## 2.9 ACTIVE FILTERS

An electric filter is often a frequency selective circuit that passes a specified band of frequencies and blocks or alternates signal and frequencies outside this band. Filters may be classified as

- Analog or digital.
- Active or passive
- Audio (AF) or Radio Frequency (RF)

1. Analog or digital filters:

Analog filters are designed to process analog signals, while digital filters process analog signals using digital technique.

2. Active or Passive:

Depending on the type of elements used in their construction, filter may be classified as passive or Active elements used in passive filters are Resistors, capacitors, inductors. Elements used in active filters are transistor, or op-amp.

## ACTIVE FILTERS OFFER THE FOLLOWING ADVANTAGES OVER PASSIVE FILTERS

1. Gain and Frequency adjustment flexibility:

Since the op-amp is capable of providing gain, the i/p signal is not attenuated as it is in a passive filter. [Active filter is easier to tune or adjust].

2. No loading problem:

Because of the high input resistance and low o/p resistance of the op-amp, the active filter does not cause loading of the source or load.
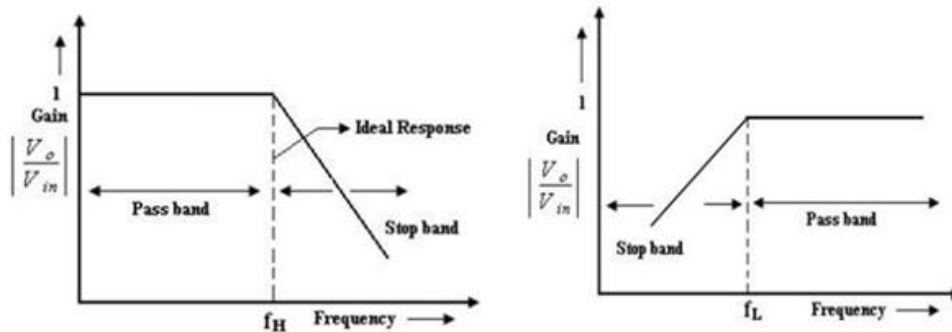
3. Cost:

Active filters are more economical than passive filter. This is because of the variety of cheaper op-amps and the absence of inductors.

The most commonly used filters are these:

1. Low pass Filters

2. High pass Filters
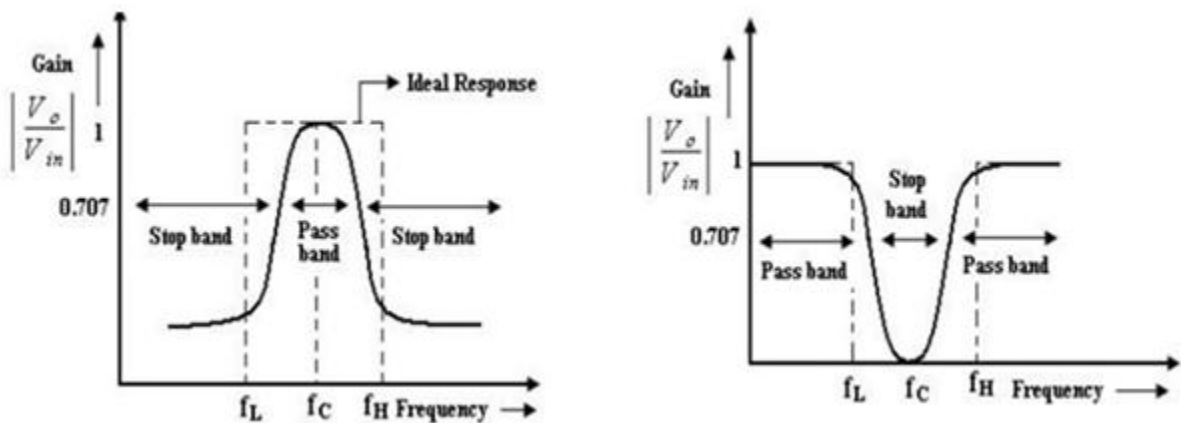
3. Band pass filters

4.      Band –reject filters

Frequency response of the active filters:



**Figure 2.9.1 a).Frequency response of LPF and HPF**

*[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]*



**Figure 2.9.1 b) Frequency response of BPF and Band reject filter**

*[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]*

## LOW PASS FILTERS

- It has a constant gain from 0 Hz to a high cutoff frequency $f_1$.
- At $f_H$ the gain in down by 3db.
- The frequency between 0 Hz and $f_H$ are known as the pass band frequencies where as the range of frequencies those beyond $f_H$, that are attenuated includes the stop band  frequencies.

## HIGH PASS FILTER

High pass filter with a stop band $0 < f < f_L$ and a pass band $f > f_L$

$f_L$ -> low cut off frequency

f -> operating frequency.
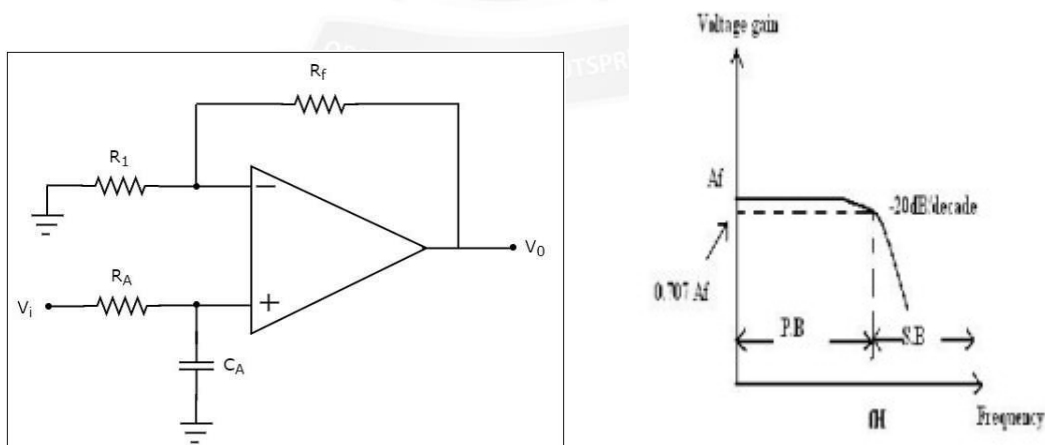
## BAND PASS FILTER

It has a pass band between 2 cut off frequencies $f_H$ and $f_L$ where $f_H > f_L$ and two, stop bands: $0 < f < f_L$ and $f > f_H$ between the band pass filter (equal to $f_H - f_L$). Band –reject filter: (Band stop or Band elimination). It performs exactly opposite to the band pass.It has a band stop between 2 cut-off frequency $f_L$ and $f_H$ and 2 pass bands: $0 < f < f_L$ and $f > f_H$ $f_C$ -> center frequency.

## FIRST ORDER LPF BUTTERWORTH FILTER

First order LPF that uses an RC for filtering op-amp shown in figure 2.9.2a) is used in the non-inverting configuration. Figure 2.9.2 b) shows the frequency response of first order LPF. Resistor $R_1$ & $R_f$ determine the gain of the filter. According to the voltage –divider rule, the voltage at the non-inverting terminal (across capacitor) C is,



**Figure 2.9.2 a) First order low pass filter  figure 2.9.2 b)frequency response**

*[source:https://www.tutorialspoint.com/linear_integrated_circuits_applications/linear_integr ated_circuits_applications_active_filters.htm]*

Gain A= $(1+R_f/R_1)$

Voltage across capacitor  $V_1 = V_i / (1+j2\pi fRC)$

Output voltage $V_0$ for non inverting amplifier $= A \, V_1$

$$= (1+R_f/R_1) \, Vi/(1+j2\pi fRC)$$

Overall gain $V_0/V_i = (1+R_f/R_1) \, Vi/(1+j2\pi fRC)$

Transfer function $H(s) = A/(jf/f_h+1)$

if $f_h = 1/2\pi RC$

$H(j\omega) = A/(j\omega RC+1) = A/(j\omega RC+1)$.

The gain magnitude and phase angle of the equation of the LPF can be obtained by converting eqn. (1) b into its equivalent polar form as follows.

1. At very low $\omega$)|frequency, $f < f_H$

$$|H(j\omega) = A$$

2. At $f = f_H$

$$|H(j\omega)| = A/\sqrt{2} = 0.707A$$

3. At $f > f_H$

$$|H(j\omega)| << A \cong 0$$

When the frequency increases by tenfold (one decade), the volt gain is divided by 10. The gain falls by 20 dB ($=20\log10$) each time the frequency is reduces by 10. Hence the rate at which the gain rolls off $f_H = 20$ dB or 6dB/octave (twofold $R_{in}$ frequency). The frequency $f = f_H$ is called the cut off frequency because the gain of the filter at this frequency is down by 3 dB ($=20 \log 0.707$).

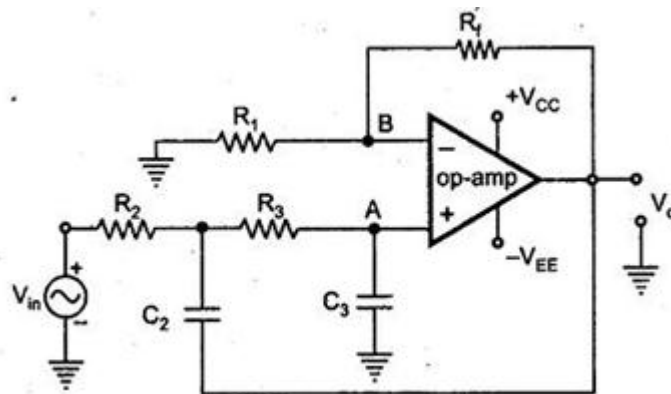**SECOND ORDER LP BUTTERWORTH FILTER**

A second order LPF having a gain 40dB/decade in stop band. A First order LPF can be converted into a II order type simply by using an additional RC network shown in figure 2.9.3

- An improved filter response can be obtained by using a second order active filter.

- A second order active filter consists of two RC pairs & has roll off rate of -40db/decade.

- The op-amp is connected as non-inverting amplifier hence

$$V_o = (1 + \frac{R_f}{R_1})V_B = A_o V_B$$

$$where, A_o = \left(1 + \frac{R_f}{R_1}\right)$$

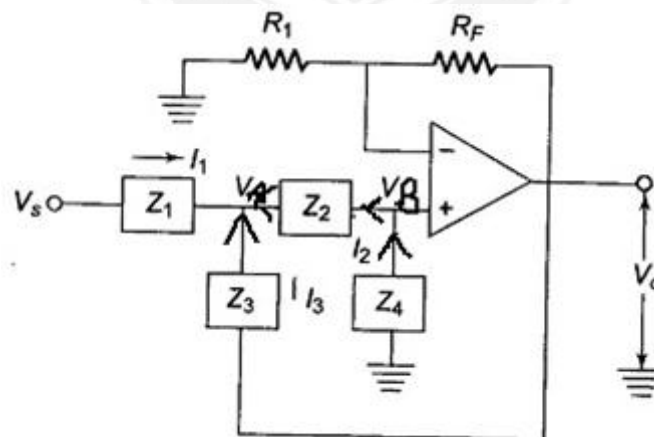$$and \ V_B \rightarrow voltage \ at \ node \ B$$



**Figure 2.9.3.Second order low pass butterworth filter**

*[source: https://www.eeeguide.com/second-order-low-pass-butterworth-filter/]*

Let us consider the General prototype second order filter circuit as in figure 2.9.4.



**Figure 2.9.4 General prototype second order filter circuit**

*[source: https://www.eeeguide.com/second-order-low-pass-butterworth-filter/]*

KCL at node A,

$$(V_i - V_A)Z_1 + (V_0 - V_A)Z_3 + (V_B - V_A)Z_2 = 0$$

$$V_i Z_1 + V_o Z_3 + V_B Z_2 - V_A(Z_1 + Z_2 + Z_3) = 0$$

$$V_i Z_1 = V_A(Z_1 + Z_2 + Z_3) - V_B Z_2 - V_o Z_3$$

$$A_o = \frac{V_o}{V_B}$$

$$V_B = \frac{V_o}{A_o}$$

$$V_i Z_1 = V_A(Z_1 + Z_2 + Z_3) - V_B Z_2 - \frac{V_o}{A_o} Z_3 \text{-----(1)}$$

$KCL\ at\ node\ B$ ,

$$(V_B - V_A)Z_2 + V_B Z_4 = 0$$

$$V_A Z_2 = V_B(Z_4 + Z_2)$$

$$V_A Z_2 = \frac{V_o}{A_o}(Z_4 + Z_2)\text{------(2)}$$

$$V_A = \frac{V_o}{A_o}\frac{(Z_2 + Z_4)}{Z_2}$$

$Sub\ V_A\ (2)\ in\ (1)$

$$V_i Z_1 = \frac{V_o}{A_o}\frac{(Z_2 + Z_4)}{Z_2}(Z_1 + Z_2 + Z_3) - V_B Z_2 - \frac{V_o}{A_o}Z_3$$

$$V_i Z_1 = V_o\left(\frac{(Z_2 + Z_4)(Z_1 + Z_2 + Z_3) - Z_3(A_o Z_2) - Z^2{}_2}{A_o Z_2}\right)$$

$$\frac{V_o}{V_i} = \frac{A_o Z_1 Z_2}{Z_1 Z_2 + Z^2{}_2 + Z_2 Z_3 + Z_1 Z_4 + Z_2 Z_4 + Z_3 Z_4 - A_o Z_2 Z_3 - Z^2{}_2}$$

$$\frac{V_o}{V_i} = \frac{A_o Z_1 Z_2}{Z_1 Z_2 + Z_4(Z_1 + Z_2 + Z_3) + Z_2 Z_3(1 - A_0)} \quad --(3)$$

$To\ make\ a\ low\ pass\ filter\ , choose\ Z_1 = Z_2 = \frac{1}{R}$ $And$ $\quad Z_3 = Z_4 =$
$SC\ from\ first\ fig.$

$\quad From\ (3), we\ get\ the\ transfer\ function\ H(s) of\ a\ low\ pass\ filter\ as$

$$H(S) = \frac{\frac{A_o}{R^2}}{\left(\frac{1}{R^2} + SC\left(\frac{1}{R} + \frac{1}{R} + SC\right) + \frac{SC}{R}(1 - A_o)\right)}$$

$After\ simplifying,\ we\ get$

$$H(S) = \frac{A_o}{S^2 C^2 R^2 + SCR(3 - A_o) + 1} \quad \text{-----(4)}$$

*From (4),*

$$H(s) = A_o, \ for \ S = 0$$

$$H(s) = \infty, \ for \ S = \infty$$

*The transfer function of the low pass second order system can be written as*

$$H(s) = \frac{A_o \omega_n^2}{S^2 + \alpha \omega_n S + \omega_n^2} \quad \text{-------(5)}$$

*Where,* $\qquad A_o \rightarrow$ *the gain*

$\omega_n \rightarrow$ *upper cutoff frequency in rad/ sec*

$\alpha \rightarrow$ *sampling coefficient*

*comparing equ (4)&(5)*

$$\omega_n = \frac{1}{RC} \quad , \alpha = (3 - A_o)$$

*The value of the damping coefficient $\alpha$ for low pass active RC filter can be determined by the value of $A_o$ chosen*

*Sub $S = j\omega$ in (5)*

$$H(j\omega) = \frac{A_o \omega_n^2}{(j\omega)^2 + \alpha \omega_n j\omega + \omega_n^2}$$

$$H(j\omega) = \frac{A_o}{\left(\frac{j\omega}{\omega_n}\right)^2 + j\alpha \frac{\omega}{\omega_n} + 1}$$

*The normalised expression for lowpass filter is*

$$H(j\omega) = \frac{A_o}{S_n^2 + \alpha S_n + 1}$$

*Where, normalised frequency* $S_n = j\left(\frac{\omega}{\omega_n}\right)$

*The expression of magnitude in db of the transfer function is*

$$20 \log|H(j\omega)| = 20 \log\left(\frac{A_o}{1 + j\alpha\left(\frac{\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2}\right)$$

$$= 20 \log\left(\frac{A_o}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)2 + \left(\alpha\frac{\omega}{\omega_n}\right)^2}\right)$$

## FIRST ORDER HP BUTTERWORTH FILTER

High pass filters are often formed simply by interchanging frequency-determining resistors and capacitors in low-pass filters. (i.e) I order HPF is formed from a I order LPF by interchanging components R & C as shown in figure 2.9.5. Similarly, II order HPF is formed from a II order LPF by interchanging R & C.
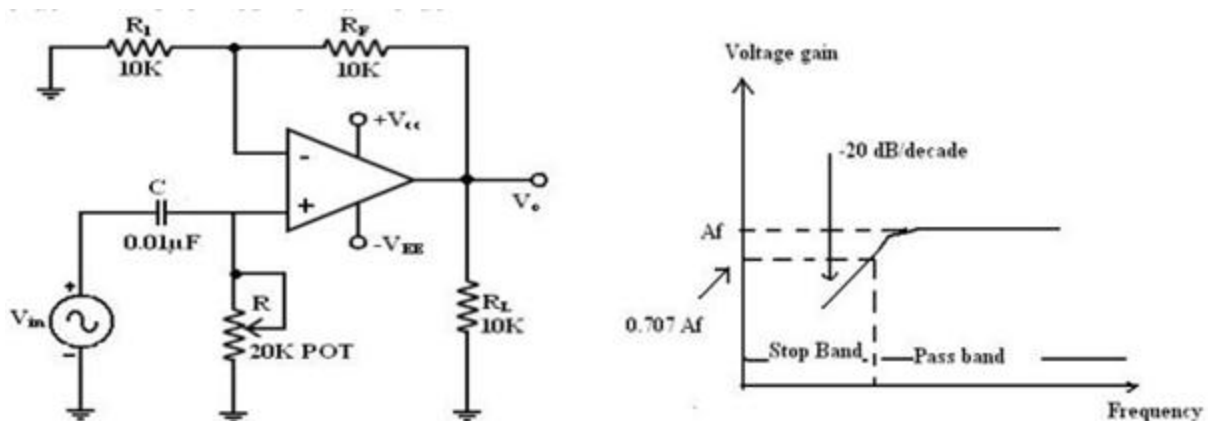


**Figure 2.9.5.first order high pass filter and its frequency response**

*[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]*

Here I order HPF with a low cut off frequency of $f_L$. This is the frequency at which the magnitude of the gain is 0.707 times its passband value.

Here all the frequencies higher than $f_L$ are passband frequencies.

The output voltage $V_0$ of the first order active high pass filter is

$$V_o = \left(1 + \frac{R_f}{R_i}\right) \frac{j2\pi fRC}{1 + j2\pi fRC} V_i$$

The gain of the filter:

$$\frac{V_o}{V_i} = A \left( \frac{j\left(\frac{f}{f_L}\right)}{1 + j\left(\frac{f}{f_L}\right)} \right)$$

Frequency response of the filter

$$|H(jf)| = \left|\frac{V_o}{V_i}\right| = \frac{A\left(\frac{f}{f_L}\right)}{\sqrt{1 + \left(\frac{f}{f_L}\right)^2}} = \frac{A}{\sqrt{1 - \left(\frac{f_L}{f}\right)^2}} \quad \text{is}$$
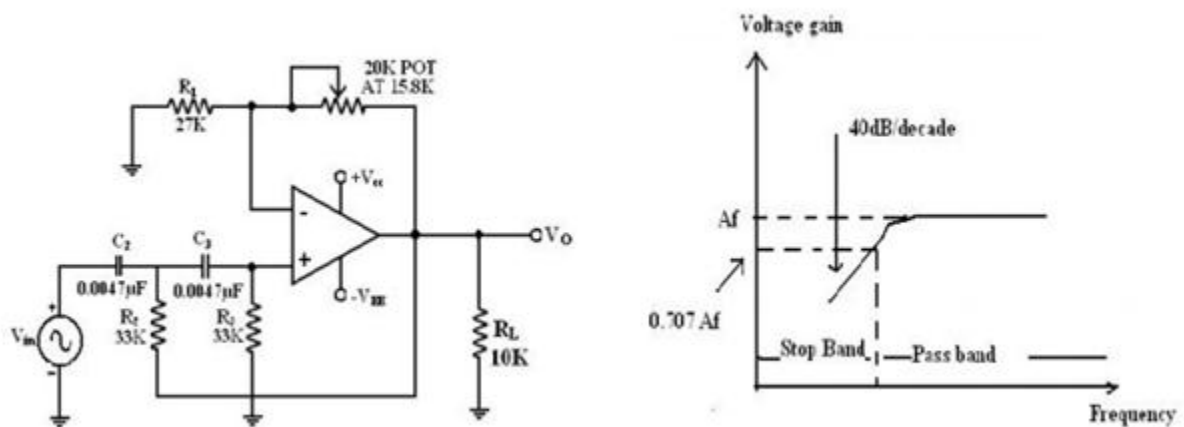
- At high frequencies $f > f_L$ gain = A.

- At f= $f_L$ gain = 0.707 A.
- At f < $f_L$ the gain decreases at a rate of -20 db /decade. The frequency below cutoff

  frequency is stop band.

## SECOND – ORDER HIGH PASS BUTTERWORTH FILTER

I order Filter, II order HPF can be formed from a II order LPF by interchanging the frequency



**Figure 2.9.6 second order high pass filter and its frequency response**

*[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]*
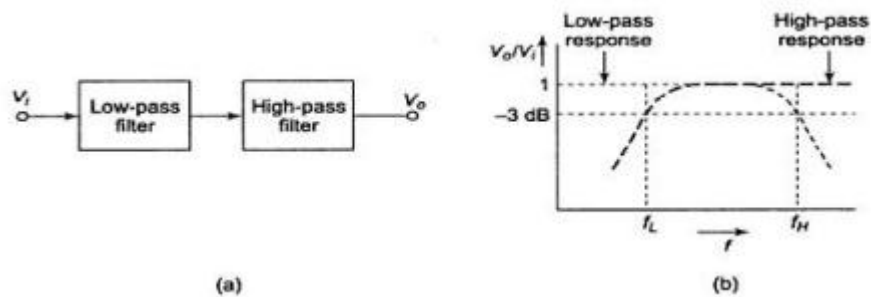
## BAND PASS FILTERS

- Filters that pass band of frequencies and attenuates others. Its high cutoff frequency and low cutoff frequency are related as $f_H > f_L$ and maximum gain at resonant frequency

$$f_r = \sqrt{f_H f_L}$$

- Figure of merit Q= $f_r / (f_H - f_L)$ = $f_r/B$ where B= bandwidth.
- 2 types of filters are Narrow band pass and wide band pass filters

## WIDE BAND PASS FILTER

It is connection of a low pass filter and a high pass filter in cascade as in figure 2.9.7.The $f_H$ of low pass filter and $f_L$ of high pass filter are related as $f_H > f_L$



**Figure 2.9.7 a)wide band pass filter b) its frequency response**

*[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]*