Method Of Images

The replacement of the actual problem with boundaries by an enlarged region or withimage charges but no boundaries is called the method of images.
Method of images is used in solving problems of one or more point charges in thepresence of boundary surfaces.

Continuity of equation:

The relation between density and the volume charge density at a point called continuity ofequation

∇•J= - ρ/ t v

Boundary Conditions for perfect Electric Fields:

Let us consider the relationship among the field components that exist at the interface between two dielectrics as shown in the figure *8.1*. The permittivity of the medium 1

and medium 2 are ε_1 and ε_2 respectively and the interface may also have a net charge density ρ_s Coulomb/m.

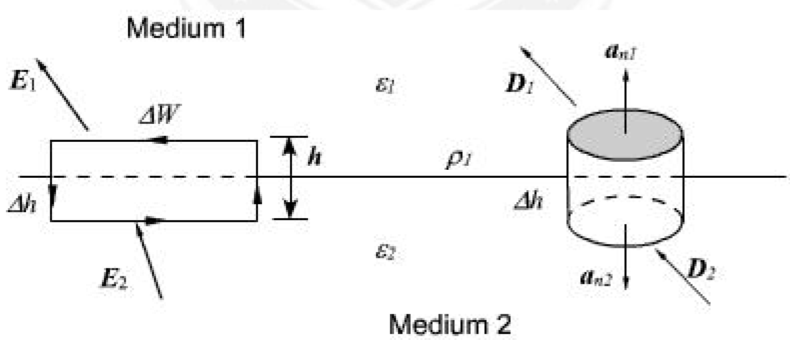

Fig *8.1*: Boundary Conditions at the interface between two dielectrics
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

We can express the electric field in terms of the tangential and normal components

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$$
$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

..........*(2.79*

where E_t and E_n are the tangential and normal components of the electric field

respectively. Let us assume that the closed path is very small so that

over the elementalpath length the
variation of E can be neglected. Moreover very near to the interface, $\Delta h \to 0$. Therefore

$$\oint \vec{E}.d\vec{l} = E_{1t}\Delta w - E_{2t}\Delta w + \frac{h}{2}\left(E_{1n} + E_{2n}\right) - \frac{h}{2}\left(E_{1n} + E_{2n}\right) = 0 \qquad \text{.......................(2.80)}$$

Thus, we have,

$E_{1t} = E_{2t}$ or $\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ i.e. the **tangential component of an electric field iscontinuous across the interface.**

For relating the flux density vectors on two sides of the interface we apply Gauss's law to asmall pillbox volume as shown in the figure. Once again as $\Delta h \to 0$, we can write

$$\oint \vec{D}\cdot d\vec{s} = (\vec{D_1}\cdot\hat{a}_{n2} + \vec{D_2}\cdot\hat{a}_{n1})\Delta s = \rho_s \Delta s \qquad \text{..................(2.81a)}$$

i.e.,

$$D_{1n} - D_{2n} = \rho_s \qquad \text{...(2.81b)}$$

.e., $\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$ .... (2.81c)

Thus we find that the **normal component of the flux density vector *D* is discontinuous across an interface by an amount of discontinuity equal to the surface charge density at the interface.**

Example

Two further illustrate these points; let us consider an example, which involves therefraction of D or E at a charge free dielectric interface as shown in the figure 8.2.

Using the relationships we have just derived, we can write

$$D_{1n} = D_1 \cos\theta_1 = D_{2n} = D_2 Cos\theta_2$$

....
……………………………………………………………(2.82a)

$$E_{1t} = E_1 \sin\theta_1 = \frac{D_1}{\varepsilon_1}\sin\theta_1 = E_{2t} = E_2 \sin\theta_2 = \frac{D_2}{\varepsilon_2}\sin\theta_2$$

....(2.82b)

$$\frac{D_1}{\varepsilon_1}\sin\theta_1 = \frac{D_2}{\varepsilon_2}\sin\theta_2$$

.......................(2.83a)

$$D_1 \cos\theta_1 = D_2 Cos\theta_2$$

.....................*(2.83b)*

Therefore,

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

.......................*(2.84)*

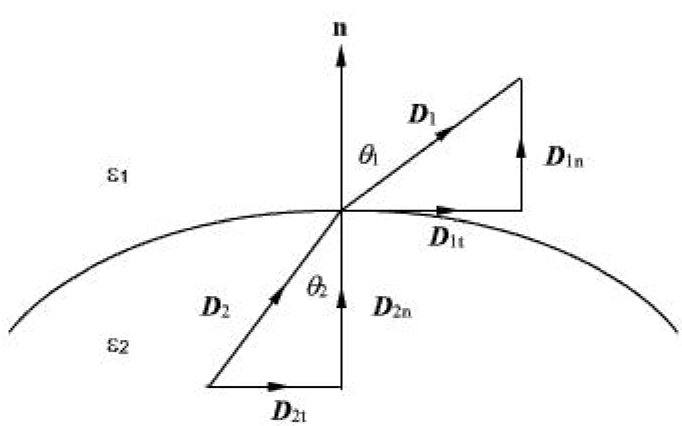

Fig *8.2*: Refraction of D or E at a Charge Free DielectricInterface

(www.brainkart.com/subject/Electromagnetic-Theory_206/)