

Histogram Processing:

histogram of a digital image with gray levels in the range $[0, L-1]$ is a discrete function of the form

$$H(r_k) = n_k$$

where r_k is the k th gray level and n_k is the number of pixels in the image having the level r_k . A normalized histogram is given by the equation

$$p(r_k) = n_k/n \text{ for } k=0,1,2,\dots,L-1$$

$P(r_k)$ gives the estimate of the probability of occurrence of gray level r_k . The sum of all components of a normalized histogram is equal to 1.

The histogram plots are simple plots of $H(r_k) = n_k$ versus r_k . In the dark image the components of the histogram are concentrated on the low (dark) side of the gray scale. In case of bright image the histogram components are biased towards the high side of the gray scale. The histogram of a low contrast image will be narrow and will be centered towards the middle of the grayscale.

The components of the histogram in the high contrast image cover a broad range of the gray scale. The net effect of this will be an image that shows a great deal of gray levels details and has high dynamic range. In the case of monochrome images, it was shown (see Fig. 3.20) to be reasonably successful at handling low-, high-, and middle-key images. Since color images are composed of multiple components, however, consideration must be given to adapting the gray-scale technique to more than one component and/or histogram.

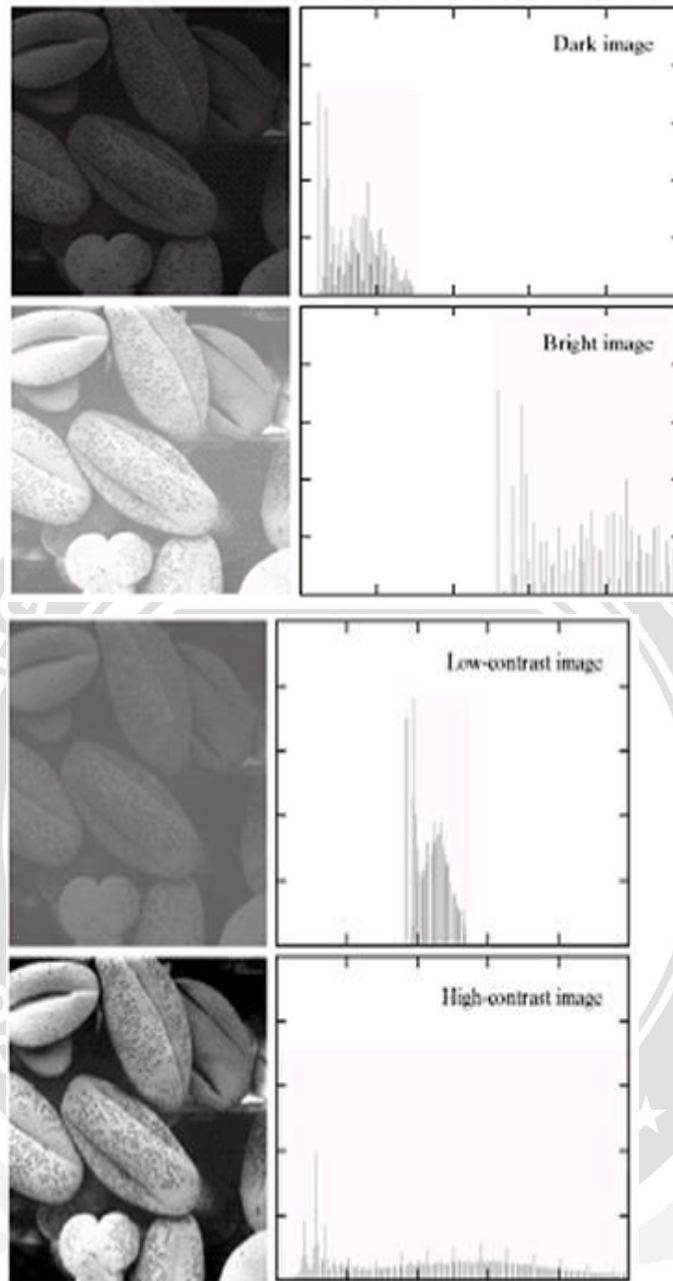


Fig 2.3.1: Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

(Source: Rafael C. Gonzalez, Richard E. Woods, 'Digital Image Processing', Pearson, Third Edition, 2010.- Page -439)

HISTOGRAM EQUALIZATION:

Histogram equalization is a common technique for enhancing the appearance of images. Suppose we have an image which is predominantly dark. Then its histogram would be skewed towards the lower end of the grey scale and all the image detail are compressed into the dark end of the histogram. If we could, stretch out the grey levels at the dark end to produce a more uniformly distributed histogram then the image

would become much clearer.

Let there be a continuous function with r being gray levels of the image to be enhanced. The range of r is $[0, 1]$ with $r=0$ representing black and $r=1$ representing white. The transformation function is of the form

$$S=T(r) \text{ where } 0<r<1$$

It produces a level s for every pixel value r in the original image.

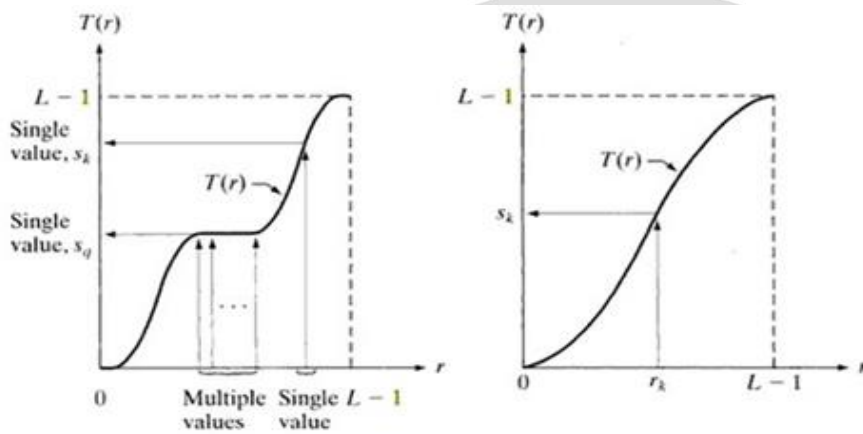


Fig 2.3.2: Recall that a function is monotonically increasing if for is a strictly monotonically Increasing function if for Similar definitions apply to monotonically Decreasing functions.

(source: Rafael C. Gonzalez, Richard E. Woods, 'Digital Image Processing', Pearson, Third Edition, 2010. – Page 440)

The transformation function is assumed to fulfill two condition $T(r)$ is single valued and monotonically increasing in the internal $0<T(r)<1$ for $0<r<1$.The transformation function should be single valued so that the inverse transformations should exist. Monotonically increasing condition preserves the increasing order from black to white in the output image. The second conditions guarantee that the output gray levels will be in the same range as the input levels. The gray levels of the image may be viewed as random variables in the interval $[0,1]$. The most fundamental descriptor of a random variable is its probability density function (PDF) $Pr(r)$ and $Ps(s)$ denote the probability density functions of random variables r and s respectively. Basic results from an

elementary probability theory states that if $P_r(r)$ and T_r are known and $T^{-1}(s)$ satisfies conditions (a), then the probability density function $P_s(s)$ of the transformed variable is given by the formula

Thus the PDF of the transformed variable s is determined by the gray levels PDF of the input image and by the chosen transformation function.

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

A transformation function of a particular importance in image processing

This is the cumulative distribution function of r .

L is the total number of possible gray levels in the image.

