STEAM NOZZLES

TECHNICAL TERMS:

1. Wet steam: The steam, which contains some water particles in superposition.

2. Dry steam / dry saturated steam:

When whole mass of steam is converted into steam then it is called as dry steam.

3. Super heated steam: When the dry steam is further heated at constant pressure, the temperature increases the above saturation temperature. The steam has obtained is called super heated steam.

4. Degree of super heat: The difference between the temperature of saturated steam and saturated temperature is called degree of superheat.

5. Nozzle: It is a duct of varying cross sectional area in which the velocity increases with the corresponding drop in pressure.

6. Coefficient of nozzle: It is the ratio of actual enthalpy drop to isentropic enthalpy drop.

7. Critical pressure ratio: There is only one value of ratio (P2/P1) which produces maximum discharge from the nozzle . then the ratio is called critical pressure ratio.

8. Degree of reaction: It is defined as the ratio of isentropic heat drop in the moving blade to isentropic heat drop in the entire stages of the reaction turbine.

9. Compounding: It is the method of absorbing the jet velocity in stages when the steam flows over moving blades. (i)Velocity compounding (ii) Pressure compounding and (iii) Velocity-pressure compounding

10. Enthalpy: It is the combination of the internal energy and the flow energy.

11. Entropy: It is the function of quantity of heat with respective to the temperature.

12. Convergent nozzle: The crossectional area of the duct decreases from inlet to the outlet side then it is called as convergent nozzle.

13. Divergent nozzle: The crossectional area of the duct increases from inlet to the outlet then it is called as divergent nozzle.

Flow of steam through nozzles:

The flow of steam through nozzles may be regarded as adiabatic expansion. - The steam has a very high velocity at the end of the expansion, and the enthalpy decreases as expansion takes place. - Friction exists between the steam and the sides of the nozzle; heat is produced as the result of the resistance to the flow. - The phenomenon of super saturation occurs in the flow of steam through nozzles. This is due to the time lag in the condensation of the steam during the expansion.

Continuity and steady flow energy equations

Through a certain section of the nozzle: $m.v = A.C$ m is the mass flow rate, v is the specific volume, A is the cross-sectional area and C is the velocity. For steady flow of steam through a certain apparatus, principle of conservation of energy states:

$$h_1 + C_1^2/2 + gz_1 + q = h_2 + C_2^2/2 + gz_2 + w$$

For nozzles, changes in potential energies are negligible, $w = 0$ and $q \cong 0$

$$h_1 + C_1^2/2 = h_2 + C_2^2/2$$

Types of Nozzles:

1. Convergent Nozzle
2. Divergent Nozzle
3. Convergent-Divergent Nozzle

Convergent Nozzle:

A typical convergent nozzle is shown in fig. in a convergent nozzle, the cross sectional area decreases continuously from its entrance to exit. It is used in a case where the back pressure is equal to or greater than the critical pressure ratio.

Divergent Nozzle:

The cross sectional area of divergent nozzle increases continuously from its entrance to exit. It is used in a case, where the back pressure is less than the critical pressure ratio.

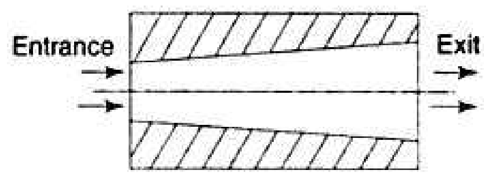


Fig. 3.1. Divergent Nozzle

Convergent-Divergent Nozzle:

In this case, the cross sectional area first decreases from its entrance to throat, and then increases from throat to exit.it is widely used in many type of steam turbines.

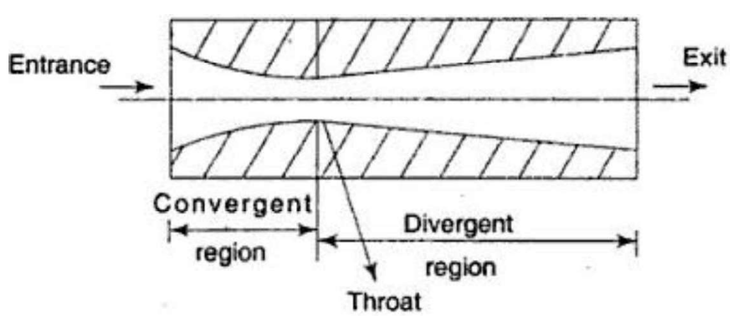
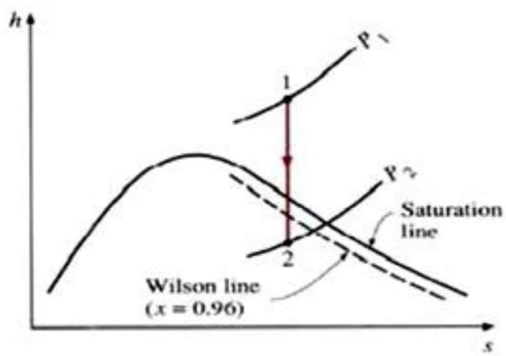


Fig. 3.2. Convergent-Divergent Nozzle

Supersaturated flow or Meta stable flow in Nozzles: As steam expands in the nozzle, its pressure and temperature drop, and it is expected that the steam start condensing when it strikes the saturation line. But this is not always the case. Owing to the high velocities, the residence time of the steam in the nozzle is small, and there may not sufficient time for the necessary heat transfer and the formation of liquid droplets. Consequently, the condensation of steam is delayed for a little while. This phenomenon is known as super saturation, and the steam that exists in the wet region without containing any liquid is known as supersaturated steam.

The locus of points where condensation will take place regardless of the initial temperature and pressure at the nozzle entrance is called the Wilson line. The Wilson line lies between 4 and 5 percent moisture curves in the saturation region on the h-s diagram for steam, and is often approximated by the 4 percent moisture line. The super saturation phenomenon is shown on the h-s chart below:



The *h-s* diagram for the isentropic expansion of steam in a nozzle.

Critical Pressure Ratio: The critical pressure ratio is the pressure ratio which will accelerate the flow to a velocity equal to the local velocity of sound in the fluid.

Critical flow nozzles are also called **sonic chokes**. By establishing a shock wave the sonic choke establish a fixed flow rate unaffected by the differential pressure, any fluctuations or changes in downstream pressure. A sonic choke may provide a simple way to regulate a gas flow.

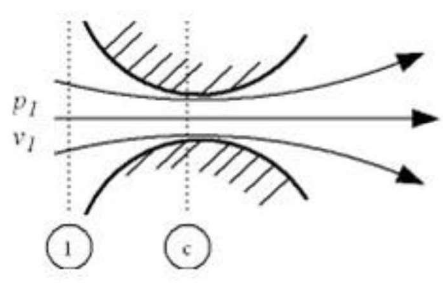


Fig. 3.3 ritical flow nozzles

The ratio between the critical pressure and the initial pressure for a nozzle can expressed as

$$Pc / p1 = (2 / (n + 1))^{n/(n-1)}$$

Where, p_c = critical pressure (Pa)

p_1 = inlet pressure (Pa)

n = index of isentropic expansion or compression or polytrophic constant

For a perfect gas undergoing an adiabatic process the index – n – is the ratio of specific heats k = cp / cv. There is no unique value for – n. Values for some common gases are

- Steam where most of the process occurs in the wet region: n = 1.135
- Steam super-heated: n = 1.30
- Air: n = 1.4
- Methane: n = 1.31
- Helium: n = 1.667

Effect of Friction on Nozzles:

1) Entropy is increased.
2) Available energy is decreased.
3) Velocity of flow at throat is decreased.
4) Volume of flowing steam is decreased.
5) Throat area necessary to discharge a given mass of steam is increased.

Most of the friction occurs in the diverging part of a convergent-divergent nozzle as the length of the converging part is very small. The effect of friction is to reduce the available enthalpy drop by about 10 to 15%. The velocity of steam will be then

$$V_2 = 44.72\sqrt{K(H_1 - H_2)}$$

Where, k is the co-efficient which allows for friction loss. It is also known as nozzle efficiency.

Velocity of Steam at Nozzle Exit:

$$V_2^2 = 2000(H_1 - H_2) + V_1^2 \quad \therefore \quad V_2 = \sqrt{2000(H_1 - H_2) + V_1^2}$$

As the velocity of steam entering the nozzle is very small, V_1 can be neglected.

$$\therefore \quad V_2 = \sqrt{2000(H_1 - H_2)} = 44.72\sqrt{(H_1 - H_2)} \text{ m/s}$$

If frictional losses are taken into account then

$$V_2 = 44.72\sqrt{(H_1 - H_2)\eta_n} \text{ m/s}$$

Mass of steam discharged through nozzle:

$$m = A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}$$

Condition for maximum discharge through nozzle: The nozzle is always designed for maximum discharge

$$\frac{m}{A} = \sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}$$

The mass flow per unit area will be maximum at the throat because the throat area is minimum.

It is seen from the above equation that the discharge through a nozzle is a function of $\frac{P_2}{P_1}$ only, as the expansion index is fixed according to the steam supplied to the nozzle.

Therefore, $\frac{m}{A}$ is maximum when

$$\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right] \text{ is minimum}$$

Values for maximum discharge:

$$m = A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}$$

we know $\frac{P_2}{P_1}=\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

Putting the value of $\frac{P_2}{P_1}$ in the above equation

$$m_{max} = A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

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$$= A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\times\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{\frac{1-n}{n-1}}-1\right]}$$

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$$= A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\times\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\frac{n+1}{2}-1\right]}$$

$$= A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\times\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left(\frac{n-1}{2}\right)}$$

$$m_{max} = A\sqrt{1000n\times\frac{P_1}{v_1}\times\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}}$$

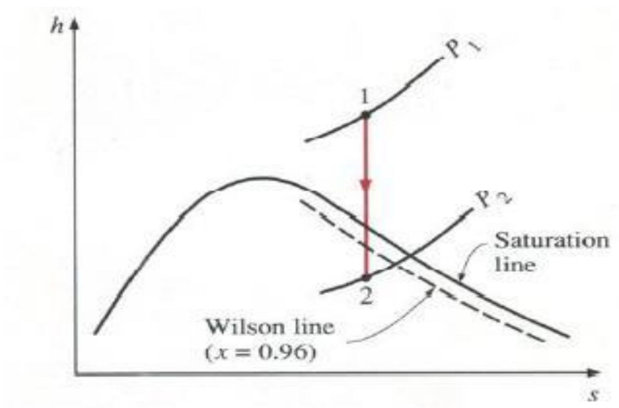
Where P_1 is the initial pressure of the steam in kpa and v_1 is the specific volume of the steam in m3/kg at the initial pressure.

STEAM NOZZLE PROBLEMS

1. Describe about supersaturated flow or metastable flow in a nozzleand state effect of super saturation

As steam expands in the nozzle, its pressure and temperature drop, and it is expected that the Steam start condensing when it strikes the saturation line. But this is not always the case. Owing to the high velocities, the residence time of the steam in the nozzle is small, and there may not Sufficient time for the necessary heat transfer and the formation of liquid droplets. Consequently, the condensation of steam is delayed for a little while. This phenomenon is known as Super saturation, and the steam that exists in the wet region without containing any liquid is Known as supersaturated steam.

The locus of points where condensation will take place regardless of the initial temperature and Pressure at the nozzle entrance is called the Wilson line. The Wilson line lies between 4 and 5 percent moisture curves in the saturation region on the h-s diagram for steam, and is often Approximated by the 4 percent moisture line. The super saturation phenomenon is shown on the h-s chart below:



The h-s diagram for the isentropic expansion of steam in a nozzle.

Effects of Supersaturation:

The following are the effects of supersaturation in a nozzle.

(a) The temperature at which the supersaturation occurs will be less than the saturation temperature corresponding to that pressure. Therefore, the density of supersaturated steam will be more than that of equilibrium condition which gives the increase in the mass of steam discharged.

(b) Supersaturation increases the specific volume and entropy of the steam.

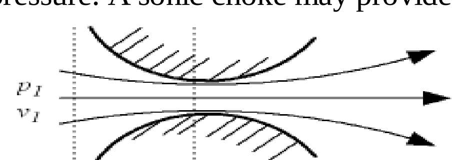
(c) Supersaturation reduces the heat drop. Thus the exit velocity of steam is reduced.

(d) Supersaturation increases the dryness fraction of the steam.

Critical Pressure Ratio:

The critical pressure ratio is the pressure ratio which will accelerate the flow to a velocity equal to the local velocity of sound in the fluid.

Critical flow nozzles are also called **sonic chokes**. By establishing a shock wave the sonic choke establish a fixed flow rate unaffected by the differential pressure, any fluctuations or changes in downstream pressure. A sonic choke may provide a simple way to regulate a gas flow.



The ratio between the critical pressure and the initial pressure for a nozzle can expressed as

$p_c / p_1 = (2 / (n + 1))^{n / (n - 1)}$

where

p_c = critical pressure (Pa)

p_1 = inlet pressure (Pa)

n = index of isentropic expansion or compression - or polytropic constant

For a perfect gas undergoing an adiabatic process the index - n - is the ratio of specific heats - k = c_p / c_v. There is no unique value for - n. Values for some common gases are

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2. Describe the Effect of Friction on Nozzles.

1) Entropy is increased.
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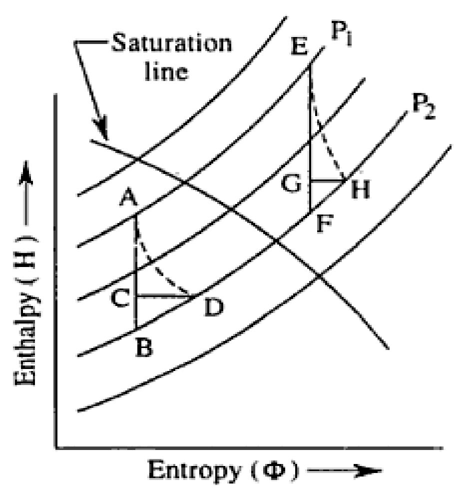
5) Throat area necessary to discharge a given mass of steam is increased.

Most of the friction occurs in the diverging part of a convergent-divergent nozzle as the length of the converging part is very small. The effect of friction is to reduce the available enthalpy drop by about 10 to 15 per cent. The velocity of steam will be then $V_2 = 44.72\sqrt{K(H_1 - H_2)}$ where K is the coefficient which allows for friction loss. It is also known as nozzle efficiency (η_n)

$$\therefore \quad V_2 = 44.72\sqrt{(H_1 - H_2)\eta_n}$$



Velocity of steam at nozzle exit:

$$V_2^2 = 2000(H_1 - H_2) + V_1^2 \quad \therefore \quad V_2 = \sqrt{2000(H_1 - H_2) + V_1^2}$$

As the velocity of steam entering the nozzle is very small, V_1 can be neglected.

$$\therefore \qquad V_2 = \sqrt{2000(H_1 - H_2)} = 44.72\sqrt{(H_1 - H_2)} \text{ m/s}$$

If frictional losses are taken into account then

$$V_2 = 44.72\sqrt{(H_1 - H_2)\eta_n} \text{ m/s}$$

3. Derive the expression for maximum discharge through a nozzle.

Mass of steam discharged through a nozzle:

$$m = A\sqrt{2000\frac{n}{n-1} \times \frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}$$

Condition for maximum discharge through nozzle:

The nozzle is always designed for maximum discharge

$$\frac{m}{A} = \sqrt{2000\frac{n}{n-1} \times \frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}$$

The mass flow per unit area will be maximum at the throat because the throat area is minimum.

It is seen from the above equation that the discharge through a nozzle is a function of $\frac{P_2}{P_1}$ only, as the expansion index is fixed according to the steam supplied to the nozzle.

Therefore, $\frac{m}{A}$ is maximum when

$$\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}} - \left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right] \text{ is minimum}$$

Values for maximum discharge:

$$m = A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{n}}-\left(\frac{P_2}{P_1}\right)^{\frac{n+1}{n}}\right]}$$

we know $\frac{P_2}{P_1}=\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

Putting the value of $\frac{P_2}{P_1}$ in the above equation

$$m_{max} = A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

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$$= A\sqrt{2000\frac{n}{n-1}\times\frac{P_1}{v_1}\times\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\left[\left(\frac{2}{n+1}\right)^{\frac{1-n}{n-1}}-1\right]}$$

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$$m_{max} = A\sqrt{1000n\times\frac{P_1}{v_1}\times\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}}$$

W3here P_1 is the initial pressure of the steam in kpa and v_1 is the specific volume of the stea

4. ***Dry saturated steam at a pressure of 15 bar enters in a nozzle and is discharged at a pressure of 1.5 bar. Find the final velocity of the steam. When the initial velocity of the steam is negligible.***

Given :

p_1 = 15 bar;

p_2 = 1.5 bar

Solution.

Final velocity of the steam

From steam tables, corresponding to a pressure of 15 bar, we find that enthalpy of dry saturated steam,

h_1 = 2789.9 kJ/Kg

and corresponding to a pressure of 1.5 bar, enthalpy of dry saturated steam,

h_2 = 2693.4 kJ/Kg

$\therefore$ Heat drop, $h_d = h_1 - h_2 = 2789.9 - 2693.4 = 96.5$ kJ/Kg

We know that final velocity of the steam,

$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{96.5} = 439.3 m/s$ Ans.

Percentage reduction in the final velocity

We know that heat drop lost in friction

= 10% = 01.

$\therefore$ Nozzle coefficient or nozzle efficiency

K = 1 – 01 = 0.9

We know that final velocity of the steam,

$V_2 = 44.72\sqrt{Kh_d} = 44.72\sqrt{0.9 \times 96.5} = 416.8 m/s$

$\therefore$ Percentage reduction in final velocity

$$= \frac{439.3 - 416.8}{439.3} = 0.051 \; or \; 5.1\%$$

5. Dry saturated steam at 10 bar is expanded isentropically in a nozzle to 0.1 bar. Using steam tables only, find the dryness fraction of the steam at exit. Also find the velocity of steam leaving the nozzle when 1. initial velocity is negligible, and 2. Initial velocity of the steam is 135 m/s

Given :

$p_1 = 10$ bar ;

$p_2 = 0.1$ bar

Solution .

Dryness fraction of the steam at exit

Let x_2 = Dryness fraction of the steam at exit.

From steam tables, corresponding to a pressure of 10 bar, we find that entropy of dry saturated steam,

$s_1 = s_{g_1} = 6.583 kJ/kgK$ s

and corresponding to a pressure of 0.1 bar,

we find that

$s_{f2} = 0.649 kJ/kgK$, And $s_{fg2} = 7.502 kJ/kgK$

Since the expansion of steam is isentropic,

Therefore

Entropy of steam at inlet (s_1) = Entropy of steam at exit (s_2)

$6.583 = s_{fg2} + x_2 s_{fg_2} = 0.649 + x_2 + 7.502$

X2 = 0.791 Ans.

1. Velocity of steam leaving the nozzle when initial velocity is negligible

From steam tables, corresponding to a pressure of 10 bar, we find that enthalpy or total heat of dry saturated steam,

$h_1 = h_{g1} = 2776.2$ kJ/kg

and Corresponding to a pressure of 0.1 bar,

h_{f2} = 191.8 kJ/kg, and h_{fg2} = 2392.9 kJ/kg

∴ Enthalpy or total heat of steam of exit,

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$= 191.8 + 0.791 \times 2392.2$$

$$= 2084.6 \text{ kJ/kg}$$

and heat drop, $h_d = h_1 - h_2 = 2776.2 - 2084.6$

$$= 691.6 \text{ kJ/kg}$$

We know that velocity of steam leaving the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{691.6} = 1176 \text{ m/s} \quad \text{Ans.}$$

Velocity of steam leaving the nozzle when initial velocity,

$$\boldsymbol{V_1 = 135 \text{ m/s}}$$

We know that velocity of steam leaving the nozzle,

$$V_2 = \sqrt{V_1^2 + 2000\,h_d} = \sqrt{(135)^2 + 2000 \times 691.69} = 1184\, m/s \quad \textbf{Ans.}$$

6. ***Dry saturated steam enters a nozzle at a pressure of 10 bar and with an initial velocity of 90 m/s. The outlet pressure is 6 bar and the outlet velocity is 435 m/s. The hear loss from the nozzle id 9 kJ/kg of stream flow.***

Calculate the dryness fraction and the area at the exit, if the area at the inlet is 1256 mm²

Given:

p_1 =10 bar; V_1 = 90 m/s;

p_3= 6 bar; V_3= 435 m/s;

Loses = 9 kJ/Kg;

A_1=1256 mm^2 = 1256 x 10^{-6} m^2

Solution.

Dryness fraction of steam

Let x_3 = Dryness fraction of steam at the exit.

From steam tables,

corresponding to a pressure of 10 bar, we find that enthalpy of dry saturated steam,

h_1 = 2776.2 kJ/kg; and $v_{g1} = 0.1943 m^3/kg$ and corresponding to a pressure of 6 bar, we find that

$$h_{\beta} = 670.4 kJ/kg;\ h_{fg3} = 2085 kK/kg;\ v_{g3} = 0.3155 m^3/kg$$

We know that for a steady flow through the nozzle,

$$h_1 + \frac{1}{1000}\left(\frac{V_1^2}{2}\right) = h_3 + \frac{1}{1000}\left(\frac{V_3^2}{2}\right) + Losses$$

$$\therefore \qquad h_3 = h_1 + \frac{2}{1000}(V_1^2 - V3^2) - Losses$$

$$= 2776.2 + \frac{1}{2000}[(90)^2 - (435)^2] - 9$$

=2776.2-99.6

= 2676.6 kJ/kg

We also know that enthalpy of wet stream (h_3),

$$2676.6 = h_{\beta} + x_3 h_{fg3} = 670.4 + x_3 \times 2085$$

$\therefore$ $x_3 = 0.962$ Ans.

Area at exit

Let A_3 Area at exit in m^2

We know that $\frac{A_1 V_1}{x_1 v_{g1}} = \frac{A_3 V_{31}}{x_{31} v_{g3}}$ or $\frac{1256x10^{-6} \times 90}{1 \times 0.1943} = \frac{A_3 \times 435}{0.962 \times 0.3155}$

…(For dry saturated steam, x_1 =1)

$A_3 = 406 \ \text{x}\ 10^{-6} m^2 = 406\ mm^2$ Ans.

7. ***Dry saturated steam at a pressure of 8 bar enters a convergent – divergent nozzle and leaves it at a pressure of 1.5 bar, If the flow is isentropic, and the corresponding expansion index is 1.135; find the ratio of cross – sectional area at exit and throat for maximum discharge.***

Solution. Given : p_1= 8 bar; p_3=1.5 bar; n=1.135

Let A_2 =Cross-sectional area at throat,

A_3 = Cross-sectional area at exit, and

m = Mass of steam discharged per second.

We know that for dry saturated steam (or when n= 1.135), critical pressure ratio,

$$\frac{p_2}{p_1} = 0.577$$

$\therefore$ $p_2=0.577p1=0.577x\ 8 = 4.616$ bar

Now complete the Mollier diagram for the expansion of steam through the nozzle, as shown in Fig.21.8.

From Mollier diagram, we find that

$h_1 = 2775$kJ/kg; $h_2 = 2650$kJ/kg; $h_3 = 2465$ kJ/kg; $x_2 = 0.965$; and $x_3 = 0.902$

From steam tables, we also find that the specific volume of steam at throat corresponding to 4.616 bar,

$$\upsilon_{g^2} = 0.405 m^3/kg$$

and specific volume of steam at exit corresponding to 1.5 bar,

$$\upsilon_{g^3} = 1.159 m^3/kg$$

Heat drop between entrance and throat,

$$h_{a2} = h_1 - h_2 = 2775 - 2650 = 125 kJ/kg$$

$\therefore$ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{125} = 500 m/s$$

and $$m = \frac{A_2 V_2}{x_2 \upsilon_{g2}}$$

or $$A_2 = \frac{m x_2 \upsilon_{g2}}{V_2} = \frac{m \times 0.965 \times 0.405}{500} = 0.000786m \qquad \ldots(i)$$

Heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = 2775 - 2465 = 310 kJ/kg$$

$\therefore$ Velocity of steam at throat,

$$V_3 = 44.72\sqrt{h_{d3}} = 44.72\sqrt{310} = 787.4 m/s$$

and $$m = \frac{A_2 V_2}{x_2 \upsilon_{g2}}$$

or $$A_3 = \frac{m x_3 \upsilon_{g3}}{V_3} = \frac{m \times 0.902 \times 1.159}{787.4} = 0.001.33m \qquad \ldots(ii)$$

$\therefore$ Ratio of cross-sectional area at exit and throat,

$$\frac{A_3}{A_2} = \frac{0.00133m}{0.000786m} = 1.7 \text{ Ans}$$

8. Derive the expression for the critical pressure ratio in a steam nozzle.

There is only one value of the ratio (called critical pressure ratio) $\frac{p_2}{p_1}$ which will produce the maximum discharge. This can be obtained by differentiating '*m*' with respect to $\left(\frac{p_2}{p_1}\right)$ and equating it to zero. Other quantities except the ratio $\frac{p_2}{p_1}$ are constant.

$$\frac{d}{d\left[p_2 / p_1\right]}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

$$\frac{2}{n}\left[\frac{p_2}{p_1}\right]^{\frac{2}{n}-1} - \frac{n+1}{n}\left[\frac{p_2}{p_1}\right]^{\frac{n+1}{n}-1} = 0$$

$$\frac{2}{n} \times \left(\frac{p_2}{p_1}\right)^{\frac{2}{n}-1} = \frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\left(\frac{p_2}{p_1}\right)^{2-n} = \left[\frac{n+1}{2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}\right]^{n}$$

$$\left[\frac{p_2}{p_1}\right]^{2-n} = \left(\frac{n+1}{2}\right)^{n}\left(\frac{p_2}{p_1}\right)$$

$$\frac{\left(\frac{p_2}{p_1}\right)^{2-n}}{\left(\frac{p_2}{p_1}\right)} = \left(\frac{n+1}{2}\right)^n$$

$$\left(\frac{p_2}{p_1}\right)^{2-n-1} = \left(\frac{n+1}{2}\right)^n$$

$$\left(\frac{p_2}{p_1}\right)^{1-n} = \left(\frac{n+1}{2}\right)^n \implies \frac{p_2}{p_1} = \left(\frac{n+1}{2}\right)^{n\times\frac{1}{1-n}}$$

$$\frac{P_2}{P_1} = \left(\frac{n+1}{2}\right)^{\frac{n}{1-n}}$$

Critical pressure ratio of steam nozzle,

$$\frac{P_2}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

9. A convergent nozzle required to discharge 2kg of steam per second. The nozzle is supplied with steam at 7 bar and 180^{0}C and discharge takes place againsta back pressure of 1 bar. The expansion up to throat is isentropic and the frictional resistance between the throat and exit is equivalent to 63 kJ/kg of steam.Take approach velocity of 75 m/s and throat pressure 4 bar, estimate

i) suitable areas for the throat and exit,and

ii) Overall efficiency of the nozzle based on enthalpy drop between the actual inlet pressure and temperature and the exit pressure.

Final velocity of the steam

From steam tables, corresponding to a pressure of 15 bar, we find that enthalpy of dry saturated steam,

h_1 = 2789.9 kJ/Kg

and corresponding to a pressure of 1.5 bar, enthalpy of dry saturated steam,

h_2 = 2693.4 kJ/Kg

∴ Heat drop, $h_d = h_1 - h_2 = 2789.9 – 2693.4 = 96.5$ kJ/Kg

We know that final velocity of the steam,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{96.5} = 439.3\,m/s$$

10.Define critical pressure ratio of a nozzle and discuss why attainment of sonic velocity determines the maximum discharge through steam nozzle.

Critical Pressure Ratio:

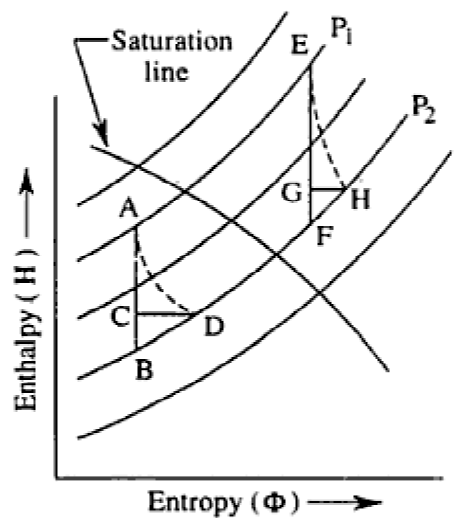
The critical pressure ratio is the pressure ratio which will accelerate the flow to a velocity equal to the local velocity of sound in the fluid.

Critical flow nozzles are also called **sonic chokes**. By establishing a shock wave the sonic choke establish a fixed flow rate unaffected by the differential pressure, any fluctuations or changes in downstream pressure. A sonic choke may provide a simple way to regulate a gas flow.

11. Explain the metastable expansion of steam in a nozzle with help of h-s diagram.

Most of the friction occurs in the diverging part of a convergent-divergent nozzle as the length of the converging part is very small. The effect of friction is to reduce the available enthalpy drop by about 10 to 15 per cent. The velocity of steam will be then $V_2 = 44.72\sqrt{K(H_1 - H_2)}$ where K is the coefficient which allows for friction loss. It is also known as nozzle efficiency (η_n)

$$\therefore \quad V_2 = 44.72\sqrt{(H_1 - H_2)\eta_n}$$



$$\theta = 29^{\circ}$$

(b) During force on the wheel

We know that driving force on the wheel,

$$F_x = m(V_w + V_{w1}) = 0.5\,(1130 + 190) = 660\text{ N}$$

(c) Arial thrust on the wheel

We know that axial thrust on the wheel,

$$F_\gamma = m(V_f - V_{f1}) = 0.5(410 - 310) = 50N$$

(d) Power development by the turbine

We know that power development by the turbine,

$$P = m(V_w + V_{w1})\,V_b$$
$$= 0.5(1130 + 190)\,375 = 247\,500\ W$$
$$= 247.5\ kW$$

12. Dry saturated steam at a pressure of 8 bar enters a convergent divergent nozzle and leaves it at a pressure of 1.5 bar. If the flow is isentropic and if the corresponding expansion index is 1.133, find the ratio of cross sectional area at exit and throat for maximum discharge.

Heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = 2775 - 2465 = 310 kJ/kg$$

∴ Velocity of steam at throat,

$$V_3 = 44.72\sqrt{h_{d3}} = 44.72\sqrt{310} = 787.4 m/s$$

and $$m = \frac{A_2 V_2}{x_2 \upsilon_{g2}}$$

or $$A_3 = \frac{m x_3 \upsilon_{g3}}{V_3} = \frac{m \times 0.902 \times 1.159}{787.4} = 0.001.33m$$

∴ Ratio of cross-sectional area at exit and throat,

$$\frac{A_3}{A_2} = \frac{0.00133m}{0.000786m} = 1.7$$

13. Dry saturated steam at a pressure of 11 bar enters a convergent divergent nozzle and leaves at at pressure of 2 bar. If the flow is adiabatic and frictionless, determine

i) The exit velocity of steam and

ii) Ratio of cross section of exit and that at throat.

From steam tables, corresponding to a pressure of 10 bar, we find that enthalpy or total heat of dry saturated steam,

$$h_1 = h_{g1} = 2776.2 \text{ kJ/kg}$$

and

corresponding to a pressure of 0.1 bar,

$$h_{f2} = 191.8 \text{ kJ/kg, and } h_{fg2} = 2392.9 \text{ kJ/kg}$$

∴ Enthalpy or total heat of steam of exit,

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$= 191.8 + 0.791 \times 2392.2$$

$$= 2084.6 \text{ kJ/kg}$$

and heat drop, $h_d = h_1 - h_2 = 2776.2 - 2084.6$

$= 691.6$ kJ/kg