Periodic Noise By Frequency Domain Filtering:

These types of filters are used for this purpose-

Band Reject Filters:

It removes a band of frequencies about the origin of the Fourier transformer.

Ideal Band reject Filter:

An ideal band reject filter is given by the expression

$$H(u,v)=\begin{cases}1 & \text{if} \quad D(u,v)< D_0 - W/2 \\ 0 & \text{if} \quad D_0 - W/2 \le D(u,v) \le D_0 + W/2 \\ 1 & \text{if} \quad D(u,v)> D_0 + W/2\end{cases}$$

D(u,v)- the distance from the origin of the centered frequency rectangle.

W- the width of the band

Do- the radial center of the frequency rectangle.

Butterworth Band Reject Filter:

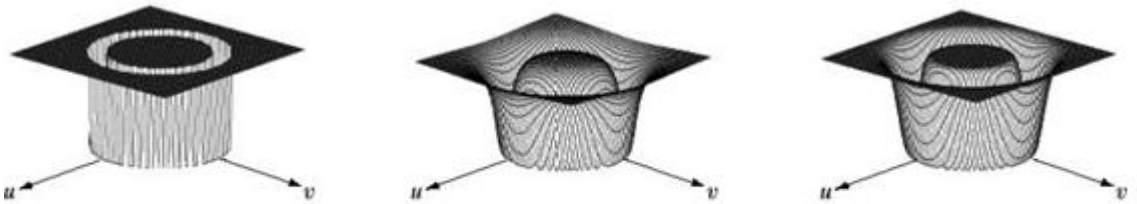
$$H(u,v)=1\Big/\left[1+\left(\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right)^{2n}\right]$$

Gaussian Band Reject Filter:

$$H(u,v)=1-\exp\left[-\frac{1}{2}\left(\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right)^2\right]$$

These filters are mostly used when the location of noise component in the frequency domain is known. Sinusoidal noise can be easily removed by using these kinds of filters because it shows two impulses that are mirror images of each other about the origin. Of the frequency transform.

Band pass Filter:



a b c

(Fig3.4.1: Perspectives plot of ideal, Butterworth , Gaussian and Notch Filter

Source: D,E. Dudgeon and RM. Mersereau, ‗Multidimensional Digital Signal Processing', Prentice Hall Professional Technical Reference, 1990Page- 312)

The function of a band pass filter is opposite to that of a band reject filter It allows a specific frequency band of the image to be passed and blocks the rest of frequencies. The transfer function of a band pass filter can be obtained from a corresponding band reject filter with transfer function Hbr(u,v) by using the equation

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

An ideal band reject filter is given by the expression

$$H(u,v) = \begin{cases} 1 & \text{if} \quad D(u,v) < D_0 - W/2 \\ 0 & \text{if} \quad D_0 - W/2 \le D(u,v) \le D_0 + W/2 \\ 1 & \text{if} \quad D(u,v) > D_0 + W/2 \end{cases}$$

D(u,v)- the distance from the origin of the centered frequency rectangle. W- the width of the band

Do- the radial center of the frequency rectangle.

Butterworth Band Reject Filter:

$$H(u,v) = 1 \Big/ \left[1 + \left(\frac{D(u,v)W}{D^2(u,v) - D_0^2} \right)^{2n} \right]$$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Notch Filters:

A notch filter rejects (or passes) frequencies in predefined neighborhoods about a center frequency.

Due to the symmetry of the Fourier transform notch filters must appear in symmetric pairs about the origin.

The transfer function of an ideal notch reject filter of radius D_0 with centers a (u_0, v_0) and by symmetry at ($-u_0$, v_0) is

$$D_1(u,v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$

$$D_2(u,v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

Ideal, Butterworth, Gaussian notch filters

$$H(u,v) = \begin{cases} 0 & \text{if} \quad D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 & \text{otherwise} \end{cases}$$

$$H(u,v) = 1 \Big/ \left[1 + \left(\frac{D_0^2}{D_1(u,v) D_2(u,v)} \right)^n \right]$$

$$H(u,v) = 1 - \exp\left[-\frac{1}{2} \left(\frac{D_1(u,v) D_2(u,v)}{D_0^2} \right) \right]$$

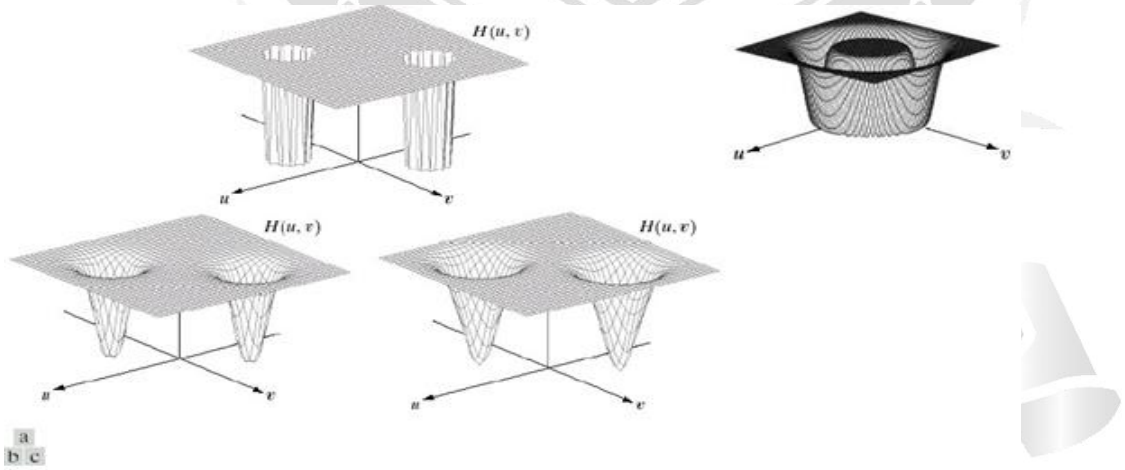


Fig 3.4.2: Perspectives plot of ideal, Butterworth , Gaussian and Notch Filter. (Source: D,E. Dudgeon and RM. Mersereau, ‗Multidimensional Digital Signal Processing', Prentice Hall Professional Technical Reference, 1990.-Page-335)

These filters cannot be applied directly on an image because it may remove too much details of an image but these are effective in isolating the effect of an

image of selected frequency bands.