UNIT V - FRICTION IN MACHINE ELEMENTS

Friction

Friction is a measure of how hard it is to slide one object over another.

1. ***Static friction.*** It is the friction, experienced by a body, when at rest.
2. ***Dynamic friction.*** It is the friction, experienced by a body, when in motion. The dynamic friction is also called ***kinetic friction*** and is less than the static friction.
It is of the following three types:
(a) ***Sliding friction.*** It is the friction, experienced by a body, when it ***slides*** over another body.
(b) Rolling friction. It is the friction, experienced between the surfaces which have ***balls*** or ***rollers*** interposed between them.
(c) ***Pivot friction.*** It is the friction, experienced by a body, due to the ***motion of rotation*** as in case of foot step bearings.

The friction may further be classified as:

1. Friction between unlubricated surfaces, and

2. Friction between lubricated surfaces.

These are discussed in the following articles.

Laws of Static Friction

Following are the laws of static friction:

1. The force of friction always acts in a direction, opposite to that in which the body tends to move.

2. The magnitude of the force of friction is exactly equal to the force, which tends the body to move.

3. The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically

$$F/R_N = \text{constant}$$

4. The force of friction is independent of the area of contact, between the two surfaces.
5. The force of friction depends upon the roughness of the surfaces.

Coefficient of friction

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = F/R_N$$

Consider that a body A of weight (W) is resting on a horizontal plane B, as shown in Fig.
If a horizontal force P is applied to the body, no relative motion will take place until the applied force P is equal to the force of friction F, acting opposite to the direction of motion. The magnitude of this force of friction is $F = \mu.W = \mu.R$

N, where R

N is the normal reaction.

In the limiting case, when the motion just begins, the body will be in equilibrium under the action of the following three forces :

1. Weight of the body (W),

2. Applied horizontal force (P), and

3. Reaction (R) between the body A and the plane B.

The reaction R must, therefore, be equal and opposite to the resultant of W and P and will be inclined at an angle _ to the normal reaction R_N. This angle _ is known as the ***limiting angle of friction***.
It may be defined as the angle which the resultant reaction R makes with the normal reaction R_N.

From, $\tan\Phi = F/R$

Angle of Repose

Consider that a body A of weight (W) is resting on an inclined plane B, If the angle of inclination of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the angle of repose.

Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as **external threads**.

- But if the threads are cut on the internal surface of a hollow rod, these are known as **internal threads**.
- The screw threads are mainly of two types i.e. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V threads are used for the purpose of tightening pieces together.

1. Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.
2. Pitch. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw.
3. Lead. It is the distance, a screw thread advances axially in one turn.
4. Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of a thread).
5. Single-threaded screw. If the lead of a screw is equal to its pitch, it is known as single threaded screw.
6. Multi-threaded screw. If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw e.g. in a double threaded screw, two threads are cut in one lead length. In such cases, all the threads run independently along the length of the rod. Mathematically,

$$\text{Lead} = \text{Pitch} \times \text{Number of threads}$$

Helix angle. It is the slope or inclination of the thread with the horizontal. Mathematically,

$$\tan\alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$

$$= p/\pi d \quad \text{...(In single-threaded screw)}$$

$$= n.p/\pi d \quad \text{...(In multi-threaded screw)}$$

α = Helix angle,

p = Pitch of the screw,

d = Mean diameter of the screw, and

n = Number of threads in one lead.

1. ***An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor.***

Solution. Given : $W = 75\text{ kN} = 75 \times 10^3\text{ N}$; $v = 300\text{ mm/min}$; $p = 6\text{ mm}$; $d_0 = 40\text{ mm}$ $\mu = \tan\phi = 0.1$

We know that mean diameter of the screw,

$$d = d_0 - p/2 = 40 - 6/2 = 37\text{ mm} = 0.037\text{ m}$$

and

$$\tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

∴ Force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 75 \times 10^3\left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1}\right] = 11.43 \times 10^3\text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 11.43 \times 10^3 \times 0.037/2 = 211.45\text{ N-m}$$

We know that speed of the screw,

$$N = \frac{\text{Speed of the nut}}{\text{Pitch of the screw}} = \frac{300}{6} = 50\text{ r.p.m.}$$

and angular speed, $\omega = 2\pi \times 50/60 = 5.24\text{ rad/s}$

∴ Power of the motor $= T.\omega = 211.45 \times 5.24 = 1108\text{ W} = 1.108$ **kW Ans.**

2. *The pitch of 50 mm mean diameter threaded screw of a screw jack is 12.5 mm. The coefficient of friction between the screw and the nut is 0.13. Determine the torque required on the screw to raise a load of 25 kN, assuming the load to rotate with the screw. Determine the ratio of the torque required to raise the load to the torque required to lower the load and also the efficiency of the machine.*

Solution. Given : $d = 50$ mm ; $p = 12.5$ mm ; $\mu = \tan \phi = 0.13$; $W = 25$ kN $= 25 \times 10^3$ N

We know that,
$$\tan \alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$$

and force required on the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan \phi - \tan \alpha}{1 + \tan \phi . \tan \alpha} \right]$$

$$= 25 \times 10^3 \left[\frac{0.08 + 0.13}{1 - 0.08 \times 0.13} \right] = 5305 \text{ N}$$

Torque required on the screw

We know that the torque required on the screw to raise the load,

$$T_1 = P \times d/2 = 5305 \times 50/2 = 132\ 625 \text{ N-mm } \textbf{Ans.}$$

Ratio of the torques required to raise and lower the load

We know that the force required on the screw to lower the load,

$$P = W \tan(\phi - \alpha) = W \left[\frac{\tan \phi - \tan \alpha}{1 + \tan \phi . \tan \alpha} \right]$$

$$= 25 \times 10^3 \left[\frac{0.13 + 0.08}{1 + 0.13 \times 0.08} \right] = 1237 \text{ N}$$

and torque required to lower the load

$$T_2 = P \times d/2 = 1237 \times 50/2 = 30\ 905 \text{ N-mm}$$

∴ Ratio of the torques required,

$$= T_1 / T_2 = 132625/30925 = 4.3 \textbf{ Ans.}$$

Efficiency of the machine

We know that the efficiency,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan \alpha (1 - \tan \alpha . \tan \phi)}{\tan \alpha + \tan \phi} = \frac{0.08(1 - 0.08 \times 0.13)}{0.08 + 0.13}$$

$$= 0.377 = 37.7\% \textbf{ Ans.}$$

Over Hauling and Self-Locking Screws

The torque required to lower the load

$$T = P \times \frac{d}{2} = W \tan(\phi - \alpha) \frac{d}{2}$$

In the above expression, if $\varphi < \alpha$, then torque required to lower the load will be negative. In other words, the load will start moving downward without the application of any torque. Such a condition is known as over hauling of screws. If however $\varphi > \alpha$, the torque required to lower the load will positive, indicating that an effort is applied to lower the load. Such a screw is known as self-locking screw. In other words, a screw will be self-locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle i.e. μ or $\tan \varphi > \tan \alpha$.

3. *A load of 10 kN is raised by means of a screw jack, having a square threaded screw of 12 mm pitch and of mean diameter 50 mm. If a force of 100 N is applied at the end of a lever to raise the load, what should be the length of the lever used? Take coefficient of friction = 0.15. What is the mechanical advantage obtained? State whether the screw is self locking.*

We know that $\tan \alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 50} = 0.0764$

∴ Effort required at the circumference of the screw to raise the load,

$$P = W \tan(\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha . \tan \phi} \right]$$

$$= 10 \times 10^3 \left[\frac{0.0764 + 0.15}{1 - 0.0764 \times 0.15} \right] = 2290 \text{ N}$$

and torque required to overcome friction,

$$T = P \times d/2 = 2290 \times 50/2 = 57\ 250 \text{ N-mm} \quad ...(i)$$

We know that torque applied at the end of the lever,

$$T = P_1 \times l = 100 \times l \text{ N-mm} \quad ...(ii)$$

Equating equations *(i)* and *(ii)*

$$l = 57\ 250/100 = 572.5 \text{ mm} \quad \textbf{Ans.}$$

Mechanical advantage

We know that mechanical advantage,

$$M.A. = \frac{W}{P_1} = \frac{10 \times 10^3}{100} = 100 \quad \textbf{Ans.}$$

Self locking of the screw

We know that efficiency of the screw jack,

$$\eta = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan \alpha (1 - \tan \alpha . \tan \phi)}{\tan \alpha + \tan \phi}$$

$$= \frac{0.0764 (1 - 0.0764 \times 0.15)}{0.0764 + 0.15} = \frac{0.0755}{0.2264} = 0.3335 \text{ or } 33.35\%$$

Friction of Pivot and Collar Bearing

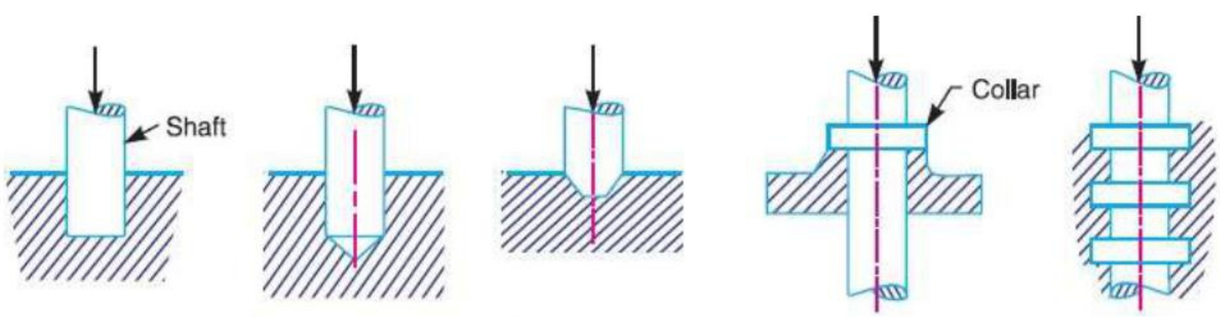
The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft.

The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface

When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig. 10.16 (c).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii

of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution. Given : $W = 20$ kN $= 20 \times 10^3$ N ; $2\,\alpha = 120°$ or $\alpha = 60°$; $p_n = 0.3$ N/mm^2

$N = 200$ r.p.m. or $\omega = 2\,\pi \times 200/60 = 20.95$ rad/s ; $\mu = 0.1$

Outer and inner radii of the bearing surface

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm.

Since the external diameter is twice the internal diameter, therefore

$$r_1 = 2\,r_2$$

We know that intensity of normal pressure (p_n),

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$$\therefore \quad (r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3 \text{ or } r_2 = 84 \text{ mm } \textbf{Ans.}$$

and

$$r_1 = 2\,r_2 = 2 \times 84 = 168 \text{ mm } \textbf{Ans.}$$

Power absorbed in friction

We know that total frictional torque (assuming uniform pressure),

$$T = \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \text{cosec}\,60° = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2}\right] \text{N-mm}$$

$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

$\therefore$ Power absorbed in friction,

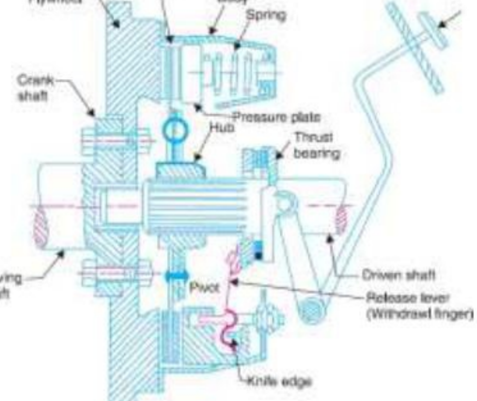
$$P = T.\omega = 301.76 \times 20.95 = 6322 \text{ W} = 6.322 \text{ kW } \textbf{Ans.}$$

Friction Clutches

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces

Single Disc or Plate Clutch

A single disc or plate clutch, as shown in Fig. 10.21, consists of a clutch plate whose both sides are faced with a friction material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel.



1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad ...(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu.p.r^2\,dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

$\therefore$ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.\,p.r^2.dr = 2\,\pi\mu\,p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\,p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (i),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. Considering uniform wear

n = Number of pairs of friction or contact surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{...(For uniform wear)}$$

...(. $p =$

$\therefore$ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

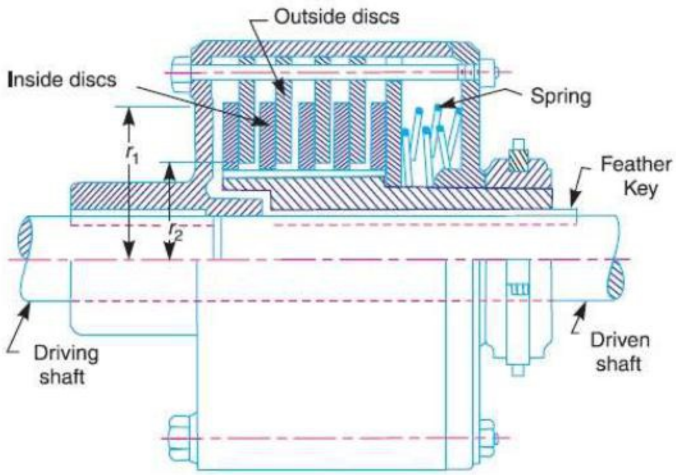
$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

e R = Mean radius of the friction surface $= \dfrac{r_1 + r_2}{2}$

s : 1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by

$$T = n.\mu.W.R$$

Multiple Disc Clutch



Example 10.22. *Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.*

Solution. Given : $W = 4$ kN $= 4 \times 10^3$ N ; $r_2 = 50$ mm ; $r_1 = 100$ mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p_{max} \times r_2 = C \quad \text{or} \quad C = 50\, p_{max}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 50\, p_{max}\,(100 - 50) = 15\,710\, p_{max}$$

$$\therefore \quad p_{max} = 4 \times 10^3/15\,710 = 0.2546 \text{ N/mm}^2$$ **Ans.**

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), therefore

$$p_{min} \times r_1 = C \quad \text{or} \quad C = 100\, p_{min}$$

We know that the total force on the contact surface (W),

$$4 \times 10^3 = 2\,\pi\, C\,(r_1 - r_2) = 2\pi \times 100\, p_{min}\,(100 - 50) = 31\,420\, p_{min}$$

$$\therefore \quad p_{min} = 4 \times 10^3/31\,420 = 0.1273 \text{ N/mm}^2$$ **Ans.**

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$ **Ans.**

Example 10.25. *A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The axial pressure is limited to 0.07 N/mm². If the coefficient of friction is 0.25, find* **1.** *Mean radius and face width of the friction lining assuming the ratio of the mean radius to the face width as 4, and* **2.** *Outer and inner radii of the clutch plate.*

Solution. Given : $P = 7.5$ kW $= 7.5 \times 10^3$ W ; $N = 900$ r.p.m or $\omega = 2\pi \times 900/60 = 94.26$ rad/s ; $p = 0.07$ N/mm² ; $\mu = 0.25$

1. *Mean radius and face width of the friction lining*

Let R = Mean radius of the friction lining in mm, and

w = Face width of the friction lining in mm,

Ratio of mean radius to the face width,

$$R/w = 4 \quad \text{...(Given)}$$

We know that the area of friction faces,

$$A = 2\pi R.w$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\pi R.w.p$$

We know that torque transmitted (considering uniform wear),

$$T = n.\mu.W.R = n.\mu\ (2\pi R.w.p)R$$

$$= n.\mu\left(2\pi R \times \frac{R}{4} \times p\right)R = \frac{\pi}{2} \times n.\mu.p.R^3 \quad \text{...}(\because w = R/4)$$

$$= \frac{\pi}{2} \times 2 \times 0.25 \times 0.07\ R^3 = 0.055\ R^3 \text{ N-mm} \quad \text{...}(i)$$

...($\because$ $n = 2$, for single plate clutch)

We also know that power transmitted (P),

$$7.5 \times 10^3 = T.\omega = T \times 94.26$$

$$\therefore \quad T = 7.5 \times 10^3/94.26 = 79.56 \text{ N-m} = 79.56 \times 10^3 \text{ N-mm} \quad \text{...}(ii)$$

From equations (*i*) and (*ii*),

$$R^3 = 79.56 \times 10^3/0.055 = 1446.5 \times 10^3 \text{ or } R = 113 \text{ mm}$$ **Ans.**

and $w = R/4 = 113/4 = 28.25$mm **Ans.**

2. *Outer and inner radii of the clutch plate*

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the width of the clutch plate is equal to the difference of the outer and inner radii, therefore

$$w = r_1 - r_2 = 28.25 \text{ mm} \quad \text{...}(iii)$$

Also for uniform wear, the mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R = 2 \times 113 = 226 \text{ mm} \quad \text{...}(iv)$$

From equations (*iii*) and (*iv*),

$r_1 = 127.125$ mm ; and $r_2 = 98.875$ **Ans.**

Example 10.28. *A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm², find the power transmitted at 500 r.p.m. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take coefficient of friction = 0.3.*

Solution. Given : $n_1 + n_2 = 5$; $n = 4$; $p = 0.127$ N/mm² ; $N = 500$ r.p.m. or $\omega = 2\pi \times 500/60 = 52.4$ rad/s ; $r_1 = 125$ mm ; $r_2 = 75$ mm ; $\mu = 0.3$

Since the intensity of pressure is maximum at the inner radius r_2, therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.127 \times 75 = 9.525 \text{ N/mm}$$

We know that axial force required to engage the clutch,

$$W = 2\,\pi\,C\,(r_1 - r_2) = 2\,\pi \times 9.525\,(125 - 75) = 2990 \text{ N}$$

and mean radius of the friction surfaces,

$$R = \frac{r_1 + r_2}{2} = \frac{125 + 75}{2} = 100 \text{ mm} = 0.1 \text{ m}$$

We know that torque transmitted,

$$T = n.\mu.W.R = 4 \times 0.3 \times 2990 \times 0.1 = 358.8 \text{ N-m}$$

∴ Power transmitted,

$$P = T.\omega = 358.8 \times 52.4 = 18\,800 \text{ W} = 18.8 \text{ kW}$$ **Ans.**

Example 10.29. *A multi-disc clutch has three discs on the driving shaft and two on the driven shaft. The outside diameter of the contact surfaces is 240 mm and inside diameter 120 mm. Assuming uniform wear and coefficient of friction as 0.3, find the maximum axial intensity of pressure between the discs for transmitting 25 kW at 1575 r.p.m.*

Solution. Given : $n_1 = 3$; $n_2 = 2$; $d_1 = 240$ mm or $r_1 = 120$ mm ; $d_2 = 120$ mm or $r_2 = 60$ mm ; $\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; $N = 1575$ r.p.m. or $\omega = 2\,\pi \times 1575/60 = 165$ rad/s

Let T = Torque transmitted in N-m, and

W = Axial force on each friction surface.

We know that the power transmitted (P),

$$25 \times 10^3 = T.\omega = T \times 165 \quad \text{or} \quad T = 25 \times 10^3/165 = 151.5 \text{ N-m}$$

Number of pairs of friction surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

and mean radius of friction surfaces for uniform wear,

We know that torque transmitted (T),

$$151.5 = n.\mu.W.R = 4 \times 0.3 \times W \times 0.09 = 0.108\,W$$

$$\therefore \quad W = 151.5/0.108 = 1403 \text{ N}$$

Let p = Maximum axial intensity of pressure.

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear

$$p.r_2 = C \quad \text{or} \quad C = p \times 60 = 60\,p \text{ N/mm}$$

We know that the axial force on each friction surface (W),

$$1403 = 2\,\pi.C\,(r_1 - r_2) = 2\,\pi \times 60\,p\,(120 - 60) = 22\,622\,p$$

$$\therefore \quad p = 1403/22\,622 = 0.062 \text{ N/mm}^2$$ **Ans.**

Belt, Rope Drives

The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

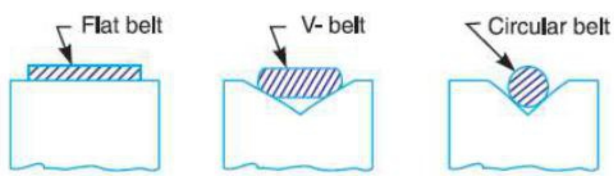
1. The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used. It may be noted that

(a) The shafts should be properly in line to insure uniform tension across the belt section.
(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.
(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thus increasing the friction load on the bearings

The belt drives are usually classified into the following three groups :

1. Light drives. These are used to transmit small powers at belt speeds upto about 10 m/s, as in agricultural machines and small machine tools.
2. Medium drives. These are used to transmit medium power at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.
3. Heavy drives. These are used to transmit large powers at belt speeds above 22 m/s, as in compressors and generators

Types of Belts



(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

11.8. Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Fig. 11.7. Consider a pulley 1 driving the pulley 2. Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4.

Let d_1 = Diameter of the pulley 1,

N_1 = Speed of the pulley 1 in r.p.m.,

d_2, d_3, d_4, and N_2, N_3, N_4 = Corresponding values for pulleys 2, 3 and 4.

We know that velocity ratio of pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \quad \text{...}(i)$$

Similarly, velocity ratio of pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{d_3}{d_4} \quad \text{...}(ii)$$

Multiplying equations (*i*) and (*ii*),

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{d_1}{d_2} \times \frac{d_3}{d_4}$$

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{...}(\because N_2 = N_3, \text{ being keyed to the same shaft})$$

A little consideration will show, that if there are six pulleys, then

$$\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}$$

$$\frac{\text{Speed of last driven}}{\text{Speed of first driver}} = \frac{\text{Product of diameters of drivers}}{\text{Product of diameters of drivens}}$$

Slip

The frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage

An engine, running at 150 rpm drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive

Solution. Given : $N_1 = 150$ r.p.m. ; $d_1 = 750$ mm ; $d_2 = 450$ mm ; $d_3 = 900$ mm ; $d_4 = 150$ mm

The arrangement of belt drive is shown in Fig. 11.10.

Let N_4 = Speed of the dynamo shaft .

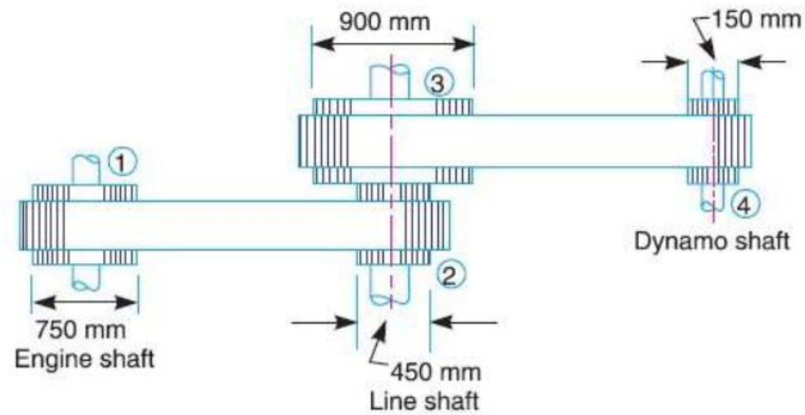


Fig. 11.10

1. *When there is no slip*

We know that $\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$ or $\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$

∴ $N_4 = 150 \times 10 = 1500$ r.p.m. **Ans.**

2. *When there is a slip of 2% at each drive*

We know that $$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

∴ $N_4 = 150 \times 9.6 = 1440$ r.p.m. **Ans.**

Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep.

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

A shaft which rotates at a constant speed of 160 r.p.m. is connected by belting to a parallel shaft 720 mm apart, which has to run at 60, 80 and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for 1. a crossed belt, and 2. an open belt. Neglect belt thickness and slip.

1. *For a crossed belt*

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2}$$

or
$$r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm} \quad \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \quad \text{or} \quad r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2r_3$$

We know that for a crossed belt drive,

$$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = 40 + 106.7 = 146.7 \text{ mm} \quad \text{...}(i)$$

$\therefore$
$$r_3 + 2\,r_3 = 146.7 \quad \text{or} \quad r_3 = 146.7/3 = 48.9 \text{ mm} \quad \textbf{Ans.}$$

and
$$r_4 = 2\,r_3 = 2 \times 48.9 = 97.8 \text{ mm} \quad \textbf{Ans.}$$

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or} \quad r_6 = r_5 \times \frac{N_5}{N_6} = r_5 \times \frac{160}{100} = 1.6r_5$$

From equation (*i*),

$$r_5 + 1.6\,r_5 = 146.7 \quad \text{or} \quad r_5 = 146.7/2.6 = 56.4 \text{ mm} \quad \textbf{Ans.}$$

and
$$r_6 = 1.6\,r_5 = 1.6 \times 56.4 = 90.2 \text{ mm} \quad \textbf{Ans.}$$

2. *For an open belt*

We know that for pulleys 1 and 2,

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \quad \text{or} \quad r_2 = r_1 \times \frac{N_1}{N_2} = 40 \times \frac{160}{60} = 106.7 \text{ mm} \quad \textbf{Ans.}$$

and for pulleys 3 and 4,

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \quad \text{or} \quad r_4 = r_3 \times \frac{N_3}{N_4} = r_3 \times \frac{160}{80} = 2r_3$$

We know that length of belt for an open belt drive,

$$L = \pi(r_1 + r_2) + \frac{(r_2 - r_1)^2}{x} + 2x$$

$$= \pi(40 + 106.7) + \frac{(106.7 - 40)^2}{720} + 2 \times 720 = 1907 \text{ mm}$$

Since the length of the belt in an open belt drive is constant, therefore for pulleys 3 and 4, length of the belt (L),

$$= \pi(r_3 + 2r_3) + \frac{(2r_3 - r_3)^2}{720} + 2 \times 720$$

$$= 9.426\, r_3 + 0.0014\, (r_3)^2 + 1440$$

or $$0.0014\, (r_3)^2 + 9.426\, r_3 - 467 = 0$$

$$\therefore \quad r_3 = \frac{-9.426 \pm \sqrt{(9.426)^2 + 4 \times 0.0014 \times 46}}{2 \times 0.0014}$$

$$= \frac{-9.426 \pm 9.564}{0.0028} = 49.3 \text{ mm} \quad \textbf{Ans.}$$

and $$r_4 = 2\, r_3 = 2 \times 49.3 = 98.6 \text{ mm} \quad \textbf{Ans.}$$

Now for pulleys 5 and 6,

$$\frac{N_6}{N_5} = \frac{r_5}{r_6} \quad \text{or}$$

$$r_6 = \frac{N_5}{N_6} \times r_5 = \frac{160}{100} \times r_5 = 1.6\, r_5$$

and length of the belt (L),

$$1907 = \pi(r_5 + r_6) + \frac{(r_6 - r_5)^2}{x} + 2x$$

$$= \pi(r_5 + 1.6r_5) + \frac{(1.6r_5 - r_5)^2}{720} + 2 \times 720$$

$$= 8.17\, r_5 + 0.0005\, (r_5)^2 + 1440$$

or $$0.0005\, (r_5)^2 + 8.17\, r_5 - 467 = 0$$

$$\therefore \quad r_5 = \frac{-8.17 \pm \sqrt{(8.17)^2 + 4 \times 0.0005 \times 467}}{2 \times 0.0005}$$

$$= \frac{-8.17 \pm 8.23}{0.001} = 60 \text{ mm} \quad \textbf{Ans.}$$

Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25 ?

Solution. Given : $d_1 = 450$ mm $= 0.45$ m or $r_1 = 0.225$ m ; $d_2 = 200$ mm $= 0.2$ m or $r_2 = 0.1$ m ; $x = 1.95$ m ; $N_1 = 200$ r.p.m. ; $T_1 = 1$ kN $= 1000$ N ; $\mu = 0.25$

We know that speed of the belt,

$$v = \frac{\pi d_1 . N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

Length of the belt

We know that length of the crossed belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{m} \quad \textbf{Ans.}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667 \quad \text{or} \quad \alpha = 9.6^\circ$$

$$\therefore \quad \theta = 180^\circ + 2\,\alpha = 180^\circ + 2 \times 9.6^\circ = 199.2^\circ$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad} \quad \textbf{Ans.}$$

Power transmitted

Let T_2 = Tension in the slack side of the belt.

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8692$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8692}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{...(Taking antilog of 0.378)}$$

$$\therefore \quad T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.714 = 2740 \text{ W} = 2.74 \text{ kW} \quad \textbf{Ans.}$$

Example 11.15. *An open belt running over two pulleys 240 mm and 600 mm diameter connects two parallel shafts 3 metres apart and transmits 4 kW from the smaller pulley that rotates at 300 r.p.m. Coefficient of friction between the belt and the pulley is 0.3 and the safe working tension is 10N per mm width. Determine : 1. minimum width of the belt, 2. initial belt tension, and 3. length of the belt required.*

Solution. Given : d_2 = 240 mm = 0.24 m ; d_1 = 600 mm = 0.6 m ; x = 3 m ; P = 4 kW = 4000 W; N_2 = 300 r.p.m. ; μ = 0.3 ; T_1 = 10 N/mm width

1. Minimum width of belt

We know that velocity of the belt,

$$v = \frac{\pi d_2 . N_2}{60} = \frac{\pi \times 0.24 \times 300}{60} = 3.77 \text{ m/s}$$

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

$\therefore$ Power transmitted (P),

$$4000 = (T_1 - T_2)\, v = (T_1 - T_2)\, 3.77$$

or

$$T_1 - T_2 = 4000 / 3.77 = 1061 \text{ N} \qquad \text{...}(i)$$

We know that for an open belt drive,

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{d_1 - d_2}{2x} = \frac{0.6 - 0.24}{2 \times 3} = 0.06 \quad \text{or} \quad \alpha = 3.44°$$

and angle of lap on the smaller pulley,

$$\theta = 180° - 2\alpha = 180° - 2 \times 3.44° = 173.12°$$

$$= 173.12 \times \pi / 180 = 3.022 \text{ rad}$$

We know that

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 3.022 = 0.9066$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.9066}{2.3} = 0.3942 \quad \text{or} \quad \frac{T_1}{T_2} = 2.478 \quad \text{...(ii)}$$

...(Taking antilog of 0.3942)

From equations (*i*) and (*ii*),

$$T_1 = 1779 \text{ N}, \text{ and } T_2 = 718 \text{ N}$$

Since the safe working tension is 10 N per mm width, therefore minimum width of the belt,

$$b = \frac{T_1}{10} = \frac{1779}{10} = 177.9 \text{mm} \quad \textbf{Ans.}$$

2. *Initial belt tension*

We know that initial belt tension,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1779 + 718}{2} = 1248.5 \text{N} \quad \textbf{Ans.}$$

3. *Length of the belt required*

We know that length of the belt required,

$$L = \frac{\pi}{2}(d_1 - d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}$$

$$= \frac{\pi}{2}(0.6 + 0.24) + 2 \times 3 + \frac{(0.6 - 0.24)^2}{4 \times 3}$$

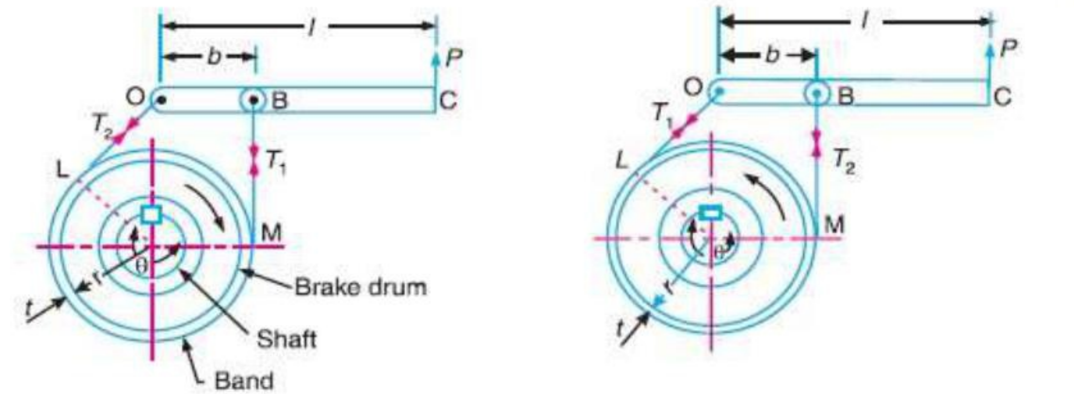
$$= 1.32 + 6 + 0.01 = 7.33 \text{ m} \quad \textbf{Ans.}$$

Friction in brakes- Band and Block brakes.

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc.

Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 19.11, is called a simple band brake in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

Example 19.6. *A band brake acts on the 3/4th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of 225 N-m. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force is applied at 500 mm from the fulcrum and the coefficient of friction is 0.25, find the operating force when the drum rotates in the* **(a)** *anticlockwise direction, and* **(b)** *clockwise direction.*

Solution. Given : $d = 450$ mm or $r = 225$ mm $= 0.225$ m ; $T_B = 225$ N-m ; $b = OB = 100$ mm $= 0.1$ m ; $l = 500$ mm $= 0.5$ m ; $\mu = 0.25$

Let P = Operating force.

(a) Operating force when drum rotates in anticlockwise direction

The band brake is shown in Fig. 19.11. Since one end of the band is attached to the fulcrum at O, therefore the operating force P will act upward and when the drum rotates anticlockwise, as shown in Fig. 19.11 (*b*), the end of the band attached to O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. First of all, let us find the tensions T_1 and T_2.

Drums for band brakes.

We know that angle of wrap,

$$\theta = \frac{3}{4}\text{th of circumference} = \frac{3}{4} \times 360° = 270°$$

$$= 270 \times \pi / 180 = 4.713 \text{ rad}$$

and

$$2.3 \log\left(\frac{T_1}{T_1}\right) = \mu.\theta = 0.25 \times 4.713 = 1.178$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{1.178}{2.3} = 0.5123 \text{ or } \frac{T_1}{T_2} = 3.253 \quad \ldots (i)$$

. . . (Taking antilog of 0.5123)

We know that braking torque (T_B),

$$225 = (T_1 - T_2)\, r = (T_1 - T_2)\, 0.225$$

$$\therefore \quad T_1 - T_2 = 225 / 0.225 = 1000 \text{ N} \quad \ldots (ii)$$

From equations (*i*) and (*ii*), we have

$$T_1 = 1444 \text{ N; and} \quad T_2 = 444 \text{ N}$$

Now taking moments about the fulcrum O, we have

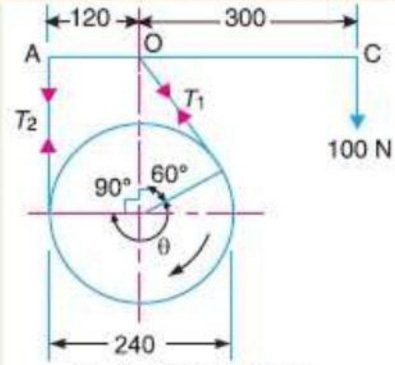
$$P \times l = T_2.b \quad \text{or} \quad P \times 0.5 = 444 \times 0.1 = 44.4$$

$$\therefore \quad P = 44.4 / 0.5 = 88.8 \text{ N } \textbf{Ans.}$$

Example 19.7. *The simple band brake, as shown in Fig. 19.12, is applied to a shaft carrying a flywheel of mass 400 kg. The radius of gyration of the flywheel is 450 mm and runs at 300 r.p.m.*

If the coefficient of friction is 0.2 and the brake drum diameter is 240 mm, find :

1. *the torque applied due to a hand load of 100 N,*
2. *the number of turns of the wheel before it is brought to rest, and*
3. *the time required to bring it to rest, from the moment of the application of the brake.*



All dimensions in mm.

Fig. 19.12

Solution. Given : $m = 400$ kg ; $k = 450$ mm $= 0.45$ m ; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s ; $\mu = 0.2$; $d = 240$ mm $= 0.24$ m or $r = 0.12$ m

1. ***Torque applied due to hand load***

First of all, let us find the tensions in the tight and slack sides of the band *i.e.* T_1 and T_2 respectively.

From the geometry of the Fig. 19.12, angle of lap of the band on the drum,

$$\theta = 360° - 150° = 210° = 210 \times \frac{\pi}{180} = 3.666 \text{ rad}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.2 \times 3.666 = 0.7332$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.7332}{2.3} = 0.3188 \quad \text{or} \quad \frac{T_1}{T_2} = 2.08 \qquad \ldots (i)$$

... (Taking antilog of 0.3188)

Taking moments about the fulcrum O,

$$T_2 \times 120 = 100 \times 300 = 30\,000 \quad \text{or} \quad T_2 = 30\,000/120 = 250 \text{ N}$$

$\therefore$ $T_1 = 2.08 T_2 = 2.08 \times 250 = 520$ N ... [From equation (*i*)]

We know that torque applied,

$$T_B = (T_1 - T_2) r = (520 - 250)\, 0.12 = 32.4 \text{ N-m} \quad \textbf{Ans.}$$

2. ***Number of turns of the wheel before it is brought to rest***

Let n = Number of turns of the wheel before it is brought to rest.

We know that kinetic energy of rotation of the drum

$$= \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 = \frac{1}{2} \times 400(0.45)^2 (31.42)^2 = 40\,000 \text{ N-m}$$

This energy is used to overcome the work done due to the braking torque (T_B).

$$\therefore \quad 40\,000 = T_B \times 2\pi n = 32.4 \times 2\pi n = 203.6\, n$$

or $n = 40\,000 / 203.6 = 196.5$ **Ans.**

Time required to bring the wheel to rest

We know that the time required to bring the wheel to rest = n / N = 196.5 / 300 = 0.655 min = 39.3 s **Ans**

Example 19.13. *The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at 'C' is shown in Fig. 19.26. The distance 'CO' is 75 mm, O being the centre of the drum. The internal radius of the brake drum is 100 mm. The friction lining extends over an arc AB, such that the angle AOC is 135° and angle BOC is 45°. The brake is applied by means of a force at Q, perpendicular to the line CQ, the distance CQ being 150 mm.*

The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of 'θ' with OC and may be taken as equal to $p_1 \sin\theta$, where p_1 is the maximum intensity of normal pressure.

The coefficient of friction may be taken as 0.4 and the braking torque required is 21 N-m. Calculate the force Q required to operate the brake when **1.** *The drum rotates clockwise, and* **2.** *The drum rotates anticlockwise.*

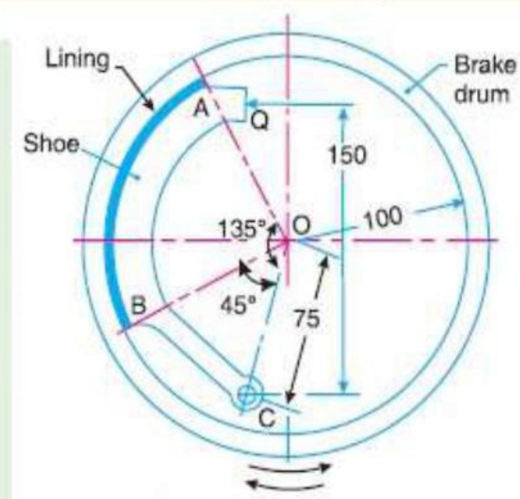


Fig. 19.26

Solution. Given : $OC = 75$ mm ; $r = 100$ mm ;

$\theta_2 = 135° = 135 \times \pi/180 = 2.356$ rad ; $\theta_1 = 45° = 45 \times \pi/180 = 0.786$ rad ; $l = 150$ mm ; $\mu = 0.4$; $T_B = 21$ N-m $= 21 \times 10^3$ N-mm

1. ***Force 'Q' required to operate the brake when drum rotates clockwise***

We know that total braking torque due to shoe (T_B),

$$21 \times 10^3 = \mu . p_1 . b . r^2 (\cos\theta_1 - \cos\theta_2)$$

$$= 0.4 \times p_1 \times b (100)^2 (\cos 45° - \cos 135°) = 5656\ p_1 . b$$

$$\therefore \quad p_1 . b = 21 \times 10^3 / 5656 = 3.7$$

Total moment of normal forces about the fulcrum C,

$$M_N = \frac{1}{2} p_1 . b . r . OC \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2) \right]$$

$$= \frac{1}{2} \times 3.7 \times 100 \times 75 \left[(2.356 - 0.786) + \frac{1}{2}(\sin 90° - \sin 270°) \right]$$

$$= 13\,875\,(1.57 + 1) = 35\,660 \text{ N-mm}$$

and total moment of friction force about the fulcrum C,

$$M_F = \mu . p_1 . b . r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OC}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right]$$

$$= 0.4 \times 3.7 \times 100 \left[100 (\cos 45° - \cos 135°) + \frac{75}{4}(\cos 270° - \cos 90°) \right]$$

$$= 148 \times 141.4 = 20\,930 \text{ N-mm}$$

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N + M_F = 35\,660 + 20\,930 = 56\,590$$

$$\therefore \quad Q = 56\,590 / 150 = 377 \text{ N}$$ **Ans.**

2. Force 'Q' required to operate the brake when drum rotates anticlockwise

Taking moments about the fulcrum C, we have

$$Q \times 150 = M_N - M_F = 35\,660 - 20\,930 = 14\,730$$

$$\therefore \quad Q = 14\,730/150 = 98.2 \text{ N}$$ **Ans.**