IDFT PROBLEMS

1. Find the IDFT of the sequence

X(k)={5,0,1-j,0,1,0,1+j,0} here N=8

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi nk}{N}} \quad 0 \le n \le N-1$$

$$x(n) = \frac{1}{8}\sum_{k=0}^{7} X(k)e^{j\frac{2\pi kn}{8}} \quad 0 \le n \le 7$$

- n=0

$$x(0) = \frac{1}{8}[X(0) + X(1) + X(2) + X(3) + X(4) + X(5) + X(6) + X(7)]$$

$$x(0) = \frac{1}{8}[5 + 0 + 1 - j + 0 + 1 + 0 + 1 + j + 0]$$

$$x(0) = \frac{1}{8}[8] \qquad \boxed{x(0) = 1}$$

- n=1,

$$x(1) = \frac{1}{8}\sum_{k=0}^{7} X(k)e^{j\frac{2\pi k(1)}{8}} = \frac{1}{8}\sum_{k=0}^{7} X(k)e^{j\frac{\pi k}{4}}$$

$$x(1) = \frac{1}{8}\begin{bmatrix} X(0) + X(1)e^{j\frac{\pi}{4}} + X(2)e^{j\frac{2\pi}{4}} + X(3)e^{j\frac{3\pi}{4}} + X(4)e^{j\frac{4\pi}{4}} \\ + X(5)e^{j\frac{5\pi}{4}} + X(6)e^{j\frac{6\pi}{4}} + X(7)e^{j\frac{7\pi}{4}} \end{bmatrix}$$

$$\boxed{x(1) = 0.75}$$

- Similarly $x(2) = 0.5$ $x(3) = 0.25$ $x(4) = 1$
  $x(5) = 0.75$ $x(6) = 0.5$ $x(7) = 0.25$
- Final Answer:
- x(n)={1,0.75,0.5,0.25,1,0.75,0.5,0.25}

Find the IDFT of $X(k) = \{4, 2, 0, 4\}$ directly.

Soln:

Given DFT is $X(k) = \{4, 2, 0, 4\}$. The IDFT of $X(k)$, i.e. $x(n)$ is given by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)W_N^{-nk} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j(2\pi/N)nk}$$

i.e. $$x(n) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j(\pi/2)nk}$$

$$\therefore \quad x(0) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{0} = \frac{1}{4}[X(0)+X(1)+X(2)+X(3)]$$

$$= \frac{1}{4}[4+2+0+4] = 2.5$$

$$x(1) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j(\pi/2)k} = \frac{1}{4}\left[X(0)+X(1)e^{j(\pi/2)}+X(2)e^{j\pi}+X(3)e^{j(3\pi/2)}\right]$$

$$= \frac{1}{4}[4+2(0+j)+0+4(0-j)] = 1-j0.5$$

$$x(2) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j\pi k} = \frac{1}{4}\left[X(0)+X(1)e^{j\pi}+X(2)e^{j2\pi}+X(3)e^{j3\pi}\right]$$

$$= \frac{1}{4}[4+2(-1+j0)+0+4(-1+j0)] = -0.5$$

$$x(3) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j(3\pi/2)k} = \frac{1}{4}\left[X(0)+X(1)e^{j(3\pi/2)}+X(2)e^{j3\pi}+X(3)e^{j(9\pi/2)}\right]$$

$$= \frac{1}{4}[4+2(0-j)+0+4(0+j)] = 1+j0.5$$

$$x_3(n) = \{2.5,\ 1-j0.5,\ -0.5,\ 1+j0.5\}$$

Find the IDFT of $X(k) = \{1, 0, 1, 0\}$.

Soln:

Given $X(k) = \{1, 0, 1, 0\}$

Let the IDFT of $X(k)$ be $x(n)$.

$$\therefore \quad x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-nk} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j(2\pi/N)nk}$$

$$x(0) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{0} = \frac{1}{4}[X(0)+X(1)+X(2)+X(3)] = \frac{1}{4}[1+0+1+0] = 0.5$$

$$x(1) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j(\pi/2)k} = \frac{1}{4}\left[X(0)+X(1)e^{j(\pi/2)}+X(2)e^{j\pi}+X(3)e^{j(3\pi/2)}\right]$$

$$= \frac{1}{4}[1+0+e^{j\pi}+0] = \frac{1}{4}[1+0-1+0] = 0$$

$$x(2) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j\pi k} = \frac{1}{4}\left[X(0)+X(1)e^{j\pi}+X(2)e^{j2\pi}+X(3)e^{j3\pi}\right]$$

$$= \frac{1}{4}[1+0+e^{j2\pi}+0] = \frac{1}{4}[1+0+1+0] = 0.5$$

$$x(3) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{j(3\pi/2)k} = \frac{1}{4}\left[X(0)+X(1)e^{j(3\pi/2)}+X(2)e^{j3\pi}+X(3)e^{j(9\pi/2)}\right]$$

$$= \frac{1}{4}[1+0+e^{j3\pi}+0] = \frac{1}{4}[1+0-1+0] = 0$$

The IDFT of $X(k) = \{1, 0, 1, 0\}$ is $x(n) = \{0.5, 0, 0.5, 0\}$.