 25) = 0.2$

$\therefore \quad \exp(-1.143 \times 10^{-2}h) = 0.2$

or, $\quad 1.143 \times 10^{-2}h = 1.6094$, and $h = 140W/m^2K$.