

Image Enhancement In Frequency Domain

BLURRING/NOISE REDUCTION:

Noise characterized by sharp transitions in image intensity. Such transitions contribute significantly to high frequency components of Fourier transform. Intuitively, attenuating certain high frequency components result in blurring and reduction of image noise.

IDEAL LOW-PASS FILTER:

Cuts off all high-frequency components at a distance greater than a certain distance from origin (cutoff frequency).

$$H(u,v) = 1, \text{ if } D(u,v) \leq D_0$$

$$0, \text{ if } D(u,v) > D_0$$

Where D_0 is a positive constant and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle; that is

$$D(u,v) = [(u-P/2)^2 + (v-Q/2)^2]^{1/2}$$

Where P and Q are the padded sizes from the basic equations

Wraparound error in their circular convolution can be avoided by padding these functions with zeros,

VISUALIZATION: IDEAL LOW PASS FILTER:

As shown in fig. below

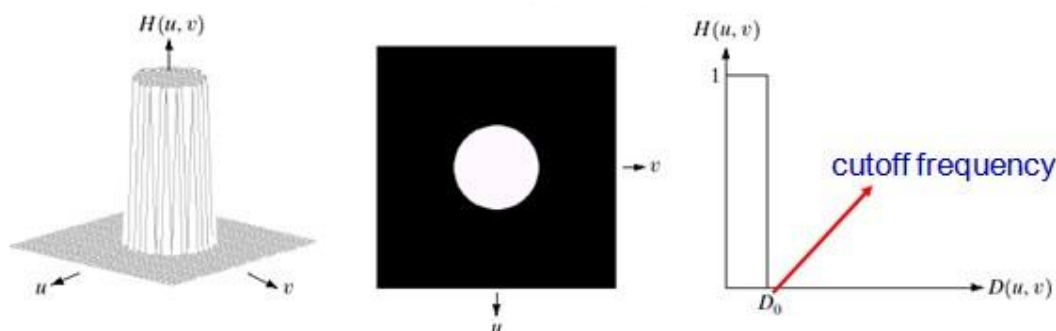


Fig : 2.2.1 ideal low pass filter 3-D view and 2-D view and line graph.

(Source: Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, Pearson, Third Edition, 2010 – Page-270)

EFFECT OF DIFFERENT CUTOFF FREQUENCIES:

Fig. below(a) Test pattern of size 688x688 pixels, and (b) its Fourier spectrum.

The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160 and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8 and 99.2% of the padded image power respectively.

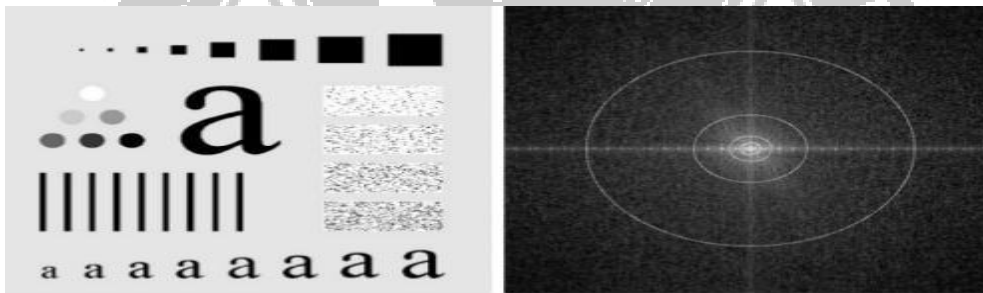


Fig: (a) Test pattern of size 688x688 pixels (b) its Fourier spectrum



Source: Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, Pearson, Third Edition, 2010

Fig: 2.2.2 (a) original image, (b)-(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160 and 460, as shown in fig.2.2.2(b).

The power removed by these filters was 13, 6.9, 4.3, 2.2 and 0.8% of the total, respectively

(Source: Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, Pearson, Third Edition, 2010. Page- 271)

BUTTERWORTH LOW-PASS FILTER:

Transfer function of a Butterworth lowpass filter (BLPF) of order n , and with cutoff frequency at a distance D_0 from the origin, is defined as

$$H(u,v) = \frac{1}{1 + [D(u,v) / D_0]^{2n}}$$

Transfer function does not have sharp discontinuity establishing cutoff between passed and filtered frequencies.

Cut off frequency D_0 defines point at which $H(u,v) = 0.5$

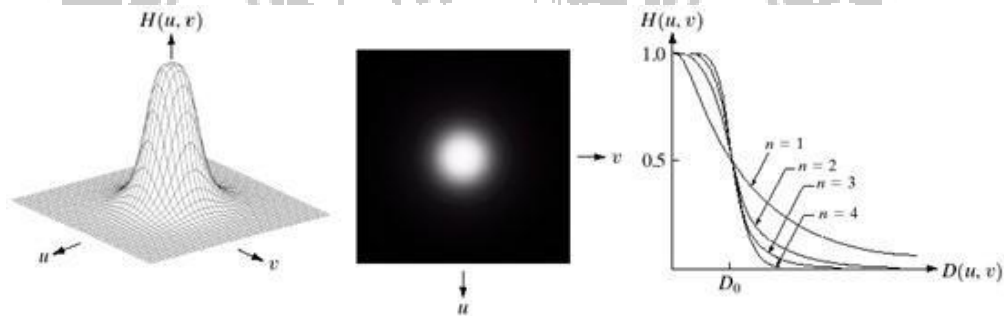


Fig. (a) perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of order 1 through 4.

Unlike the ILPF, the BLPF transfer function does not have a sharp discontinuity that gives a clear cutoff between passed and filtered frequencies

Source: Rafael C. Gonzalez, Richard E. Woods, 'Digital Image Processing', Pearson, Third Edition, 2010-Page-274

BUTTERWORTH LOW-PASS FILTERS OF DIFFERENT FREQUENCIES:



Fig. 2.2.3(a) Original image.(b)-(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii

(Source: Rafael C. Gonzalez, Richard E. Woods, 'Digital Image Processing', Pearson, Third Edition , 2010- Page-275)

Fig. shows the results of applying the BLPF of eq. to fig.(a), with $n=2$ and D_0 equal to the five radii in fig.(b) for the ILPF, we note here a smooth transition in blurring as a function of increasing cutoff frequency. Moreover, no ringing is visible in any of the images processed with this particular BLPF, a fact attributed to the filter's smooth transition between low and high frequencies.

A BLPF of order 1 has no ringing in the spatial domain. Ringing generally is imperceptible in filters of order 2, but can become significant in filters of higher order.

Fig.shows a comparison between the spatial representation of BLPFs of various orders (using a cutoff frequency of 5 in all cases). Shown also is the

intensity profile along a horizontal scan line through the center of each filter. The filter of order 2 does show mild ringing and small negative values, but they certainly are less pronounced than in the ILPF. A butter worth filter of order 20 exhibits characteristics similar to those of the ILPF (in the limit, both filters are identical).

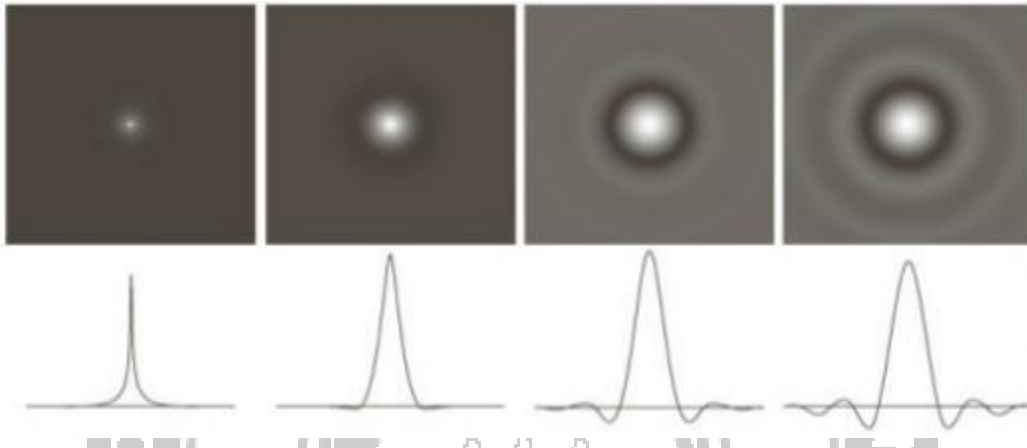


Fig.2.2.4 (a)-(d) Spatial representation of BLPFs of order 1, 2, 5 and 20 and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 x 1000 and the cutoff frequency is 5) Observe how ringing increases as a function of filter order.

Source: Rafael C. Gonzalez, Richard E. Woods, 'Digital Image Processing', Pearson, Third Edition, 2010- Page- 276

GAUSSIAN LOWPASS FILTERS:

$$H(u, v) = e^{-D^2(u, v)/2D_0^2}$$

The form of these filters in two dimensions is given by

- This transfer function is smooth, like Butterworth filter.
- Gaussian in frequency domain remains a Gaussian in spatial domain
- Advantage: No ringing artifacts.

Where D_0 is the cutoff frequency. When $D(u, v) = D_0$, the GLPF is down

to 0.607 of its maximum value. This means that a spatial Gaussian filter, obtained by computing the IDFT of above equation, will have no ringing.

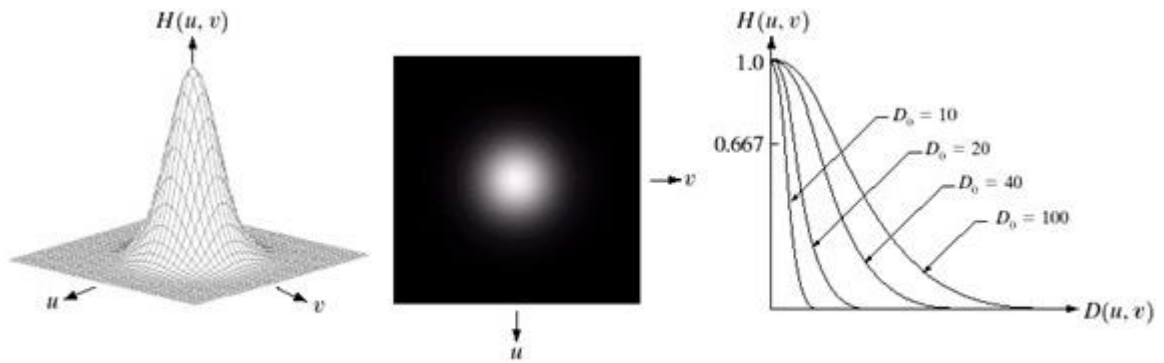


Fig.2.2.5(a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c). Filter radial cross sections for various values of D_0

Source: Rafael C. Gonzalez, Richard E. Woods, 'Digital Image Processing', Pearson, Third Edition, 2010. – Page- 277



Fig.2.2.6 Original image. (b)-(f) Results of filtering using GLPFs with cutoff frequencies at the radii

Source: Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, Pearson, Third Edition , 2010. –Page- 278



Fig. 2.2.7(a) Original image (784x 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and(c).

Fig. shows an application of low pass filtering for producing a smoother, softer- looking result from a sharp original. For human faces, the typical objective is to reduce the sharpness of fine skin lines and small blemishes.

Source: Rafael C. Gonzalez, Richard E. Woods, *Digital Image Processing*, Pearson, Third Edition , 2010. – Page-280