UNIT– II BALANCING

2.1 INTRODUCTION:

Balancing is the process of eliminating or at least reducing the ground forces and/or moments. It is achieved by changing the location of the mass centres of links. Balancing of rotating parts is a well known problem. A rotating body with fixed rotation axis can be fully balanced i.e. all the inertia forces and moments. For mechanism containing links rotating about axis which are not fixed, force balancing is possible, moment balancing by itself may be possible, but both not possible. We generally try to do force balancing. A fully force balance is possible, but any action in force balancing severe the moment balancing.

2.2 BALANCING OF ROTATING MASSES:

The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

2.2.1 Static balancing:

The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of the rotation. This is the condition for static balancing.

2.2.2 Dynamic balancing:

The net couple due to dynamic forces acting on the shaft is equal to zero. The algebraic sum of the moments about any point in the plane must be zero.

2.2.3 Various cases of balancing of rotating masses:

- Balancing of a single rotating mass by single mass rotating in the same plane.
- Balancing of a single rotating mass by two masses rotating in the different plane.
- Balancing of a several masses rotating in single plane.
- Balancing of a several masses rotating in different planes.

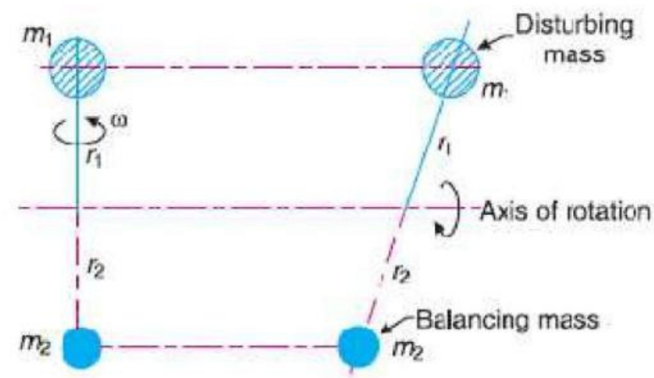
2.3 BALANCING OF A SINGLE ROTATING MASS BY SINGLE MASS ROTATING IN THE SAME PLANE:

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \qquad \ldots (i)$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.



Balancing of a single rotating mass by a single mass rotating in the same plane.

Let r_2 – Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

$\therefore$ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 . \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Notes : 1. The product $m_2.r_2$ may be split up in any convenient way. But the radius of rotation of the balancing mass (m_2) is generally made large in order to reduce the balancing mass m_2.

2. The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

2.4 BALANCING OF A SINGLE ROTATING MASS BY TWO MASSES ROTATING IN THE DIFFERENT PLANE:

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing***.
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1)** and **(2)** together give ***dynamic balancing***. The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

We shall now discuss both the above cases one by one.

Let l_1 = Distance between the planes *A* and *L*,

l_2 = Distance between the planes *A* and *M*, and

l = Distance between the planes *L* and *M*.

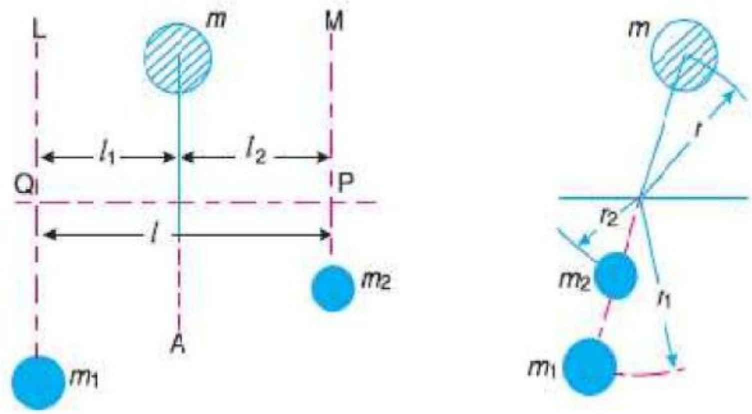


Fig. 21.2. Balancing of a single rotating mass by two rotating masses in different planes when the plane of single rotating mass lies in between the planes of two balancing masses.

We know that the centrifugal force exerted by the mass *m* in the plane *A*,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane *L*,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane *M*,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \qquad \text{or} \qquad m \cdot \omega^2 \cdot r = m_1 \, \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \qquad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \qquad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \qquad \text{or} \qquad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \qquad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \qquad \text{or} \qquad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \qquad \ldots (ii)$$

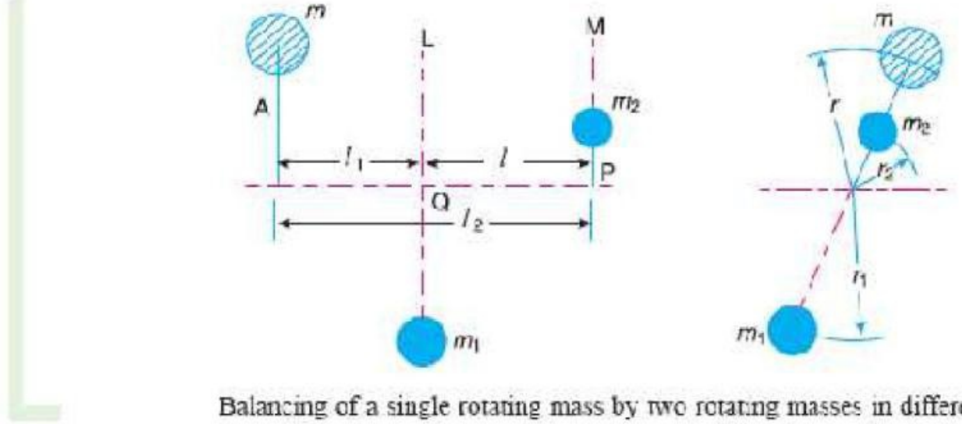
Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \qquad \text{or} \qquad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\therefore \qquad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \qquad \text{or} \qquad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \qquad (iii)$$

It may be noted that equation (*i*) represents the condition for static balance, but in order to achieve dynamic balance, equations (*ii*) or (*iii*) must also be satisfied.

2. *When the plane of the disturbing mass lies on one end of the planes of the balancing masses*



Balancing of a single rotating mass by two rotating masses in different planes, when the plane of single rotating mass lies at one end of the planes of balancing masses

In this case, the mass m lies in the plane A and the balancing masses lie in the planes L and M, as shown in Fig. 21.3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1} \qquad \text{or} \qquad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

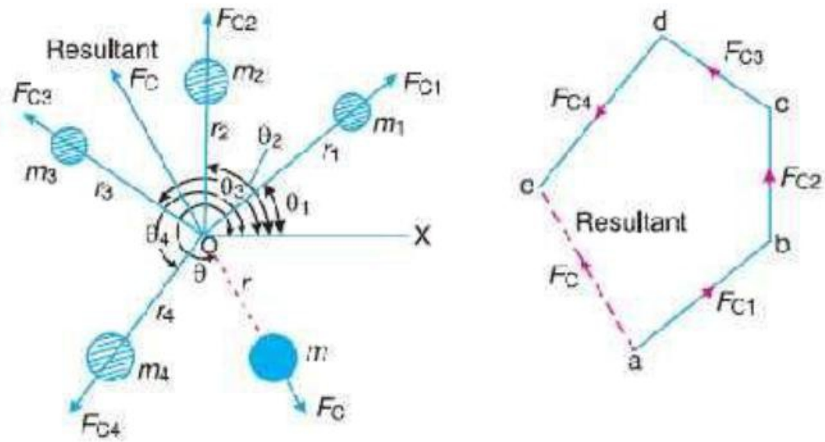
$$\therefore \qquad m \cdot r + m_2 \cdot r_2 = m_1 . r_1 \qquad \ldots (iv)$$

Now, to find the balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

2.5 BALANCING OF A SEVERAL MASSES ROTATING IN SAME PLANE:

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. . Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



(*a*) Space diagram. (*b*) Vector diagram.

Fig. 21.4. Balancing of several masses rotating in the same plane.

1. *Analytical method*

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV. We know that

   Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in *opposite direction*.
6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and

r = Its radius of rotation.

2. *Graphical method*

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig.
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).
4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig.
5. The balancing force is, then, equal to the resultant force, but in *opposite direction.*
6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

or

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

2.6 BALANCING OF SEVERAL MASSES ROTATING DIFFERENT PLANE:

When several masses revolve in different planes, they may be transferred to a ***reference plane*** (briefly written as ***R.P.***), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.
2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.

Let us now consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in

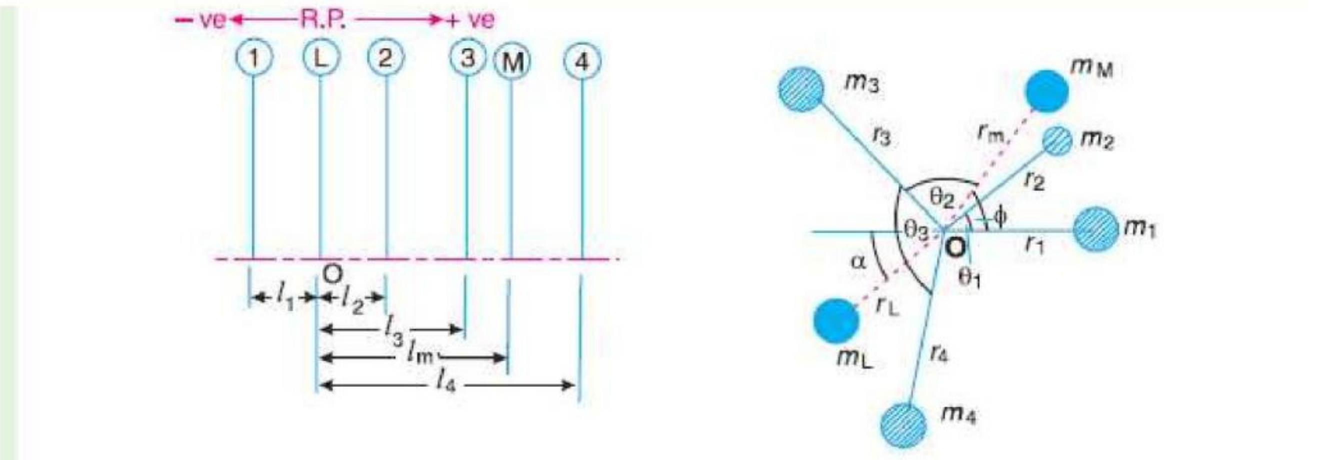
Diesel engine.

Fig. (*a*). The relative angular positions of these masses are shown in the end view [Fig. 21.7 (*b*)]. The magnitude of the balancing masses m_L and m_M in planes *L* and *M* may be obtained as discussed below :

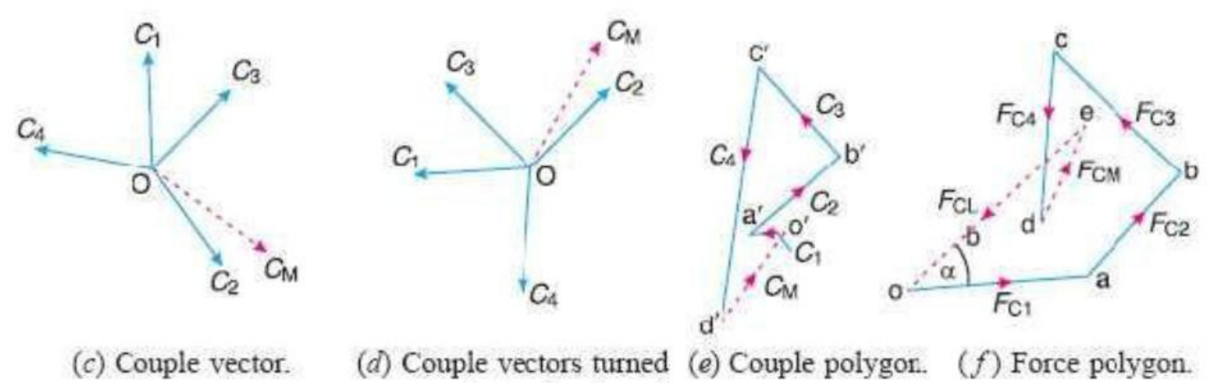
1. Take one of the planes, say *L* as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative***, and those to the right as ***positive***.
2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Table 21.1

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.



(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

Fig. Balancing of several masses rotating in different planes.

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is propor-

tional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. (*c*). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (*d*). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. (*e*). The vector $d'o'$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

From this expression, the value of the balancing mass m_M in the plane *M* may be obtained, and the angle of inclination ϕ of this mass may be measured from Fig. 21.7 (*b*).

6. Now draw the force polygon as shown in Fig. (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

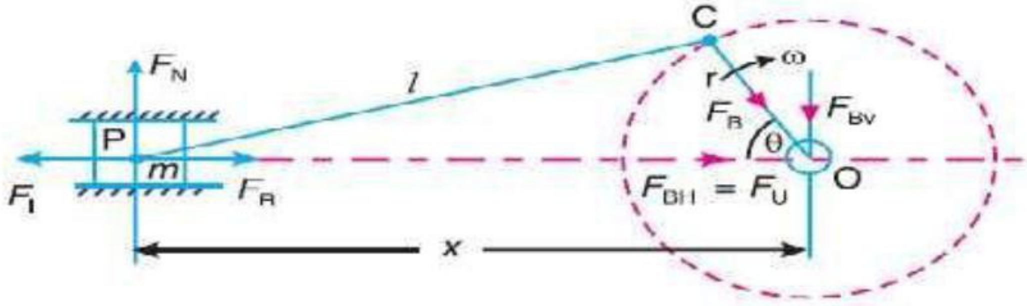
$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

From this expression, the value of the balancing mass m_L in the plane *L* may be obtained the angle of inclination α of this mass with the horizontal may be measured from Fig. (*b*).

2.7 BALANCING OF RECIPROCATING MASSES:

Mass balancing encompasses a wide array of measures employed to obtain partial or complete compensation for the inertial forces and moments of inertia emanating from the crankshaft assembly. All masses are externally balanced when no free inertial forces or moments of inertia are transmitted through the block to the outside. However, the remaining internal forces and moments subject the engine mounts and block to various loads as well as deformities and vibratory stresses. The basic loads imposed by gas-based and inertial forces

2.7.1 Primary and secondary unbalanced forces of reciprocating parts:



Reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts,

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Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC,

r = Radius of the crank OC,

θ = Angle of inclination of the crank with the line of stroke PO,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

We have already discussed in Art. 15.8 that the acceleration of the reciprocating parts is approximately given by the expression,

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

∴ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

We have discussed in the previous article that the horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

∴ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as ***primary unbalanced force*** and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called ***secondary unbalanced force***.

2.8 BALANCING OF SINGLE CYLINDER ENGINE:

A single cylinder engine produces three main vibrations. In describing them we will assume that the cylinder is vertical. Firstly, in an engine with no balancing counterweights, there would be an enormous vibration produced by the change in momentum of the piston, gudgeon pin, connecting rod and crankshaft once every revolution. Nearly all single-cylinder crankshafts incorporate balancing weights to reduce this. While these weights can balance the crankshaft completely, they cannot completely balance the motion of the piston, for two reasons. The first reason is that the balancing weights have horizontal motion as well as vertical motion, so balancing the purely vertical motion of the piston by a crankshaft weight adds a horizontal vibration. The second reason is that, considering now the vertical motion only, the smaller piston end of the connecting rod (little end) is closer to the larger crankshaft end (big end) of the connecting rod in mid-stroke than it is at the top or bottom of the stroke, because of the connecting rod's angle. So during the 180° rotation from mid-stroke through top-dead-center and back to mid-stroke the minor contribution to the piston's up/down movement from the connecting rod's change of angle has the same direction as the major contribution to the piston's up/down movement from the up/down movement of the crank pin. By contrast, during the 180° rotation from mid-stroke through bottom-dead-center and back to mid-stroke the minor contribution to the piston's up/down movement from the connecting rod's change of angle has the opposite direction of the major contribution to the piston's up/down movement from the up/down movement of the crank pin. The piston therefore travels faster in the top half of the cylinder than it does in the bottom half, while the motion of the crankshaft weights is sinusoidal. The vertical motion of the piston is therefore not quite the same as that of the balancing weight, so they can't be made to cancel out completely.

Secondly, there is a vibration produced by the change in speed and therefore kinetic energy of the piston. The crankshaft will tend to slow down as the piston speeds up and absorbs energy, and to speed up again as the piston gives up energy in slowing down at the top and bottom of the stroke. This vibration has twice the frequency of the first vibration, and absorbing it is one function of the flywheel.

Thirdly, there is a vibration produced by the fact that the engine is only producing power during the power stroke. In a four-stroke engine this vibration will have half the frequency of the first vibration, as the cylinder fires once every two revolutions. In a two-stroke engine, it will have the same frequency as the first vibration. This vibration is also absorbed by the flywheel.

2.9 BALANCING OF INERTIAL FORCES IN THE MULTI-CYLINDER ENGINE:

In multi-cylinder engines the mutual counteractions of the various components in the Crank shaft assembly are one of the essential factors determining the selection of the Crank shafts configuration and with it the design of the engine itself. The inertial forces are Balanced if the common centre of gravity for all moving crankshaft-assembly components lies at the crankshaft's midpoint, i.e. if the crankshaft is symmetrical (as viewed from the front). The crankshaft's symmetry level can be defined using geometrical representations of 1st- and 2ndorder forces (star diagrams). The 2nd order star diagram for the four-cylinder in-line engine is asymmetrical, meaning that this order is characterized by substantial free inertial Forces. These forces can be balanced using two countershafts rotating in opposite directions at double the rate of the crankshaft (Lanchester system).

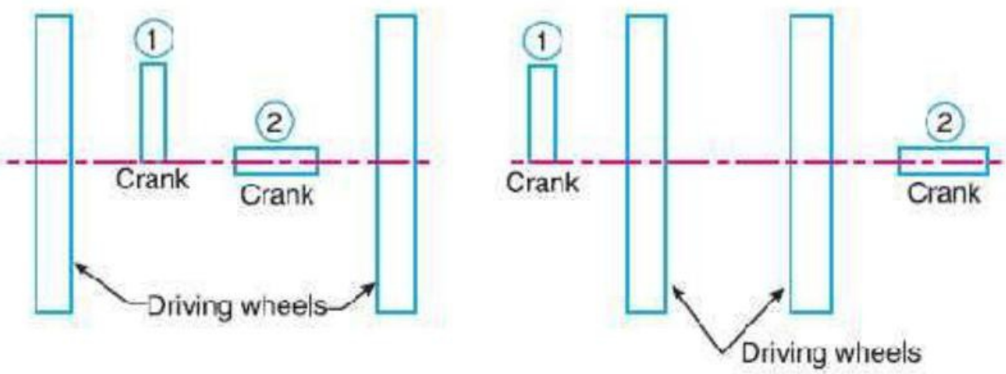
2.10 PARTIAL BALANCING OF LOCOMOTIVES:

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives may be classified as :

1. Inside cylinder locomotives ; and 2. Outside cylinder locomotives.

In the ***inside cylinder locomotives***, the two cylinders are placed in between the planes of two driving wheels as shown in Fig. (*a*) ; whereas in the ***outside cylinder locomotives***, the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in Fig. (*b*). The locomotives may be

(*a*) Single or uncoupled locomotives ; and (*b*) Coupled locomotives.



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

2.10.1 Variation of Tractive force:

The resultant unbalanced force due to the cylinders, along the line of stroke, is known as tractive force.

2.10.2 Swaying Couple:

The couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clock wise and anticlockwise directions. Hence the couple is known as swaying couple.

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line *YY* between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.***

Let a = Distance between the centre lines of the two cylinders.

∴ Swaying couple

$$= (1-c)m.\omega^2.r\cos\theta\times\frac{a}{2} - (1-c)m.\omega^2.r\cos(90^\circ+\theta)\frac{a}{2}$$

$$= (1-c)m\,\omega^2.r\times\frac{a}{2}(\cos\theta+\sin\theta)$$

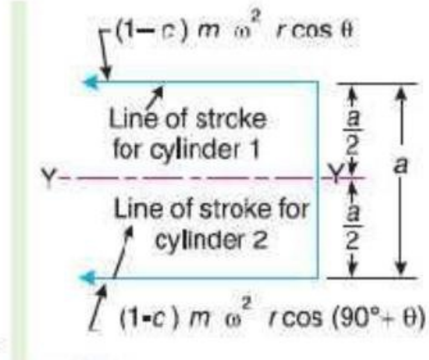


Fig. Swaying couple.

The swaying couple is maximum or minimum when $(\cos\theta+\sin\theta)$ is maximum or minimum. For $(\cos\theta+\sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta+\sin\theta)=0 \quad\text{or}\quad -\sin\theta+\cos\theta=0 \quad\text{or}\quad -\sin\theta=-\cos\theta$$

$$\therefore \quad \tan\theta=1 \quad\text{or}\quad \theta=45^\circ \quad\text{or}\quad 225^\circ$$

Thus, the swaying couple is maximum or minimum when $\theta = 45^\circ$ or 225°.

∴ Maximum and minimum value of the swaying couple

$$= \pm(1-c)m.\omega^2.r\times\frac{a}{2}(\cos 45^\circ+\sin 45^\circ) = \pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

2.10.3 Hammer blow:

The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as Hammer blow.

We have already discussed that the maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as ***hammer blow***.

We know that the unbalanced force along the perpendicular to the line of stroke due to the balancing mass B, at a radius b, in order to balance reciprocating parts only is $B.\omega^2.b \sin\theta$. This force will be maximum when $\sin\theta$ is unity, *i.e.* when $\theta = 90°$ or $270°$.

$\therefore$ Hammer blow $= B.\omega^2.b$ (Substituting $\sin\theta = 1$)

The effect of hammer blow is to cause the variation in pressure between the wheel and the rail. This variation is shown in Fig. 22.6, for one revolution of the wheel.

Let P be the downward pressure on the rails (or static wheel load).

$\therefore$ Net pressure between the wheel and the rail

$$= P \pm B.\omega^2.b$$

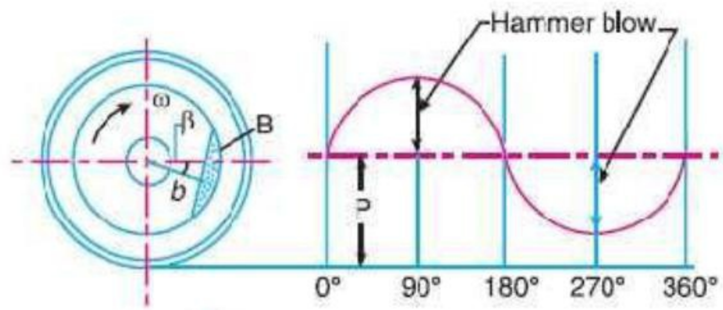


Fig. Hammer blow.

If $(P - B.\omega^2.b)$ is ***negative***, then the wheel will be lifted from the rails. Therefore the limiting condition in order that the wheel does not lift from the rails is given by

$$P = B.\omega^2.b$$

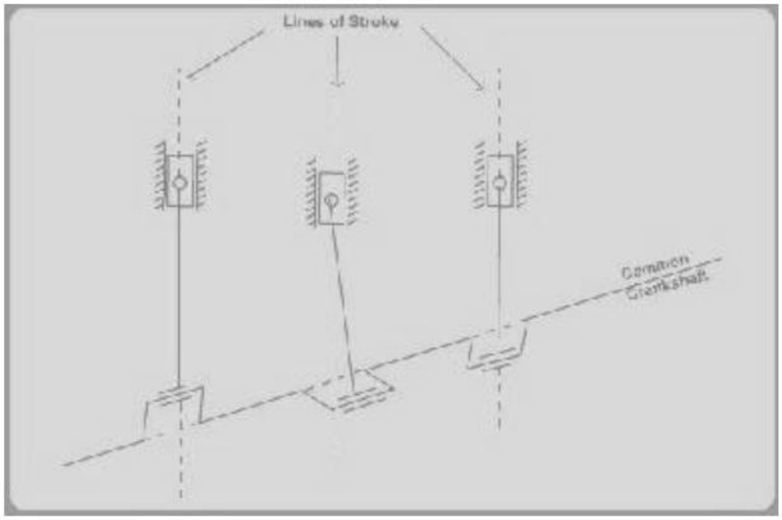
and the permissible value of the angular speed,

$$\omega = \sqrt{\frac{P}{B.b}}$$

2.11 BALANCING OF INLINE ENGINES:

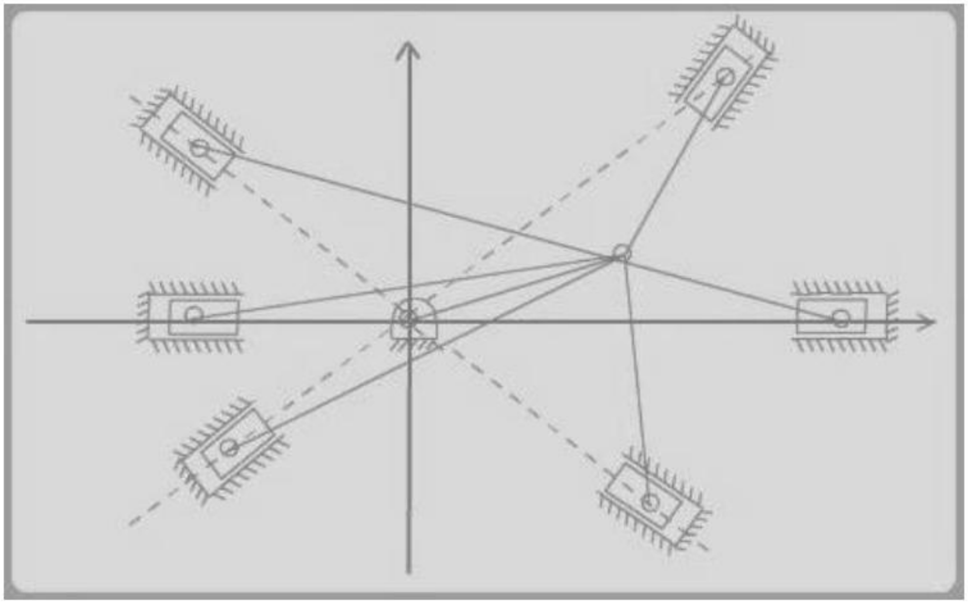
An in-line engine is one wherein all the cylinders are arranged in a single line, one behind the other as schematically indicated in Fig. Many of the passenger cars found on Indian roads such as Maruti 800, Zen, Santro, Honda City, Honda CR-V, and Toyota Corolla all have four cylinder in-line engines. Thus this is a commonly employed engine and it is of interest to us to understand the analysis of its state of balance.

For the sake of simplicity of analysis, we assume that all the cylinders are identical viz., r, ℓ, and m_{rec} m_{rot} are same. Further we assume that the rotating masses have been balanced out for all cylinders and we are left with only the forces due to the reciprocating masses.



2.12 BALANCING OF RADIAL ENGINES:

A radial engine is one in which all the cylinders are arranged circumferentially as shown in Fig.These engines were quite popularly used in aircrafts during World War II. Subsequent developments in steam/gas turbines led to the near extinction of these engines. However it is still interesting to study their state of balance in view of some elegant results we shall discuss shortly. Our method of analysis remains identical to the previous case i.e., we proceed with the assumption that all cylinders are identical and the cylinders are spaced at uniform interval $\left(\frac{2\pi}{n}\right)$ around the circumference.



2.13 SOLVED PROBLEMS

1. A shaft has three eccentrics, each 75 mm diameter and 25 mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60 mm apart. The distance of the centres from the axis of rotation are 12 mm, 18 mm and 12 mm and their angular positions are 120° apart. The density of metal is 7000 kg/m³. Find the amount of out-of-balance force and couple at 600 r.p.m. If the shaft is balanced by adding two masses at a radius 75 mm and at distances of 100 mm from the central plane of the middle eccentric, find the amount of the masses and their angular positions. (AU-MAY/JUNE-2013)

Solution. Given : $D = 75$ mm $= 0.075$ m ; $t = 25$ mm $= 0.025$ m ; $r_A = 12$ mm $= 0.012$ m ; $r_B = 18$ mm $= 0.018$ m ; $r_C = 12$ mm $= 0.012$ mm ; $\rho = 7000$ kg/m³ ; $N = 600$ r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad/s ; $r_L = r_M = 75$ mm $= 0.075$ m

We know that mass of each eccentric,

$$m_A = m_B = m_C = \text{Volume} \times \text{Density} = \frac{\pi}{4} \times D^2 \times t \times \rho$$

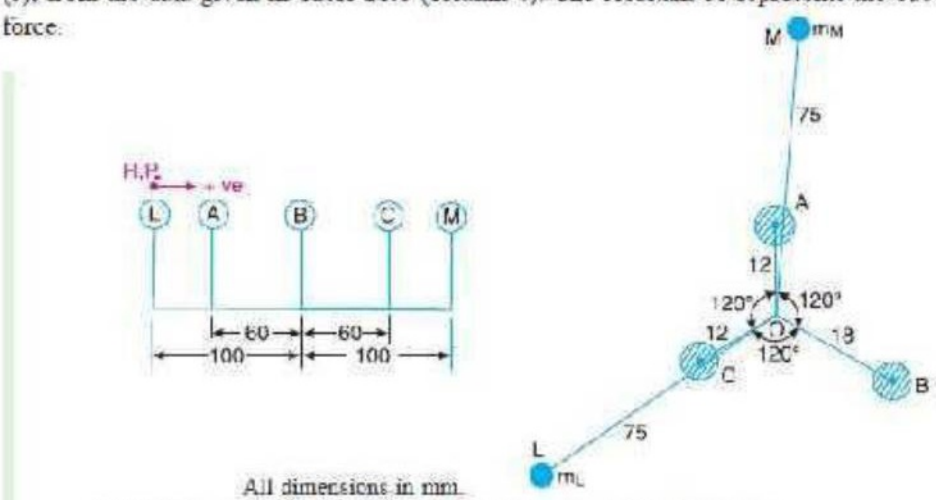
$$= \frac{\pi}{4}(0.075)^2(0.025)7000 = 0.77 \text{ kg}$$

Let L and M be the planes at distances of 100 mm from the central plane of middle eccentric. The position of the planes and the angular position of the three eccentrics is shown in Fig. 21.12 (*a*) and (*b*) respectively. Assuming L as the reference plane and mass of the eccentric A in the vertical direction, the data may be tabulated as below :

Plane (1)	*Mass* (m) kg (2)	*Radius* (r) m (3)	*Cent. force* ÷ ω^2 (m.r) kg-m (4)	*Distance from plane L.*(l) m (5)	*Couple* ÷ ω^2 (m.r.l) kg-m² (6)
L (R.P.)	m_L	0.075	$75 \times 10^{-3}\, m_L$	0	0
A	0.77	0.012	9.24×10^{-3}	0.04	0.3696×10^{-3}
B	0.77	0.018	13.86×10^{-3}	0.1	1.386×10^{-3}
C	0.77	0.012	9.24×10^{-3}	0.16	1.4784×10^{-3}
M	m_M	0.075	$75 \times 10^{-3}\, m_M$	0.20	$15 \times 10^{-3}\, m_M$

Out-of-balance force

The out-of-balance force is obtained by drawing the force polygon, as shown in Fig. 21.12 (*c*), from the data given in Table 21.6 (column 4). The resultant *oc* represents the out-of-balance force.

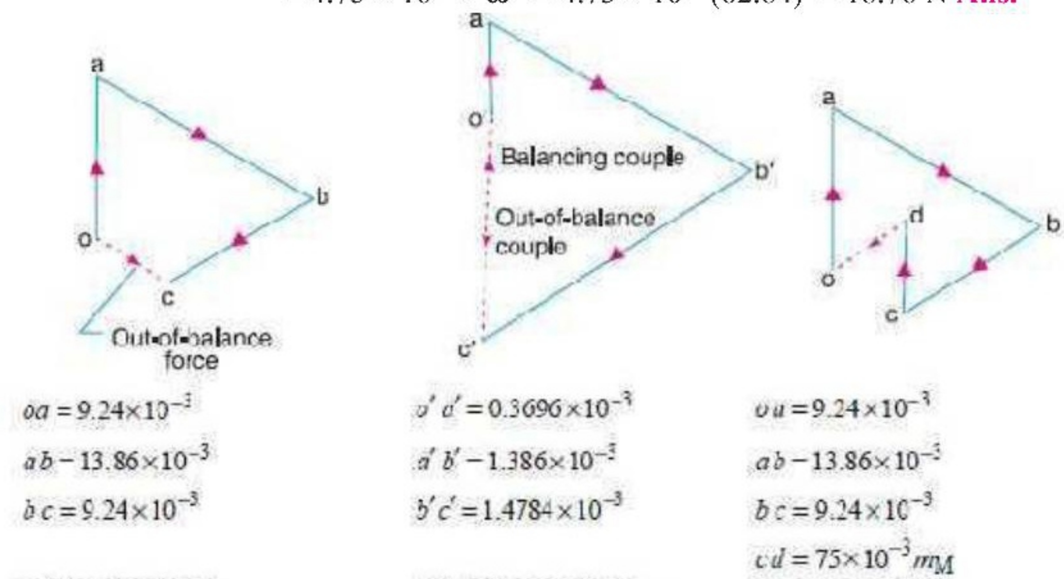


(*a*) Position of planes (*b*) Angular position of masses

Since the centrifugal force is proportional to the product of mass and radius (*i.e.* *m.r*), therefore by measurement.

Out-of-balance force = vector $oc = 4.75 \times 10^{-3}$ kg-m

$$= 4.75 \times 10^{-3} \times \omega^2 = 4.75 \times 10^{-3} (62.84)^2 = 18.76 \text{ N}$$ **Ans.**



(*c*) Force polygon. (*d*) Couple polygon. (*e*) Force polygon.

Fig. 21.12

Out-of-balance couple

The out-of-balance couple is obtained by drawing the couple polygon from the data given in Table 21.6 (column 6), as shown in Fig. 21.12 (d). The resultant o ' c' represents the out-ofbalance couple. Since the couple is proportional to the product of force and distance (m.r.l), there-fore by measurement,

Out-of-balance couple = vector $o'c' = 1.1 \times 10^{-3}$ kg-m^2

$$= 1.1 \times 10^{-3} \times \omega^2 = 1.1 \times 10^{-3} (62.84)^2 = 4.34 \text{ N-m}$$ **Ans.**

Amount of balancing masses and their angular positions

The vector $c'o'$ (in the direction from c' to o'), as shown in Fig. 21.12 (*d*) represents the balancing couple and is proportional to $15 \times 10^{-3}\, m_M$, *i.e.*

$15 \times 10^{-3}\, m_M$ = vector $c'o'$ = 1.1×10^{-3} kg-m^2 or $m_M = 0.073$ kg **Ans.**

Draw *OM* in Fig. 21.12 (*b*) parallel to vector $c'o'$. By measurement, we find that the angular position of balancing mass (m_M) is 5° from mass *A* in the clockwise direction. **Ans.**

In order to find the balancing mass (m_L), a force polygon as shown in Fig. 21.12 (*e*) is drawn. The closing side of the polygon *i.e.* vector *do* (in the direction from *d* to *o*) represents the balancing force and is proportional to $75 \times 10^{-3}\, m_L$. By measurement, we find that,

$75 \times 10^{-3}\, m_L$ = vector $do = 5.2 \times 10^{-3}$ kg-m or $m_L = 0.0693$ kg **Ans.**

Draw *OL* in Fig. 21.12 (*b*), parallel to vector *do*.

By measurement, we find that the angular position of mass (m_L) is 124° from mass *A* in the clockwise direction. **Ans.**

2.(i) A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively.Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

(AU-NOV/DEC-2012) ***(8)***

Solution. Given : r_A = 100 mm = 0.1 m ; r_B = 125 mm = 0.125 m ; r_C = 200 mm = 0.2 m ; r_D = 150 mm = 0.15 m ; m_B = 10 kg ; m_C = 5 kg ; m_D = 4 kg

1. The position of planes is shown in Fig. 21.10 (*a*). Assuming the plane of mass *A* as the reference plane (*R.P.*), the data may be tabulated as below :

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent. Force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from plane A (l)* m (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m² (6)
A(R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

First of all, the angular setting of masses *C* and *D* is obtained by drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass *B* in the horizontal direction *OB* as shown in Fig. 21.10 (*b*). Now the couple polygon as shown in Fig.(*c*) is drawn as discussed below :

1. Draw vector *o′ b′* in the horizontal direction (*i.e.* parallel to *OB*) and equal to 0.75 kgm², to some suitable scale.

2. From points *o′* and *b′*, draw vectors *o′ c′* and *b′ c′* equal to 1.2 kg-m² and 1.08 kg-m² respectively. These vectors intersect at *c′*.

3. Now in Fig. 21.10 (*b*), draw *OC* parallel to vector *o′ c′* and *OD* parallel to vector *b′ c′*. By measurement, we find that the angular setting of mass *C* from mass *B* in the anticlockwise direction, *i.e.* ∠ *BOC* = 240° **Ans.**

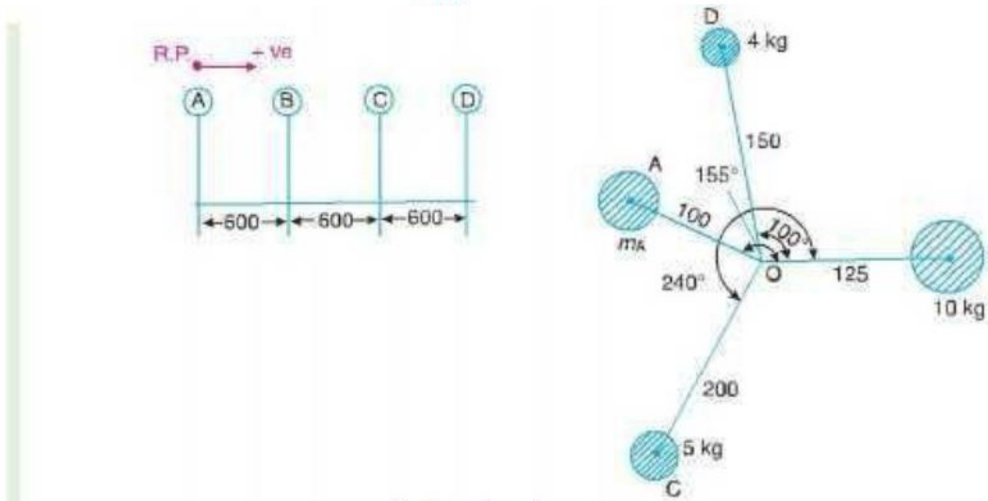
and angular setting of mass *D* from mass *B* in the anticlockwise direction, *i.e.* ∠ *BOD* = 100° **Ans.**

In order to find the required mass *A* (m_A) and its angular setting, draw the force polygon to some suitable scale, as shown in Fig. 21.10 (*d*), from the data given in Table 21.4 (column 4). Since the closing side of the force polygon (vector *do*) is proportional to 0.1 m_A, therefore by measurement,

$0.1\, m = 0.7$ kg-m² or mA = 7 kg **Ans.**

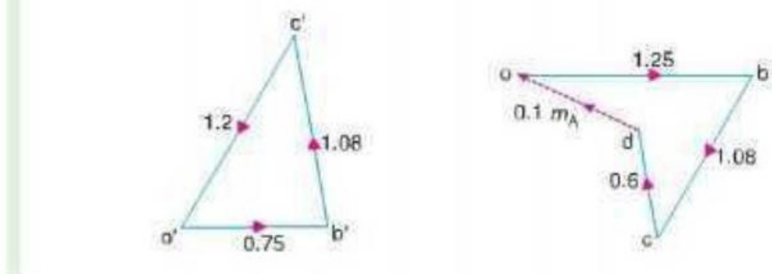
Now draw OA in Fig. 21.10 (b), parallel to vector do. By measurement, we find that the angular setting of mass A from mass B in the anticlockwise direction, *i.e.*

$$\angle\ BOA = 155° \textbf{ Ans.}$$



All dimensions in mm

(a) Position of planes. (b) Angular position of masses.



(c) Couple polygon. (d) Force polygon.

Fig. 21.10

2(ii) Derives the expressions for the following: (i) Variation in tractive force and (ii) Swaying couple. (8) (AU-NOV/DEC-2009)

Variation in tractive force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as ***tractive force***. Let the crank for the first cylinder be inclined at an angle θ with the line of stroke, as shown in Fig. 22.4.

Since the crank for the second cylinder is at right angle to the first crank, therefore the angle of inclination for the second crank will be (90° + θ).

Let m = Mass of the reciprocating parts per cylinder, and

c = Fraction of the reciprocating parts to be balanced.

$$= (1-c)m\omega^2.r\cos\theta$$

Similarly, unbalanced force along the line of stroke for cylinder 2,

$$= (1-c)m.\omega^2 \cdot r\cos(90° + \theta)$$

∴ As per definition, the tractive force,

F_1 = Resultant unbalanced force along the line of stroke

$$= (1-c)m\omega^2.r\cos\theta + (1-c)m\omega^2.r\cos(90° + \theta)$$

$$= (1-c)m\omega^2.r(\cos\theta - \sin\theta)$$

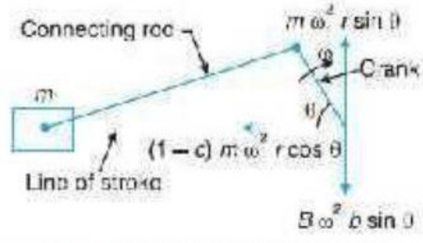


Fig. 22.4 Variation of tractive force.

We know that unbalanced force along the line of stroke for cylinder 1

The tractive force is maximum or minimum when (cos θ − sin θ) is maximum or minimum. For (cos θ − sin θ) to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0 \quad \text{or} \quad -\sin\theta - \cos\theta = 0 \quad \text{or} \quad -\sin\theta = \cos\theta$$

$$\therefore \quad \tan\theta = -1 \quad \text{or} \quad \theta = 135° \quad \text{or} \quad 315°$$

Thus, the tractive force is maximum or minimum when θ = 135° or 315°.

∴ Maximum and minimum value of the tractive force or the variation in tractive force

$$= \pm(1-c)\,m.\omega^2.r(\cos 135° - \sin 135°) = \pm\sqrt{2}\,(1-c)\,m.\omega^2.r$$

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line *YY* between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.*** *a* = Distance between the centre lines of the two cylinders.

Let a = Distance between the centre lines of the two cylinders.

∴ Swaying couple

$$= (1-c)m\omega^2.r\cos\theta \times \frac{a}{2} - (1-c)m\omega^2.r\cos(90° + \theta)\frac{a}{2}$$

$$= (1-c)m.\omega^2.r \times \frac{a}{2}(\cos\theta + \sin\theta)$$

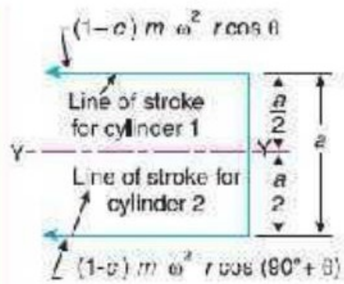


Fig. 22.5. Swaying couple.

The swaying couple is maximum or minimum when $(\cos\theta + \sin\theta)$ is maximum or minimum. For $(\cos\theta + \sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0 \quad \text{or} \quad \sin\theta + \cos\theta = 0 \quad \text{or} \quad -\sin\theta = -\cos\theta$$

$$\therefore \quad \tan\theta = 1 \quad \text{or} \quad \theta = 45° \quad \text{or} \quad 225°$$

Thus, the swaying couple is maximum or minimum when θ = 45° or 225°.

∴ Maximum and minimum value of the swaying couple

$$= \pm(1-c)m\omega^2.r \times \frac{a}{2}(\cos 45° + \sin 45°) = \pm\frac{a}{\sqrt{2}}(1-c)m\omega^2.r$$

3. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A

at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm,
find their magnitudes and angular positions. (AU-MAY/JUNE-2012)

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm =0.08m ;
$r_B = 70$ mm = 0.07 m ; $r_C = 60$ mm = 0.06 m ; $r_D = 80$ mm = 0.08 m ; $r_X = r_Y = 100$ mm = 0.1 m

Let m_X = Balancing mass placed in plane *X*, and m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (*a*) and (*b*) respectively.

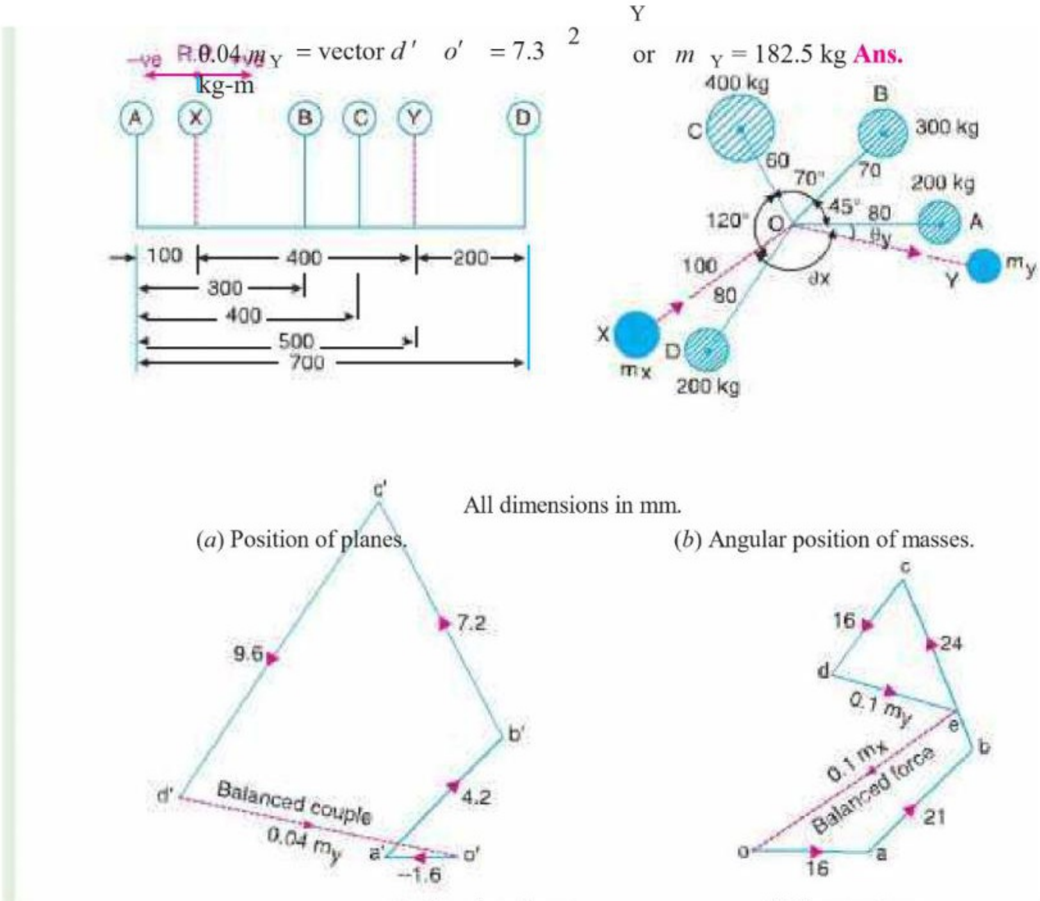
Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve. The data may be tabulated as shown in Table 21.2.

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent.force* ÷ ω^2 *(m.r)* kg-m (4)	*Distance from Plane x(l)* m (5)	*Couple* ÷ ω^2 *(m.r.l)* kg-m² (6)
A	200	0.08	16	– 0.1	– 1.6
X(R.P.)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

The balancing masses m_X and m_Y and their angular positions may be determined graphi-cally as discussed below :

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector *d 'o'* represents the balanced couple. Since the

$0.04\, m_Y$ = vector $d'\, o'$ = 7.3 kg-m^2 or m_Y = 182.5 kg **Ans.**



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon

balanced couple is proportional to 0.04 *m* , therefore by measurement,

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 21.8 (*b*), parallel to vector d' o'. By measurement, the angular position of m_Y is θ_Y □ 12□ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans**.

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (*d*). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

0.1 m_X □ vector *eo* □ 35.5 kg-m or m_X = **355 kg Ans.**

The angular position of the mass m_X is obtained by drawing Om_X in Fig. 21.8 (*b*), parallel to vector *eo*. By measurement, the angular position of m_X is θ_X □ 145□ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans**.

4. Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	**—**	**30**	**50**	**40**
Radius (mm)	**180**	**240**	**120**	**150**

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. **The magnitude and the angular position of mass A ; and**
2. **The position of planes A and D. (AU-NOV/DEC-2011)**

Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ;
m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ;
$\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$

1. ***The magnitude and the angular position of mass A***

Let m_A = Magnitude of Mass *A*,

x = Distance between the planes *B* and *D,* and y = Distance between the planes *A* and *B*. The position of the planes and the angular position of the masses is shown in Fig. 21.9 (*a*) and (*b*) respectively.

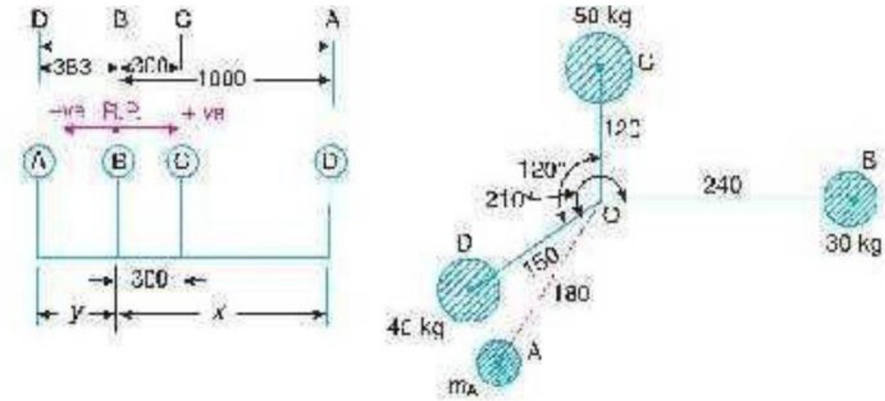
Assuming the plane *B* as the reference plane (*R.P.*) and the mass *B* (m_B) along the horizon-tal line as shown in Fig. 21.9 (*b*), the data may be tabulated as below :

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent.force ÷ ω^2 (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	m_A	0.18	0.08 m_A	– y	– 0.18 $m_A y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

The magnitude and angular position of mass *A* may be determined by drawing the force polygon from the data given in Table 21.3 (Column 4), as shown in Fig. 21.9 (*c*), to some suitable scale. Since the masses are to be completely balanced, therefore the force polygon must be a closed figure. The closing side (*i.e.* vector *do*) is proportional to 0.18 m_A. By measurement,

$$0.18\ m_A = \text{Vector } do = 3.6 \text{ kg-m} \quad \text{or} \quad m_A = 20 \text{ kg} \textbf{ Ans.}$$

In order to find the angular position of mass *A*, draw *OA* in Fig. 21.9 (*b*) parallel to vector *do*. By measurement, we find that the angular position of mass *A* from mass *B* in the anticlockwise direction is $\angle AOB$ = **236° Ans.**



(*c*) Force polygon. (*d*) Couple polygon.

Fig. 21.9.

2. *Position of planes A and D*

The position of planes *A* and *D* may be obtained by drawing the couple polygon, as shown in Fig. 21.9 (*d*), from the data given in Table 21.3 (column 6). The couple polygon is drawn as discussed below :

1. Draw vector $o'c'$ parallel to *OC* and equal to 1.8 kg-m^2, to some suitable scale.

2. From points c' and o', draw lines parallel to *OD* and *OA* respectively, such that they intersect at point d'. By measurement, we find that

$$6\,x = \text{vector } c'd' = 2.3 \text{ kg-m}^2 \text{ or } x = 0.383 \text{ m}$$

We see from the couple polygon that the direction of vector $c'd'$ is opposite to the direction of mass *D*. Therefore the plane of mass *D* is 0.383 m or 383 mm towards left of plane *B* and not towards right of plane *B* as already assumed. **Ans.** Again by measurement from couple polygon,

$$-\,0.18\, m_A . y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

$$-\,0.18 \times 20\, y = 3.6 \qquad \text{or } y = -\,1 \text{ m}$$

The negative sign indicates that the plane *A* is not towards left of *B* as assumed but it is 1 m or 1000 mm towards right of plane *B*. **Ans.**

5. An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.

The whole of the rotating and 2/3 of the recipro-cating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m. (AU-NOV/DEC-2008) Solution. Given : $a = 0.7$ m; $l_B = l_C = 0.6$ m or

$r_B = r_C = 0.3$ m; $m_1 = 150$ kg; $m_2 = 180$ kg; $c = 2/3$; $r_A = r_D = 0.6$ m; $N = 300$ r.p.m. or ω

□ 2□□ 300 / 60 = 31.42 rad/s

We know that the equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2 = 150 + \frac{2}{3} \times 180 = 270 \text{ kg}$$

Magnitude and direction of the balancing masses Let m_A and m_D = Magnitude of the balancing masses θ_A and θ_D = Angular position of the balancing masses m_A and m_D from the first crank *B*.

The magnitude and direction of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 22.7 (*a*). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder *B* in the horizontal direc-tion, draw *OC* and *OB* at right angles to each other as shown in Fig. 22.7 (*b*).

Tabulate the data as given in the following table. Assume the plane of wheel *A* as the reference plane.

Plane (1)	*mass. (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* ÷ ω^2 *(m.r) kg-m* (4)	*Distance from plane A (l) m* (5)	*Couple* ÷ ω^2 *(m.r.l) kg-m²* (6)
A (*R.P.*)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6m_D$	1.5	0.9 m_D

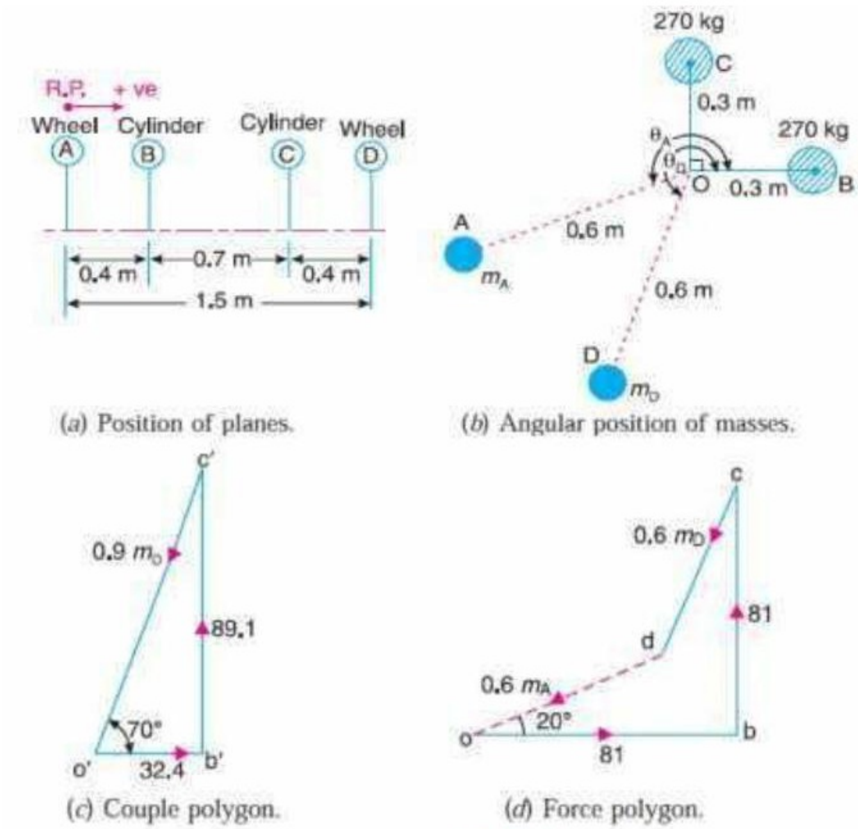
3. Now, draw the couple polygon from the data given in Table 22.1 (column 6), to some suitable

scale, as shown in Fig 22.7 (*c*). The closing side *c* '*o*' represents the balancing couple and it is proportional to 0.9 m_D. Therefore, by measurement,

$$0.9\ m_D = \text{vector } c'o' = 94.5 \text{ kg-m}^2 \quad \text{or} \quad m_D = 105 \text{ kg} \quad \textbf{Ans.}$$

$$D - \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses.



(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.7

4. To determine the angular position of the balancing mass *D*, draw *OD* in Fig. 22.7 (*b*) parallel to vector *c' o'*. By measurement,

$$\theta_D = 250°$$

Ans.

5. In order to find the balancing mass *A*, draw the force polygon from the data given in Table 22.1 (column 4), to some suitable scale, as shown in Fig. 22.7 (*d*), The vector *do* represents the balancing force and it is proportional to 0.6 m_A. Therefore by measurement, $0.6 m_A$ = vector *do* = 63 kg-m or m_A = 105 kg **Ans.**

6. To determine the angular position of the balancing mass *A*, draw *OA* in Fig. 22.7 (*b*) parallel to vector *do*. By measurement,

$$\theta_A = 200°$$

Ans.

Fluctuation in rail pressure

We know that each balancing mass105 kg

∴ Balancing mass for rotating masses,

$$B = \frac{c\,m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

∴ Fluctuation in rail pressure or hammer blow

$$= B\omega^2.b = 46.6\ (31.42)^2\ 0.6 = 27\ 602 \text{ N. } \textbf{Ans.} \qquad \ldots (\because b = r_A = r_D)$$

Variation of tractive effort

We know that maximum variation of tractive effort

,

$$= \pm\sqrt{2}(1-c)\,m_2.\omega^2.r = \pm\sqrt{2}\left(1-\frac{2}{3}\right)180(31.42)^2 0.3\,\text{N}$$

$$= + 25\,127 \text{ N Ans.} \qquad \ldots (\because\ r = r_B = r_C)$$

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2 0.3 \text{ N-m}$$

$$= 8797 \text{ N-m Ans.}$$

6. The following data apply to an outside cylinder uncoupled locomotive :
Mass of rotating parts per cylinder = 360 kg ; Mass of reciprocating parts per cylinder = 300 kg ; Angle between cranks = 90° ; Crank radius = 0.3 m ; Cylinder centres = 1.75 m ; Radius of balance masses = 0.75 m ; Wheel centres = 1.45 m. If whole of the rotating and two-thirds of reciprocating parts are to be balanced in planes of the driving wheels, find :

1.Magnitude and angular positions of balance masses,

2.Speed in kilometres per hour at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and

3.Swaying couple at speed arrived at in (2) above. (AU-NOV/DEC-2013) Solution : Given : $m_1 = 360$ kg ; $m_2 = 300$ kg ; $\angle AOD = 90°$; $r_A = r_D = 0.3$ m ; $a = 1.75$ m ; $r_B = r_C = 0.75$ m ; $c = 2 / 3$.

We know that the equivalent mass of the rotating parts to be balanced per cylinder,

$$m = m_A = m_D = m_1 + c.m_2 = 360 + \frac{2}{3} \times 300 = 560 \text{ kg}$$

1. *Magnitude and angular position of balance masses*

Let m_B and m_C = Magnitude of the balance masses, and

θ_B and θ_C = angular position of the balance masses m_B and m_C from the crank A.

The magnitude and direction of the balance masses may be determined, graphically, as discussed below :

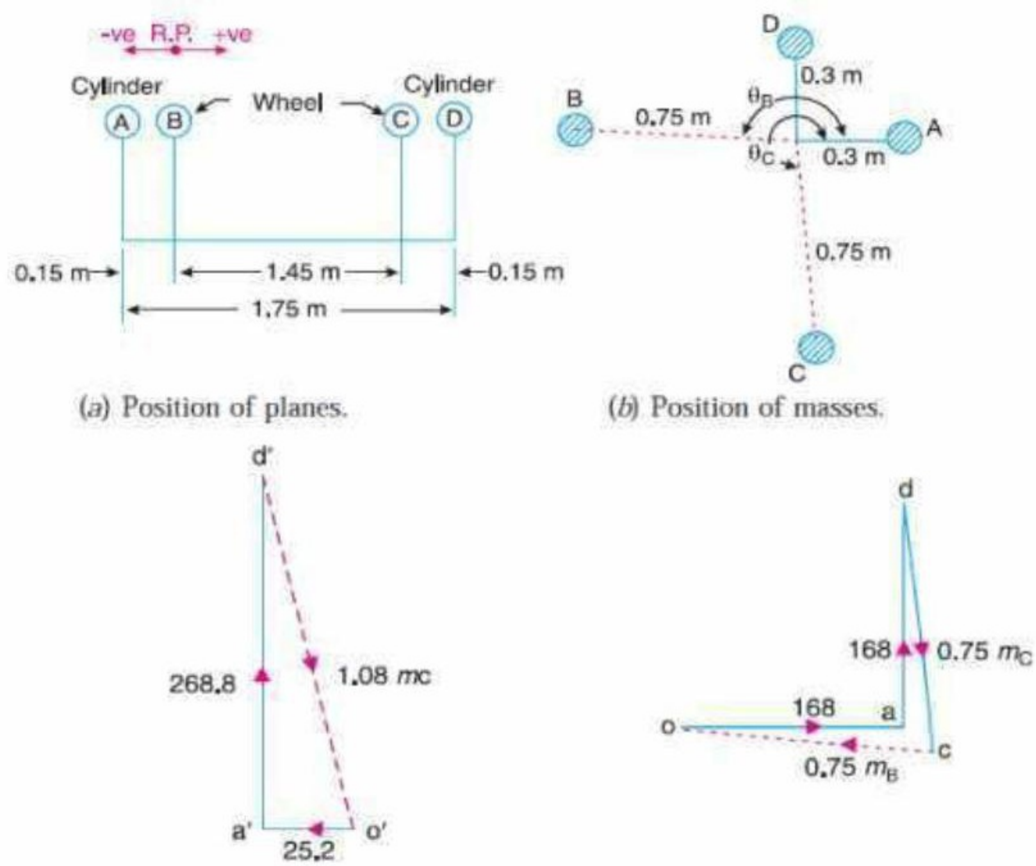
1. First of all, draw the positions of the planes of the wheels and the cylinders as shown in Fig. 22.11 (*a*). Since the cranks of the two cylinders are at right angles, therefore assum-ing the position of the cylinder A in the horizontal direction, draw OA and OD at right angles to each other as shown in Fig. 22.11 (*b*).

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m² (6)
A	560	0.3	168	– 0.15	– 25.2
B (R.P)	m_B	0.75	0.75 m_B	0	0
C	m_C	0.75	0.75 m_C	1.45	1.08 m_C
D	560	0.3	168	1.6	268.8

2. Assuming the plane of wheel *B* as the reference plane, the data may be tabulated as be-low:

3. Now draw the couple polygon with the data given in Table 22.4 column (6), to some suitable scale as shown in Fig. 22.11(*c*). The closing side *d* ' *o*' represents thebalancing couple and it is proportional to 1.08 m_C. Therefore, by measurement,

$$1.08\, m_C = 269.6 \text{ kg-m}^2 \quad \text{or} \quad m_C = 249 \text{ kg} \quad \textbf{Ans.}$$



(*a*) Position of planes. (*b*) Position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.11

4. To determine the angular position of the balancing mass *C*, draw *OC* parallel to vector *d*' *o* ' as shown in Fig. 22.11 (*b*). By measurement, $\theta_C = 275°$ **Ans.**

5. In order to find the balancing mass *B*, draw the force polygon with the data given in Tablecolumn (4), to some suitable scale, as shown in Fig. 22.11 (*d*). The vector *co* represents the balancing force and it is proportional to 0.75 m_B. Therefore, by measurement, 0.75 m_B = 186.75 kg-m or m_B = 249 kg **Ans.**

4. To determine the angular position of the balancing mass *B*, draw *OB* parallel to vector *oc* as shown Fig. 22.11 (*b*). By measurement, $\theta_B = 174.5°$ **Ans.**

2. ***Speed at which the wheel will lift off the rails***

Given : $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$; $D = 1.8$ m

Let ω = Angular speed at which the wheels will lift off the rails in rad/s,
and v = Corresponding linear speed in km/h.

We know that each balancing mass, $m_B = m_C = 249$ kg

∴ Balancing mass for reciprocating parts,

$$B = \frac{c.m_1}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 249 = 89 \text{ kg}$$

We know that $\omega = \sqrt{\frac{P}{B.b}} = \sqrt{\frac{30 \times 10^3}{89 \times 0.75}} = 21.2 \text{ rad/s}$ ...(∵ $b = r_B = r_C$)

and $v = \omega \times D/2 = 21.2 \times 1.8/2 = 19.08 \text{ m/s}$

$= 19.08 \times 3600/1000 = 68.7$ km/h **Ans.**

3. *Swaying couple at speed ω = 21.1 rad/s*

We know that the swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{1.75\left[1-\frac{2}{3}\right]}{\sqrt{2}} \times 300(21.2)^2 0.3 \text{ N-m}$$

$= 16\,687$ N m $= 16.687$ kN m **Ans.**

7. The three cranks of a three cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 meter and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank. If 40% of the reciprocating parts are to be balanced, find :

1. **the magnitude and the position of the balancing masses required at a radius of 0.6 m ;**

And

2. **the hammer blow per wheel when the axle makes 6 r.p.s. (AU-MAY/JUNE-2013)**

Solution. Given : ∠ AOB = ∠ BOC = ∠ COA = 120° ; $l_A = l_B = l_C = 0.6$ m or $r_A = r_B = r_C = 0.3$ m ; $m_I = 300$ kg ; $m_O = 260$ kg ; $c = 40\% = 0.4$; $b_1 = b_2 = 0.6$ m ; $N = 6$ r.p.s.

$= 6 \times 2\pi = 37.7$ rad/s

Since 40% of the reciprocating masses are to be balanced, therefore mass of the reciprocat-ing parts to be balanced for each outside cylinder,

$m_A = m_C = c \times m_O = 0.4 \times 260 = 104$ kg and mass of the reciprocating parts to be balanced for inside cylinder, m_B

$= c \times m_I = 0.4 \times 300 = 120$ kg

1. ***Magnitude and position of the balancing masses***

Let B_1 and B_2 = Magnitude of the balancing masses in kg, θ_1 and θ_2 = Angular position of the balancing masses B_1 and B_2 from crank

A.

The magnitude and position of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the position of planes and cranks as shown in Fig. 22.8 (*a*) and (*b*) respectively. The position of crank *A* is assumed in the horizontal direction.

2. Tabulate the data as given in the following table. Assume the plane of balancing mass B_1 (*i.e.* plane 1) as the reference plane.

Table 22.2

Plane (1)	*Mass* (m)kg (2)	*Radius* (r) m (3)	*Cent. force* ÷ ω^2 (m.r) kg-m (4)	*Distance from plane 1* (l) m (5)	*Couple* ÷ ω^2 (m.r.l) kg-m^2 (6)
A	104	0.3	31.2	0.2	6.24
1 (*R.P.*)	B_1	0.6	0.6 B_1	0	0
B	120	0.3	36	0.8	28.8
2	B_2	0.6	0.6 B_2	1.6	0.96 B_2
C	104	0.3	31.2	1.8	56.16

3. Now draw the couple polygon with the data given in Table 22.2 (column 6), to some suitable scale, as shown in Fig. 22.8 (*c*). The closing side *c ' o'* represents the balancing couple and it is proportional to 0.96 B_2. Therefore, by measurement,

$$0.96\ B_2 = \text{vector } c'\ o' = 55.2 \text{ kg-m}^2 \text{ or } B_2 = 57.5 \text{ kg} \quad \textbf{Ans.}$$

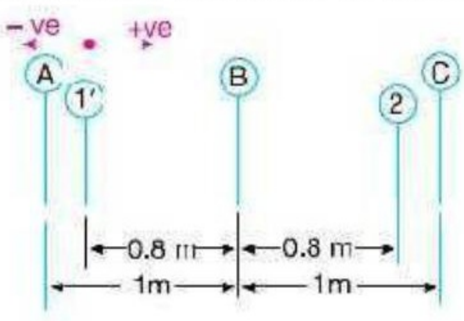
4. To determine the angular position of the balancing mass B_2, draw OB_2 parallel to vector *c ' o'* as shown in Fig. 22.8 (*b*). By measurement,

$$\theta_2 = 24° \quad \textbf{Ans.}$$

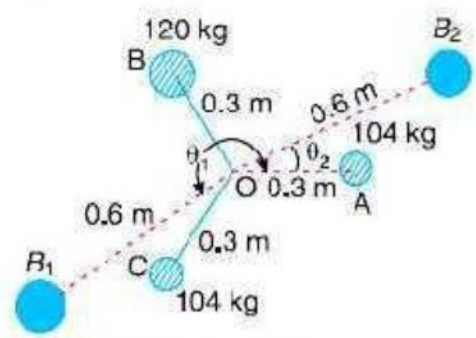
5. In order to find the balance mass B_1, draw the force polygon with the data given in Table 22.2(column 4), to some suitable scale, as shown in Fig. 22.8 (*d*). The closing side *co* represents the balancing force and it is proportional to 0.6 B_1. Therefore, by measurement,

$$0.6\ B_1 = \text{vector } co = 34.5 \text{ kg-m or } B_1 = 57.5 \text{ kg} \quad \textbf{Ans.}$$

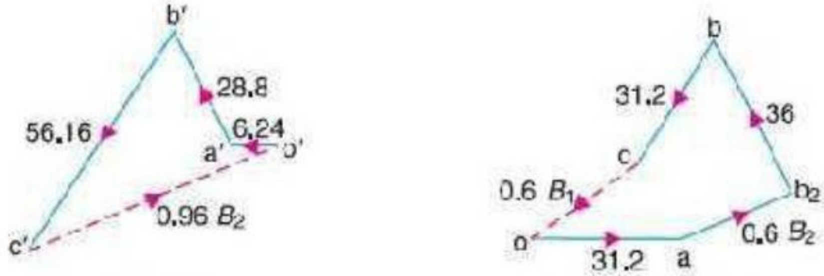
6. To determine the angular position of the balancing mass B_1, draw OB_1 parallel to vector *co*, as shown in Fig. 22.8 (*b*). By measurement, $\theta_1 = 215°$ **Ans.**



(*a*) Position of planes.



(*b*) Position of cranks.



(c) Couple polygon.

(d) Force polygon.

Fig. 22.8

2. *Hammer blow per wheel*

We know that hammer blow per wheel

$$= B_1.\omega^2.b_1 = 57.5\ (37.7)^2\ 20.6 = 49\ 035\ \text{N}\ \textbf{Ans.}$$

8. The following data refer to two cylinder locomotive with cranks at 90° :

Reciprocating mass per cylinder = 300 kg ; Crank radius = 0.3 m ; Driving wheel diameter = 1.8 m ; Distance between cylinder centre lines = 0.65 m ; Distance between the driving wheel central planes = 1.55 m.

Determine : 1. the fraction of the reciprocating masses to be balanced, if the hammer blow is not to exceed 46 kN at 96.5 km. p.h. ; 2. the variation in tractive effort ; and 3. the maximum swaying couple. (AU-MAY/JUNE-2009)

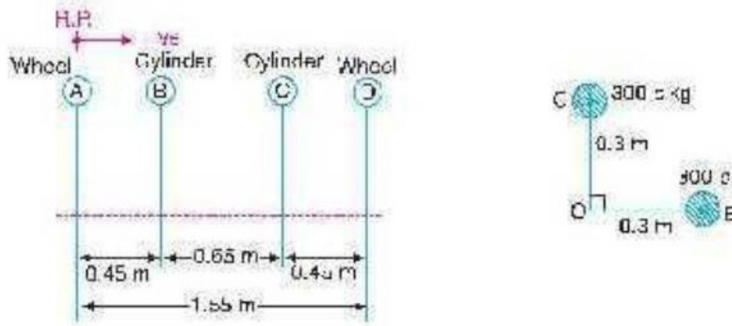
Solution. Given : $m = 300$ kg ; $r = 0.3$ m ; $D = 1.8$ m or $R = 0.9$ m ; $a = 0.65$ m ; Hammer blow $= 46$ kN $= 46 \times 10^3$ N ; $v = 96.5$ km/h $= 26.8$ m/s

1. ***Fraction of the reciprocating masses to be balanced***

Let c = Fraction of the reciprocating masses to be balanced, and

B = Magnitude of balancing mass placed at each of the driving wheels at radius b.

We know that the mass of the reciprocating parts to be balanced $= c.m = 300c$ kg



(*a*) Position of planes. (*b*) Position of cranks.

Fig. 22.9

The position of planes of the wheels and cylinders is shown in Fig. 22.9 (*a*), and the position of

Plane (1)	*Mass* (*m*) kg (2)	*Radius* (*r*) m (3)	*Cent. force* ÷ ω^2 (*m.r*) kg-m (4)	*Distance from plane A* (*l*) m (5)	*Couple* ÷ ω^2 (*m.r.l*) kg-m² (6)
A (*R.P.*)	*B*	*b*	*B.b*	0	0
B	300 *c*	0.3	90 *c*	0.45	40.5 *c*
C	300 *c*	0.3	90 *c*	1.1	99 *c*
D	*B*	*b*	*B.b*	1.55	1.55 *B.b*

cranks is shown in Fig 22.9 (*b*). Assuming the plane of wheel *A* as the reference plane, the data may be tabulated as below :

Now the couple polygon, to some suitable scale, may be drawn with the data given in Table 22.3 (column 6), as shown in Fig. 22.10. The closing side of the polygon (vector $c'\,o'$) represents the balancing couple and is proportional to 1.55 $B.b$.

From the couple polygon,

$$1.55\ B.b = \sqrt{(40.5c)^2 + (99\,c)^2} = 107c$$

$\therefore\ B.b = 107\ c / 1.55 = 69\ c$

We know that angular speed,

$\omega = v/R = 26.8/0.9 = 29.8$ rad/s $\therefore$ Hammer blow,

$$46 \times 10^3 = B.\ \omega^2 .b$$

$$= 69\ c\ (29.8)^2 = 61\ 275\ c$$

$$\therefore \quad c = 46 \times 10^3/61\ 275 = 0.751$$

Ans.

2. Variation in tractive effort

We know that variation in tractive effort,

$$= \pm\sqrt{2}(1-c)\,m\omega^2.r = \pm\sqrt{2}(1-0.751)\ 300\ (29.8)^2\ 0.3$$

$$= 28\ 140\ \text{N} = 28\ 14\ \text{kN}$$ **Ans.**

3.Maximum swaying couple

We know the maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m.\omega^2\ r = \frac{0.65\ (1\ \ 0.751)}{\sqrt{2}} \times 300\ (29.8)^2\ 0.3 = 9148\ \text{N-m}$$

$$= 9.148\ \text{kN-m}$$ **Ans.**

2.14 REVIEW QUESTIONS

1. Discuss how a single revolving mass is balanced by two masses revolving in different planes.
2. How the different masses rotating in different planes are balanced ?
3. In order to have a complete balance of the several revolving masses in different planes?
4. What are in-line engines ? How are they balanced ? It is possible to balance them completely ?
5. The primary unbalanced force is maximum when the angle of inclination of the crank with the lineof stroke is _____________
6. In order to facilitate the starting of locomotive in any position, the cranks of a locomotive, with twocylinders, are placed at . . . . . . to each other
7. When the primary direct crank of a reciprocating engine makes an angle θ with the line of stroke, then the secondary direct crank will make an angle of . . . . . with the line of stroke.

2.15 TUTORIAL PROBLEMS:

1. Four masses *A*, *B*, *C* and *D* revolve at equal radii and are equally spaced along a shaft. The mass *B* is 7 kg and the radii of *C* and *D* make angles of 90° and 240° respectively with the radius of *B*. Find the magnitude of the masses *A*, *C* and *D* and the angular position of *A* so that the system may be completely balanced.

[Ans. 5 kg ; 6 kg ; 4.67 kg ; 205° from mass *B* in anticlockwise direction]

2. A rotating shaft carries four masses *A*, *B*, *C* and *D* which are radially attached to it. The mass centres are 30 mm, 38 mm, 40 mm and 35 mm respectively from the axis of rotation. The masses *A*, *C* and *D* are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of *A* and *B* is 400 mm and between *B* and *C* is 500 mm. The masses *A* and *C* are at right angles to each other. Find for a complete balance,

1. the angles between the masses *B* and *D* from mass *A*,

2. the axial distance between the planes of rotation of *C* and *D*,
3. the magnitude of mass *B*. **[Ans. 162.5°, 47.5° ; 511 mm : 9.24 kg]**

3. A three cylinder radial engine driven by a common crank has the cylinders spaced at 120°. The stroke is 125 mm, length of the connecting rod 225 mm and the mass of the reciprocating parts per cylinder 2 kg. Calculate the primary and secondary forces at crank shaft speed of 1200 r.p.m.

[Ans. 3000 N ; 830 N]

4. The pistons of a 60° twin *V*-engine has strokes of 120 mm. The connecting rods driving a common crank has a length of 200 mm. The mass of the reciprocating parts per cylinder is 1 kg and the

speed of the crank shaft is 2500 r.p.m. Determine the magnitude of the primary and secondary forces. **[Ans. 6.3 kN ; 1.1 kN]**

5. A twin cylinder *V*-engine has the cylinders set at an angle of 45°, with both pistons connected to the single crank. The crank radius is 62.5 mm and the connecting rods are 275 mm long. The reciprocating mass per line is 1.5 kg and the total rotating mass is equivalent to 2 kg at the crank radius. A balance mass fitted opposite to the crank, is equivalent to 2.25 kg at a radius of 87.5 mm. Determine for an engine speed of 1800 r.p.m. ; the maximum and minimum values of the primary and secondary forces due to the inertia of reciprocating and rotating masses.

[Ans. Primary forces : 3240 N (max.) and 1830 N (min.) Secondary forces : 1020 N (max.) and 470 N (min.)]