



**ROHINI**  
COLLEGE OF ENGINEERING & TECHNOLOGY

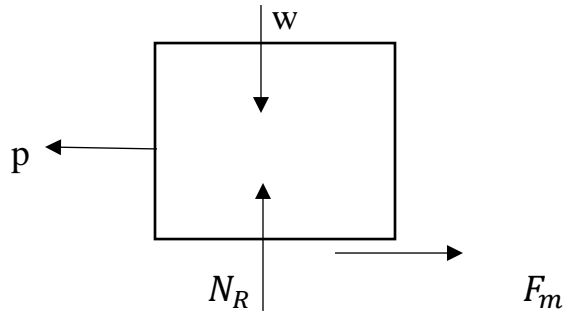
## Unit –V

### FRICTION

Friction:

When the two surface contact with each other and one surface tends to move with respect to another surface.

The tangent force developed in the contact surface and in the opposite reaction.



Types of Friction:

- Dry Friction or coulomb friction
- Fluid friction

Dry friction:

It is referred to the friction which is developed b/w two dry surface

Dry friction types

1. Static friction
2. Dynamic friction

Static friction:

It is the friction which experience by object when it is rest.

Dynamic friction:

It is the friction which experience by object in moving condition.

Dynamic friction further classified by two types

1. Sliding friction
2. Rolling friction

Coefficient of Friction:

It is the ratio b/w limiting friction and the normal reaction [normal friction] is called coefficient of friction. it is denoted by letter  $\mu$

$$\mu = \frac{F_N}{R}$$

### Coulomb Law of Friction [Dry Law]

1. Law of static friction
2. Law of dynamic friction

#### Law of static friction:

- \* The frictional force is always act opposite direction to be movement of the object or body
- \* The frictional force does not depends upon the size and shape of the object
- \* The frictional force depends the degree of roughness of the contact surface b/w the two object
- \* The frictional force is equal to force applied to the body or object
- \* Limiting friction is directly propositional to the normal friction

$$F_N \propto N_R$$

#### Coulomb Law of Dynamic Friction:

- \* The frictional force is always opposite to the movement of the object.
- \* The magnitude of the dynamic friction the contact ratio to normal reaction b/w two system
- \* The coefficient of kinetic friction less then coefficient of static friction

#### Angle of friction:

Angle of friction is Angle made by the resultant with the normal reaction and frictional force.

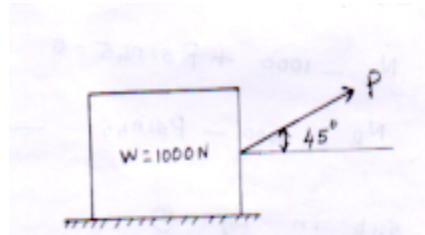
$$\tan \phi = \frac{F_N}{N_R}$$

$$\phi = \tan^{-1} \left[ \frac{F_N}{N_R} \right]$$

Problem based on Friction:

1. A body weighting 1000N is lying on a horizontal plane. Determine the necessary force to move the body along the plane if the force is applied at angle of  $45^\circ$  to the horizontal with the coefficient of friction 0.24

Given:

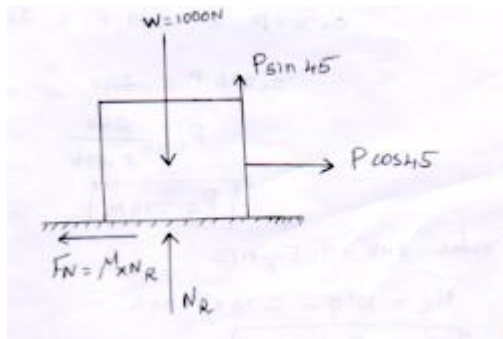


Coefficient of friction  $\mu = 0.24$

Weight of body  $w = 1000\text{ N}$

To find

Soln



Sum of X Direction Force +  $\longrightarrow$  -  $\longleftarrow$

$$\sum F_x = 0$$

$$P \cos 45 - F_N = 0$$

$$P \cos 45 - \mu \times N_R = 0$$

$$P \cos 45 - 0.24 \times N_R = 0 \text{ ----- (1)}$$

Sum of Y direction force  $\uparrow + \downarrow -$

$$\sum F_y = 0$$

$$N_R - 1000 + P \sin 45 = 0$$

$$N_R = 1000 + P \sin 45 = 0$$

$N_R$  value sub in Eqn(1)

$$P \cos 45 - 0.24 \times [1000 - P \sin 45] = 0$$

$$P \cos 45 - 240 + [P \sin 45 \times 0.24] = 0$$

$$0.707P + 0.169P = 240$$

$$0.876P = 240$$

$$P = \frac{240}{0.876}$$

$$P = 273N$$

P value sub in eqn (2)

$$N_R = 1000 - 273 \times \sin 45$$

$$N_R = 806.95 N$$

Problem: 2

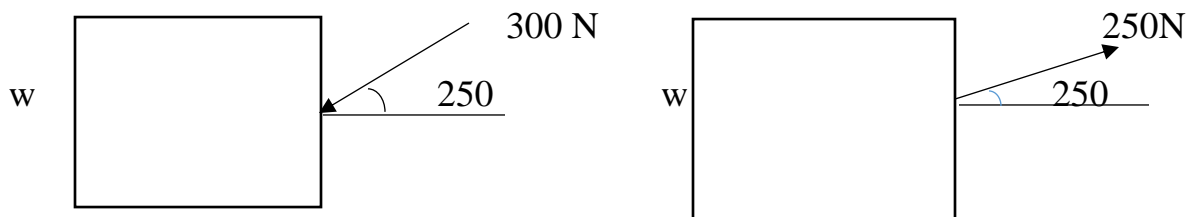
A pull of 250N inclined at  $25^\circ$  to the horizontal plane is required just to move a body kept on a rough horizontal plane. But the push required just to move the body is 300N. If the push is inclined at  $25^\circ$  to the horizontal. Find the weight of the body and the coefficient of friction b/w the body and the plane.

Given:

$$P_1 = \text{pull load} = 250 N \text{ at } 25^\circ$$

$$P_2 = \text{push load} = 300 N \text{ at } 25^\circ$$

To find

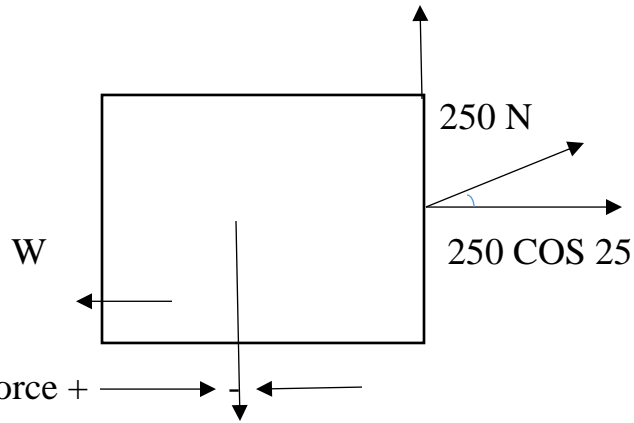


Weight and coefficient of friction

Soln

Case (i)

Free body diagram



Sum of x directional force + —————> —————> —————>

$$\sum F_x = 0$$

$$250 \cos 25^\circ - F_N = 0$$

$$250 \cos 25^\circ - \mu \times N_{R1} = 0$$

$$226.57 - \mu \times N_{R1} = 0$$

$$-\mu \times N_{R1} = -226.57$$

$$\mu \times N_{R1} = 226.57 \text{----- (1)}$$

Sum of Y directional force —————> —————> —————>  
↓ - ↑ +

$$\sum F_y = 0$$

$$N_{R1} - W + 250 \sin 25 = 0$$

$$N_{R1} = W - 250 \sin 25$$

$$N_{R1} = W - 105.65 \text{----- (2)}$$

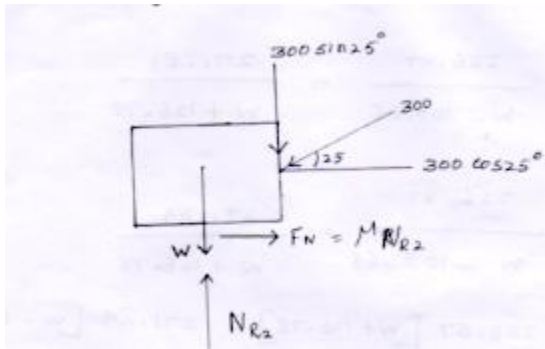
Sub eqn (1), from eqn (1)

$$\mu N_{R1} = 226.57$$

$$\mu = \frac{226.57}{N_{R1}}$$

$$\mu = \frac{226.57}{W - 105.65} \text{----- (3)}$$

## Case (2) Free body diagram



Sum of X Directional force  $\sum F_x = 0$

$$F_N - 300 \cos 25^\circ = 0$$

$$\mu N_{R2} = 300 \cos 25^\circ$$

$$\mu N_{R2} = 271.89 \text{ N} \text{----- (4)}$$

Sum of vertical force[Y direction]  $\sum F_y = 0$

$$N_{R2} - W - 300 \sin 25^\circ = 0$$

$$N_{R2} = W + 300 \sin 25^\circ$$

$$N_{R2} = W + 126.78$$

$N_{R2}$  value sub in Eqn (4)

$$\mu N_{R2} = 271.89$$

$$\mu = \frac{271.89}{N_{R2}}$$

$$\mu = \frac{271.89}{W + 126.78} \text{----- (5)}$$

Eqn (3) = Eqn (5)

$$\frac{226.57}{W - 105.65} = \frac{271.89}{W + 126.78}$$

$$226.57[W + 126.78] = 271.89[W - 105.65]$$

$$226.57W + 28.72 \times 10^3 = 271.89W - 28.72 \times 10^3$$

$$226.57W - 271.89W = -28.72 \times 10^3 - 28.72 \times 10^3$$

$$-45.32W = -57.44 \times 10^3$$

$$W = \frac{57.44 \times 10^3}{45.32}$$

$$W = 1267.43N$$

W value sub in eqn (3)

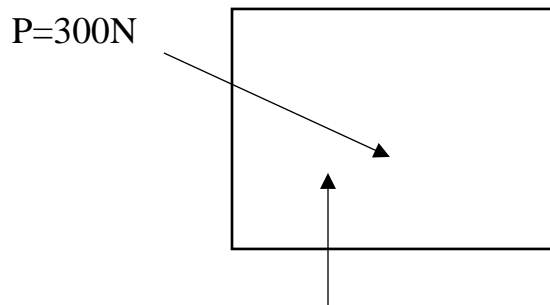
$$\mu = \frac{226.57}{W - 105.65}$$

$$\mu = \frac{226.57}{1267.43 - 105.65}$$

$$\mu = 0.195$$

### Problem 3

Calculate the static coefficient of friction  $\mu_s$  b/w the block shown in fig having a mass of 75kg and the surface. Also find the magnitude and direction of the friction force if the force P applied is inclined at  $45^\circ$  to the horizontal and  $\mu_s = 0.30$



Given:

$$\text{Weight} = 75 \text{ kg} = 75 \times 9.81 = 735.75N$$

To find

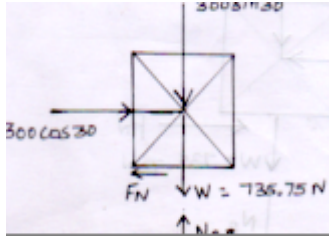
Case (i) Coefficient of friction  $\mu_s$

Case (ii) Frictional force ' $F_N$ ', Direction  $\emptyset$

Soln

Case (i) free body diagram





Sum of all the X direction force  $\sum F_x = 0$

$$300 \cos 30 - F_N = 0$$

$$300 \cos 30 - \mu N_R = 0$$

$$\mu N_R = -300 \cos 30$$

$$\mu N_R = 259.8 \text{----- (1)}$$

Sum of all the Y direction force  $\sum F_y = 0$

$$-300 \sin 30 - 735.75 + N_R = 0$$

$$N_R = 300 \sin 30 + 735.75$$

$$N_R = 885.75 \text{ N}$$

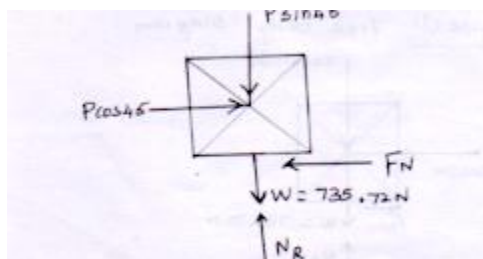
$N_R$  value sub in eqn(i)

$$\mu N_R = 259.8$$

$$\mu = \frac{259.8}{N_R} = \frac{259.8}{885.75}$$

$$\mu = 0.29$$

Case (ii) free body diagram



Sum of all the horizontal force [X direction]  $\sum F_x = 0$

$$P \cos 45 - F_N = 0$$

$$-F_N = -P \cos 45$$

$$F_N = P \cos 45$$

$$\mu N_R = p \cos 45$$

$$N_R = \frac{p \cos 45}{\mu} = \frac{p \cos 45}{0.3}$$

$$N_R = 2.35p$$

$$\text{Sum of all the Direction force } \sum F_y = 0$$

$$-p \sin 45 - 732.72 + N_R = 0$$

$$-p \times 0.7 - 735.72 + 2.35p = 0$$

$$-0.7p + 2.35p = 735.72$$

$$1.64p = 735.72$$

$$p = \frac{735.72}{1.64}$$

$$p = 447.81N$$

$$N_R = 2.35p$$

$$N_R = 2.35 \times 447.81$$

$$N_R = 1052.37N$$

$$F_N = \mu \times R = 0.3 \times 1052.37$$

$$F_N = 315.71N$$

$$\text{Direction } \emptyset$$

$$\emptyset = \tan^{-1} \left[ \frac{F_N}{N_R} \right]$$

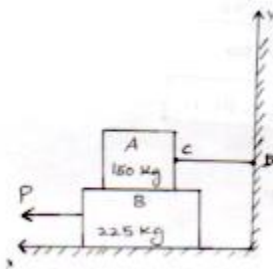
$$\emptyset = \tan^{-1} \left[ \frac{315.71}{1052.37} \right]$$

$$\emptyset = 16^\circ 41'$$

Problem 4:

Determine the smallest force  $P$  required to move the block B shown in fig below (i) block A is restrained by cable CD as shown in fig. (ii) Cable CD is removed. Take

$$\mu_s = 0.30 \text{ and } \mu_k = 0.25$$



Given:

$$W_A = 150kg = 150 \times 9.81 = 1471.5N$$

$$W_B = 225kg = 225 \times 9.81 = 2207.25N$$

$$\mu_s = 0.3$$

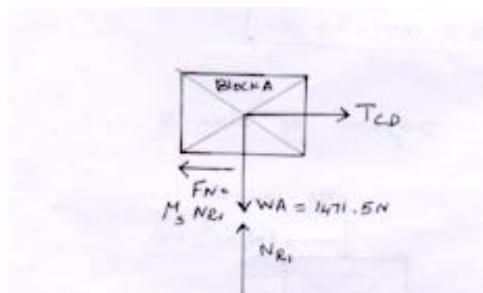
$$\mu_k = 0.25$$

To find

Force  $P$

Soln

Block A is restrained by cable CD



Sum of X direction force  $\sum F_x = 0$

$$T_{CD} - F_N = 0$$

$$T_{CD} - \mu_s N_{R1} = 0$$

$$T_{CD} = \mu_s N_{R1}$$

$$T_{CD} = 0.3 N_{R1} \text{----- (1)}$$

Sum of vertical [Y direction force]  $\Sigma F_y = 0$

$$N_{R1} - W_A = 0$$

$$N_{R1} = W_A$$

$$N_{R1} = 1471.5N$$

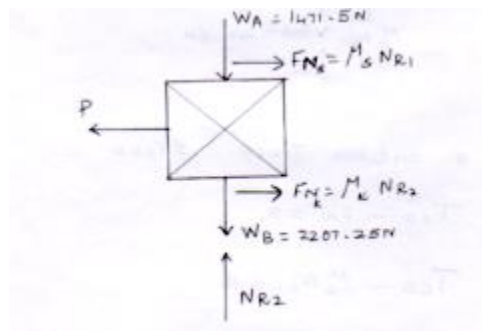
$N_{R1}$  value sub in eqn(i)

$$T_{CD} = 0.3 N_{R1}$$

$$T_{CD} = 0.3 \times 1471.3$$

$$T_{CD} = 441.45 N$$

Free body diagram of block B



Sum of X direction force

$$F_{Ns} + F_{Nk} - p = 0$$

$$\mu_s N_{R1} + \mu_k N_{R2} - p = 0$$

$$p = \mu_s N_{R1} + \mu_k N_{R2} \text{----- (1)}$$

Sum of Y direction force

$$-1471.5 - 2207.25 + N_{R2} = 0$$

$$N_{R2} = 1471.5 + 2207.25$$

$$N_{R2} = 3678.75N$$

$N_{R2}$  value sub in Eqn (1)

$$p = \mu_s N_{R1} + \mu_k N_{R2}$$

$$p = 0.3 \times 1471.5 + 0.25 \times 3678.75$$

$$p = 1361.14 \text{ N}$$

(i) Cable CD is removed

Both block is removed

Both the block will consider as a single body

$$p = F_N$$

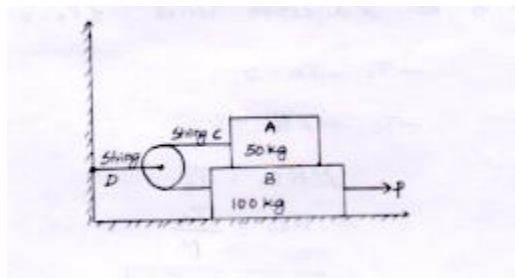
$$p = \mu_k N_{R2} = 0.25 \times 3678.75$$

$$p = 919.68 \text{ N}$$

Problem-6

Two blocks A and B of mass 50 kg and 10 kg respectively are connected by a string c which passes through a frictionless pulley connected with the fixed wall by another string D as shown in fig. Find the force P required to pull the block B. also find the tension in the string D.

Take coefficient of friction at all contact surface as 0.3°



Given:

$$\text{Weight of block A } W_A = 50 \text{ kg} = 50 \times 9.81 = 490.5 \text{ N}$$

$$\text{Weight of block B } W_B = 100 \text{ kg} = 100 \times 9.81 = 981 \text{ N}$$

$$\text{Coefficient of friction } \mu = 0.3$$

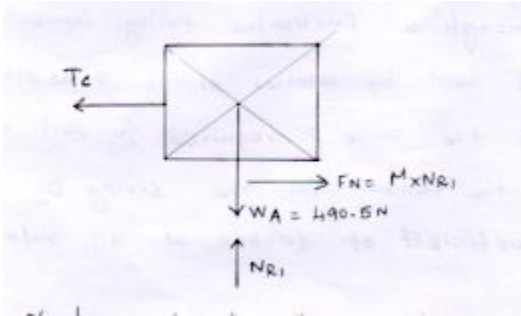
To find

(i) Force P

(ii) Tension in string  $T_D$

Soln

Free body diagram of block A



Sum of x direction forces  $\sum F_x = 0$

$$-T_c - F_N = 0$$

$$-T_c + \mu N_{R1} = 0$$

$$\mu N_{R1} = T_c$$

$$N_{R1} = \frac{T_c}{\mu}$$

$$N_{R1} = \frac{T_c}{0.3} \text{----- (1)}$$

Sum of Y direction force  $\sum F_Y = 0$

$$-W_A + N_{R1} = 0$$

$$N_{R1} = W_A$$

$$N_{R1} = 490.5 \text{ N}$$

$N_{R1}$  value sub in Eqn (1)

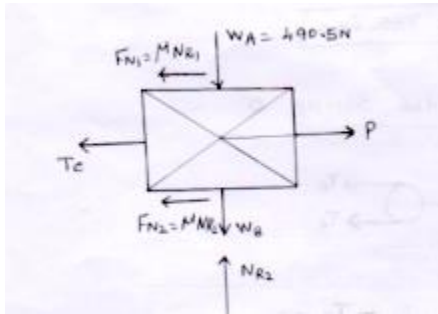
$$N_{R1} = \frac{T_c}{0.3}$$

$$490.5 = \frac{T_c}{0.3}$$

$$T_c = 147.15 \text{ N}$$

Consider block B

free body diagram



Sum of X direction force  $\sum F_x = 0$

$$p - T_c - F_{N1} - F_{N2} = 0$$

$$p - \mu N_{R1} + \mu N_{R2} = 0$$

$$p - 147.15 - 0.3 \times 490.15 - 0.3 \times N_{R2} = 0$$

$$p - 147.15 - 147.04 - 0.3 \times N_{R2} = 0$$

$$p - 294.19 - 0.3N_{R2} = 0$$

$$p = 294.19 - 0.3N_{R2} = 0$$

$$P = 294.19 + N_{R2} \text{----- (2)}$$

Sum of Y direction force  $\sum F_y = 0$

$$N_{R2} - W_B - W_A = 0$$

$$N_{R2} - 981 - 490.5 = 0$$

$$N_{R2} = 1471.5N$$

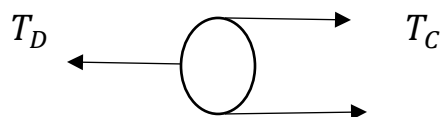
$N_{R2}$  value sub in eqn(2)

$$p = 294.19 + 0.3N_{R2}$$

$$p = 294.19 + 0.3 \times 1471.5$$

$$p = 735.64N$$

Tension in the string D:



$$\Sigma F_x = 0$$

$$T_c + T_c - T_D = 0$$

$$2T_c - T_D = 0$$

$$2 \times 147.15 - T_D = 0$$

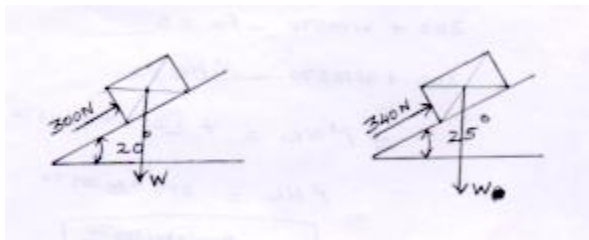
$$294.3 - T_D = 0$$

$$-T_D = -294.3$$

$$T_D = 294.3N$$

Problem -7

A force of 300 n is required just to move a block up a plane inclined at  $20^\circ$ . To the horizontal, the force being applied parallel to the plane shown in fig. if the inclination of the plane is increased to  $25^\circ$ , the force required just to move the block up is 340 N, [the force is acting parallel to the plane]. Determine the weight of the block and coefficient of friction.



Given:

Case (i)

Weight of body  $w=?$

Force on body  $P= 300\text{ N}$  at  $20^\circ$  inclined on plane horizontal

Case (ii)

Force on body  $P= 340\text{ N}$  at  $25^\circ$

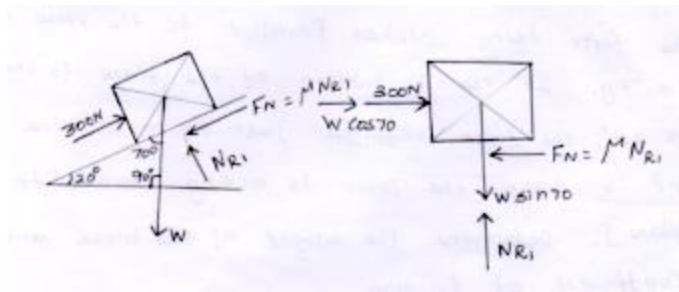
To find:

Weight of body & coefficient of friction

Soln:

Case (i) free body diagram





Sum of X directional force  $\sum F_x = 0$

$$300 + w \cos 70 - F_N = 0$$

$$300 + w \cos 70 - \mu N_{R1} = 0$$

$$-N_{R1} = -[300 + w \cos 70]$$

$$\mu N_{R1} = 300 + w \cos 70$$

$$\mu = \frac{300 + w \cos 70}{N_{R1}} \text{----- (1)}$$

Sum of all Y direction force  $\sum F_y = 0$

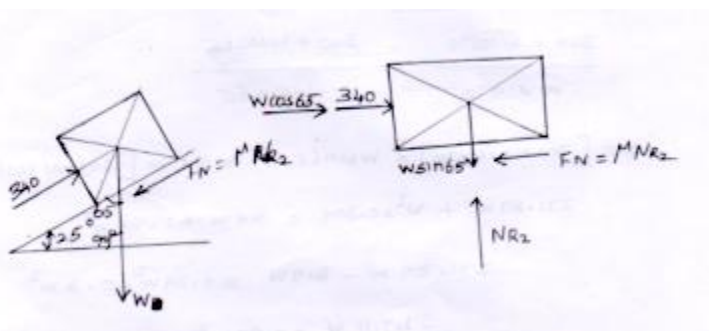
$$N_{R1} - w \sin 70 = 0$$

$$N_{R1} = w \sin 70 \text{----- (2)}$$

$N_{R1}$  value sub in eqn (1)

$$\mu = \frac{300 + w \cos 70}{w \sin 70} \text{----- (3)}$$

Case (ii) consider block 2



Sum of all the X direction force  $\sum F_x = 0$

$$340 + w \cos 65 - F_N = 0$$

$$340 + w \cos 65 - \mu N_{R2} = 0$$

$$-\mu N_{R2} = -[340 + w \cos 65]$$

$$\mu = \frac{340 + w \cos 65}{N_{R2}} \text{----- (4)}$$

Sum of all the Y direction force  $\Sigma F_Y = 0$

$$N_{R2} - w \sin 65 = 0$$

$$N_{R2} = w \sin 65 \text{----- (5)}$$

Eqn (5) sub in eqn (4)

$$\mu = \frac{340 + w \cos 65}{w \sin 65} \text{----- (6)}$$

Eqn(3) = eqn (6)

$$\frac{300 + w \cos 70}{w \sin 70} = \frac{340 + w \cos 65}{w \sin 65}$$

$$[300 + w \cos 70] \times w \sin 65 = w \sin 70 [340 + w \cos 65]$$

$$271.89w + w^2 \times 0.309 = 319w + 0.39w^2$$

$$271.86w - 319w = 0.39w^2 - 0.3w^2$$

$$-47.11 w = 0.09w^2$$

$$-47.11 w = 0.09w$$

$$w = \frac{-47.11}{0.09}$$

$$\text{ans } w = -523.47 \text{ N}$$

w value sub in eqn(3)

$$\mu = \frac{300 + w \cos 70}{w \sin 70}$$

$$\mu = \frac{300 + (-523.47) \cos 70}{(-523.47) \sin 70}$$

$$\text{Ans } \mu = -0.24$$

### Problem 8

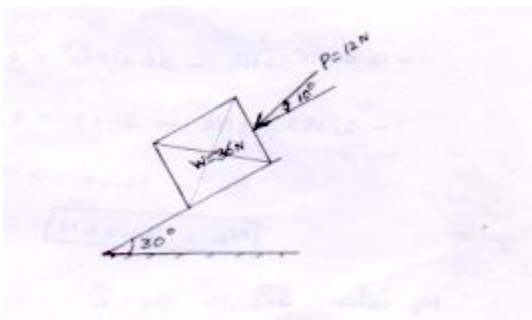
A block weighting 360 n is resting on a rough inclined plane having an inclination of  $30^\circ$ . A force of 12 N is applied at an angle of  $10^\circ$  up and the block is just on the point of moving down the plane. Determine the coefficient of friction

Given:

Block weight  $w=36\text{ N}$

Inclination of the plane  $\theta = 30^\circ$

Force on block  $P=12\text{ N}$  at  $10^\circ$

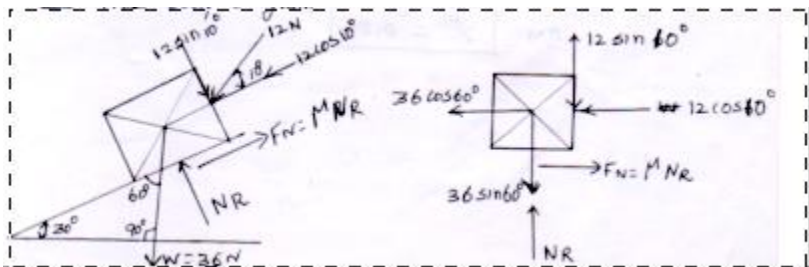


To find

Coefficient of friction  $\mu$

Soln:

Free body diagram



Sum of X direction force  $\sum F_X = 0$

$$-12 \cos 10 - 36 \cos 60 + F_N = 0$$

$$-11.87 - 18 + \mu N_R = 0$$

$$-29.87 + \mu N_R = 0$$

$$\mu N_R = 29.87$$

$$\mu = \frac{29.87}{N_R} \text{----- (1)}$$

Sum of all the Y direction force  $\Sigma F_y = 0$

$$-12 \sin 10^\circ + N_R - 36 \sin 60^\circ = 0$$

$$-2.083 + N_R - 31.17 = 0$$

$$N_R - 33.25 = 0$$

$$N_R = 33.25 \text{ N}$$

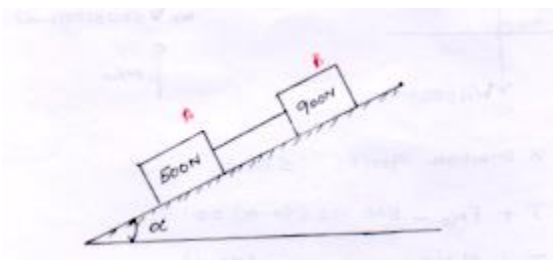
$N_R$  value sub in Eqn (1)

$$\mu = \frac{29.87}{N_R} = \frac{29.87}{33.25}$$

$$\text{ans } \mu = 0.89$$

Problem 9

Two block of weight 500 N and 900 N connected by a rod are kept in an inclined plane is shown in fig. the rod is parallel to the plane. The coefficient of friction b/w 500 N block and the plane is 0.3 and that b/w 900 N block and the plane is 0.4 find the inclination of the plane with the horizontal and the rod when the motion down the plane is just about the start.



Given:

Weight of block A  $W_A = 500 \text{ N}$

Weight of block B  $W_B = 900 \text{ N}$

Coefficient of friction  $\mu_A = 0.3$

Coefficient friction at block B  $\mu_B = 0.4$

To find

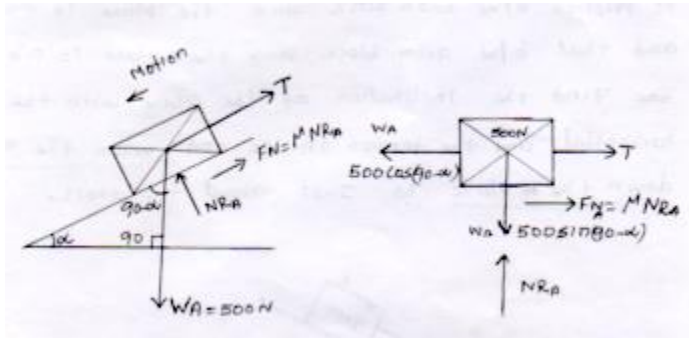
1. Inclination of the plane  $\alpha$

2. Tension

Soln:

Consider block 'A'

Free body diagram



Sum of X direction force  $\Sigma F_X = 0$

$$T + F_{N_A} - 500 \cos(90 - \alpha) = 0$$

$$T + \mu_A N_{RA} = 500 \cos(90 - \alpha)$$

$$T + \mu_A N_{RA} = 500 \sin \alpha \text{----- (1)}$$

Sum of Y direction force  $\Sigma F_Y = 0$

$$N_{RA} - 500 \sin(90 - \alpha) = 0$$

$$N_{RA} = 500 \sin(90 - \alpha)$$

$$N_{RA} = 500 \cos \alpha \text{----- (2)}$$

$$T + 0.3 N_{RA} = 500 \sin \alpha$$

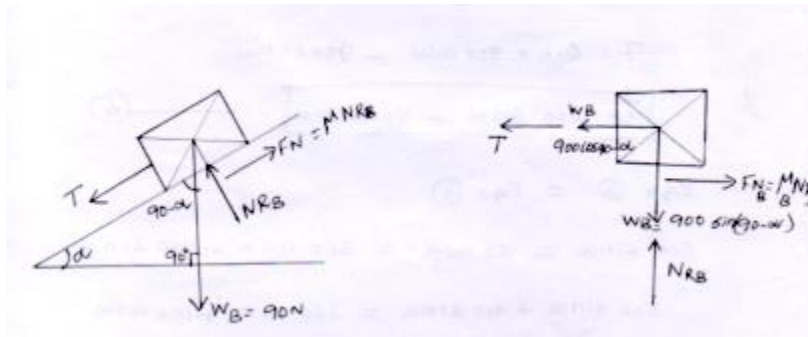
$N_{RA}$  value sub in Eqn (1)

$$T + 0.3 \times 500 \cos \alpha = 500 \sin \alpha$$

$$T + 150 \cos \alpha = 500 \sin \alpha$$

$$T = 500 \sin \alpha - 150 \cos \alpha \text{----- (3)}$$

Consider block B



Sum of X direction force  $\Sigma F_x = 0$

$$F_{NB} - T - 900 \cos(90 - \alpha) = 0$$

$$\mu_B N_{RB} - T - 900 \sin \alpha = 0$$

$$0.4 N_{RB} - T - 900 \sin \alpha = 0$$

$$-T = -0.4 N_{RB} + 900 \sin \alpha$$

$$-T = -[0.4 N_{RB} - 900 \sin \alpha]$$

$$T = [0.4 N_{RB} - 900 \sin \alpha] \text{----- (4)}$$

Sum of vertical [Y direction force]  $\Sigma F_y = 0$

$$N_{RB} - 900 \sin(90 - \alpha) = 0$$

$$N_{RB} = 900 \sin(90 - \alpha)$$

$$N_{RB} = 900 \cos \alpha \text{----- (5)}$$

$N_{RB}$  value sub in Eqn(4)

$$T = 0.4 \times N_{RB} - 900 \sin \alpha$$

$$T = 0.4 \times 900 \cos \alpha - 900 \sin \alpha$$

$$T = 360 \cos \alpha - 900 \sin \alpha \text{----- (6)}$$

Eqn (3) = eqn(6)

$$500 \sin \alpha - 150 \cos \alpha = 360 \cos \alpha - 900 \sin \alpha$$

$$500 \sin \alpha + 900 \sin \alpha = 360 \cos \alpha + 150 \cos \alpha$$

$$1400 \sin \alpha = 510 \cos \alpha$$

$$\frac{\sin \alpha}{\cos \alpha} = \frac{510}{1400}$$

$$\tan \alpha = \frac{510}{1400}$$

$$\alpha = \tan^{-1} \left[ \frac{510}{1400} \right]$$

$$\text{Ans } \alpha = 20^\circ$$

$\alpha$  value sub in Eqn (3)

$$T = 500 \sin \alpha - 150 \cos \alpha$$

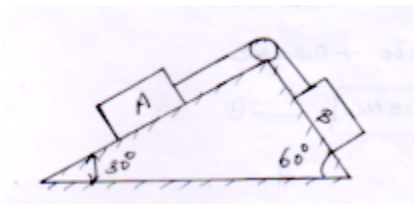
$$T = 500 \sin 20 - 150 \cos 20$$

$$T = 171 - 140$$

$$T = 31 \text{ N}$$

Problem 10

Two block A and B are placed on inclined planes as shown in fig. the block a weight 1000N. Determine the minimum weight of the block b for maintaining the equilibrium of the system. Assume that the blocks are connected by an inextensible string passing over a frictionless pulley. Coefficient of friction  $\mu_A$  b/w the block A and the plane is 0.25. Assume the same value for  $\mu_B$



Given:

Weight of block A  $W_A = 1000 \text{ N}$

Coefficient of friction in block A & B  $\mu_A = \mu_B = 0.25$

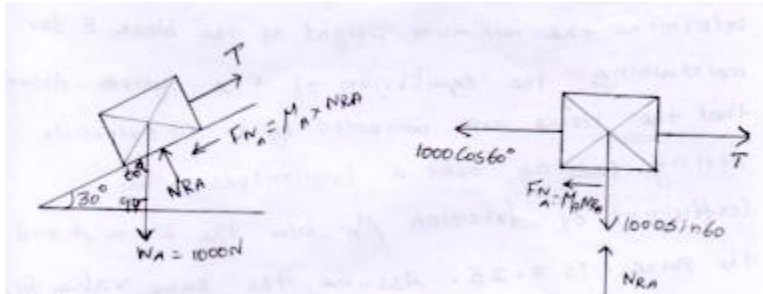
To find

Weight of block 'B'  $W_B$

Solution:

Consider block 'A'

Free body diagram



Sum of X direction force  $\Sigma$

$$T - 1000 \cos 60 - F_{NA} = 0$$

$$T - 1000 \cos 60 - \mu_A N_{RA} = 0$$

$$T = 1000 \cos 60 + 0.25 N_{RA}$$

$$T = 500 + 0.25 N_{RA} \text{----- (1)}$$

Sum of all Y direction forces

$$N_{RA} - W_A = 0$$

$$N_{RA} - 100 \sin 60 = 0$$

$$N_{RA} = 1000 \sin 60$$

$$N_{RA} = 866 \text{ N}$$

$N_{RA}$  value sub in eqn (1)

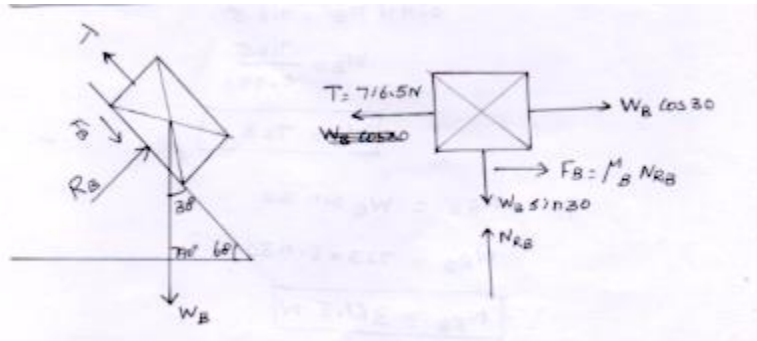
$$T = 500 + 0.25 N_{RA}$$

$$T = 500 + 0.25 \times 866$$

$$T = 716.5 \text{ N}$$



Consider block B



Sum of X direction  $\Sigma F_x = 0$

$$-716.5 + W_B \cos 30 + \mu_B N_{RB} = 0$$

$$-716.5 + W_B \cos 30 + 0.25 \times N_{RB} = 0$$

$$W_B \cos 30 = 716.5 - 0.2 \times N_{RB} \text{----- (2)}$$

Sum of Y direction Force  $\Sigma F_y = 0$

$$N_{RB} = W_B \sin 30 = 0$$

$$N_{RB} = W_B \sin 30$$

$N_{RB}$  value sub in Eqn (2)

$$W_B \cos 30 = 716.5 - 0.2 \times N_{RB}$$

$$W_B \cos 30 = 716.5 - 0.2 \times [W_B \sin 30]$$

$$W_B \cos 30 = 716.5 - 0.125[W_B]$$

$$0.866W_B = 716.5 - 0.125W_B$$

$$0.866W_B + 0.125W_B = 716.5$$

$$0.991W_B = 716.5$$

$$W_B = \frac{716.5}{0.991}$$

$$W_B = 723 \text{ N}$$

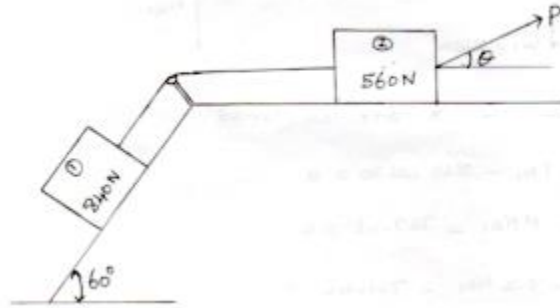
$$N_{RB} = W_B \sin 30$$

$$N_{RB} = 723 \times \sin 30$$

$$N_{RB} = 361.5 \text{ N}$$

Problem: 11

Determine the least value of  $P$  required to cause the motion impend the system shown in fig below. Assume coefficient of friction on all the contact surface as 0.2



Given:

Weight of block (1)  $W_1 = 840 \text{ N}$

Weight of block (2)  $W_2 = 560 \text{ N}$

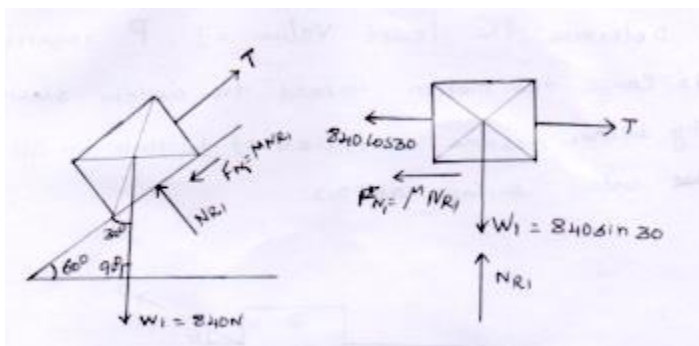
Coefficient of friction  $\mu = 0.2$

To find

Load on block (2)  $P$

Soln

Free body diagram of block



Sum of all the X direction forces

$$T - F_{N1} - 840 \cos 30 = 0$$

$$T - \mu N_{R1} - 727.46 = 0$$

$$T - 0.2 N_{R1} - 721.46 = 0$$

$$T = 0.2 N_{R1} + 721.46 \text{----- (1)}$$

Sum of All the Y direction force

$$N_{R1} - w_1 \sin 30 = 0$$

$$N_{R1} - 840 \sin 30 = 0$$

$$N_{R1} - 420 = 0$$

$$N_{R1} = 420 \text{ N}$$

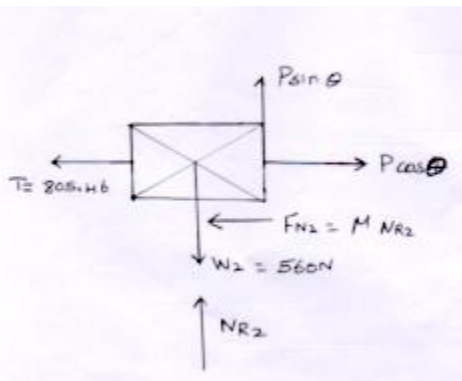
$N_{R1}$  value sub in eqn (1)

$$T = 0.2 \times 420 + 721.46$$

$$T = 805.46 \text{ N}$$

Consider block (2)

Free body diagram



Sum of X direction Force  $\Sigma F_X = 0$

$$-T - F_{N2} + p \cos \theta = 0$$

$$-805.46 - \mu N_{R2} + P \cos \theta = 0$$

Problem 12(H.W)

Determine whether the block shown in fig below having a mass of 40 kg is equilibrium and find the magnitude and direction of the friction force. Take  $\mu_s = 0.40$  and  $\mu_k = 0.3$

## Simple Contact Friction:

The friction force is the resisting force developed at the contact surface of two bodies due to their roughness and when the surface of one body moves over the surface of another body.

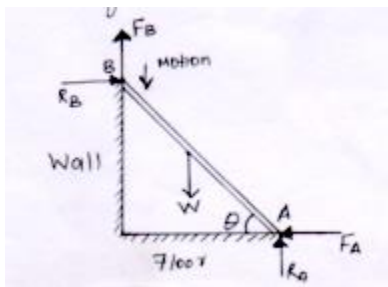
Some of important Engg application of simple contact friction are,

- Ladder friction
- Wedge friction
- Screw friction
- Belt friction

### 1. Ladder friction:

A ladder is a device used for climbing on roof or wall

Consider a ladder AB of length 'l' and weight 'w' resting on the ground at A and leaning against a rough wall at B as shown in fig.



Let the angle of ladder with horizontal be  $\theta$  when the angle  $\theta$  exceeds angle of friction (or) when the weight of the man on the ladder makes instability to the ladder, the ladder slips down.

During sliding the upper end of the ladder tends to slip downwards, hence the friction force at B,  $F_B$  will act upwards. If the coefficient of friction of wall surface B is  $\mu_B$

$$\text{Then } \mu_B = \frac{F_B}{N_{RB}} \quad (\text{or}) \quad F_B = \mu_B N_{RB}$$

At the same time the lower end of the ladder A tends to slip away from the wall, hence the frictional force at A,  $F_A$  will act towards the wall if the coefficient of friction of floor surface A is  $\mu_A$

$$\mu_A = \frac{F_A}{N_{RA}} \quad (\text{or}) \quad F_A = \mu_A N_{RA}$$

Condition:

$$\Sigma F_X = 0$$

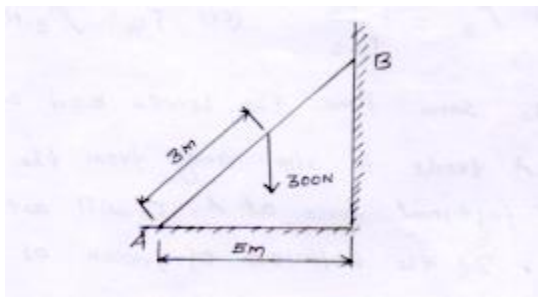
$$\Sigma F_Y = 0$$

$$\Sigma M = 0$$

$$F = \mu N_R$$

Problem 1:

A ladder is 8m long and weight 300N. The center of gravity of the ladder is 3m along the length of from the bottom end. The ladder rests against a vertical wall at B and on the horizontal floor at A as shown below. Determine the safe height upto which a man weighting 900N can climb without making the ladder slip. The coefficient of friction b/w ladder and floor is 0.4 and ladder top and wall is 0.3



Given:

Length of ladder  $l = 8\text{m}$

Weight of ladder  $w = 300\text{N}$  at 3m length from lower end

$$\mu_A = 0.4$$

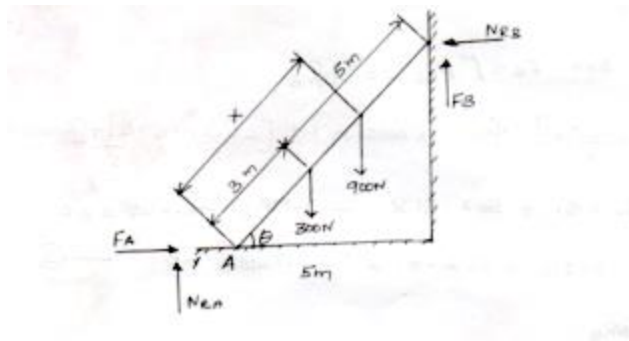
$$\mu_B = 0.3$$

To find

Safe height 'H'

Soln:

Free body diagram



Sum of X direction force  $\Sigma F_x = 0$

$$F_A - N_{RB} = 0$$

$$F_A - N_{RB} = 0 \quad (1)$$

Sum of Y direction force  $\Sigma F_y = 0$

$$N_{RA} + F_B - 300 - 900 = 0$$

$$N_{RA} + F_B = 300 + 900 = 0$$

$$N_{RA} + F_B = 1200 \quad (2)$$

$\Sigma M_A = 0$  Moment about 'A'

Now find out the  $\theta$

$$\theta = \cos^{-1} \left( \frac{5}{8} \right)$$

$$\theta = 51^\circ 19'$$

$$\Sigma M_A = 0 \quad \begin{array}{c} \uparrow \\ - \\ \downarrow \\ + \end{array}$$

$$(300 \times 3 \cos \theta) + (900 \times \cos \theta) + [F_B \times 5] + [-N_{RB} \times 8 \sin \theta] = 0$$

$$[300 \times 3 \cos 51^\circ 19'] + [900 \times \cos 51^\circ 19'] + [-F_B \times 5] + [-N_{RB} \times \sin 51^\circ 19'] = 0$$

$$\text{Sub } F_B = \mu N_{RB} = 0.3 N_{RB}$$

$$[300 \times 3 \cos 51^\circ 19'] + [900 \times \cos 51^\circ 19'] + [-0.3 N_{RB} \times 5] + [-N_{RB} \times 8 \sin 51^\circ 19'] = 0$$

$$562.51 + 562.51 \times -1.5 N_{RB} - 6.24 N_{RB} = 0$$

$$562.51 + 562.21 \times -7.74 N_{RB} = 0 \text{-----} (3)$$

To find  $N_{RB}$

From eqn (2)

$$N_{RB} = F_A \quad F_A = \mu \times N_{RA}$$

$$N_{RA} + F_B = 1200 \text{-----} (2)$$

$$F_A = 0.4 \times N_{RA}$$

$$\frac{F_A}{0.4} + 0.3 \times N_{RB} = 1200$$

$$N_{RA} = \frac{F_A}{0.4}$$

$$\frac{N_{RB}}{0.4} + 0.3 \times N_{RB} = 1200$$

$$2.5 N_{RB} + 0.3 N_{RB} = 1200$$

$$2.8 N_{RB} = 1200$$

$$N_{RB} = \frac{1200}{2.8}$$

$$N_{RB} = 428.57 N$$

$N_{RB}$  value sub in Eqn(3)

$$562.51 + 562.51 \times -7.74 N_{RB} = 0$$

$$562.51 + 562.51 \times -7.74 \times 428.57 = 0$$

$$562.51 \times = [7.74 \times 428.57] - [562.51]$$

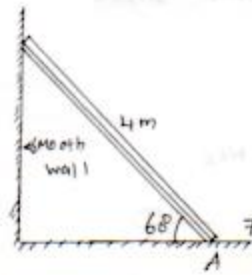
$$562.51 \times = 2754.63$$

$$X = \frac{2754.63}{562.51}$$

Ans  $X = 4.89m$  from the floor of Ladder

Problem 2

A ladder of weight 1000 N and length 4m rest as shown in fig. if the 750 n weight is applied at distance of 3m from the top of ladder, it is at the point of sliding. Determine the coefficient of friction b/w ladder and the floor.



Given:

Length of Ladder  $l = 4\text{m}$

Weight of ladder  $w = 1000\text{N}$

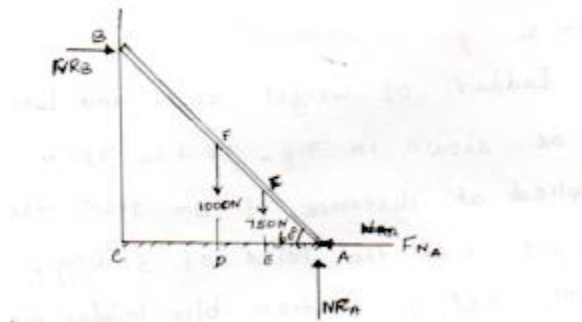
Weight of man  $= 750\text{ N}$  at  $3\text{m}$  from top

To find:

Coefficient of friction

Soln:

Free body diagram



Sum of X direction force  $\Sigma F_x = 0$

$$N_{RB} - F_{NA} = 0$$

$$N_{RB} - \mu_A N_{RA} = 0$$

$$-\mu_A N_{RA} = -N_{RB}$$

$$\mu_A = \frac{N_{RB}}{N_{RA}} \text{----- (1)}$$

Sum of all Y direction force



$$N_{RA} - 1000 - 750 = 0$$

$$N_{RA} - 1750 = 0$$

$$N_{RA} = 1750N$$

Take moment about 'A'  $\downarrow + \uparrow -$

$$\Sigma M_A = 0$$

$$\Sigma M_A = [N_{RB} \times BC] + [-1000 \times AD] + [-750 \times AE] = 0$$

$$BC = 4 \sin 60^\circ \quad AD = 2 \cos 60^\circ \quad AE = -1 \cos 60^\circ$$

$$[N_{RB} \times 4 \sin 60^\circ] + [-1000 \times 2 \cos 60^\circ] + [-750 \times -1 \cos 60^\circ] = 0$$

$$3.46N_{RB} - 1000 - 375 = 0$$

$$3.46N_{RB} - 1375 = 0$$

$$N_{RB} = \frac{1375}{3.46}$$

$$N_{RB} = 397.39 N$$

$N_{RA}$  &  $N_{RB}$  value sub in Eqn (1)

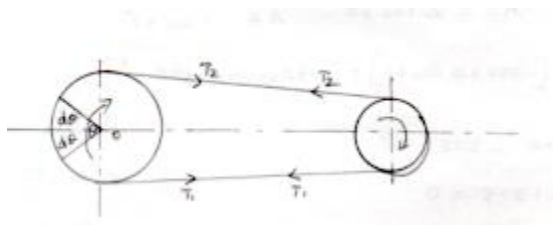
$$\mu = \frac{N_{RB}}{N_{RA}} = \frac{397.39}{1750}$$

$$\text{ans } \mu = 0.22$$

Belt friction:

Power is transmitted through a belt that is running round the two pulgs.This is used in laths, diesel engine, and rise mills etc for power transmission.

The power is transmission is due to the friction existing b/w the belt and the pulley surface. The friction is called belt friction.



$$\frac{T_2}{T_1} = e^{MQ} \longrightarrow \text{Tension Ratio}$$

$T_2$  = tension in tight side

$T_1$  = tension in slack side

$\theta$  = angle of contact

$\mu$  = coefficient of friction

$T_2 > T_1$

$\theta$  value sub in radian

$\theta$  in radian =  $\frac{\pi}{180} \times \theta \text{ value in degree}$

Power  $P = [T_2 - T_1] \times V$

$$V = \frac{\pi d N}{60} \text{ m/s}$$

N = speed of drum

D = diameter of drum

V = belt speed (or) velocity of belt

#### Problem 1

A flat belt develops a tight side tension of 2000 N during power transmission the coefficient of friction b/w pulley and belt is 0.3, the angle of lap on smaller pulley is  $165^\circ$  and the belt speed is 18 m/s. determine the power that can be transmitted, if the belt is assumed to be perfectly elastic and without mass.

Given data:

Tension in tight side  $T_1 = 2000 \text{ N}$

Coefficient of friction  $\mu = 0.3$

Angle of contact  $\theta = 165^\circ$

Velocity of belt  $V = 18 \frac{\text{m}}{\text{s}}$

To find:

Power 'P'

Soln:

$$\text{Power } P = [T_2 - T_1] \times V$$

$$T_2 = 2000, V = 18 \text{ m/s given}$$

By Tension Ratio

$$\frac{T_2}{T_1} = e^{\mu \theta}, \mu = 0.3$$

$$\theta = 165^\circ$$

$$\theta = 165^\circ \times \pi / 180$$

$$\theta = 2.87 \text{ radian}$$

$$\frac{2000}{T_1} = e^{0.3 \times 2.87}$$

$$T_1 = \frac{2000}{e^{0.3 \times 2.87}} = \frac{2000}{e^{0.861}} = \frac{2000}{2.36}$$

$$T_1 = 845.47 \text{ N}$$

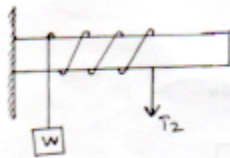
$$\text{Power } P = [T_2 - T_1] \times V$$

$$P = [2000 - 845.47] \times 18$$

$$P = 20 \times 10^3 \text{ W}$$

## Problem 2

A rope is wrapped three time around a rod shown in fig



Determine the force T required on the free end of the rope. To support a load of  $W = 20 \text{ kN}$ . Take  $\mu$  as 0.3.

Given Data:

$$w = T_2$$

$$T_2 = T_1 \quad T_1 = T$$

$$T_1 = T$$

Soln:

$$\frac{T_2}{T_1} = e^{MQ}$$

$$\theta = 18.84 \text{ radian}$$

$$\frac{T_2}{T_1} = e^{0.3 \times 18.84}$$

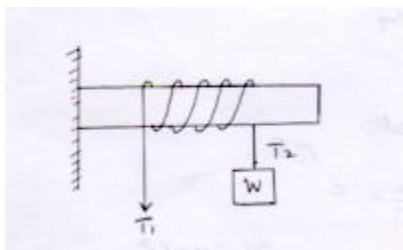
$$\frac{20}{\pi} = e^{0.3 \times 18.84}$$

$$T_1 = \frac{20}{e^{0.3 \times 18.84}}$$

$$T_1 = 0.07 \text{ KN}$$

$$T = T_1 = 0.07 \text{ KN}$$

A wire rope is wrapped three and a half times around a cylinder as shown in below. Determine the force  $T_1$  exerted on the free end of rope that is required to support a 1kN weight the coefficient of friction b/w the rope and the cylinder is 2.5



Given:

Coefficient of friction  $\mu = 0.25$

No of turns=3.5

$$T_2 = w = 1KN$$

To find:

Force exerted  $T_1$

Soln:

Tension ratio

Tension Ratio

$$\frac{T_2}{T_1} = e^{M\theta}$$

$$\theta = 360 \times \left[\frac{\pi}{180}\right] \times 3.5$$

$$\theta = 21.99 \text{ radian}$$

$$\frac{T_2}{T_1} = e^{0.25 \times 21.99}$$

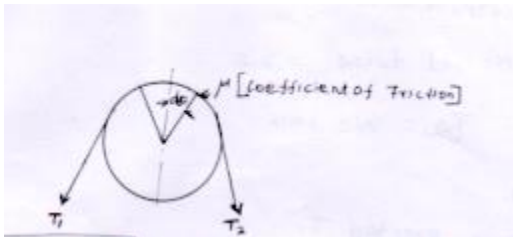
$$\frac{1}{T_1} = e^{0.25 \times 21.99}$$

$$T_1 = \frac{1}{e^{0.25 \times 21.99}}$$

$$T_1 = 4.09 \times 10^3 \text{ KN}$$

$$T = T_1 = 4.09 \text{ KN}$$

Derive the ratio b/w the two belt tension forces



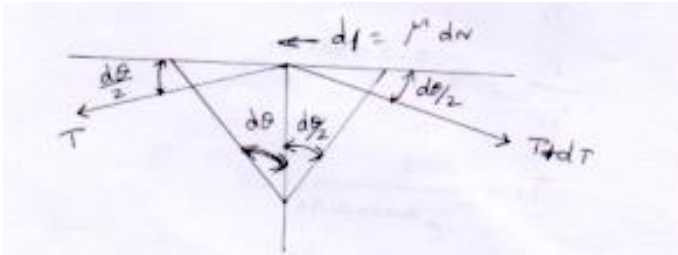
$$\frac{T_2}{T_1} = e^{M\theta}$$

$T_1$  = Tension in tight side

$T_2$  = Tension in slack side

$\theta$  = Angle of contact

$\mu$  = Coefficient of friction



$$\Sigma F_x = 0$$

$$-T \cos \frac{d\theta}{2} + (T + dT) \cos \frac{d\theta}{2} - df = 0$$

$$-T \cos \frac{d\theta}{2} + T \cos \frac{d\theta}{2} + (dT) \cos \frac{d\theta}{2} - df = 0$$

$$df = \mu dN \quad \& \cos \frac{d\theta}{2} = 1$$

$$dT = -\mu dN = 0$$

$$dt = \mu dN \text{----- (1)}$$

$$\Sigma F_y = 0$$

$$dN + ([T + dT]) \sin \frac{d\theta}{2} - T \sin \frac{d\theta}{2} = 0$$

$$dN - T \sin \frac{d\theta}{2} - dT \sin \frac{d\theta}{2} - T \sin \frac{d\theta}{2} = 0$$

$$dN = 2T \frac{d\theta}{2} - dT d\theta = 0$$

$$dN - T d\theta = 0$$

$$dN - T d\theta \text{----- (2)}$$

$dN$  value sub in Eqn (1)

$$dT = \mu dN \text{----- (1)}$$

$$dT = \mu \times dN$$

$$\frac{dT}{T} = \mu d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^\theta \mu d\theta$$

$$[\log T]_{T_1}^{T_2} = [\mu]_0^\theta$$

$$\log T_2 - \log T_1 = \mu\theta = \log \frac{T_2}{T_1} = \mu\theta$$

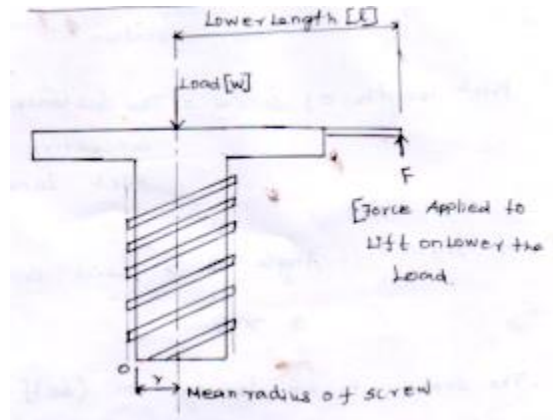
$$\text{Flat for belt } \frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{T_2}{T_1} = e^{\mu\theta \operatorname{cosec} \frac{B}{2}}$$

Problem 4:

A three turns of cables is used to hold the 100N weight with a tensile force of 50 N at the other end of shown in fig. find out the coefficient of friction b/w the cable and shaft surface

## Screw friction



$w = \text{load active } w \tan[\theta + \phi]$

It is used to raise and lower the load

Ex, screw jack

Taking moment about the axis of the screw

$$W \tan(\phi + \theta) \times r - F \times l = 0$$

$$W \times r \times \tan(\phi + \theta) = Fl$$

Force applied to raise (or) Lower the load F

$$F = \frac{wr \tan(\phi + \theta)}{l}$$

$\theta = \text{Lead angle}$

$\phi = \text{Friction angle}$

$$\tan \phi = \mu$$

$$\theta = \tan^{-1}(\mu)$$

Lead of screw = the height lifted for one full form rotation

Pitch length of screw = The distance b/w the two connective thread head screw pitch length

If friction angle  $\phi > \text{lead angle } \theta$

$$\phi > \theta$$



The screw is in locking or (self-locking) of the screw

$$\mu = \frac{\tan \theta}{\tan(\phi + \theta)} \times 100$$

$$\text{Mean radius} = r = \frac{\text{pitch length}}{2}$$

$$r = \frac{\text{dia meter}}{2}$$

Problem: 1

A screw jack has a square thread of mean radius 5cm and pitch length of 1.5 cm. length of lever 50 cm. it is used to raise and lower the load of 25 KN.  $\mu = 0.2$  find

- (i) Force applied to raise the Load
- (ii) Force applied to the lower the load
- (iii) Threaded efficiency

Given

$$\text{Load } W = 25 \text{ KN} = 25 \times 10^3 \text{ N}$$

$$\text{Mean radius } r = 5 \text{ cm}$$

$$\text{Pitch length} = 1.5 \text{ cm}$$

$$\text{Length of lever (l)} = 50 \text{ cm } \mu = 0.2$$

Soln

- (i) Force applied to raise the road

$$F = \frac{W r \tan[\phi + \theta]}{l}$$

$$\tan \phi = \mu$$

$$\phi = \tan^{-1}(\mu)$$

$$\phi = \tan^{-1}(0.2)$$

$$\theta = \text{Lead angle}$$

$$\tan\theta = \frac{1.5}{2 \times \pi \times 5}$$

$$\theta = \tan^{-1}\left(\frac{1.5}{2 \times \pi \times 5}\right)$$

$$\theta = 2^{\circ}4'$$

$$F = \frac{25 \times 10^3 \times 5}{50} \tan[11^{\circ}55' + 2^{\circ}4']$$

$$F = 653.54 \text{ N}$$

(ii) Force applied to lower the load

$$F = \frac{w \tan(\phi - \theta)}{l}$$

$$F = \frac{25 \times 10^3 \times 5 \times \tan[11^{\circ}55' - 2^{\circ}4']}{50}$$

$$F = 0.104 \text{ KN}$$

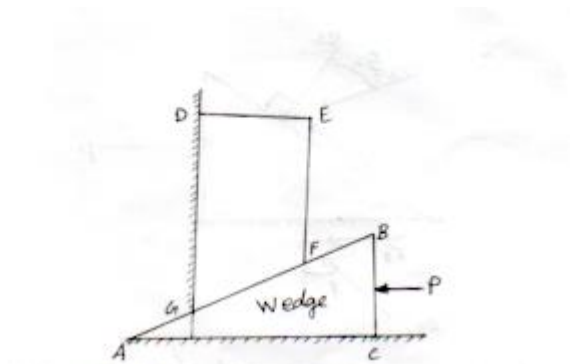
(iii) Efficiency:  $\eta$

$$\eta = \frac{\tan\theta}{\tan(\theta + \phi)} \times 100$$

$$\eta = \frac{\tan 2^{\circ}4'}{\tan(2^{\circ}4' + 11^{\circ}55')}$$

$$\eta = 36$$

Wedge Friction

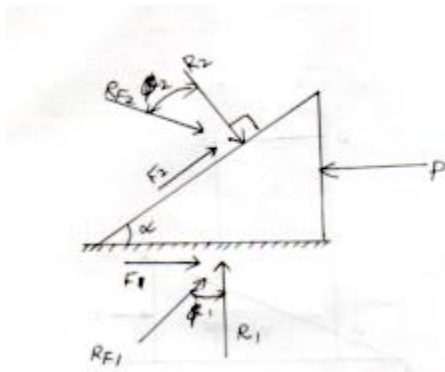


A wedge is a piece of wood or metal, usually of a triangular (or) trapezoidal shape in cross section used for Lifting Loads (or) for slight adjustment like tightening keys for shift

When the force P is applied, sliding take place on the edge AC, DG, and Gf. Hence the reactive force components, normal reactions and the frictional forces are also developed on these sliding surface

Now Let as see the direction of these reactive forces, and draw the free body diagram of wedge and body.

Equilibrium of wedge



Normal force and Frictional force are combined to a single resultant force ' $R_F$ '

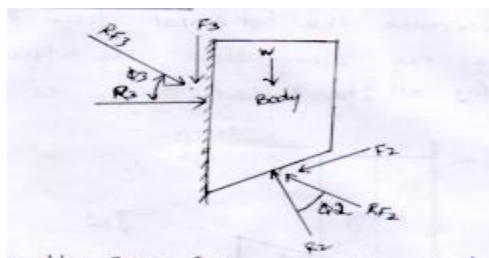
$R_{F1}$  &  $R_{F2}$  are drawn on wedge

$$\text{Where } R_{F1} = \sqrt{F_1^2 + R_1^2}$$

$$R_{F2} = \sqrt{F_2^2 + R_2^2}$$

$F_1$  &  $F_2$  are Limiting friction  $\phi_1, \phi_2$  means of angle of friction

It is defined as the in b/w the angle of [Line of action of the normal reaction  $N_1$  &  $N_2$ ] and Resultant force  $R_{F1}$  &  $R_{F2}$



Equilibrium of body

When the force 'P' push the wedge, the body tends to move upwards, hence the frictional force  $F_3$  on the surface AD is acting download.

Then the normal reaction  $R_3$  and  $F_3$  are replaced by a single resultant  $R_{F3}$ , which makes on angle of  $\phi_3$  with line of action of normal reaction  $R_3$

Concurrent forces  $R_1 R_2$ , self weight(w)

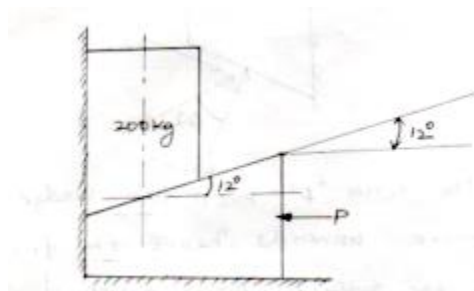
Note (i) always draw the free body diagram of wedge first then draw the force body diagram of the below

(ii) while solving, if the Load is given, solve free body of block first and if the force 'P' is given. solve the free body of wedge

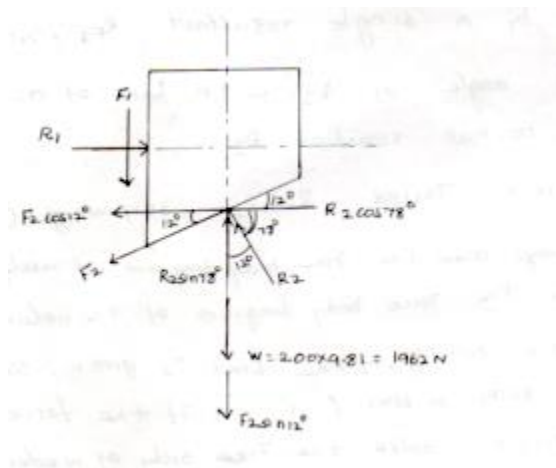
(iii) self-weight of the wedge is neglected

Problem: 1

Determine the horizontal force P required to raise the 200 kg block the efficient of friction for all the contact surface is 0.25



Free body Diagram of block



Sum of all X direction force

$$\Sigma F_X = 0 \quad + \longrightarrow \quad \longleftarrow -$$

$$R_1 - R_2 \cos 78 - F_2 \cos 12^\circ = 0$$

$$R_1 - 0.207R_2 - \mu R_2 \cos 12^\circ = 0$$

$$R_1 - 0.207R_2 - 0.25 \times R_2 \times \cos 12^\circ = 0$$

$$R_1 - 0.207R_2 - 0.24R_2 = 0$$

$$R_1 = 0.207R_2 + 0.24R_2$$

$$R_1 = 0.45R_2$$

Sum of all Y direction force

$$\Sigma F_Y = 0 \quad \downarrow - \quad \uparrow +$$

$$-1962 - F_1 - F_2 \sin 12 + R_2 \sin 78 = 0$$

$$-1962 - 0.25R_1 - 0.25R_2 \sin 12^\circ + R_2 \sin 78^\circ = 0$$

$$-1962 - 0.25R_1 - 0.051R_2 + 0.97R_2 = 0$$

$$-1962 - 0.25 \times [0.45R_2] - 0.051R_2 + 0.97R_2 = 0$$

$$-1962 - 0.112R_2 - 0.051R_2 + 0.97R_2 = 0$$

$$-1962 + 0.807R_2 = 0$$

$$0.807R_2 = 1962$$

$$R_2 = \frac{1962}{0.867}$$

$$R_2 = 2431.22 \text{ N}$$

$R_2$  Value sub in eqn (1)

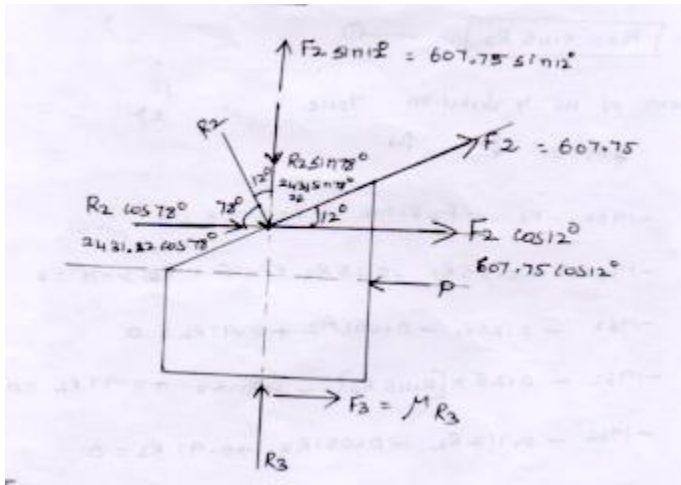
$$R_1 = 0.45 R_2 \quad \longrightarrow 0.45 \times 2431.22$$

$$R_1 = 1094 \text{ N}$$

$$F_1 = \mu R_1 = 0.25 \times 1094 = 273.51 \text{ N}$$

$$F_2 = \mu R_2 = 0.25 \times 2431 = 607.75 \text{ N}$$

Free body diagram of wedge



$$\sum F_x = 0$$

$$-P + F_3 + 2431.22 \cos 78^\circ + 607.75 \cos 12^\circ = 0$$

$$-P + \mu R_3 + 278 + 594.46 = 0$$

$$-P = -\mu R_3 - 2378 - 594.46$$

$$-P = -[\mu R_3 + 2378 + 594.46]$$

$$P = 0.25 \times R_3 + 2378 + 594.46 \text{----- (2)}$$

$$\sum F_y = 0$$

$$R_3 - R_2 \sin 78^\circ + 607.75 \sin 12^\circ = 0$$

$$R_3 - 2431.51 \sin 78^\circ + 607.75 \sin 12^\circ = 0$$

$$R_3 - 505.47 + 126.35 = 0$$

$$R_3 = 505.47 - 126.35$$

$$R_3 = 379.11 \text{ N}$$

$R_3$  value sub in Eqn (2)

$$P = 0.25 \times R_3 + 2378 + 594.46$$

$$P = 0.25 \times 379.11 + 2378 + 594.46$$

$$P = 3067.23 \text{ N}$$