4.4. ANALYSIS OF INDETERMINATE RIGID FRAMES BY FLEXIBILITY METHOD.

4.4.1.NUMERICAL PROBLEMS ON RIGID FRAMES;

PROBLEM NO:01

Analysis the rigid portal frame ABCD shown in fig,by using Flexibility method.

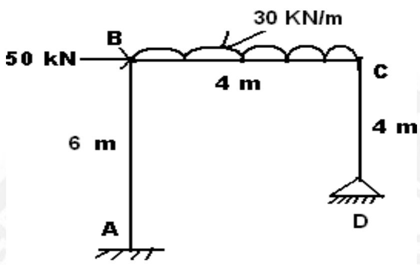


Fig.

Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 2) – 3 = 2

  Release at B and C by apply hinge

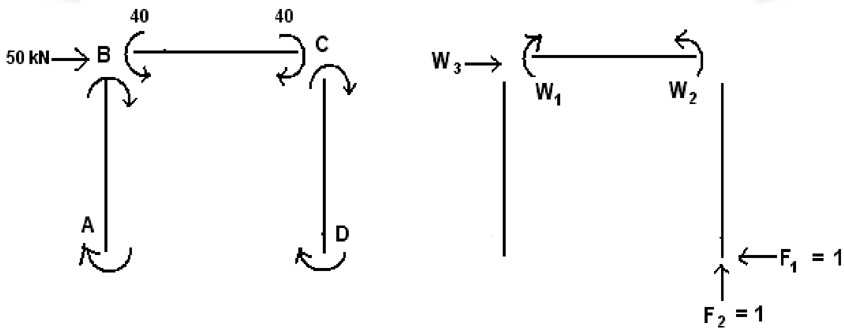
  Apply a unit force at B joint.

- **Fixed End Moments:**

  MFBC = $-Wl^2/12 = -30x4^2/12 = -40$ kNm

  MFCB = $Wl^2/12 = 30x4^2/12 = 40$ kNm

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  $B = B_W \cdot B_X$

$$B = \begin{bmatrix} 0 & 0 & -6 & -2 & 4 \\ 0 & 0 & 0 & 4 & -4 \\ 1 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}$$

$$\mathbf{F_X = B_X{}^T \,.\, F \,.\, B_X}$$

$$= \frac{1}{EI}\begin{bmatrix} -2 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 4 & -4 \\ -4 & 4 \\ 4 & 0 \\ -4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 141.28 & -104 \\ -104 & 117.28 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.0204 & 0.0181 \\ 0.0181 & 0.0246 \end{bmatrix}$$

$\mathbf{F_W = B_X^T . F . B_W}$

$$= \frac{1}{EI}\begin{bmatrix} -2 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.33 & -0.67 & 0 & 0 \\ 0 & 0 & -0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.33 & -0.67 \\ 0 & 0 & 0 & 0 & -0.67 & 1.33 \end{bmatrix}\begin{bmatrix} 0 & 0 & -6 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} -8 & -5.32 & 48 \\ 5.32 & 0 & -72 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $\mathbf{X = - F_X^{-1} . F_W . W}$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.0204 & 0.0181 \\ 0.0181 & 0.0246 \end{bmatrix}\begin{bmatrix} -8 & -5.32 & 48 \\ 5.32 & 0 & -72 \end{bmatrix}\begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

$$= -\begin{bmatrix} 0.0669 & 0.1085 & 0.3240 \\ 0.0139 & 0.0963 & 0.9024 \end{bmatrix}\begin{bmatrix} 40 \\ -40 \\ 50 \end{bmatrix}$$

$$= -\begin{bmatrix} -14.536 \\ -41.824 \end{bmatrix}$$

$$X = \begin{bmatrix} 14.536 \\ 41.824 \end{bmatrix}$$

- **Internal Force (P):**

$$\mathbf{P} = \begin{bmatrix} W \\ X \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -6 & -2 & 4 \\ 0 & 0 & 0 & 4 & -4 \\ 1 & 0 & 0 & -4 & 4 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 40 \\ -40 \\ 50 \\ 14.536 \\ 41.824 \end{bmatrix}$$

$$P = \begin{bmatrix} -161.776 \\ -109.152 \\ 149.152 \\ 58.144 \\ -98.144 \\ 0 \end{bmatrix}$$

- **Final Moments (M):**

$$\mathbf{M = \mu + P}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -40 \\ 40 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -161.776 \\ -109.152 \\ 149.152 \\ 58.144 \\ -98.144 \\ 0 \end{bmatrix}$$

$$M = \begin{bmatrix} -161.776 \\ -109.152 \\ 109.152 \\ 98.144 \\ -98.144 \\ 0 \end{bmatrix}$$

PROBLEM NO:02

Analysis the rigid portal frame ABCD shown in fig,by using Flexibility method.And sketch the bending moment diagram.

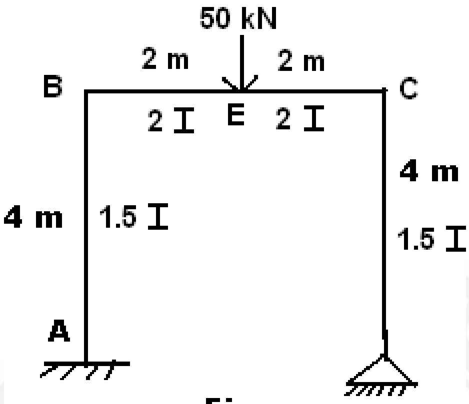


Fig.

Solution:

- **Static indeterminacy:**

  Degree of redundancy = (3 + 2) – 3 = 2

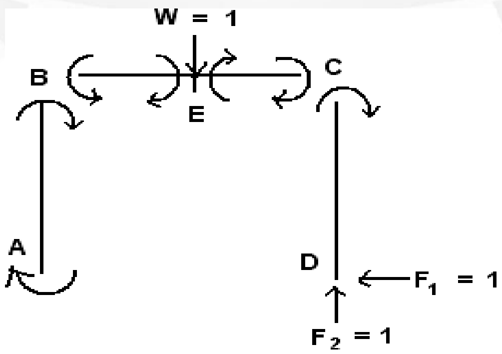
  Release at D by apply horizontal and vertical supports

  Apply a unit force at E joint.

- **Fixed End Moments:**

  MFAB = MFBA = MFBC = MFCB = MFCD = MFDC = 0

- **Equivalent Joint Loads:**



- **Flexibility Co-efficent Matrix (B):**

  $\mathbf{B = B_W \cdot B_X}$

$$B = \begin{bmatrix} -2 & 0 & 4 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \\ 0 & 4 & -2 \\ 0 & -4 & 2 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- **Flexibility Matrix (F):**

$$F = \frac{L}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$F = \frac{1}{EI} \begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix}$$

$$\mathbf{F_X = B_X^T \cdot F \cdot B_X}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & 4 & -4 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 4 & -4 \\ -4 & 4 \\ 4 & -2 \\ -4 & 2 \\ 4 & 0 \\ -4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_x = \frac{1}{EI}\begin{bmatrix} 60.48 & -37.28 \\ -37.28 & 53.2 \end{bmatrix}$$

$$F_x^{-1} = EI\begin{bmatrix} 0.0291 & 0.0203 \\ 0.0203 & 0.033 \end{bmatrix}$$

$$\mathbf{F_W = B_X^T \,.\, F \,.\, B_W}$$

$$= \frac{1}{EI}\begin{bmatrix} 0 & 4 & -4 & 4 & -4 & 4 & -4 & 0 \\ 4 & -4 & 4 & -2 & 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.89 & -0.44 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.44 & 0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.33 & -0.17 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.17 & 0.33 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.33 & -0.17 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.17 & 0.33 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.89 & -0.44 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.44 & 0.89 \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F_W = \frac{1}{EI}\begin{bmatrix} 14.64 \\ -24.60 \end{bmatrix}$$

- **Displacement Matrix (X):**

  $$\mathbf{X = - F_X^{-1} \,.\, F_W \,.\, W}$$

$$= -\frac{EI}{EI}\begin{bmatrix} 0.0291 & 0.0203 \\ 0.0203 & 0.033 \end{bmatrix}\begin{bmatrix} 14.64 \\ -24.60 \end{bmatrix} 50$$

$$= -\begin{bmatrix} -0.0734 \\ -0.5146 \end{bmatrix} 50$$

$$= -\begin{bmatrix} -3.67 \\ -25.73 \end{bmatrix}$$

$$X = \begin{bmatrix} 3.67 \\ 25.73 \end{bmatrix}$$

- **Internal Force (P):**

$$P = [\,^{W}_{X}]$$

$$= \begin{bmatrix} -2 & 0 & 4 \\ 2 & 4 & -4 \\ -2 & -4 & 4 \\ 0 & 4 & -2 \\ 0 & -4 & 2 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 50 \\ 3.67 \\ 25.73 \end{bmatrix}$$

$$P = \begin{bmatrix} 2.92 \\ 11.76 \\ -11.76 \\ -36.78 \\ 36.78 \\ 14.68 \\ -14.68 \\ 0 \end{bmatrix}$$

The final moments also same, since there are no external forces acting on the members.