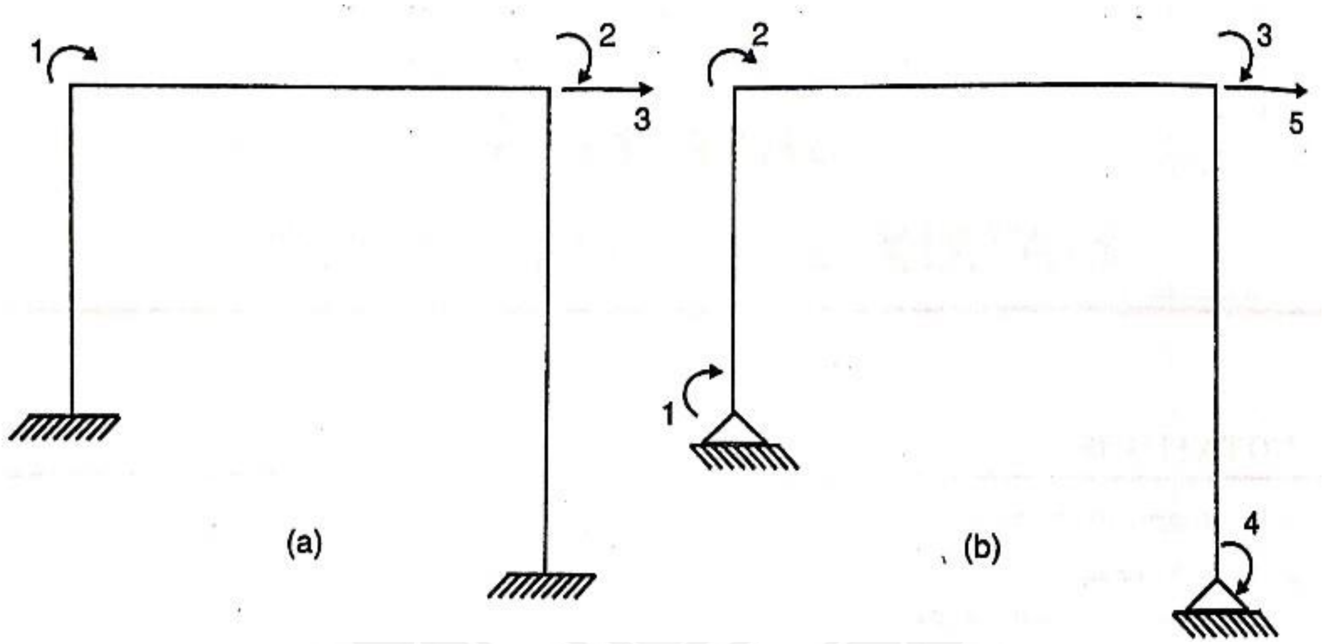
5.2. FORMATION OF STIFFNESS MATRICES

The *n x n* stiffness matrix of a structure with a specified set of *n* co-ordinates is determined by applying one unit displacement at a time and determining the forces at each co-ordinate to sustain that displacement**.**

For example if we want to determine the 3 x 3 stiffness matrix for the structure in this fig.5.1,.



- Find the forces at 1,2 and 3 when displacements at 1 is unity and displacements at 2 and 3 are zero i.e., find P1,P2 and P3 when $\delta_1 = 1$ and $\delta_2 = \delta_3 = 0$.These 3 Forces constitute the first column of the stiffness matrix $[k_1]$.
- Find the 3 forces at 1,2 and 3 when $\delta_2 = 1$ and $\delta_1 = \delta_3 = 0$.These 3 Forces constitute the second column of the stiffness matrix $[k_1]$.
- Find forces at 1,2 and 3 when $\delta_3 = 1$and $\delta_1 = \delta_2 = 0$.These 3 forces make the third column of $[k_1]$.

Example 5.2.1

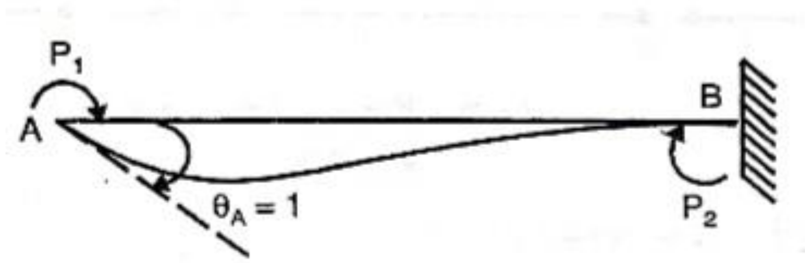
Determine the 2 x 2 stiffness matrix of the beam system shown in fig.5.7

$l = 3$, $A = 1$, $E = 2$, $I = 3$

Fig. 5.7

Solution:

Step 1.To find the first column of [k] apply a unit displacement at 1 only and restrained 2 from rotating



If $\theta = 1$,

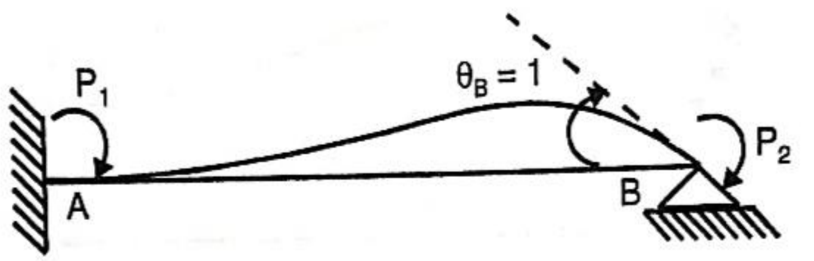
$P_1 = 4EI\theta_A / L = 4 \times 2 \times 3 / 3 = 8$

$P_2 = 2EI\theta_A / L = 2 \times 2 \times 3 / 3 = 4$

Hence;

$$\begin{Bmatrix} k_{11} \\ k_{21} \end{Bmatrix} = \begin{Bmatrix} 8 \\ 4 \end{Bmatrix}$$

Step 2.To get the second column of [k] apply a unit rotation at B and restrain A



$P_2 = 4Ei\theta_B / L = 4 \times 2 \times 3 / 3 = 8$

$P_1 = 2Ei\theta_B / L = 2 \times 2 \times 3 / 3 = 4$

Hence;

$$\begin{Bmatrix} k_{12} \\ k_{22} \end{Bmatrix} = \begin{Bmatrix} 4 \\ 8 \end{Bmatrix} \quad \text{and } [k] = \begin{bmatrix} 8 & 4 \\ 4 & 8 \end{bmatrix}$$