UNIFORMLY SPACED ARRAY WITH NON UNIFORM EXCITATION AMPLITUDE

Binomial Array

In order to increase the directivity of an array its total length need to be increased. In this approach, number of minor lobes appears which are undesired for narrow beam applications. In has been found that number of minor lobes in the resultant pattern increases whenever spacing between elements is greater than $\lambda/2$. As per the demand of modern communication where narrow beam (no minor lobes) is preferred, it is the greatest need to design an array of only main lobes. The ratio of power density of main lobe to power density of the longest minor lobe is termed side lobe ratio. A particular technique used to reduce side lobe level is called tapering. Since currents/amplitude in the sources of a linear array is non-uniform, it is found that minor lobes can be eliminated if the centre element radiates more strongly than the other sources. Therefore tapering need to be done from centre to end radiators of same specifications. The principle of tapering are primarily intended to broadside array but it is also applicable to end-fire array. Binomial array is a common example of tapering scheme and it is an array of n-isotropic sources of non-equal amplitudes. Using principle of pattern multiplication, John Stone first proposed the binomial array in 1929, where amplitude of the radiating sources arc arranged according to the binomial expansion. That is. if minor lobes appearing in the array need to be eliminated, the radiating sources must have current amplitudes proportional to the coefficient of binomial series, i.e. proportional to the coefficient of binomial series, i.e.

$$(1+x)^n = 1 + (n-1)x + \frac{(n-1)(n-2)}{!2}x^2 + \frac{(n-1)(n-2)(n-3)}{!3}x^3 \pm \cdots$$

...(1) where n is the number of radiating sources in the array.

For an array of total length $n\lambda/2$, the relative current in the nth element from the one end is given by

$$= \frac{n!}{r!(n-r)!}$$

where r = 0, 1, 2, 3, and the above relation is equivalent to what is known as Pascal's triangle. For example, the relative amplitudes for the array of 1 to 10 radiating sources are as follows:

No. of sources	*Pascal's triangle*
$n = 1$	1
$n = 2$	1 1
$n = 3$	1 2 1
$n = 4$	1 3 3 1
$n = 5$	1 4 6 4 1
$n = 6$	1 5 10 10 5 1
$n = 7$	1 6 15 20 15 6 1
$n = 8$	1 7 21 35 35 21 7 1
$n = 9$	1 8 28 56 70 56 28 8 1
$n = 10$	1 9 36 84 126 126 84 36 9 1

Since in binomial array the elements spacing is less than or equal to the half-wave length, the HPBW of the array is given by

$$\text{HPBW} = \frac{10.6}{\sqrt{n-1}} = \frac{1.06}{\sqrt{\frac{2L}{\lambda}}} = \frac{0.75}{\sqrt{L_\lambda}}$$

and directivity

$$D_0 = 1.77\sqrt{n} = 1.77\sqrt{1 + 2L_\lambda}$$

Using principle of multiplication, the resultant radiation pattern of an n-source binomial array is given by

In particular, if identical array of two point sources is superimposed one above other, then three effective sources with amplitude ratio 1:2:1 results. Similarly, in case three such elements are superimposed in same fashion, then an array of four sources is obtained whose current amplitudes are in the ratio of 1:3:3:1.

The far-field pattern can be found by substituting n = 3 and 4 in the above expression and they take shape as shown in Fig. 14(a) and (b).

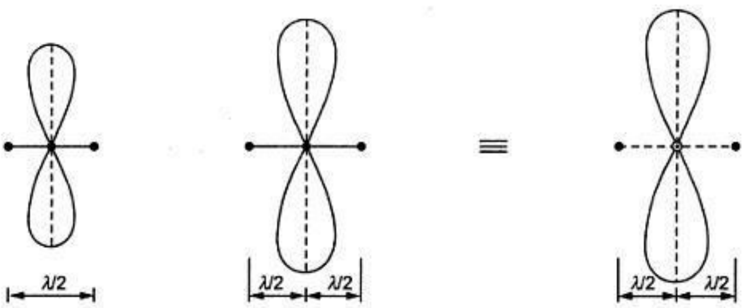

Fig. 14(a) Radiation pattern of 2-element array with amplitude ratio 1:2:1.

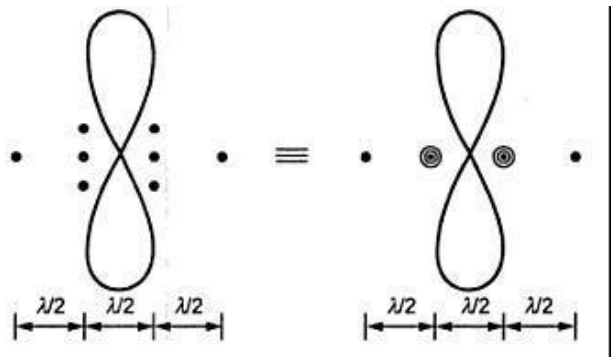

Fig 14(b) Radiation pattern of 3-element array with amplitude ratio 1:3:3:1.

It has also been noticed that binomial array offers single beam radiation at the cost of directivity, the directivity of binomial array is greater than that of uniform array for the same length of the array. In other words, in uniform array secondary lobes appear, but principle lobes are narrower than that of the binomial array.

Disadvantages of Binomial Array

(a) The side lobes are eliminated but the directivity of array reduced.

(b) As the length of array increases, larger current amplitude ratios are required.

Array of n Elements with Equal Spacing and Currents Equal in Magnitude but with Progressive Phase Shift - End Fire Array

Consider n number of identical radiators supplied with equal current which are not in phase as shown in the Fig. 11. Assume that there is progressive phase lag of βd radians in each radiator.

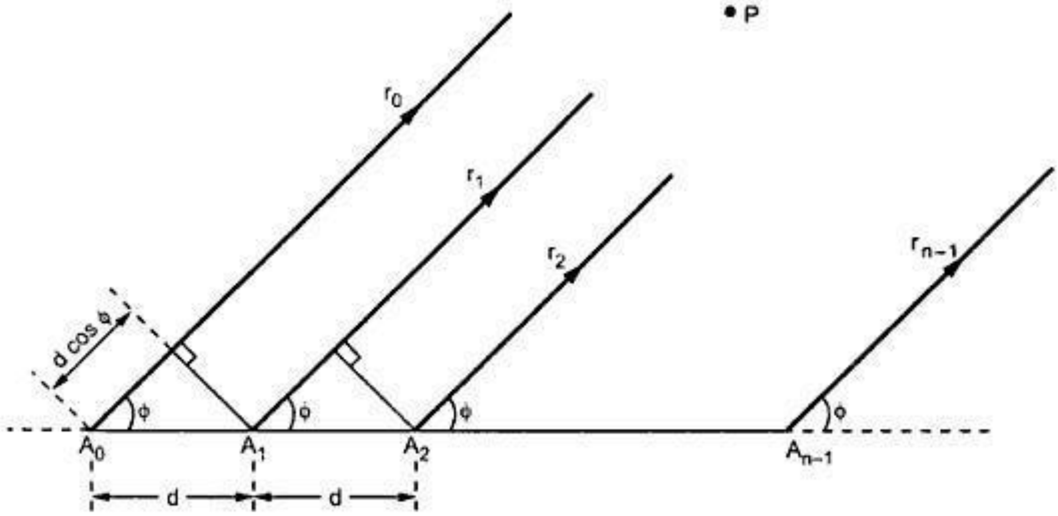

Fig.11 End fire array

Consider that the current supplied to first element A_0 be I_0. Then the current supplied to A_1 is given by,

$$I_1 = I_0 \cdot e^{-j\beta d}$$

Similarly the current supplied to A_2 is given by,

$$I_2 = I_1 \cdot e^{-j\beta d} = \left[I_0 \cdot e^{-j\beta d}\right] e^{-j\beta d} = I_0 \cdot e^{-j2\beta d}$$

Thus the current supplied to last element is

$$I_{n-1} = I_0\, e^{-j(n-1)\beta d}$$

The electric field produced at point P, due to A_0 is given by,

$$E_0 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right] e^{-j\beta r_0} \quad \text{... (1)}$$

The electric field produced at point P, due to A_1 is given by,

$$E_1 = \frac{I\, dL \sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_1}\right] e^{-j\beta r_1} \cdot e^{-j\beta d}$$

But $r_1 = r_0 - d\cos\phi$

$$\therefore \quad E_1 = \frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta(r_0-d\cos\phi)}\cdot e^{-j\beta d}$$

$$\therefore \quad E_1 = \left[\frac{I\,dL\sin\theta}{4\pi\omega\varepsilon_0}\left[j\frac{\beta^2}{r_0}\right]e^{-j\beta r_0}\right]e^{j\beta d\cos\phi}\cdot e^{-j\beta d}$$

$$\therefore \quad E_1 = E_0\cdot e^{j\beta d(\cos\phi-1)} \qquad \text{... (2)}$$

Let υ = βd(cosυ -1)

$$\therefore \quad E_1 = E_0\,e^{j\psi} \qquad \text{... (3)}$$

The electric field produced at point P, due to A2 is given by,

$$E_2 = E_0\cdot e^{j2\psi} \qquad \text{... (4)}$$

Similarly electric field produced at point P, due to An-1 is given by,

$$E_{n-1} = E_0\,e^{j(n-1)\psi} \qquad \text{... (5)}$$

The resultant field at point p is given by,

$$E_T = E_0 + E_1 + E_2 + \ldots + E_{n-1}$$

$$\therefore \quad E_T = E_0 + E_0\,e^{j\psi} + E_0\,e^{j2\psi} + \ldots + E_0\,e^{j(n-1)\psi}$$

$$\therefore \quad E_T = E_0\left[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}\right] \qquad \text{... (6)}$$

$$E_T = E_0\cdot\frac{1-e^{jn\psi}}{1-e^{j\psi}}$$

$$\therefore \quad \frac{E_T}{E_0} = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}\cdot e^{j\frac{(n-1)}{2}\psi} \qquad \text{... (7)}$$

Considering only magnitude we get,

$$\therefore \quad \boxed{\left|\frac{E_T}{E_0}\right| = \frac{\sin\frac{n\psi}{2}}{\sin\frac{\psi}{2}}} \qquad \text{...(8)}$$

Properties of End Fire Array 1. Major lobe

For the end fire array where currents supplied to the antennas are equal in amplitude but the phase changes progressively through array, the phase angle is given by,

υ = βd(cosυ -1) **...(9)**

In case of the end fire array, the condition of principle maxima is given by,

υ = = 0 i.e.

$$\boxed{\beta d(\cos\phi - 1) = 0} \quad ...(10)$$

i.e. cosυ

= 1

0

i.e. υ = 0 **...(11)**

0

Thus υ = 0 indicates the direction of principle maxima.

2. Magnitude of the major lobe

The maximum radiation occurs when υ= 0. Thus we can write,

$$|\text{Major lobe}| = \lim_{\psi \to 0} \left\{ \frac{\frac{d}{d\psi}\left(\sin n\frac{\psi}{2}\right)}{\frac{d}{d\psi}\left(\sin\frac{\psi}{2}\right)} \right\} = \lim_{\psi \to 0} \left\{ \frac{\left(\cos n\frac{\psi}{2}\right)\left(n\frac{\psi}{2}\right)}{\left(\cos\frac{\psi}{2}\right)\left(\frac{\psi}{2}\right)} \right\}$$

$$\therefore \quad \boxed{|\text{Major lobe}| = n} \quad ...(12)$$

where, n is the number of elements in the array.

3. Nulls

The ratio of total electric field to an individual electric field is given by,

$$\left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

Equating ratio of magnitudes of the fields to zero,

$$\therefore \quad \left|\frac{E_T}{E_0}\right| = \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}} = 0$$

The condition of minima is given by,

$$\boxed{\sin n\frac{\psi}{2} = 0, \quad \text{but } \sin\frac{\psi}{2} \neq 0} \quad ...(13)$$

Henc

e we can write,

$$\sin n\frac{\Psi}{2} = 0$$

$$\text{i.e. } n\frac{\Psi}{2} = \sin^{-1}(0) = \pm m\pi, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Substituting value of υ from equation (9), we get,

$$\therefore \frac{n\beta d(\cos\phi - 1)}{2} = \pm m\pi$$

But β= 2π/λ

$$\therefore \frac{nd}{\lambda}(\cos\phi - 1) = \pm m \quad \ldots(14)$$

Note that value of (cosυ-1) is always less than 1. Hence it is always negative.

Hence only considering -ve values, R.H.S., we get

$$\frac{nd}{\lambda}(\cos\phi - 1) = -m$$

$$\text{i.e. } \cos\phi - 1 = -\frac{m\lambda}{nd}$$

$$\boxed{\phi_{min} = \cos^{-1}\left[1 - \frac{m\lambda}{nd}\right]} \quad \ldots(15)$$

where, n= number of elements in the array d= spacing

between elements in meter

λ= wavelength in meter

m= constant= 1, 2 , 3....

Thus equation (15) gives direction of nulls

Consider equation(14),

$$\cos\phi_{min} - 1 = \pm\frac{m\lambda}{nd}$$

Expressing term on L.H.S. in terms of halfangles, we get,

$$2\sin^2\frac{\phi_{min}}{2} = \pm\frac{m\lambda}{nd} \quad \ldots\left(\cos\theta - 1 = 2\sin^2\frac{\theta}{2}\right)$$

$$\therefore \quad \sin^2 \frac{\phi_{min}}{2} = \pm \frac{m\lambda}{2nd}$$

$$\therefore \quad \boxed{\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right]} \qquad \ldots(16)$$

4. Side Lobes Maxima

The directions of the subsidary maxima or side lobes maxima can be obtained if in equation (8),

$$\sin\left(n\frac{\psi}{2}\right) = \pm 1$$

$$\therefore \quad \boxed{n\frac{\psi}{2} = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots} \qquad \ldots(17)$$

Hence $\sin(n\upsilon/2)$, is not considered. Because if $n\upsilon/2 = \pm\pi/2$ then $\sin n\upsilon/2 = 1$ which is the direction of principle maxima.

Hence we can skip $\sin n\upsilon/2 = \pm\pi/2$ value Thus, we get

$$\frac{n\psi}{2} = \pm(2m+1)\frac{\pi}{2}, \text{ where } m = 1, 2, 3, \ldots\ldots$$

Putting value of υ from equation (9) we get

$$\frac{n\beta d(\cos\phi - 1)}{2} = \pm(2m+1)\frac{\pi}{2}$$

$$\therefore \ n\beta d(\cos\phi - 1) = \pm(2m+1)\pi$$

Now equation for υ can be written as, But $\beta = 2\pi/\lambda$

$$n\left(\frac{2\pi}{\lambda}\right)d(\cos\phi - 1) = \pm(2m+1)\pi$$

$$\text{i.e. } \cos\phi - 1 = \pm(2m+1)\frac{\lambda}{2nd}$$

Note that value of $(\cos\upsilon - 1)$ is always less than 1. Hence it is always negative. Hence only considering -ve values, R.H.S., we get

$$\cos\phi - 1 = -(2m+1)\frac{\lambda}{2nd}$$

i.e. $$\cos\phi = 1-(2m+1)\frac{\lambda}{2nd}$$

i.e. $$\boxed{\phi = \cos^{-1}\left[1-\frac{(2m+1)\lambda}{2nd}\right]} \qquad ...(18)$$

5. Beamwidth of Major Lobe

Beamwidth is defined as the angle between first nulls. Alternatively beamwidth is the angle equal to twice the angle between first null and the major lobe maximum direction.

From equation (16) we get,

$$\phi_{min} = 2\sin^{-1}\left[\pm\sqrt{\frac{m\lambda}{2nd}}\right] \qquad ...(19)$$

$$\therefore \quad \sin\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

ϕ_{min} is very low

Hence $\sin \phi_{min}/2 \approx \phi_{min}/2$

$$\frac{\phi_{min}}{2} = \pm\sqrt{\frac{m\lambda}{2nd}}$$

$$\phi_{min} = \pm\sqrt{\frac{4m\lambda}{2nd}} = \pm\sqrt{\frac{2m\lambda}{nd}} \qquad ...(20)$$

But $nd \approx (n-1)d$ if n is very large. This L= (nd) indicates total length of the array. So equation (20) becomes,

$$\phi_{min} = \pm\sqrt{\frac{2m\lambda}{L}} = \pm\sqrt{\frac{2m}{L/\lambda}} \qquad ...(21)$$

BWFN is given by,

$$\boxed{BWFN = 2\phi_{min} = \pm 2\sqrt{\frac{2m}{L/\lambda}}} \qquad ...(22)$$

BWFN in degree is expressed as

$$BWFN = \pm 2\sqrt{\frac{2m}{L/\lambda}} \times 57.3 = \pm 114.6\sqrt{\frac{2m}{L/\lambda}} \text{ degree}$$

For m=1,

$$\boxed{BWFN = \pm 2\sqrt{\frac{2}{L/\lambda}} \text{ rad} = 114.6\sqrt{\frac{2}{L/\lambda}} \text{ degree}} \qquad ...(23)$$

6. Directivity

The directivity in case of endfire array is defined as,

$$G_{Dmax} = \frac{\text{Maximum radiation intensity}}{\text{Average radiation intensity}} = \frac{U_{max}}{U_{avg}} = \frac{U_{max}}{U_0} \qquad ...(23)$$

where, U_0 is average radiation intensity which is given by, For endfire array, U_{max} = 1and $U_0 = \frac{\pi}{2n\beta d}$

$$\therefore \quad G_{Dmax} = \frac{1}{\frac{\pi}{2n\beta d}} = \frac{2n\beta d}{\pi}$$

$$\therefore \quad G_{Dmax} = 2n\left(\frac{2\pi}{\lambda}\right)\cdot\frac{d}{\pi}$$

$$\therefore \quad \boxed{G_{Dmax} = 4\left(\frac{nd}{\lambda}\right)} \qquad ...(24)$$

The total length of the array is given by, L = (n - 1) d ≈ nd, if n is very large. Hence the directivity can be expressed in terms of the total length of the array as,

$$\therefore \quad \boxed{G_{Dmax} = 4\left(\frac{L}{\lambda}\right)} \qquad ...(25)$$

Multiplication of patterns

In the previous sections we have discussed the arrays of two isotropic point sources radiating field of constant magnitude. In this section the concept of array is extended to non-isotropic sources. The sources identical to point source and having field patterns of definite shape and orientation. However, it is not necessary that amplitude of individual sources is equal. The simplest case of non-isotropic sources is when two short dipoles are superimposed over the two isotopic point sources separated by a finite distance. If the field pattern of each source is given by

$$E_0 = E_1 = E_2 = E' \sin\theta$$

Then the total far-field pattern at point P becomes

$$E_T = 2E_0 \cos\left(\frac{\psi}{2}\right) = 2E' \sin\theta \cos\left(\frac{\psi}{2}\right) \Rightarrow E_{Tn} = \sin\theta \cos\left(\frac{\psi}{2}\right) \quad \text{...(1)}$$

$$E_{Tn} = E(\theta) \times \cos\left(\frac{\psi}{2}\right)$$

where

$$\psi = \left(\frac{2\pi d}{\lambda}\cos\theta + \alpha\right)$$

Equation (1) shows that the field pattern of two non-isotropic point sources (short dipoles) is equal to product of patterns of individual sources and of array of point sources. The pattern of array of two isotropic point sources, i.e., cos υ/2 is widely referred as an array factor. That is

E_T= E (Due to reference source) x Array factor

This leads to the principle of pattern multiplication for the array of identical elements. In general, the principle of pattern multiplication can he stated as follows:

The resultant field of an array of non-isotropic hut similar sources is the product of the fields of individual source and the field of an array of isotropic point sources, each located at the phase centre of individual source and hating the relative amplitude and phase. The total phase is addition of the phases of the individual source and that of isotropic point sources. The same is true for their respective patterns also.

The normalized total field (i.e., E_{Tn}), given in Eq. (1), can re-written as

$$E = E_1(\theta) \times E_2(\theta)$$

where $E_1(\theta) = \sin\theta$ = Primary pattern of array

$$E_2(\theta) = \cos\left(\frac{2\pi d}{\lambda}\cos\theta + \alpha\right)$$ = Secondary pattern of array.

Thus the principle of pattern multiplication is a speedy method of sketching the field pattern of complicated array. It also plays an important role in designing an array. There is no restriction on the number of elements in an array; the method is valid to any number of identical elements which need not have identical magnitudes, phase and spacing between then). However, the array factor varies with the number of elements and their arrangement, relative magnitudes, relative phases and element spacing. The array of elements having identical amplitudes, phases and spacing provides a simple array factor. The array factor does not depend on the directional characteristic of the array elements; hence it can be formulated by using pattern multiplication techniques. The proper selection of the individual radiating element and their excitation are also important for the performance of array. Once the array factor is derived using the point-source array, the total field of the actual array can be obtained using Eq. (2).