1.7 SPECIFIC ENERGY AND SPECIFIC FORCE

The total energy of a channel flow referred to datum is given by,

$$\mathbf{H = z + y + \frac{v^2}{2g}}$$

If the datum coincides with the channel bed at the cross-section, the resulting expression is know as specific energy and is denoted by E. Thus, specific energy is the energy at a cross-section of an open channel flow with respect to the channel bed. The concept of specific energy, introduced by Bakmeteff, is very useful in defining critical water depth and in the analysis of open channel flow. It may be noted that while the total energy in a real fluid flow always decreases in the downstream direction, the specific energy is constant for a uniform flow and can either decrease or increase in a varied flow, since the elevation of the bed of the channel relative to the elevation of the energy line, determines the specific energy.

Specific energy at a cross-section is,

$$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}$$

Here, cross-sectional area A depends on water depth y and can be defined as, A = A(y). show us that, there is a functional relation between the three variables as,

$$f\ (E,\ y, Q) = 0$$

In order to examine the functional relationship on the plane, two cases are introduced

1. Q = Constant = Q1 → E = f (y, Q1).

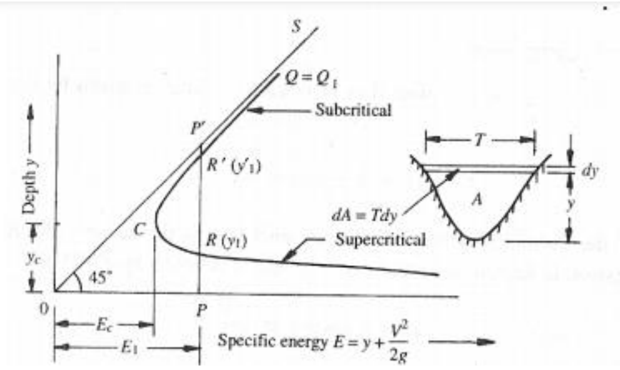
Variation of the specific energy with the water depth at a cross-section for a given discharge Q1.

2. E = Constant = E1 → E1 = f (y,Q) Variation of the discharge with the water depth at across-section for a given specific energy E1.

Constant Discharge Situation

Since the specific energy,

$$\mathbf{E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}}$$



For a channel of known geometry, E = f (y, Q). Keeping Q = constant = Q1, the variation of E with y is represented by a cubic parabola. (Figure 5.1). It is seen that there are two positive roots for the equation E indicating that any particular discharge Q1 can be passed in a given channel at two depths and still maintain the same specific energy E1. The depths of flow can be either PR = y1 or PR` = y`1. These two possible depths having the same specific energy are known as alternate depths. In Fig. (5.1), a line (OS) drawn such that E = y (i.e. at 450 to the abscissa) is the asymptote of the upper limb of the specific energy curve. It may be noticed that the intercept P`R` and P`R represents the velocity head. Of the two alternate depths, one (PR = y1) is smaller and has a large velocity head while the other (PR`= y`1) has a larger depth and consequently a smaller velocity head. For a given Q, as the specific energy is increased the difference between the two alternate depths increases. On the other hand, if E is decreased, the difference (y`1 – y1) will decrease and a certain value E = Ec, the two depths will merge with each other (point C in Fig. 5.1). No value for y can be obtained when E < Ec , denoting that the flow under the given conditions is not possible in this region. The condition of minimum specific energy is known as the critical flow condition and the corresponding depth yc is known as critical depth.

problem 1

Calculate the Specific energy,Critical depth and the velocity of the flow of 10 m^3 in a cement lined rectangular channel 2.5m wide with 2 m depth of water. Is the given flowis sub critical or super critical

Given Data

Q = 10 m3/s

b = 2.5 m

y = 2 m

To find

1. Specific Energy
2. Critical Depth
3. Velocity for the flow

SOLUTION:

STEP 1: Specific Energy:

$$E = y + \frac{v^2}{2g}$$

$$V = \frac{Q}{A} \quad , \; V = \frac{10}{2 \times 2.5}$$

v = 2m/s

$$E = 2 + \frac{2^2}{2 \times 9.81}$$

$$E = 2.20\text{m}$$

STEP 2 : Critical Depth

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \frac{Q}{b}$$

$$q = \frac{10}{2.5} = 5\text{m}^2/\text{sec}$$

$$y_c = \left(\frac{5^2}{9.81}\right)^{1/3}$$

$$y_c = 1.18m$$

STEP 3: Velocity of flow

$$\mathrm{v_c} = \sqrt{\mathrm{y_c} \times \mathrm{g}}$$

$$\mathrm{v_c} = \sqrt{1.18 \times 9.81}$$

$$\mathrm{v_c} = 3.4\mathrm{m/s}$$

STEP 4: To find weather the flow is Sub critical of Super critical

$$F = \frac{v}{\sqrt{g \times D}}$$

$$= \frac{2}{\sqrt{9.81 \times 2}}$$

$$=0.45<1.0$$

Hence the flow is Sub critical