2.4 POWER AND IMPEDANCE MEASUREMENT ON LINE

The voltage at any point at distance 's' from the receiving end of a transmission line is given by,

$$E = \frac{E_R}{2}\left[\frac{Z_R + Z_o}{Z_R}\right]\left[e^{\sqrt{ZY}S} + \left[\frac{Z_R - Z_o}{Z_R + Z_o}\right] e^{-\sqrt{ZY}S}\right]$$

For dissipation less line,

$\gamma = \sqrt{ZY}$

$\gamma = j\omega\sqrt{LC}$

$\beta = \omega\sqrt{LC}$

$\gamma = j\beta$

$$E = \frac{E_R}{2Z_R}\left[(Z_R + Z_o)\, e^{j\beta S} + (Z_R - Z_o)\, e^{-j\beta S}\right]$$

$$E = \frac{E_R}{2Z_R}\left[Z_R\, e^{j\beta S} + Z_o\, e^{j\beta S} + Z_R\, e^{-j\beta S} - Z_o\, e^{-j\beta S}\right]$$

$$E = \frac{E_R}{2Z_R}\left[Z_R\,(e^{j\beta S} + e^{-j\beta S}) + Z_o\,(e^{j\beta S} - e^{-j\beta S})\right]$$

- $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
- $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$E = \frac{E_R}{Z_R}\left[Z_R\frac{(e^{j\beta S} + e^{-j\beta S})}{2} + Z_o\frac{(e^{j\beta S} - e^{-j\beta S})}{2}\right]$$

$$E = \frac{E_R}{Z_R}\left[Z_R\cos\beta S + jZ_o\sin\beta S\right]$$

$$E = E_R\cos\beta S + \frac{jZ_oE_R}{Z_R}\sin\beta S$$

- $I_R = \frac{E_R}{Z_R}$

$E = E_R \cos \beta S + jZ_o I_R \sin \beta S$

This is the voltage equation for the dissipation less line.

$$I = \frac{I_R}{2} \left[\frac{Z_O + Z_R}{Z_O} \right] \left[e^{\sqrt{ZY}S} - \frac{I_R}{2} \left[\frac{Z_R - Z_O}{Z_R + Z_O} \right] e^{-\sqrt{ZY}S} \right]$$

$\gamma = \sqrt{ZY} = j\,\beta$

$$I = \frac{I_R}{2Z_O} \left[(Z_R + Z_o)\, e^{j\beta S} - (Z_R - Z_o)\, e^{-j\beta S} \right]$$

$$I = \frac{I_R}{2Z_O} \left[Z_R\, e^{j\beta S} + Z_o\, e^{j\beta S} - Z_R\, e^{-j\beta S} + Z_o\, e^{-j\beta S} \right]$$

$$I = \frac{I_R}{2Z_O} \left[Z_R\, (e^{j\beta S} - e^{-j\beta S}) + Z_o\, (e^{j\beta S} + e^{-j\beta S}) \right]$$

$$I = \frac{I_R}{Z_O} \left[Z_R \frac{(e^{j\beta S} - e^{-j\beta S})}{2} + Z_o \frac{(e^{j\beta S} + e^{-j\beta S})}{2} \right]$$

$$I = \frac{I_R}{Z_O} \left[Z_O \cos \beta S + jZ_R \sin \beta S \right]$$

$$I = I_R \cos \beta S + \frac{jZ_R I_R}{Z_O} \sin \beta S$$

$$E_R = Z_R I_R$$

$$I = I_R \cos \beta S + \frac{jE_R}{Z_O} \sin \beta S$$

This is the current equation for the dissipation less line.

We know that,

$$Z_o = \sqrt{\frac{L}{C}} = R_o$$

$\text{E} = I_R \cos \beta S + \frac{\text{j}E_R}{R_O} \sin \beta S$

These are the voltage and current equation of the dissipation less line.

The above equation represents the voltage interms of receiving end voltage and current as well as current interms of receiving end voltage and current.

The voltage and current distribution is the sum of cosine and sine distribution.

Wkt,

$\beta = \frac{2\pi}{\lambda}$

Sub in above equ,

$\text{E} = E_R \cos\left(\frac{2\pi}{\lambda}\right) S + jR_o \text{I}_\text{R} \sin\left(\frac{2\pi}{\lambda}\right) S$

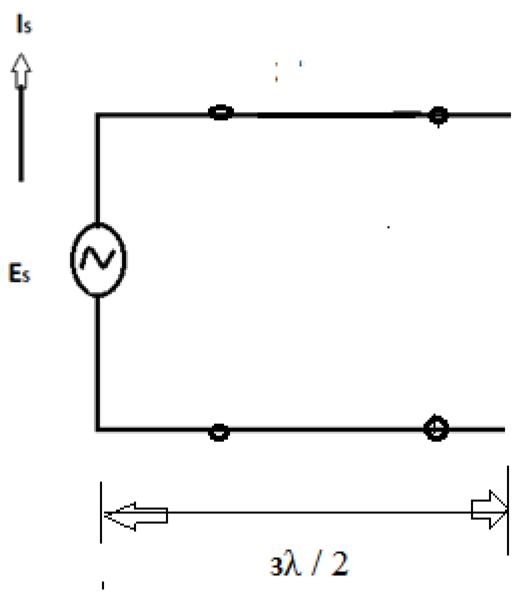
$\text{I} = I_R \cos\left(\frac{2\pi}{\lambda}\right) S + \frac{\text{j}E_R}{R_O} \sin\left(\frac{2\pi}{\lambda}\right) S$

CASE (i):

When the line is open circuited at the receiving end . $Z_R = \infty$, $I_R = 0$

$E_{OC} = E_R \cos\left(\frac{2\pi}{\lambda}\right) S$

$I_{OC} = \sin\left(\frac{2\pi}{\lambda}\right) S$



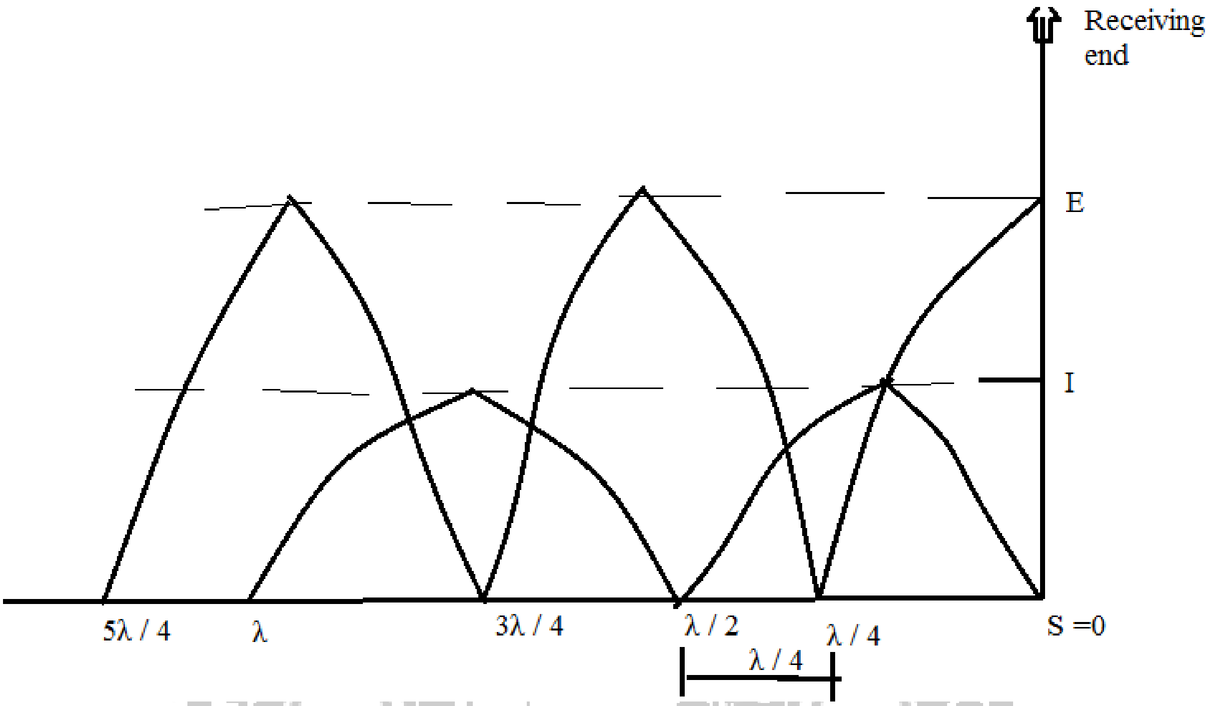


Fig: 2.4.1 Open circuit

The magnitude of voltage and current distribution for a open circuit line for 3 λ / 2 distance is draw in Fig 2.4.1. For every λ / 4 distance the voltage changes from maximum to minimum and viceversa.

CASE (ii):

If the line is short circuited Z_R = 0 , E_R = 0

$$E_{SC} = jR_o I_R \sin\left(\frac{2\pi}{\lambda}\right) S$$

$$I_{SC} = I_R \cos\left(\frac{2\pi}{\lambda}\right) S$$

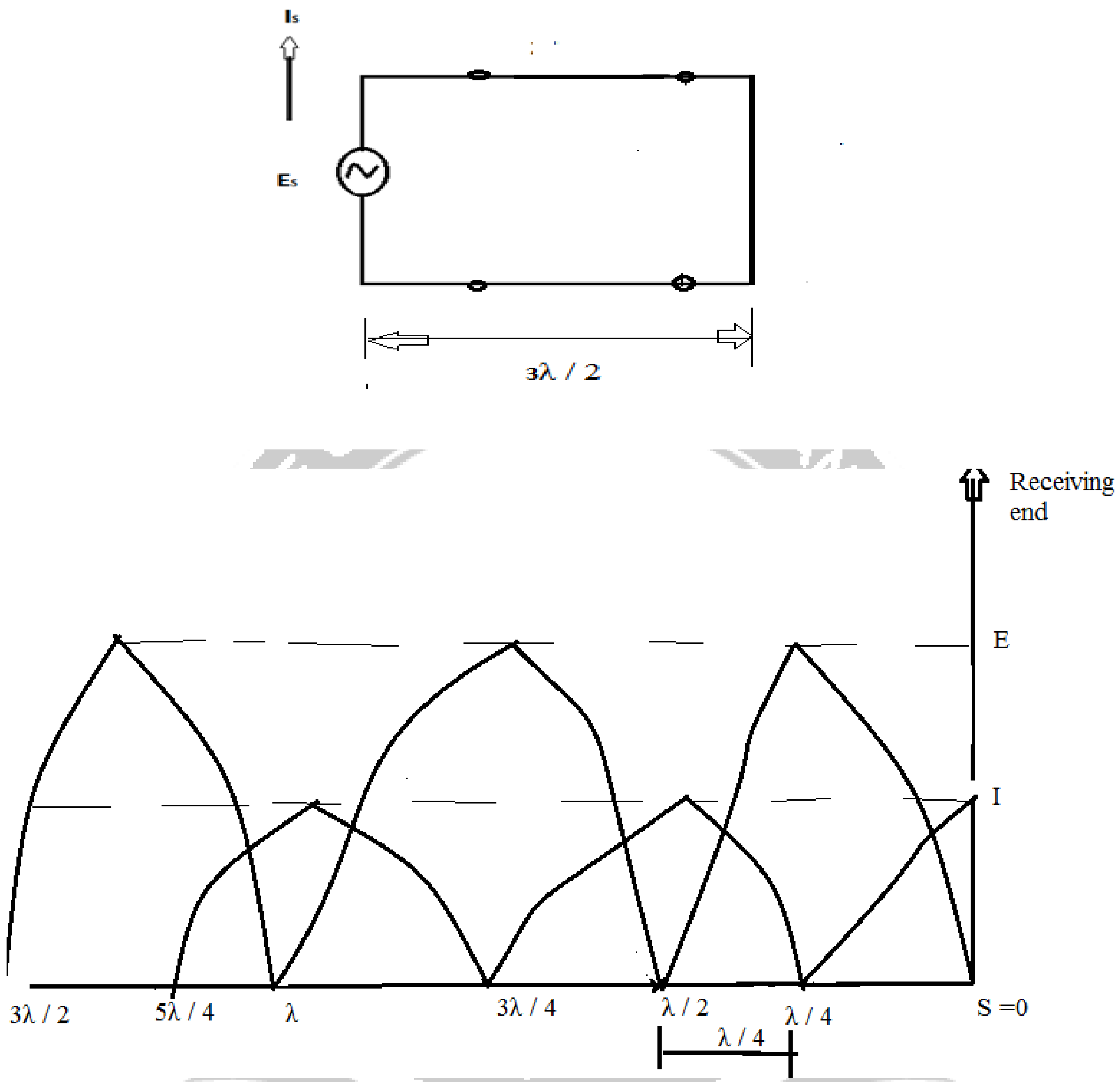


Fig: 2.4.2 Short circuit

The similarity of performance of short-circuited lines to that lines to that of series-resonant or antiresonant circuits circuits may be readily noted by comparison of the curves of Fig 2.4.2.

<u>CASE (iii):</u>

In Fig 2.4.3, when the line is terminated with an impedance $Z_R = Z_O$

Wkt,

$$K = \frac{Z_R - Z_O}{Z_R + Z_O}$$

At high frequencies $Z_O = R_O$

$K = \frac{R_O - R_O}{R_O + R_O}$

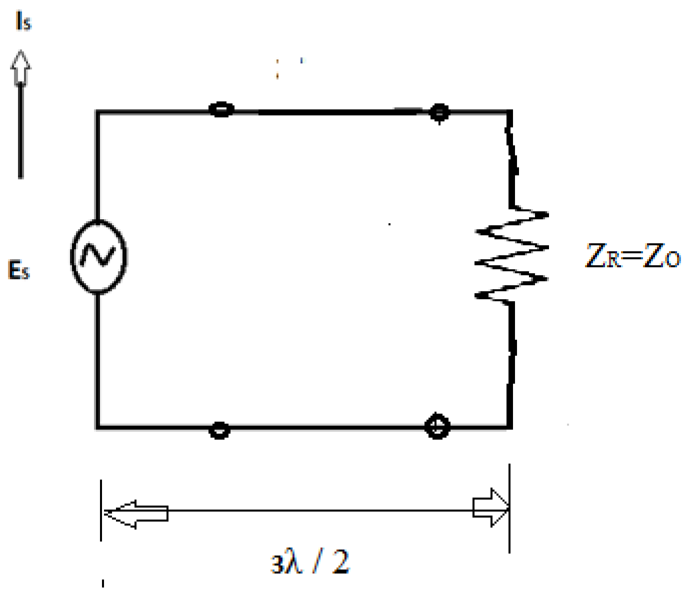
K = 0

This means that, the reflected wave is absent.

The voltage and current in the line is given by,

$E = E_R \ e^{j\,\beta S}$

$I = I_R \ e^{j\,\beta S}$



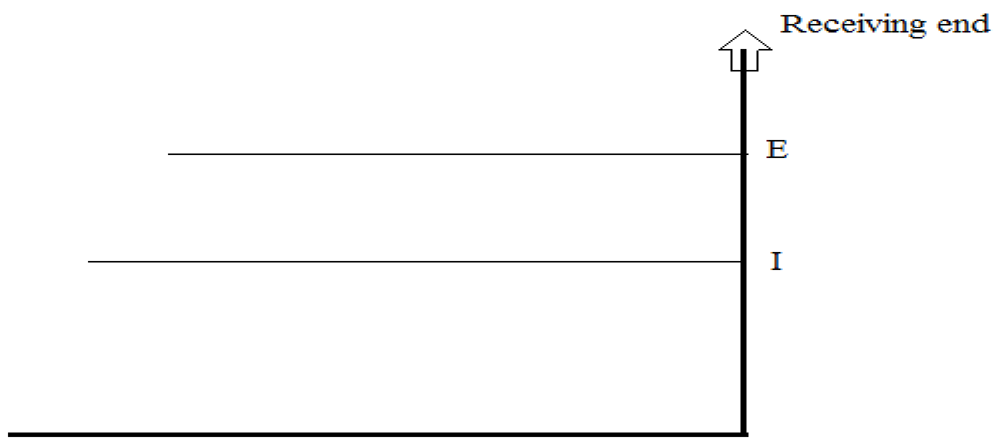


Fig: 2.4.3 Transmission line is terminated with an impedance $Z_R = Z_O$

OPEN AND SHORT CIRCUITED IMPEDANCE:

To find short circuit impedance:

In Fig 2.4.4 shows that the wave is progressing from the receiving end toward the load, the initial value equal to the reflected voltage at the load for open circuit. This is incident wave.

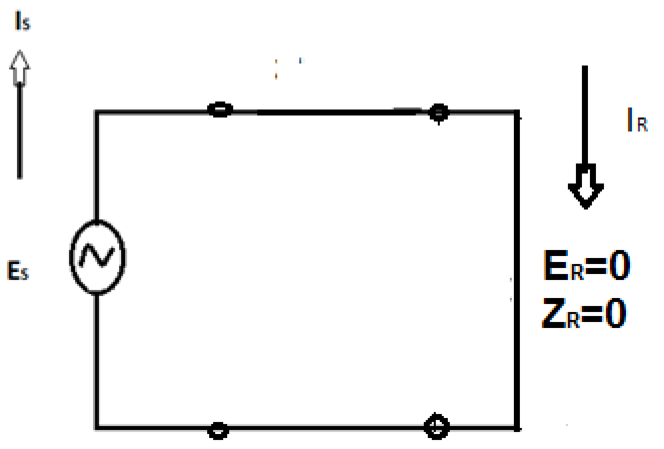


Fig: 2.4.4 Short circuit

At short circuit condition Z_R=0, we know,

$$z_{in} = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$$

Sub $Z_R = 0$,

$$z_{in} = R_O \left[\frac{jR_O \tan\beta s}{R_O}\right]$$

$$z_{in} = jR_O \tan\beta s$$

We know that,

$$\beta = \frac{2\pi}{\lambda}$$

Sub β value in Z_{in},

$$z_{sc} = jR_O \tan\left(\frac{2\pi}{\lambda}\right) s$$

$$R_s + jX_s = jR_O \tan\left(\frac{2\pi}{\lambda}\right) s$$

Equating real and imag parts,

$R_s = 0$

$X_s = R_O \tan\left(\frac{2\pi}{\lambda}\right) s$

$\frac{X_s}{R_O} = \tan\left(\frac{2\pi}{\lambda}\right) s$

Open Circuit impedance:

In Fig 2.4.5 shows that the wave is progressing from the receiving end toward the source, the initial value equal to the incident voltage at the load for open circuit. This is reflected wave.

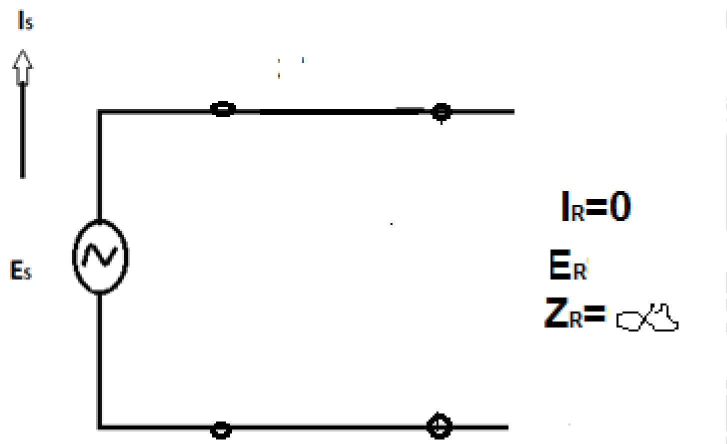


Fig: 2.4.5 Open circuit

$z_{in} = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right]$

$z_{in} = R_O \frac{Z_R}{Z_R} \left[\frac{1 + j\frac{R_O}{Z_R}\tan\beta s}{\frac{R_O}{Z_R} + j\tan\beta s}\right]$

$Z_R = \infty$ sub in above equ,

$z_{oc} = R_O \left[\frac{1}{j\tan\beta s}\right]$

$z_{oc} = \frac{-jR_O}{\tan\beta s}$

$z_{oc} = -jR_O \cot\beta s$

$z_{oc} = -jR_O \cot\beta s$

$\beta = \frac{2\pi}{\lambda}$

Sub β value in Z_{oc},

$z_{oc} = -jR_O \cot\left(\frac{2\pi}{\lambda}\right)$ s

R_s + j$X_s = -jR_O \cot\left(\frac{2\pi}{\lambda}\right)$ s

Equating real and imag parts,

$R_s = 0$

$X_s = -R_O \cot\left(\frac{2\pi}{\lambda}\right)$ s

$\frac{X_s}{R_O} = -\cot\left(\frac{2\pi}{\lambda}\right)$ s