Boundary conditions for Electrostatic fields

In our discussions so far we have considered the existence of electric field in the homogeneous medium. Practical electromagnetic problems often involve media with different physical properties. Determination of electric field for such problems requires the knowledge of the relations of field quantities at an interface between two media. The conditions that the fields must satisfy at the interface of two different media are referred to as ***boundary conditions*** .

In order to discuss the boundary conditions, we first consider the field behavior insome common material media.

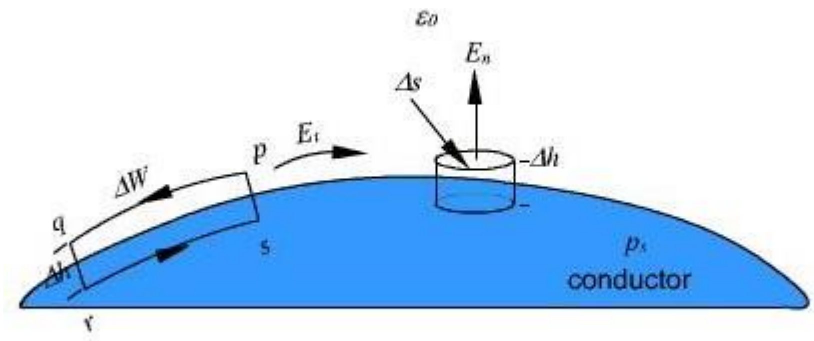

Fig *6*.1: Boundary Conditions for at the surface of a Conductor
(www.brainkart.com/subject/Electromagnetic-Theory_206/)

In general, based on the electric properties, materials can be classified into three categories: conductors, semiconductors and insulators (dielectrics). In *conductor* , electrons in the outermost shells of the atoms are very loosely held and they migrate easily from one atom to the other. Most metals belong to this group. The electrons in the atoms of *insulators* or *dielectrics* remain confined to their orbits and under normal circumstances they are not liberated under the influence of an externally applied field. The electrical properties of *semiconductors* fall between those of conductors and insulators since semiconductors have very few numbers of free charges.

The parameter *conductivity* is used characterizes the macroscopic electrical property of a material medium. The notion of conductivity is more important in dealing with the current flow and hence the same will be considered in detail later on.

If some free charge is introduced inside a conductor, the charges will experience a force due
to mutual repulsion and owing to the fact that they are free to move, the

charges will appear on the surface. The charges will redistribute themselves in such a manner that the field within the conductor is zero. Therefore, under steady condition, inside a conductor

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From Gauss's theorem it follows that

$$\rho_v = 0 \quad ....(2.51)$$

The surface charge distribution on a conductor depends on the shape of the conductor. The charges on the surface of the conductor will not be in equilibrium if there is a tangential component of the electric field is present, which would produce movement of the charges. Hence under static field conditions, tangential component of the electric field on the conductor surface is zero. The electric field on $\vec{E}$ the surface of the conductor is normal everywhere to the surface . Since the tangential component of electric field is

zero, the conductor surface is an equipotential surface. As = 0 inside the conductor, the conductor as a whole has the same potential. We may further note that charges require a $\sim 10^{-19}$

finite time to redistribute in a conductor. However, this time is very small like copper.

Let us now consider an interface between a conductor and free space as

shown in the figure 6.1

Let us consider the closed path *pqrsp* for which we can write,

$$\vec{E} \quad \oint \vec{E} \cdot d\vec{l} = 0 \quad ....(2.52)$$

For $\Delta h \rightarrow 0$ and noting that inside the conductor is zero, we can write

$$E_t \Delta w = 0 ....(2.53)$$

E_t is the tangential component of the field. Therefore we find that

$$E_t = 0 .... (2.54)$$

In order to determine the normal component E_n, the normal component $\vec{E}$ of , at the surface of the conductor, we consider a small cylindrical Gaussian surface as shown in the Fig.12. Let Δs represent the area of the top and bottom faces and Δh represents the height of the cylinder. Once again, $\Delta h \rightarrow 0$ as , we approach the surface of the $\vec{E}$ conductor. Since = 0 inside the conductor is zero,

$$\varepsilon_0 \oint_s \vec{E} \cdot d\vec{s} = \varepsilon_0 E_n \Delta s = \rho_s \Delta s$$

...........(2.55)

$$E_n = \frac{\rho_s}{\varepsilon_0} \qquad ....(2.56)$$

Therefore, we can summarize the boundary conditions at the surface of a conductor as:

$$E_t = 0 \qquad ....(2.57)$$

$$E_n = \frac{\rho_s}{\varepsilon_0} \qquad ....(2.58)$$