3.5 PRINCIPLES OF LEAST SQUARES

It is found from the probability equation that the most probable values of a series of errors arising from observations of equal weight are those for which the sum of the squares is a minimum. The fundamental law of least squares is derived from this. According to the principle of least squares, the most probable value of an observed quantity available from a given set of observations is the one for which the sum of the squares of the residual errors is a minimum. When a quantity is being deduced from a series of observations, the residual errors will be the difference between the adopted value and the several observed values,

Let *V*1, *V*2, *V*3 etc. be the observed values x = most probable value

LAW OF WEIGHTS

From the method of least squares the following laws of weights are established:

(i) The weight of the arithmetic mean of the measurements of unit weight is equal to the number of observations.

For example, let an angle A be measured six times, the following being the values:

A	Weight	A	Weight
30 º 20' 8"		30 º 20' 10"	
30 º 20' 10"		30 º 20' 09"	
30 º 20' 07"		30 º 20' 10"	

Arithmetic mean= 30 º 20' + 1/6 (8' + 10' + 7' + 10' + 9' + 10')

= 30 º 20' 09"

Weight of arithmetic mean = number of observations = 6.

(ii) The weight of the weighted arithmetic mean is equal to the sum of the individual weights.

For example, let an angle A be measured six times, the following being the values :

A Weight A Weight

30 º 20' 8"	2	30 º 20' 10"	3
30 º 20' 10"	3	30 º 20' 09"	4
30 º 20' 07"	2	30 º 20' 10"	2

Sum of weights = 2 + 3 + 2 + 3 + 4 + 2 =16

Arithmetic mean = 30 º 20' + 1/16 (8'X2 + 10' X3+ 7'X2 + 10'X3 + 9' X4+ 10'X2)

= 30 º 20' 9".

Weight of arithmetic mean = 16.

(iii) The weight of algebric sum of two or more quantities is equal to the reciprocals of the individual weights.

For Example angle A = 30 º 20' 10" , Weight 2

B = 15 º 20' 08" , Weight 3

(iv)If a quantity of given weight is multiplied by a factor, the weight of the result is obtained by dividing its given weight by the square of the factor.

(v)If a quantity of given weight is divided by a factor, the weight of the result is obtained by multiplying its given weight by the square of the factor.

(vi)If a equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of the equation.

(vii)The weight of the equation remains unchanged, if all the signs of the equation are changed or if the equation is added or subtracted from a constant.

DISTRIBUTION OF ERROR OF THE FIELD MEASUREMENT

Whenever observations are made in the field, it is always necessary to check for the closing error, if any. The closing error should be distributed to the observed quantities. For examples, the sum of the angles measured at a central angle should be 360 o , the error should be distributed to the observed angles after giving proper weight age to the observations. The following rules should be applied for the distribution of errors:

(i).The correction to be applied to an observation is inversely proportional to the weight of the observation.

(ii) The correction to be applied to an observation is directly proportional to the square of the probable error.

(iii)In case of line of levels, the correction to be applied is proportional to the length.

PROBLEMS

1.The following are the three angles x,y and z observed at a station P closing the horizon, along with their probable errors of measurement. Determine their corrected values.

Solution.

x = 78 ^{o} 12' 12" 2'

y = 136 ^{o} 48' 30" 4'

z = 144 ^{o} 59' 08" 5'

Sum of the three angles = 359 ^{o} 59' 50" Discrepancy = 10"

Hence each angle is to be increased, and the error of 10'' is to be distributed in proportion to the square of the probable error.

Let c1, c2 and c3 be the correction to be applied to the angles x, y and z respectively.

c1 : c2 : c3 = $(2)^2 : (4)^2 : (5)^2$ = 4 : 16 : 25 -----------(1)

Also, c1 + c2 + c3 = 10' ---------- (2)

From (1), c2 = 16 /4 c1 = 4c1 And

c3 = 25/4 c1

Substituting these values of c2 and c3 in (2), we get

C 1 + 4c1 + 25/4 c1 = 10" or

c1 (1 + 4 + 25/4)= 10"

c1 = 10 x 4/45 = 0'.89

c2 = 4c1 = 3".36

And c3 = 25 /4 c1 = 5".55

Check: c1 + c2 + c3 = 0."89 + 3".56 + 5".55 = 10'"

Hence the corrected angles are

x= 78 º 12' 12" + 0'.89 = 78 º 12' 12".89

y = 136 º 48' 30" + 3".56 = 136 º 48' 33".56 and

z= 144 º 59' 08" + 5".55 = 144 º 59' 13".55

Sum = 360 º 00' 00"+ 00

2.An angle A was measured by different persons and the following are the values

Angle	**Number of measurements**
65 º 30' 10''	**2**
65 º 29' 50''	**3**
65 º 30' 00''	**3**
65 º 30' 20''	**4**
65 º 30' 10''	**3**

Find the most probable value of the angle.

Solution.

As stated earlier, the most probable value of an angle is equal to its weighted arithmetic mean.

65 º 30' 10" x 2 = 131 º 00' 20"

65 º 29' 50" x 3 = 196 º 29' 30"

65 º 30' 00" x 3 = 196 º 30' 00"

65 º 30' 20" x 4 = 262 º 01' 20"

65 º 30' 10" x 3 = 196 º 30' 30"

Sum = 982 º 31' 40" weight = 2 + 3 + 3 + 4 + 3 = 15

Weighted arithmetic mean

= 982 º 31' 40"/15 = 65 º 30' 6".67

Hence most probable value of the angle = 65 º 30' 6".67