

Error Sources

- ✓ Errors in the detection mechanism can arise from various noises and disturbances associated with the signal detection system.
- ✓ The two most common samples of the spontaneous fluctuations are shot noise and thermal noise.
- ✓ Shot noise arises in electronic devices because of the discrete nature of current flow in the device.
- ✓ Thermal noise arises from the random motion of electrons in a conductor.
- ✓ The random arrival rate of signal photons produces a quantum (or shot) noise at the photodetector. This noise depends on the signal level.
- ✓ This noise is of particular importance for PIN receivers that have large optical input levels and for APD receivers.
- ✓ When using an APD, an additional shot noise arises from the statistical nature of the multiplication process. This noise level increases with increasing avalanche gain M .

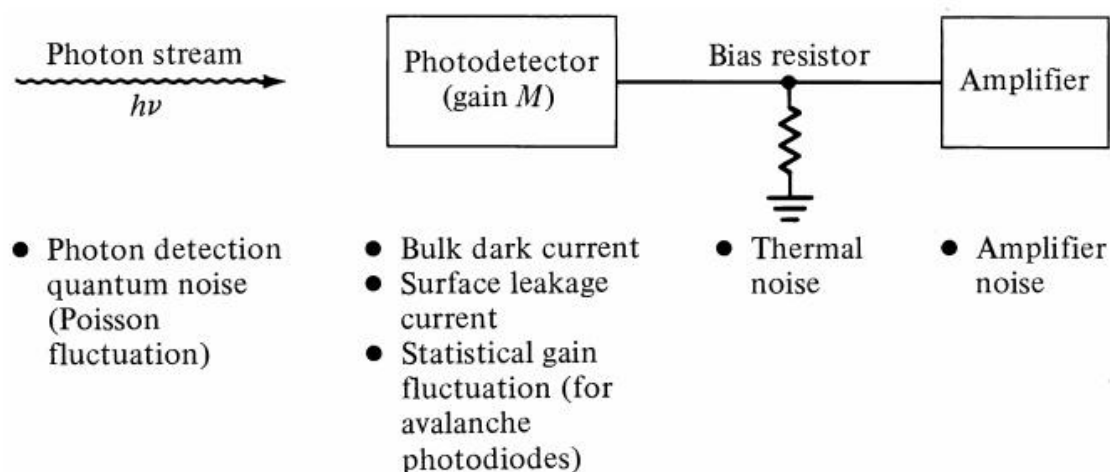


Figure 4.2 Noise sources and disturbances in the optical pulse detection mechanism.

[Source: <http://img.brainkart.com>]

- ✓ Thermal noises arising from the detector load resistor and from the amplifier electronics tend to dominate in applications with low SNR when a PIN photodiode is used.
- ✓ When an APD is used in low-optical-signal level applications, the optimum avalanche gain is determined by a design tradeoff between the thermal noise and the gain-dependent quantum noise.
- ✓ The primary photocurrent generated by the photodiode is a time-varying Poisson process.
- ✓ If the detector is illuminated by an optical signal $P(t)$, then the average number of electron-hole pairs generated in a time t is

$$\bar{N} = \frac{\eta}{h\nu} \int_0^t P(t) dt = \frac{\eta E}{h\nu}$$

where η is the detector quantum efficiency, $h\nu$ is the photon energy, and E is the energy received in a time interval.

- ✓ The actual number of electron-hole pairs n that are generated fluctuates from the average according to the Poisson distribution

$$P_r(n) = \bar{N}^n \frac{e^{-\bar{N}}}{n!}$$

where $Pr(n)$ is the probability that n electrons are emitted in an interval t

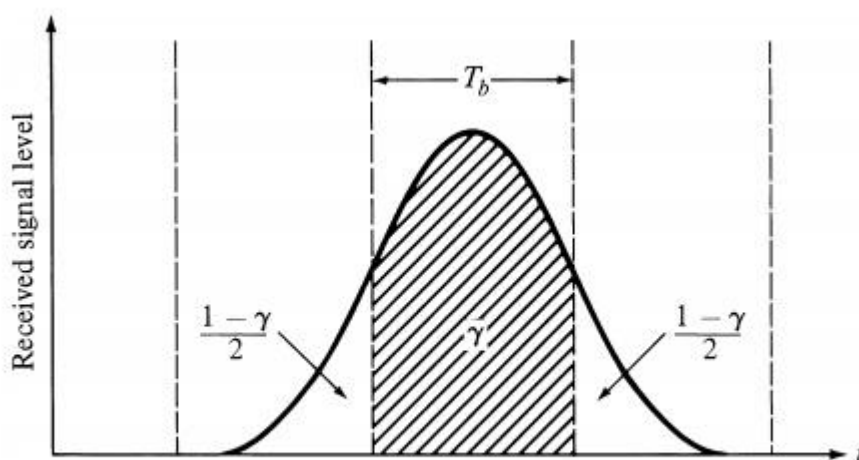


Figure 4.3 Pulse spreading in an optical signal that leads to ISI.

[Source: <http://img.brainkart.com>]

✓ For a detector with a mean avalanche gain M and an ionization rate ratio k , the excess noise factor $F(M)$ for electron injection is where the factor x ranges between 0 and 1.0 depending on the photodiode material.

$$F(M) = kM + \left(2 - \frac{1}{M}\right)(1-k)$$

Or

$$F(M) \cong M^x$$

- ✓ A further error source is attributed to *intersymbol interference* (ISI), which results from pulse spreading in the optical fiber.
- ✓ The fraction of energy remaining in the appropriate time slot is designated by g so that $1-g$ is the fraction of energy that has spread into adjacent time slots.

Receiver Configuration

- ✓ A typical optical receiver is shown in Figure 4.4. The three basic stages of the receiver are a photodetector, an amplifier, and an equalizer.
- ✓ The photo-detector can be either an APD with a mean gain M or a PIN for which $M=1$.
- ✓ The photodiode has a quantum efficiency η and a capacitance C_d .
- ✓ The detector bias resistor has a resistance R_b which generates a thermal noise current $i_b(t)$.

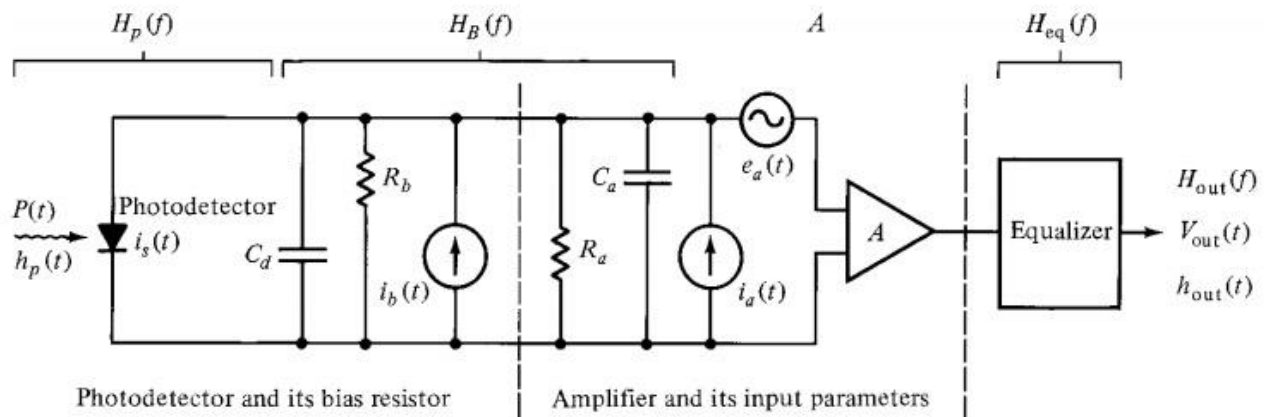


Figure 4.4 Schematic diagram of a typical optical receiver.

[Source: <http://img.brainkart.com>]

Amplifier Noise Sources:

- ✓ The input noise current source $i_a(t)$ arises from the thermal noise of the amplifier input resistance R_a ;
- ✓ The noise voltage source $e_a(t)$ represents the thermal noise of the amplifier channel.
- ✓ The noise sources are assumed to be Gaussian in statistics, flat in spectrum (which characterizes *white* noise), and uncorrelated (statistically independent).

The Linear Equalizer:

- ✓ The equalizer is normally a linear frequency shaping filter that is used to mitigate the effects of signal distortion and inter symbol interference (ISI).
- ✓ The equalizer accepts the combined frequency response of the transmitter, the fiber, and the receiver, and transforms it into a signal response suitable for the following signal processing electronics.
- ✓ The binary digital pulse train incident on the photo-detector can be described by

$$P(t) = \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

✓ Here, $P(t)$ is the received optical power, T_b is the bit period, b_n is an amplitude parameter representing the n th message digit, and $h_p(t)$ is the received pulse shape.

✓ Let the nonnegative photodiode input pulse $h_p(t)$ be normalized to have unit area

$$\int_{-\infty}^{\infty} h_p(t) dt = 1$$

then b_n represents the energy in the n th pulse.

✓ The mean output current from the photodiode at time t resulting from the pulse train given previously is

$$\langle i(t) \rangle = \frac{\eta q}{h\nu} MP(t) = R_0 \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

where $R_0 = \eta q / h\nu$ is the photodiode responsivity.

✓ The above current is then amplified and filtered to produce a mean voltage at the output of the equalizer.

PROBABILITY OF ERROR

- The digital receiver performance can be evolved by measuring of error and quantum limit.
- In practice, several standard ways are available to measuring the rate of error occurrences in a digital data stream.

- **Bit error rate or the error rate [BER]**

Bit error rate is defined by the ratio between number of errors (N_e) occurring over a certain time interval t to the number of error (N_t) during this interval.

$$BER = \frac{N_e}{N_t} = \frac{N_e}{B_t}$$

Where $B=1/T_b$ is the bit rate (is the pulse transmission rate).

- The error rates the optical fiber telecommunication system range from 10^{-9} to 10^{-12} .
- The probability distribution of signal at the equalizer output should be known to compute BER. Here the decision is made as to whether a 0 or a 1 is sent.

$$P_1(v) = \int_{-\infty}^v p(y/1) dy$$

$$P_0(v) = \int_v^{\infty} p(y/0) dy$$

$P_1(v)$ is the probability that the equalizer output voltage is less than v when logic "1" is sent.

$P_0(v)$ is the probability that the equalizer output voltage exceeds " v " when logic '0' is sent.

- The figure shows the probability distribution for received logic 0 and 1. The function $P(y/1)$ and $P(y/0)$ are the conditional probability distribution functions, that is $P(y/x)$ is the probability that the output voltage is y , given that an x was transmitted.

- If the threshold voltage is v_{th} then the error probability P_e is defined as

$$P_e = a P_1(v_{th}) + b P_0(v_{th})$$

- The weighing factors a and b determined by the priori distribution of the data and b are the probabilities that either a 1 or 0 occurs.
- To calculate the “ p_e ”, the mean and standard deviation of the output voltage $V_{out}(t)$ should be known.
- Thus, let us assume that a signal S (Which can be either a noise disturbance or a desired information bearing signal) has a Gaussian probability distribution function with a mean value m .

$$f(s) ds = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(s-m)^2 / 2\sigma^2} \cdot ds$$

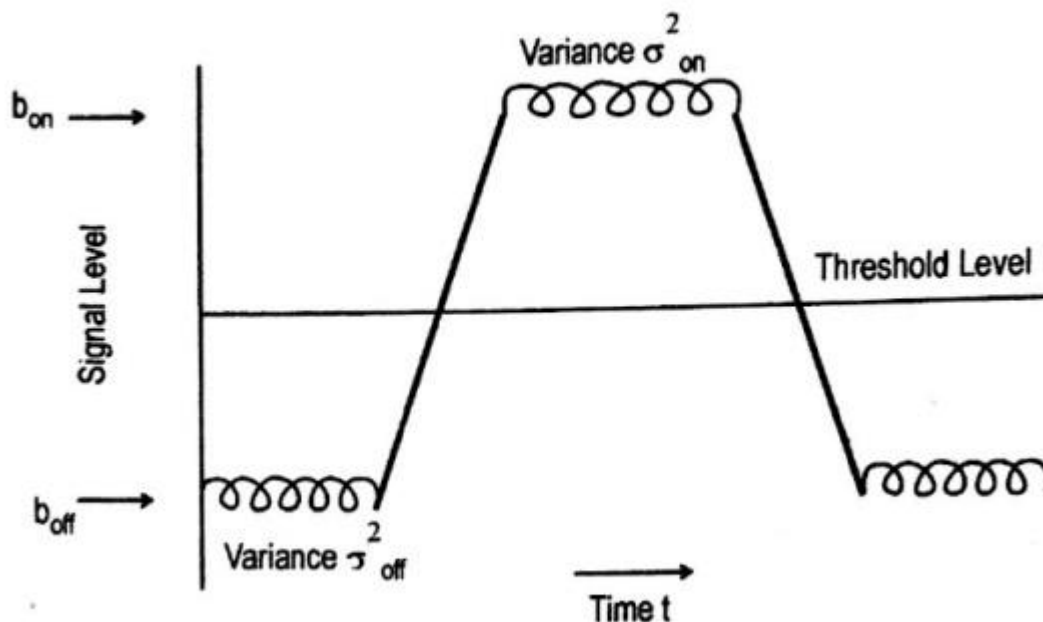


Figure 4.5 Gaussian Noise Statistics of a Binary Signal

[Source: <http://img.brainkart.com>]

- Now, we can use the probability density function to determine the probability of error for a data stream in which the 1 pulse are all of amplitude V .
- The above figure shows, the mean and variance of the Gaussian output for a 1 pulse are b_{on} and σ_{on}^2 , respectively, whereas for a 0 pulse they are b_{off} and σ_{off}^2 , respectively.

PROBABILITY OF ERROR WHEN 0 PULSE SENT

- Let us first consider the case of 0 pulse being sent. So that no pulse is present at the decoding time. The probability that noise will exceed the threshold voltage v_{th} and be 1 pulse.

$$P_0(v_{th}) = \int_{v_{th}}^{\infty} p(y/0) dy = \int_{v_{th}}^{\infty} f_0(y) dy$$

$$= \frac{1}{\sqrt{2\pi} \sigma_{off}} \int_{v_{th}}^{\infty} \exp \left[-\frac{(v - b_{off})^2}{2\sigma_{off}^2} \right] dv$$

- Where the subscript 0 denotes the presence of a 0 bit.

PROBABILITY OF ERROR WHEN 1 PULSE SENT

- Transmitted pulse 1 is misinterpreted as a 0 by the electronics following the equalizer.

$$P_1(v_{th}) = \int_{-\infty}^{v_{th}} p(y/1) dy = \int_{-\infty}^{v_{th}} f_1(v) dv$$

$$= \frac{1}{\sqrt{2\pi} \sigma_{on}} \int_{-\infty}^{v_{th}} \exp \left[-\frac{(b_{on} - v)^2}{2\sigma_{on}^2} \right] dv$$

Where the subscript 1 denotes the presence of a 1 bit.

QUANTUM LIMIT

- An ideal photodetector which has unity quantum efficiency and which produces no dark current, that is no electron hole pairs are generated in the absence of an optical pulse.

- This condition, it is possible to find the minimum received optical power required for a specific bit error rate performance in a digital system. This minimum received power level is known as the quantum limit.
- Assume an optical pulse of energy E falls on the photodetector in a time interval τ . This can only be interpreted by the receiver as a 0 pulse if no electron hole pairs are generated with the pulse with the pulse present.
- The probability that $n=0$ electrons are emitted in a time interval τ is

$$P_r(0) = e^{-\bar{N}}$$

where the average number of electron-hole pairs, $\bar{N} = \eta E / h\nu$.

- Thus for a given probability $P_r(0)$, we can find the minimum energy E required at a specific wavelength λ .