Chapter-3 Flow in Closed Conduits (Pipes)

3.0 PARAMETERS INVOLVED IN THE STUDY OF FLOW THROUGH CLOSED CONDUITS

In the previous chapter, the energy level changes along the flow was discussed. The losses due to wall friction in flows was not discussed. In this chapter the determination of drop in pressure in pipe flow systems due to friction is attempted.

Fluids are conveyed (transported) through closed conduits in numerous industrial processes. It is found necessary to design the pipe system to carry a specified quantity of fluid between specified locations with minimum pressure loss. It is also necessary to consider the initial cost of the piping system.

The flow may be laminar with fluid flowing in an orderly way, with layers not mixing macroscopically. The momentum transfer and consequent shear induced is at the molecular level by pure diffusion. Such flow is encountered with very viscous fluids. Blood flow through the arteries and veins is generally laminar. Laminar condition prevails upto a certain velocity in fluids flowing in pipes.

The flow turns turbulent under certain conditions with macroscopic mixing of fluid layers in the flow. At any location the velocity varies about a mean value. Air flow and water flow in pipes are generally turbulent.

The flow is controlled by (*i*) pressure gradient (*ii*) the pipe diameter or hydraulic mean diameter (*iii*) the fluid properties like viscosity and density and (*iv*) the pipe roughness. The velocity distribution in the flow and the state of the flow namely laminar or turbulent also influence the design. Pressure drop for a given flow rate through a duct for a specified fluid is the main quantity to be calculated. The inverse-namely the quantity flow for a specified pressure drop is to be also worked out on occasions.

The basic laws involved in the study of incompressible flow are (*i*) Law of conservation of mass and (*ii*) Newton's laws of motion.

Besides these laws, modified Bernoulli equation is applicable in these flows.

3.1 BOUNDARY LAYER CONCEPT IN THE STUDY OF FLUID FLOW

When fluids flow over surfaces, the molecules near the surface are brought to rest due to the viscosity of the fluid. The adjacent layers also slow down, but to a lower and lower extent. This slowing down is found limited to a thin layer near the surface. The fluid beyond this layer is not affected by the presence of the surface. The fluid layer near the surface in which there is a general slowing down is defined as boundary layer. The velocity of flow in this layer increases from zero at the surface to free stream velocity at the edge of the boundary layer. The development of the boundary layer in flow over a flat plate and the velocity distribution in the layer are shown in Fig. 3.1.1.

Pressure drop in fluid flow is to overcome the viscous shear force which depends on the velocity gradient at the surface. Velocity gradient exists only in the boundary layer. The study thus involves mainly the study of the boundary layer. The boundary conditions are (*i*) at the wall surface, (zero thickness) the velocity is zero. (*ii*) at full thickness the velocity equals the free stream velocity (*iii*) The velocity gradient is zero at the full thickness. Use of the concept is that the main analysis can be limited to this layer.

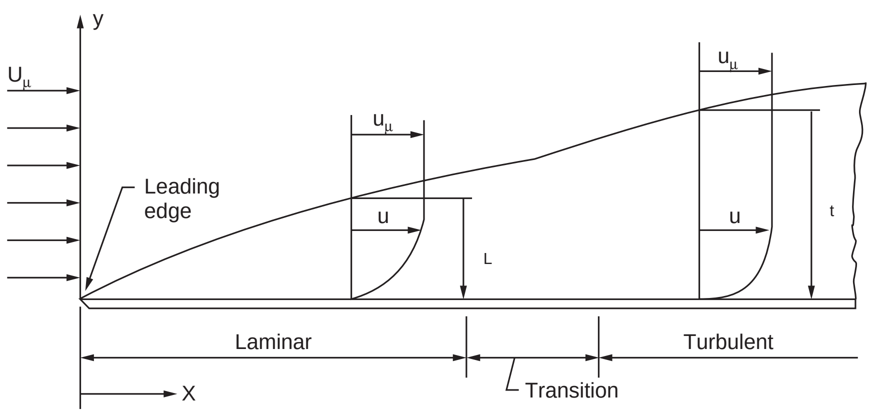


Figure 3.1.1 Boundary Layer Development (flat-plate)

BOUNDARY LAYER DEVELOPMENT OVER A FLAT PLATE

The situation when a uniform flow meets with a plane surface parallel to the flow is shown in Fig. 3.1.1. At the plane of entry (leading edge) the velocity is uniform and equals free stream velocity. Beyond this point, the fluid near the surface comes to rest and adjacent layers are retarded to a larger and larger depth as the flow proceeds.

The thickness of the boundary layer increases due to the continuous retardation of flow. The flow initially is laminar. There is no intermingling of layers. Momentum transfer is at the molecular level, mainly by diffusion. The viscous forces predominate over inertia forces. Small disturbances are damped out. Beyond a certain distance, the flow in the boundary layer becomes turbulent with macroscopic mixing of layers. Inertia forces become predominant. This change occurs at a value of Reynolds number (given Re = ux/v, where v is the kinematic viscosity) of about 5×10^5 in the case of flow over flat plates. Reynolds number is the ratio of inertia and viscous forces. In the turbulent region momentum transfer and consequently the shear forces increase at a more rapid rate.

3.3 DEVELOPMENT OF BOUNDARY LAYER IN CLOSED CONDUITS (PIPES)

In this case the boundary layer develops all over the circumference. The initial development of the boundary layer is similar to that over the flat plate. At some distance from the entrance, the boundary layers merge and further changes in velocity distribution becomes impossible. The velocity profile beyond this point remains unchanged. The distance upto this point is known as entry length. It is about 0.04 Re × *D*. The flow beyond is said to be fully developed. The velocity profiles in the entry region and fully developed region are shown in Fig. 3.3.1*a*. The laminar or turbulent nature of the flow was first investigated by Osborn Reynolds in honour of

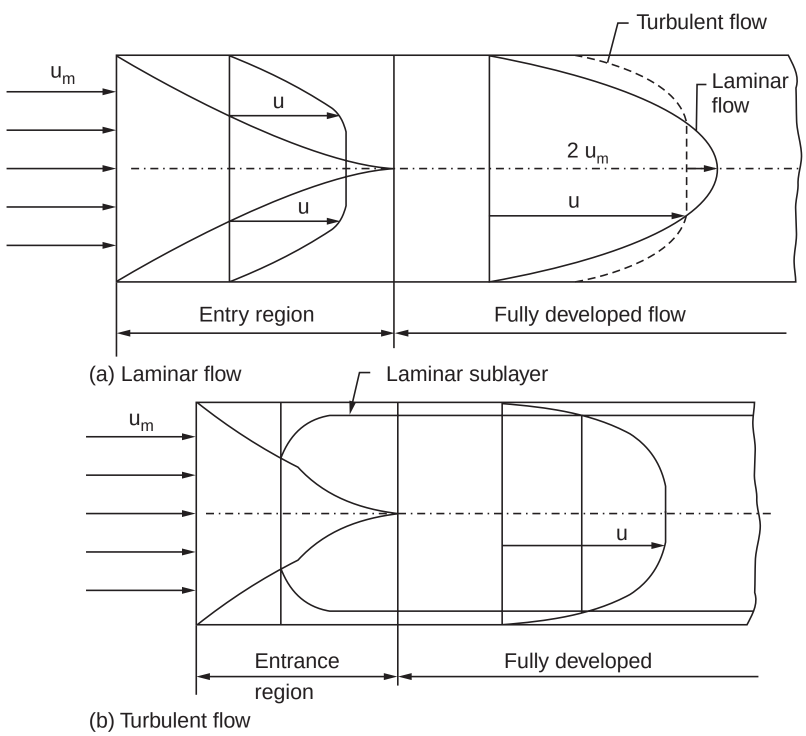


Figure 3.3.1 Boundary layer development (pipe flow)

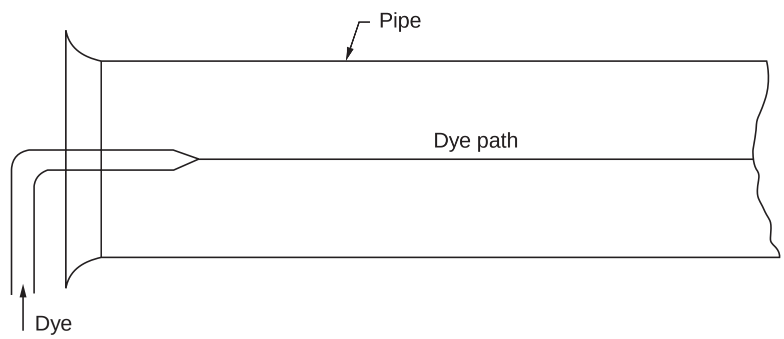
whom the dimensionless ratio of inertia to viscous forces is named. The flow was observed to be laminar till a Reynolds number value of about 2300. The Reynolds number is calculated on the basis of diameter (ud/v). In pipe flow it is not a function of length. As long as the diameter is constant, the Reynolds number depends on the velocity for a given flow. Hence the value of velocity determines the nature of flow in pipes for a given fluid. The value of the flow Reynolds number is decided by the diameter and the velocity and hence it is decided at the entry itself. The development of boundary layer in the turbulent range is shown in Fig. 2.3.1*b*. In this case, there is a very short length in which the flow is laminar. This length, x, can be calculated using the relation $ux/v = 2000$. After this length the flow in the boundary layer turns turbulent. A very thin laminar sublayer near the wall in which the velocity gradient is linear is present all through. After some length the boundary layers merge and the flow becomes fully developed. The entry length in turbulent flow is about 10 to 60 times the diameter.

The velocity profile in the fully developed flow remains constant and is generally more flat compared to laminar flow in which it is parabolic.

3.4 FEATURES OF LAMINAR AND TURBULENT FLOWS

In laminar region the flow is smooth and regular. The fluid layers do not mix macroscopically (more than a molecule at a time). If a dye is injected into the flow, the dye will travel along a straight line. Laminar flow will be maintained till the value of Reynolds number is less than of the critical value (2300 in conduits and 5×10^5 in flow over plates). In this region the viscous forces are able to damp out any disturbance.

The friction factor, f for pipe flow defined as $4\tau_s/(\rho u^2/2g_o)$ is obtainable as $f = 64/\text{Re}$ where τ_s is the wall shear stress, u is the average velocity and Re is the Reynolds number. In the case of flow through pipes, the average velocity is used to calculate Reynolds number. The dye path is shown in Fig. 7.4.1.



In turbulent flow there is considerable mixing between layers. A dye injected into the flow will quickly mix with the fluid. Most of the air and water flow in conduits will be turbulent. Turbulence leads to higher frictional losses leading to higher pressure drop. The friction factor is given by the following empirical relations.

$$f = 0.316/\text{Re}^{0.25} \quad \text{for Re} < 2 \times 10^4 \tag{3.4.1}$$

$$f = 0.186/\text{Re}^{0.2} \quad \text{for Re} > 2 \times 10^4$$

These expressions apply for smooth pipes. In rough pipes, the flow may turn turbulent below the critical Reynolds number itself. The friction factor in rough pipe of diameter D, with a roughness height of ε, is given by

$$f = 1.325/[\ln \{(\varepsilon/3.7D) + 5.74/\mathrm{Re}^{0.9}\}]^2$$

3.5 HYDRAULICALLY "ROUGH" AND "SMOOTH" PIPES

In turbulent flow, a thin layer near the surface is found to be laminar. As no fluid can flow up from the surface causing mixing, the laminar nature of flow near the surface is an acceptable assumption. The thickness of the layer δ_l is estimated as

$$\delta_l = 32.8v/u\sqrt{f} \tag{3.5.1}$$

If the roughness height is ε and if $\delta_1 > 6\varepsilon$, then the pipe is considered as hydraulically smooth. Any disturbance caused by the roughness is within the laminar layer and is smoothed out by the viscous forces. So the pipe is hydraulically smooth. If $\delta_l < 6\varepsilon$, then the pipe is said to be hydraulically rough. The disturbance now extends beyond the laminar layer. Here the inertial forces are predominant. So the disturbance due to the roughness cannot be damped out. Hence the pipe is hydraulically rough.

It may be noted that the relative value of the roughness determines whether the surface is hydraulically rough or smooth.

3.6 CONCEPT OF "HYDRAULIC DIAMETER": (D_h)

The frictional force is observed to depend on the area of contact between the fluid and the surface. For flow in pipes the surface area is not a direct function of the flow. The flow is a direct function of the sectional area which is proportional to the square of a length parameter. The surface area is proportional to the perimeter. So for a given section, the hydraulic diameter which determines the flow characteristics is defined by equation 3.6.1 and is used in the calculation of Reynolds number.

$$D_h = 4A/P \tag{3.6.1}$$

where D_h is the hydraulic diameter, A is the area of flow and P is the perimeter of the section. This definition is applicable for any cross section. For circular section $D_h = D$, as the equals $(4\pi D^2/4\pi D)$. For flow through ducts the length parameter in Reynolds number is the hydraulic diameter. *i.e.,*

$$\mathrm{Re} = D_h \times u/v \tag{3.6.2}$$

Example 3.1 *In model testing, similarity in flow through pipes will exist if Reynolds numbers are equal. Discuss how the factors can be adjusted to obtain equal Reynolds numbers.*

Reynolds number is defined as $Re = uD\rho/\mu$. For two different flows

$$\frac{u_1 D_1 \rho_1}{\mu_1} = \frac{u_2 D_2 \rho_2}{\mu_2} \quad \text{or} \quad \frac{u_1 D_1}{v_1} = \frac{u_2 D_2}{v_2}$$

As the kinematic viscosities v_1 and v_2 are fluid properties and cannot be changed easily (except by changing the temperature) the situation is achieved by manipulating $u_2 D_2$ and $u_1 D_1$

$$\frac{v_2}{v_1} = \frac{u_2 D_2}{u_1 D_1} \quad \text{(A)}$$

this condition should be satisfied for flow similarity in ducts. Reynolds number will increase directly as the velocity, diameter and density. It will vary inversely with the dynamic viscosity of the fluid.

Reynolds number can be expressed also by **Re = G.D**/μ where G is the mass velocity in kg /m^{2}s. So Reynolds number in a given pipe and fluid can be increased by increasing mass velocity. For example if flow similarity between water and air is to be achieved at 20 °C then (using v values in eqn. A)

$$\frac{1.006 \times 10^{-6}}{15.06 \times 10^{-6}} = \frac{\text{Velocity of water} \times \text{diameter in water flow}}{\text{Velocity of air} \times \text{diameter of air flow}}$$

If diameters are the same, the air velocity should be about 15 times the velocity of water for flow similarity. If velocities should be the same, the diameter should be 15 times that for water. For experiments generally both are altered by smaller ratios to keep $u \times D$ constant.

3.7 VELOCITY VARIATION WITH RADIUS FOR FULLY DEVELOPED LAMINAR FLOW IN PIPES

In pipe flow, the velocity at the wall is zero due to viscosity and the value increases as the centre is approached. The variation if established will provide the flow rate as well as an average velocity.

Consider an annular element of fluid in the flow as shown in Fig. 3.7.1*a*. The dimensions are: inside radius = r; outside radius = $r + dr$, length = dx.

Surface area = $2\pi r dx$

Assuming steady fully developed flow, and using the relationship for force balance, the velocity being a function of radius only.

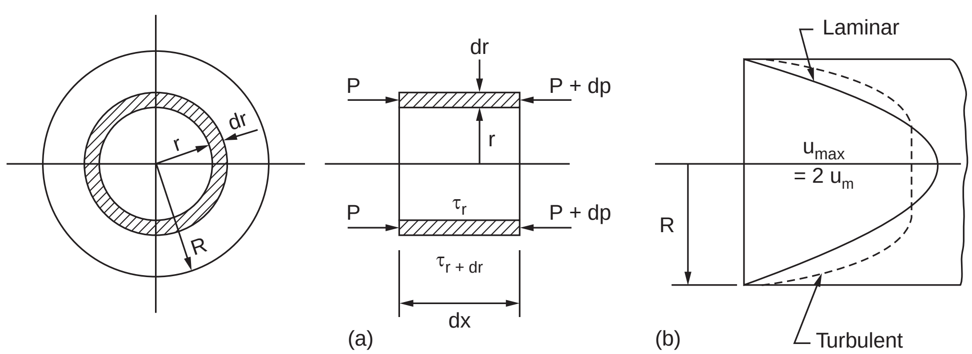


Figure 3.7.1

Net pressure force = $dp\ 2\pi\ rdr$

Net shear force $= \frac{d}{dr}\left(\mu \frac{du}{dr} 2\pi r dx\right) dr$, Equating the forces and reordering

$$\frac{d}{dr}\left(r \frac{du}{dr}\right) = \frac{1}{\mu}\frac{dp}{dx} r$$

Integrating $\quad r\frac{du}{dr} = \frac{1}{\mu}\frac{dp}{dx}\frac{r^2}{2} + C$, at $r = 0 \quad \therefore \quad C = 0$

Integrating again and after simplification,

$$u = \frac{1}{\mu}\frac{dp}{dx}\frac{r^2}{4} + B$$

at $r = R$, $u = 0$ (at the wall)

$$\therefore \quad B = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}$$

$$\therefore \quad u = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

The velocity is maximum at $r = 0$,

$$\therefore \quad u_{max} = -\frac{1}{\mu}\frac{dp}{dx}\frac{R^2}{4} \tag{3.72}$$

At a given radius, dividing 3.7.1 by (7.7.2), we get 3.7.3, which represents parabolicdistribution.

$$\therefore \quad \frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2$$

If the average velocity is u_{mean} then the flow is given by $Q = \pi R^2 u_{mean}$ (A)

The flow Q is also given by the integration of small annular flow streams as in the element considered

$$Q = \int_0^R 2\pi u r dr \text{ but } u = u_{max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Substituting and integrating between the limits 0 to R, and using equation A

$$Q = \frac{\pi R^2}{u} u_{max} = \pi R^2 u_{mean} \quad \therefore \quad 2\ u_{mean} = u_{max}$$

The average velocity is half of the maximum velocity

$$\therefore \quad \frac{u}{u_{mean}} = 2\left[1 - \left(\frac{r}{R}\right)^2\right]$$

In turbulent flow the velocity profile is generally represented by the equation

$$\frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{(1/n)}$$, where n varies with Reynolds number.

The average velocity is 0.79 u_{max} for n = 6 and 0.87 u_{max} for n = 10.

3.8 DARCY–WEISBACH EQUATION FOR CALCULATING PRESSURE DROP

In the design of piping systems the choice falls between the selection of diameter and the pressure drop. The selection of a larger diameter leads to higher initial cost. But the pressure drop is lower in such a case which leads to lower operating cost. So in the process of design of piping systems it becomes necessary to investigate the pressure drop for various diameters of pipe for a given flow rate. Another factor which affects the pressure drop is the pipe roughness. It is easily seen that the pressure drop will depend directly upon the length and inversely upon the diameter. The velocity will also be a factor and in this case the pressure drop will depend in the square of the velocity (refer Bernoulli equation).

Hence we can say that

$$\Delta p \propto \frac{LV^2}{2D} \qquad (3.8.1)$$

The proportionality constant is found to depend on other factors. In the process of such determination Darcy defined or friction factor f as

$$f = 4\,\tau_0/(\rho u_m^{\,2}/2g_0) \qquad (3.8.2)$$

This quantity is dimensionless which may be checked.

Extensive investigations have been made to determine the factors influencing the friction factor.

It is established that in laminar flow f depends only on the Reynolds number and it is given by

$$f = \frac{64}{\text{Re}} \qquad (3.8.3)$$

In the turbulent region the friction factor is found to depend on Reynolds number for smooth pipes and both on Reynolds number and roughness for rough pipes. Some empirical equations are given in section 3.4 and also under discussions on turbulent flow. The value of friction factor with Reynolds number with roughness as parameter is available in Moody diagram, given in the appendix. Using the definition of Darcy friction factor and conditions of equilibrium, expression for pressure drop in pipes is derived in this section. Consider an elemental length L in the pipe. The pressures at sections 1 and 2 are P_1 and P_2.

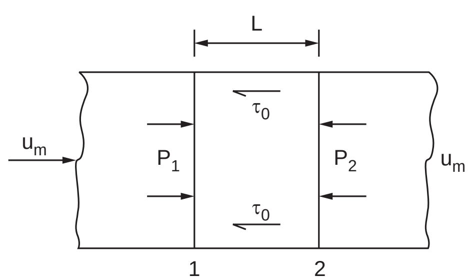


Figure 3.8.1

The other force involved on the element is the wall shear τ_0.

Net pressure force in the element is $(P_1 - P_2)$

Net shear force in the element is $\tau_0 \pi DL$

Force balance for equilibrium yields

$$(P_1 - P_2) \frac{\pi D^2}{4} = \tau_0 \pi DL \tag{3.8.4}$$

From the definition friction factor

$$f = 4 \tau_0 / (\rho u_m^2 / 2 g_0)$$

$$\tau_0 = \frac{f \rho u_m^2}{8 g_0}$$

Substituting and letting $(P_1 - P_2)$ to be ΔP.

$$\Delta P . \frac{\pi D^2}{4} = \frac{f \rho u_m^2}{8 g_0} . \pi DL$$

This reduces to $\quad \Delta P = \dfrac{f L u_m^2 \rho}{2 g_0 D}$

This equation known as Darcy-Weisbach equation and is generally applicable in most of the pipe flow problems. As mentioned earlier, the value of f is to be obtained either from equations or from Moody diagram. The diameter for circular tubes will be the hydraulic diameter D_h defined earlier in the text.

It is found desirable to express the pressure drop as head of the flowing fluid.

In this case as $\quad h = \dfrac{P}{\gamma} = \dfrac{P g_0}{\rho g}$

$$\Delta h = h_f = \frac{f L u_m^2}{6)2g D} \qquad (3.3.8.$$

The velocity term can be replaced in terms of volume flow and the equation obtained is found useful in designs as Q is generally specified in designs.

$$u_m = \frac{4 Q}{\pi D^2}, u_m^2 = \frac{16 Q^2}{\pi^2 D^4}$$

Substituting in (7.8.6), we get

$$h_f = \frac{8 f L Q^2}{\pi^2 g D^5}$$

It is found that $h_f \propto \dfrac{Q^2}{D^5}$

Another coefficient of friction C_f is defined as $C_f = f/4$

In this case $$h_f = \frac{4C_f\ L\,u_m^{\ 2}}{2\,g\,D}$$

Now a days equation 7.8.5 are more popularly used as value of *f* is easily available.

3.9 HAGEN–POISEUILLE EQUATION FOR FRICTION DROP

In the case of laminar flow in pipes another equation is available for the calculation of pressure drop. The equation is derived in this section.

Refer to section (7.7) equation (7.7.1)

$$u = -\frac{1}{\mu}\frac{dp}{dL}\frac{R^2}{4}\left[1-\left(\frac{r}{R}\right)^2\right]$$

$\frac{dP}{dL}$ can be approximated to $\Delta\,P/L$ as the pressure drop is uniform along the length L under steady laminar flow

Using eqn (7.7.2), $$u_{max} = -\frac{dP}{dL}\frac{1}{\mu}\cdot\frac{R^2}{4} = 2u_m$$

$$\therefore \quad -\frac{dP}{dL} = \frac{8\,u_m\,\mu}{R^2}$$

$$\therefore \quad -\frac{dP}{dL} = \frac{8u_m\mu}{R^2} = \frac{32u_m\mu}{D^2}$$, Substituting for $-\frac{dP}{dL}$ as $\frac{\Delta P}{L}$

$$\Delta P = \frac{32\,\mu\,u_m L}{D^2} \qquad (3.9.1)$$

This can also be expressed in terms of volume flow rate Q as

$$Q = \frac{\pi\,D^2}{4}\,.\,u_m$$

$\therefore \quad u_m = 4Q/\pi D^2$, substituting

$$\Delta P = 128\,\mu\,L\,Q/\pi\,D^4 \qquad (3.9.2)$$

Converting ΔP as head of fluid

$$h_f = \frac{32\,\upsilon u_m\,Lg_0}{gD^2}$$

This equation is known as Hagen-Poiseuille equation

g_0 is the force conversion factor having a value of unity in the SI system of unit. Also $(\mu / \rho) = \nu$.

Equations 3.9.1, 3.9.2 and 3.9.3 are applicable for laminar flow only whereas Darcy-Weisbach equation (3.8.6) is applicable for all flows

Example 3.2 *Using the Darcy-Weisbach equation and the Hagen Poiseuille equation obtain an expression for friction factor f, in terms of Reynolds number in laminar region.*

The equation are $h_f = \dfrac{fLu_m^{\,2}}{2gD}$ and $h_f = \dfrac{32\,u_m\,vL}{gD^2}$

equating and simplifying a very useful relationship is obtained, namely

$$f = \frac{2 \times 32\,\nu}{u_m D} = \frac{64}{\text{Re}}, \text{ as } \left(\frac{u_m D}{v}\right) = \text{Re}$$

In the laminar flow region the friction factor can be determined directly in terms of Reynolds number.

3.10 SIGNIFICANCE OF REYNOLDS NUMBER IN PIPE FLOW

Reynolds number is the ratio of inertia force to viscous force. The inertia force is proportional to the mass flow and velocity *i.e.,* ($\rho u.u$). The viscous force is proportional to $\mu(du/dy)$ or $\mu\, u/D$, dividing

$$\frac{\text{inertia force}}{\text{viscous force}} = \frac{\rho uuD}{\mu u} = \frac{\rho uD}{\mu} = \frac{uD}{v}$$

Viscous force tends to keep the layers moving smoothly one over the other. Inertia forces tend to move the particles away from the layer. When viscous force are sufficiently high so that any disturbance is smoothed down, laminar flow prevails in pipes. When velocity increases, inertia forces increase and particles are pushed upwards out of the smoother path. As long as Reynolds number is below 2,300, laminar flow prevails in pipes. The friction factor in flow is also found to be a function of Reynolds number (in laminar flow, f = 64/Re).

Example 3.3. *Lubricating Oil at a velocity of 1 m/s (average) flows through a pipe of 100 mm ID. Determine* ***whether the flow is laminar or turbulent.*** *Also* ***determine the friction factor*** *and the pressure drop over 10 m length. What should be the velocity for the flow to turn turbulent? Density = 930 kg/m³ . Dynamic viscosity μ = 0.1 Ns/m² (as N/m² is call Pascal, μ can be also expressed as Pa.s).*

$$\textbf{Re} = \frac{uD\rho}{\mu} = \frac{1 \times 0.1 \times 930}{1 \times 0.1} = \textbf{930, so the flow is laminar}$$

Friction factor, **f** = 64/930 = **0.06882**

$\mathbf{h_f} = f\,L\,u_m^{\,2}/2gD = (64/930) \times 10 \times 1^2/(2 \times 9.81 \times 0.1)$

= **0.351 m head of oil.**

or $\Delta P = 0.351 \times 0.93 \times 9810 =$ **3200 N/m²**

At transition Re = 2000 (can be taken as 2300 also)

Using (7.9.1) (Hagen-Poiseuille eqn.)

$$\mathbf{\Delta P} = \frac{32 \times \mu \times u_m \times L}{D^2} = \frac{32 \times 0.1 \times 1 \times 10}{0.1^2}$$

= **3200 N/m².** (same as by the other equation)

To determine velocity on critical condition

$$2300 = 4\ m \times 0.1 \times 930/0.1$$

$\therefore$ $$\mathbf{u_m = 2.47\ m/s.}$$

3.11 VELOCITY DISTRIBUTION AND FRICTION FACTOR FOR TURBULENT FLOW IN PIPES

The velocity profile and relation between the mean and maximum velocity are different in the two types of flow. In laminar flow the velocity profile is parabolic and the mean velocity is half of the maximum velocity. Such a relation is more complex in turbulent flow. For example one such available relation is given by

$$\frac{u_m}{u_{max}} = \frac{1}{1 + 1.33\sqrt{f}} \qquad (3.11.1)$$

The friction factor f is a complex function of Reynolds number.

A sample velocity variation is given in equation (7.12.2).

$$u = (1 + 1.33\sqrt{f})\ u_m - 2.04\sqrt{f}\ u_m \log (R/(R-1)) \qquad (3.11.2).$$

For higher values of f the velocity variation will be well rounded at the centre compared to low values of f.

A new reference velocity called shear velocity is defined as below.

$$u^* = \sqrt{\frac{\tau_0\ g_0}{\rho}} \qquad (3.11.3)$$

Several other correlation using the reference velocity are listed below.

$$\frac{u}{u^*} = 5.75 \log \frac{Ru^*}{\nu} + 5.5 \qquad (3.11.4)$$

For rough pipes,

$$\frac{u}{u^*} = 5.75 \ln \frac{(R-r)}{\varepsilon} + 8.5 \qquad (3.11.5)$$

where ε is the roughness dimension.

The mean velocity u_m is obtained for smooth and rough pipes as

$$\frac{u_m}{u^*} = 5.75 \log \frac{(R-r)\,u^*}{\nu} + 7.5 \qquad (3.11.6)$$

and $$\frac{u_m}{u^*} = 5.75 \log \frac{R}{\varepsilon} + 4.75 \qquad (3.11.7)$$

The laminar sublayer thickness is used for defining smooth pipe. The thickness of this layer is given by

$$\delta_t = 11.6\ \nu/u^*$$

In the case of turbulent show the wall shear force is given by the following equation.

$$\tau_0 = \frac{f}{4} \cdot \frac{\rho u_m^2}{2}$$

Similar to velocity profile, several correlations are available for friction factor. These correlations together with correlations for velocity profile are useful in numerical methods of solution.

The friction factor for very smooth pipes can be calculated by assuming one seventh power law leading to,

$$f = 0.316/\text{Re}^{0.25} \text{ for Re} < 2 \times 10^4 \tag{3.11.10}$$

For all ranges either of the following relations can be used

$$f = 0.0032 + (0.221/\text{Re}^{0.237}) \tag{3.11.11}$$

$$1/\sqrt{f} = 1.8 \log \text{Re} - 1.5186 \tag{3.11.12}$$

For rough pipes of radius R

$$\frac{1}{\sqrt{f}} = 2 \log \frac{R}{\varepsilon} + 1.74 \tag{3.11.13}$$

Charts connecting f, Re and ε/D are also available and can be used without appreciable error. As in laminar flow the frictional loss of head is given by

$$h_f = f L u_m^2/2 g D_h$$

Also $$h_f = \frac{8 fL}{g\pi^2} \frac{Q^2}{D^5}$$

The value of f is to be determined using the approximate relations or the chart.

3.12 MINOR LOSSES IN PIPE FLOW

Additional frictional losses occur at pipe entry, valves and fittings, sudden decrease or increase in flow area or where direction of flow changes.

The frictional losses other than pipe friction are called minor losses. In a pipe system design, it is necessary to take into account all such losses.

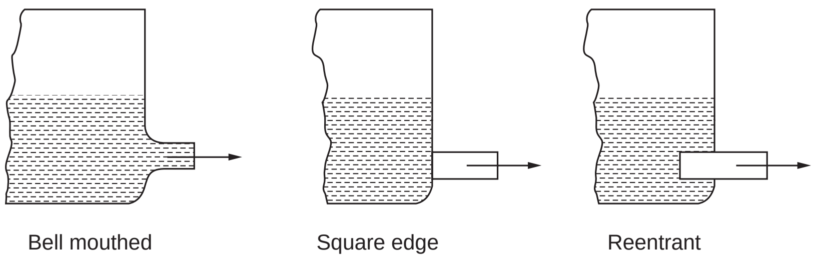
These losses are generally expressed as $h_f = C u_m^2/2g$ where C is constant, the value of which will depend on the situation and is called the loss coefficient. The expression is applicable both for laminar and turbulent flows.

(*i*) Loss of head at entrance: At the entrance from the reservoir into the pipe, losses take place due to the turbulence created downstream of the entrance. Three types of entrances are known.

(a) Bell mouthed: This is a smooth entrance and turbulence is suppressed to a great extent and $C = 0.04$ for this situation.

(b) Square edged entrance: Though it is desirable to provide a bell mouthed entrance it will not be always practicable. Square edged entrance is used more popularly. The loss coefficient, $C = 0.5$ in this case.

(c) Reentrant inlet: The pipe may sometimes protrude from the wall into the liquid. Such an arrangement is called reentrant inlet. The loss coefficient in this case is about 0.8.



(*ii*) Loss of head at submerged discharge: When a pipe with submerged outlet discharges into a liquid which is still (not moving) whole of the dynamic head $u^2/2g$ will be lost. The loss coefficient is 1.0. The discharge from reaction turbines into the tail race water is an example. The loss is reduced by providing a diverging pipe to reduce the exit velocity.

(*iii*) Sudden contraction: When the pipe section is suddenly reduced, loss coefficient depends on the diameter ratio. The value is 0.33 for $D_2/D_1 = 0.5$. The values are generally available in a tabular statement connecting D_2/D_1 and loss coefficient. Gradual contraction will reduce the loss. For gradual contraction it varies with the angle of the transition section from 0.05 to 0.08 for angles of 10° to 60°.

(*iv*) Sudden expansion: Here the sudden expansion creates pockets of eddying turbulence leading to losses. The loss of head h_f is given by

$$\text{Loss of head} = (u_1 - u_2)^2 / 2g. \tag{3.12.1}$$

where u_1 and u_2 are the velocities in the smaller and larger sections. Gradual expansion will reduce the losses.

(*v*) Valves and fittings : Losses in flow through valves and fittings is expressed in terms of an equivalent length of straight pipe.

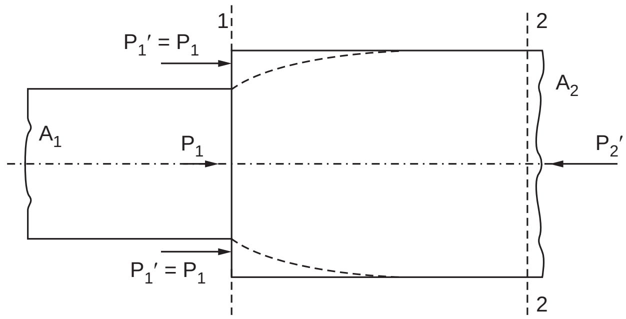
For gate valves $L = 8D$, and for globe valves it is 340 D. For 90° bends it is about 30 D.

3.13 EXPRESSION FOR THE LOSS OF HEAD AT SUDDEN EXPANSION IN PIPE FLOW

The situation is shown in Fig 3.13.1.

Using Bernoulli equation and denoting the ideal pressure at section 2 as P_2 (without losses), datum remaining unaltered,

$$\frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{u_1^2}{2g} - \frac{u_2^2}{2g} \quad \text{or} \quad \frac{P_2}{\rho} = \frac{P_1}{\rho} + \frac{u_1^2}{2} - \frac{u_2^2}{2} \tag{1}$$



Applying conservation of momentum principle to the fluid between section 1 and 2, and denoting the actual pressure at section 2 as P_2,

The pressure forces are (here the pressure on the annular section of fluid at 1 is assumed as P_1)

$$(P_1 A_1 - P_2' A_2).$$

The change in momentum is given by

$$(\rho A_2 u_2 u_2 - \rho A_1 u_1 u_1)$$

noting $A_1 u_1 = A_2 u_2$, replacing $A_1 u_1$ by $A_2 u_2$ and equating the net forces on the element to the momentum change,

$$P_1 A_1 - P_2' A_2 = \rho A_2 u_2^2 - \rho A_2 u_2 u_1$$

Dividing by ρ and A_2 allthrough

$$\frac{P_1}{\rho} - \frac{P_2'}{\rho} = u_2^2 - u_1 u_2$$

or

$$\frac{P_2'}{\rho} = \frac{P_1}{\rho} - (u_2^2 - u_1 u_2) \qquad (2)$$

Subtracting on either side of equations 1 and 2 (ideal and real)

$$\frac{P_2 - P_2'}{\rho} = \frac{P_1}{\rho} + \frac{u_1^2}{2} - \frac{u_2^2}{2} - \frac{P_1}{\rho} - (u_2^2 - u_1 u_2)$$

$$\therefore \qquad \frac{P_2 - P_2'}{\rho} = \frac{u_1^2}{2} - \frac{u_2^2}{2} + (u_2^2 - u_1 u_2)$$

Multiplying both sides by 2

$$\frac{2(P_2 - P_2')}{\rho} = u_1^2 - u_2^2 + 2u_2^2 - 2u_1 u_2 = (u_1 - u_2)^2$$

Dividing the both sides by g and simplifying

$$\frac{P_2 - P_2'}{\rho g} = \frac{(u_2 - u_2)^2}{2g}$$

But

$$\frac{P_2 - P_2'}{\rho g} = h_f \text{ (head loss)}$$

$$\therefore \qquad h_f = \frac{(u_1 - u_2)^2}{2g}.$$

3.14 LOSSES IN ELBOWS, BENDS AND OTHER PIPE FITTINGS

Fittings like valves, elbows etc. introduce frictional losses either by obstruction or due to secondary flows. The losses may be accounted for by a term equivalent length which will depend on the type of fitting or in terms of $(u^2/2g)$ or dynamic head. In the case of bends, the loss is due to the variation of centrifugal force along different stream lines which causes secondary flows. In large bends fitting curved vanes will reduce the loss. The loss will vary with radius of the bend. Globe valves are poorer compared to gate valves with regard to pressure drop.

3.15 ENERGY LINE AND HYDRAULIC GRADE LINE IN CONDUIT FLOW

The plot of the sum of pressure head and dynamic head along the flow path is known as energy line. This refers to the total available energy of the system at the location. The line will dip due to losses. For example in straight constant area pipe the line will slope proportional to the head drop per m length. There will be sudden dips if there are minor losses due to expansions, fittings etc.

Hydraulic grade line is the plot of pressure head along the flow path. Hydraulic grade line will be at a lower level and the difference between the ordinates will equal the dynamic head *i.e.*, $u^2/2g$. This line will dip sharply if velocity increases and will slope upwards if velocity decreases. This line will also dip due to frictional losses. Flow will be governed by hydraulic grade line.

Introduction of a pump in the line will push up both the lines. Specimen plot is given in Fig. 3.15.1 (pump is not indicated in figure).

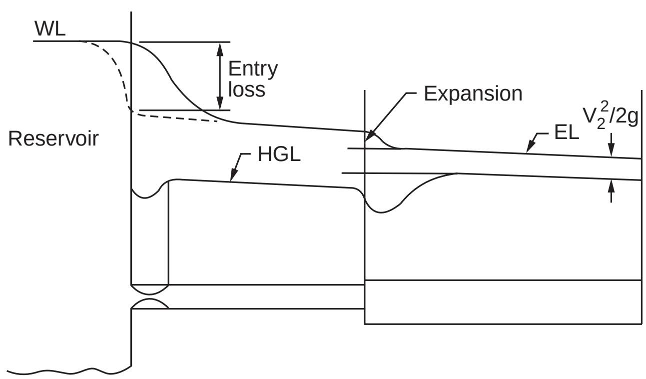


Figure 3.15.1 Energy and Hydraulic grade lines

Example 3.4. *A pump takes in water from a level 5 m below its centre line and delivers it at a height of 30 m above the centre line, the rate of flow being 3 m³/hr. The diameter of the pipe line allthrough is 50 mm (ID). The fittings introduce losses equal to 10 m length of pipe in addition to the actual length of 45 m of pipe used.* ***Determine the head to be developed by the pump****.*

The head to be developed will equal the static head, friction head and the dynamic head.

Static head = 30 + 5 = 35 m, Friction head = $f L u_m^2/2g D$,

$$L = 45 + 10 = 55 \text{ m}, D = 0.05 \text{ m},$$

$$u_m = (3/3600)\ 4/\pi \times 0.05^2 = 0.4244 \text{ m/s}.$$

Assuming the temperature as 20 °*C*,

$$v = 1.006 \times 10^{-6} \text{ m}^2/\text{s}.$$

∴ $$\text{Re} = u_m D/v = 0.05 \times 0.4244/1.006 \times 10^{-6} = 21093$$

∴ The flow is turbulent

Assuming smooth pipe, $f = 0.316/\text{Re}^{0.25} = 0.316/21093^{0.25} = 0.02622$

(check $f = 0.0032 + 0.221/\text{Re}^{0.237} = 0.0241$), using the value 0.02622,

Frictional loss of head $= 0.02622 \times 55 \times 0.4244^2/2 \times 9.81 \times 0.05 = 0.265$ m

Dynamic head $= u_m^2/2g = \dfrac{0.4244^2}{2 \times 9.81} = 0.0092$ m

∴ Total head $= 35 + 0.265 + 0.0092 = 35.2562$ m.

Head to be developed by the pump = **35.2562 m head of water.**

Example 3.5 *A pipe 250 mm dia, 4000 m long with f = 0.021 discharges water from a reservoir at a level 5.2 m below the water reservoir level.* ***Determine the rate of discharge.***

The head available should equal the sum of frictional loss and the dynamic head.

Frictional Head = $f L u_m^2/2g D$, Dynamic Head = $u_m^2/2g$

When long pipes are involved, minor losses may often be neglected.

$$5.2 = [(0.021 \times 4000 \times u_m^2)/(2 \times 9.81 \times 0.25)] + [u_m^2 / (2 \times 9.81)]$$

$$= 17.176\ u_m^2$$

∴ $$u_m = 0.55 \text{ m/s}.$$

∴ **Flow rate** = $(\pi \times 0.25^2/4) \times 0.55 = 0.027$ m^3/s. or **97.23 m^3/hr.**

3.16 CONCEPT OF EQUIVALENT LENGTH

For calculation of minor losses it is more convenient to express the pressure drop in fittings, expansion–contraction and at entry in terms of a length of pipe which will at that discharge rate lead to the same pressure drop. This length is known as equivalent length, L_e. From the relation knowing *C, f* and *D*.

$$L_e\, u_m^2/2g D = C\, u_m^2/2g \quad \therefore \quad L_e \text{ can be calculated}$$

As a number of fittings at various positions may be involved causing minor losses in a pipe system, this is a convienent way to estimate minor losses.

3.17 CONCEPT OF EQUIVALENT PIPE OR EQUIVALENT LENGTH

When pipes of different friction factors are connected in series (or in parallel) it is convenient to express the losses in terms of one of the pipes (Refer to 3.11.15). The friction loss h_f for pipe 1 with L_1 and f_1 is given by

$$h_{f1} = 8\, f_1 L_1\, Q^2/g\, \pi^2\, D_1^{\,5}$$

For the same pressure loss and flow rate Q and discharge through another pipe of diameter D_2 with f_2, the equivalent pipe will have a length L_2. Hence

$$h_{f2} = 8\, f_2 L_2\, Q^2/g\, \pi^2\, D_2^{\,5}$$

Cancelling common terms

$$\frac{f_1\, L_1}{D_1^{\,5}} = \frac{f_2\, L_2}{D_2^{\,5}} \quad \text{or} \quad L_2 = L_1 \frac{f_1}{f_2} \frac{D_2^{\,5}}{D_1^{\,5}}$$

If pipes are in series a common diameter can be chosen and the equivalent length concept can be used conveniently to obtain the solution.

In parallel arrangement, as the pressure loss is the same, then

$$\frac{8\, f_1\, L_1\, Q_1^{\,2}}{g\, \pi^2\, D_1^{\,5}} = \frac{8\, f_2\, L_2\, Q_2^{\,2}}{g\, \pi^2\, D_2^{\,5}}$$

$$\therefore \qquad \frac{Q_2}{Q_1} = \left[\frac{f_1}{f_2} \frac{L_1}{L_2} \left(\frac{D_2}{D_1}\right)^5\right]^{0.5}$$

The idea of using equivalent length thus helps to reduce tediusness in calculations.

Example 3.6. *Three pipes of 400 mm, 350 mm and 300 mm diameter* ***are connected in series*** *between two reservoirs with a difference in level of 12 m. The friction factors are 0.024, 0.021 and 0.019 respectively. The lengths are 200 m, 300 m and 250 m respectively. Determine the flow rate neglecting minor losses.*

This problem can be solved using

$$12 = \frac{8\, f_1\, L_1\, Q^2}{\pi^2\, g\, D_1^{\,5}} + \frac{8\, f_2\, L_2\, Q^2}{\pi^2\, g\, D_2^{\,5}} + \frac{8\, f_3\, L_3\, Q^2}{\pi^2\, g\, D_3^{\,5}}$$

Solving $\qquad Q^2 = 0.04 \quad \therefore \quad \mathbf{Q = 0.2\ m^3/s}$

Using equivalent length concept and choosing 0.4 m pipe as the base.

Refer 7.17.1 $\qquad L_{2e} = 300\ (0.021/0.024) \times (0.4/0.35)^5 = 511.79$ m

$L_{3e} = 250(0.019/0.024) \times (0.4/0.3)^5 = 834.02$ m

Total length $\qquad = 200 + 511.79 + 834.02 = 1545.81$ m

$12 = (8 \times 0.024 \times 1545.81 \times Q^2)/(9.81 \times \pi^2 \times 0.4^5)$

$Q^2 = 0.04$; $\mathbf{Q = 0.2\ m^2/s.}$

Example 3.7. *Two reservoirs are connected by three* ***pipes in parallel*** *with the following details of pipes:*

Pipe No.	*Length, m*	*Diameter, m*	*Friction factor*
1	600	0.25	0.021
2	800	0.30	0.019
3	400	0.35	0.024

The total flow is 24,000 *l*/min. Determine the flow in each pipe and also the level difference between the reservoirs.

Let the flows be designated as Q_1, Q_2, Q_3

Then $$Q_1 + Q_2 + Q_3 = 24000/(60 \times 1000) = 0.4 \text{ m}^3/\text{s}$$

Using equation (3.17.2), Considering pipe 1 as base

$$\frac{Q_2}{Q_1} = \left[\frac{f_1}{f_2}\frac{L_1}{L_2}\left(\frac{D_2}{D_1}\right)^5\right]^{0.5} = \left[\frac{0.021}{0.019} \times \frac{600}{800} \times \left(\frac{0.3}{0.25}\right)^5\right]^{0.5} = 1.4362$$

$\therefore$ $$Q_2 = 1.4362\ Q_1$$

$$\frac{Q_3}{Q_1} = \left[\frac{f_1}{f_3}\frac{L_1}{L_3}\left(\frac{D_3}{D_1}\right)^5\right]^{0.5} = \left[\frac{0.021}{0.024} \times \frac{600}{400} \times \left(\frac{0.35}{0.25}\right)^5\right]^{0.5} = 2.6569$$

$\therefore$ $$Q_3 = 2.6569\ Q_1$$

$$\text{Total flow} = 0.4 = Q_1 + 1.4362\ Q_1 + 2.6569\ Q_1 = 5.0931\ Q_1$$

$\therefore$ $$\mathbf{Q_1 = 0.07854\ m^3/s}$$

$\therefore$ $$\mathbf{Q_2 = 1.4362\ Q_1 = 0.11280\ m^3/s}$$

$\therefore$ $$\mathbf{Q_3 = 2.6569\ Q_1 = 0.20867\ m^3/s}$$

$$\text{Total} = 0.4001 \text{ m}^3/\text{s}$$

Head loss or level difference (Ref para 7.11, eqn 7.11.15

Pipe 1 $\mathbf{h_f} = 8\, f\, L\, Q^2/\pi^2\, g\, D^5$

$8 \times 0.021 \times 600 \times 0.07854^2/\ \pi^2 \times 9.81 \times 0.25^5 =$ **6.576 m**

Check with other pipes

Pipe 2, $h_f = 8 \times 0.019 \times 800 \times 0.1128^2/\pi^2 \times 9.81 \times 0.3^5 =$ **6.576 m**

Example 3.8. *Water is drawn from two reservoirs at the same water level through pipe 1 and 2 which join at a common point.* $D_1 = 0.4$ *m,* $L_1 = 2000$ *m,* $f_1 = 0.024$*,* $D_2 = 0.35$ *m,* $L_2 = 1500$ *m,* $f_2 = 0.021$*. The water from the common point is drawn through pipe 3 of 0.55 m dia over a length of 1600 m to the supply location. The total head available is 25.43 m. Determine the flow rate through the system. The value of* $f_3 = 0.019$*.*

Pipes 1 and 2 meet at a common location. The two reservoir levels are equal. So, the head drops are equal (refer para 7.17, eqn. 7.17.2. Let the flow in pipe 1 be Q_1 and that in pipe 2 be Q_2.

Then $$\frac{Q_2}{Q_1} = \left[\frac{f_1}{f_2}\frac{L_1}{L_2}\left(\frac{D_2}{D_1}\right)^5\right]^{0.5}$$

Here $f_1 = 0.024$, $f_2 = 0.021$, $L_1 = 2000$ m, $L_2 = 1500$ m, $D_1 = 0.4$ m, $D_2 = 0.35$ m

$\therefore$ $$Q_2 = Q_1\left[\frac{0.024}{0.021} \times \frac{2000}{1500} \times \left(\frac{0.35}{0.4}\right)^5\right]^{0.5} = 0.8841\ Q_1$$

$\therefore$ $$Q_1 + Q_2 = 1.8841\ Q_1$$

This flow goes through pipe 3. The total head drop equals the sum of the drops in pipe 1 and in pipe 3.

$$25.43 = \frac{8 \times 0.024 \times 2000\, Q_1^2}{\pi^2 \times 9.81 \times 0.4^5} + \frac{8 \times 0.019 \times 1600 \times 1.8841^2\, Q_1^2}{\pi^2 \times 9.81 \times 0.55^5}$$

$$= 387.31\, Q_1^2 + 177.17\, Q_1^2 = 564.48\, Q_1^2$$

∴ $\mathbf{Q_1 = 0.2123\ m^3/s,\ Q_2 = 0.1877\ m^3/s}$

$\mathbf{Q_3} = Q_1 + Q_2 = \mathbf{0.4\ m^3/s}$, check for pressure at common point

$$\mathbf{h_{f1}} = \frac{8 \times 0.024 \times 2000 \times 0.2123^2}{\pi^2 \times 9.81 \times 0.4^5} = \mathbf{17.46\ m}$$

$$\mathbf{h_{f2}} = \frac{8 \times 0.021 \times 1500 \times 0.1877^2}{\pi^2 \times 9.81 \times 0.35^5} = \mathbf{17.46\ m}$$

both are equal as required. Check for drop in the third pipe

$$\mathbf{h_{f3}} = \frac{8 \times 0.019 \times 1600 \times 0.4^2}{\pi^2 \times 9.81 \times 0.55^5} = \mathbf{7.99\ m}$$

Total head = 7.99 + 17.46 = **25.45 m , checks.**

3.18 FLUID POWER TRANSMISSION THROUGH PIPES

High head and medium head hydal plants convey water from a high level to the power house through pressure pipe called penstock pipes. The choices of the pipe diameter depends on the expected efficiency of transmission and also on the economical aspect of the cost of pipe. Higher efficiencies can be obtained by the use of larger diameter pipes, but this will prove to be costly. It is desirable to maximise the power transmitted as compared to an attempt to increase efficiency. Applications are also there in hydraulic drives and control equipments.

3.18.1 Condition for Maximum Power Transmission

Consider that the head available is h and the frictional loss is h_f (neglecting minor losses) left the pipe diameter be D and the flow velocity be u.

Net head available = $h - h_f$. Quantity flow = $\pi D^2 u/4$

$$h_f = f L u^2/2gD, \text{ Power = mass flow × net head}$$

Power, $$P = \frac{\pi D^2}{4} u\rho\, [h - (fLu^2/2gD)] = \frac{\pi D^2}{4} \rho \left[uh - \frac{f L u^3}{2g D} \right]$$

Differetiating P with respect to u, for maximum power,

$$\frac{dP}{du} = \frac{\pi D^2 \rho}{4} [h - 3(fLu^2/2gD)] = \frac{\pi D^2 \rho}{4} [h - 3h_f]$$

Equating to zero $\mathbf{h_f = h/3}$ (3.18.1)

For maximum power generation frictional loss will equal one third of available head and the corresponding transmission efficiency is 66.67%. If the available rate of flow is known the velocity and then the diameter can be determined or if the diameter is fixed the flow rate can be obtained. The friction factor for the pipe can be fixed as this is nearly constant above a certain value of Reynolds number. For maximum power when flow rate is specified, pipe diameter is fixed and when diameter is specified the flow rate will be fixed.

Example 3.9 *In a hydroelectric plant the head available is 450 m of water. 25 cm penstock pipe with friction factor of 0.014 is used. Determine* ***the maximum power*** *that can be developed. The length of the pipe line is 3600 m.*

Using equation 7.18.1, h_f = $h/3$ = 450/3 = 150 m

150 = (0.014 × 3600 × u^2)/(2 × 9.81 × 0.25)

solving, u = 3.82 m/s, flow rate = ($\pi D^2/4$) × u = 0.18755 m^3 /s

Power developed = $Q\rho g \times (h - h_f)$ = 0.18755 × 1000 × 9.81 (450 – 150) = 551963 *W*

= **551.963 kW**

Example 3.10. *Determine for the data in example 7.9 the* ***power transmitted for u = 4.5 m/s and u = 3 m/s.***

(*i*) u = 4.5 m/s, h_f = (0.014 × 3600 × 4.5^2)/(2 × 9.81 × 0.25) = 208.07 m

Power = (π × 0.25^2/4) × 4.5 × 1000 × 9.81 (450 – 208.07) = 524246 *W* = **524.25 kW**

(*ii*) u = 3 m/s, h_f = (0.014 × 3600 × 3^2)/(2 × 9.81 × 0.25) = 92.48 m

Power = (π × 0.25^2/4) × 3 × 1000 × 9.81 (450 – 92.48) = 516493 *W* = **516.49 kW**

This brings out clearly that the maximum power for a given diameter and head is when the frictional drop equals one third of available head.

Example 3.11 *In a hydrosystem the flow availability was estimated as 86.4 × 10^3 m^3/day. The head of fall was estimated as 600 m. The distance from the dam to the power house considering the topography was estimated as 3000 m. The available pipes have friction factor 0.014.* ***Determine the pipe diameter for transmitting maximum power,*** *and also calculate the* ***velocity and power transmitted.***

Refer Eqn 7.18.1. The frictional drop is equal to one third of available head.

∴ h_f = 600/3 = 200 m

$h_f = fLu^2/2g\,D$, Here both u and D are not specified.

But Q = area × velocity

∴ $u = 4Q/\pi D^2$ ∴ $u^2 = 16\,Q^2/\pi^2 D^4$

∴ $$h_f = \frac{fL\,16\,Q^2}{2\pi^2\,gD^5} = 200, \quad Q = 86.4 \times 10^3/(24 \times 3600) = 1\ m^3/s$$

200 = (0.014 × 3000 × 16 × 1^2)/($2\pi^2$ × 9.81) D^5,

D^5 = 0.01735, **D = 0.4445 m**

Velocity = 4 × 1/(π × 0.4445^2) = **6.444 m/s**

Power = 1000 × 9.81 × 400 = **3.924 × 10^6 *W* or 3.924 *MW***

Check for frictional loss

h_f = (0.014 × 3000 × 6.444^2)/(2 × 9.81 × 0.4445) = 200 m (checks)

3.19 NETWORK OF PIPES

Complex connections of pipes are used in city water supply as well as in industrial systems. Some of these are discussed in the para.

3.19.1 Pipes in Series—Electrical Analogy

Series flow problem can also be solved by use of resistance network. Consider equation 3.11.15. For given pipe specification the equation can be simplified as

$$h_f = 8\,f\,L\,Q^2/\pi^2 g\,D^5 = R\,Q^2$$

Note: The dimension for R is s^2/m^5. For flow in series Q is the same through all pipes. This leads to the relation

$$h_{f1} + h_{f2} + h_{f3} + \ldots\ldots\ldots\, h_{fn} = h_f = (R_1 + R_2 + R_3 + \ldots. + R_n)\,Q^2$$

The R values for the pipe can be calculated. As the total head is also known Q can be evaluated. The length L should include minor losses in terms of equivalent lengths. The circuit is shown in Fig. 3.19.1.

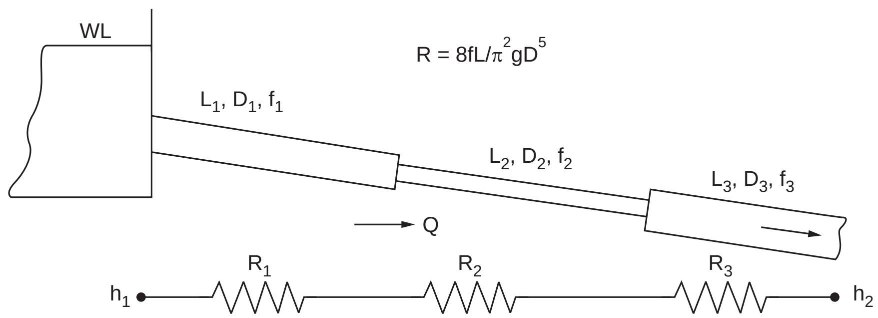


Figure 3.19.1 Equivalent circuit for series flow

Example 3.12. *A reservoir at a level with respect to datum of 16 m supplies water to a ground level reservoir at a level of 4 m. Due to constraints pipes of different diameters were to be used. Determine the flow rate.*

No.	Diameter, m	Length including minor losses, m	f
1	0.30	220	0.02
2	0.35	410	0.018
3	0.45	300	0.013
4	0.40	600	0.015

The resistance values are calculated using Eqn. 3.11.15

Pipe 1. $\mathbf{R_1} = \dfrac{8 \times 0.02 \times 220}{\pi^2 \times 9.81 \times 0.3^5} = \mathbf{149.61}$, Pipe 2. $\mathbf{R_2} = \dfrac{8 \times 0.018 \times 410}{\pi^2 \times 9.81 \times 0.35^5} = \mathbf{116.1}$

Pipe 3. $\mathbf{R_3} = \dfrac{8 \times 0.013 \times 300}{\pi^2 \times 9.81 \times 0.45^5} = \mathbf{17.463}$, Pipe 4. $\mathbf{R_4} = \dfrac{8 \times 0.015 \times 600}{\pi^2 \times 9.81 \times 0.4^5} = \mathbf{72.62}$

$$h = [R_1 + R_2 + R_3 + R_4]\,Q^2$$

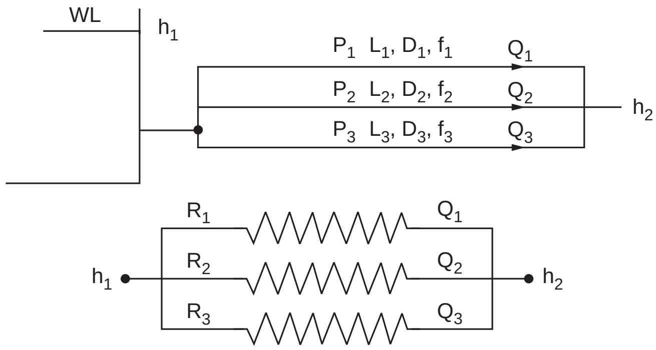
$$(16 - 4) = (149.61 + 17.463 + 72.62)\,Q^2 = 355.793\,Q^2$$

∴ $Q = 0.18365 \text{ m}^3/\text{s}$

Check $\Sigma h = \Sigma (R_1 Q_1^2) = 5.046 + 3.916 + 0.589 + 2.449 = 12$ m

3.19.2 Pipes in Parallel

Such a system is shown in Fig. 3.19.2



Case (*i*) The head drop between locations 1 and 2 are specified: The total flow can be determined using

$$h_f = \frac{8 f_1 L_1 Q_1^2}{\pi^2 g D_1^5} = \frac{8 f_2 L_2 Q_2^2}{\pi^2 g D_2^5} = \frac{8 f_3 L_3 Q_3^2}{\pi^2 g D_3^5}$$

As h_f and all other details except flow rates Q_1, Q_2 and Q_3 are specified, these flow rates can be determined.

Total flow $Q = Q_1 + Q_2 + Q_3$

The process can be extended to any number of connections.

Case (*ii*) Total flow and pipe details specified. One of the methods uses the following steps:

1. Assume by proper judgement the flow rate in pipe 1 as Q_1.
2. Determine the frictional loss.
3. Using the value find Q_2 and Q_3.
4. Divide the total Q in the proportion $Q_1 : Q_2 : Q_3$ to obtain the actual flow rates.

Case (*iii*) Electrical analogy is illustrated or in problem Ex. 7.13 and Ex. 7.14.

Example 3.13 *The details of a* ***parallel pipe system*** *for water flow are given below.*

No.	length, m	Diameter, m	Friction factor
1	800	0.2	0.022
2	1200	0.3	0.02
3	900	0.4	0.019

1. If the frictional drop between the junctions is 15 m of water, determine the total flow rate
2. If the total flow rate is 0.66 m^3/s, determine the individual flow and the friction drop.

The system is shown in Fig. Ex. 7.13.

Case (*i*) Let the flows be Q_1, Q_2 and Q_3. Total flow $Q = Q_1 + Q_2 + Q_3$, using equation 7.11.15

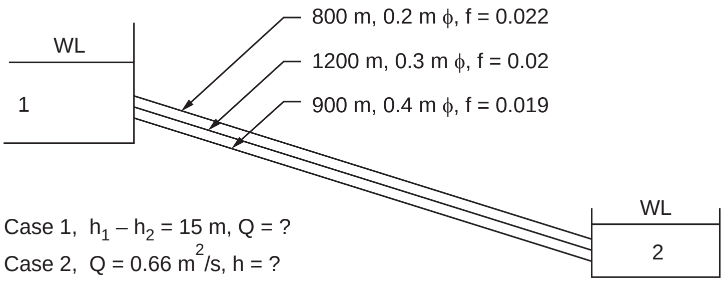


Figure Ex. 3.13

The flow rates are calculated individually with h_f = 15 m and totalled.

$$15 = \frac{8\, f_1 L_1 Q_1^2}{\pi^2\, g\, D_1^5} = \frac{8 \times 0.022 \times 800 \times Q_1^2}{\pi^2 \times 9.81 \times 0.2^5} \quad \textit{solving } \mathbf{Q_1 = 0.05745\ m^3/s}$$

$$15 = \frac{8\, f_2 L_2 Q_2^2}{\pi^2\, g\, D_2^5} = \frac{8 \times 0.02 \times 1200 \times Q_2^2}{\pi^2 \times 9.81 \times 0.3^5} \quad \textit{solving } \mathbf{Q_2 = 0.1355\ m^3/s}$$

$$15 = \frac{8\, f_3 L_3 Q_3^2}{\pi^2\, g\, D_3^5} = \frac{8 \times 0.019 \times 900 \times Q_3^2}{\pi^2 \times 9.81 \times 0.4^5} \quad \textit{solving } \mathbf{Q_3 = 0.32971\ m^3/s}$$

$$\mathbf{Q} = Q_1 + Q_2 + Q_3 = \mathbf{0.522\ m^3/s}$$

Case (*ii*) Total flow is 0.66 m^3/s . Already for 15 m head individual flows are available. Adopting method 2 the total flow is divided in the ratio of $Q_1 : Q_2 : Q_3$ as calculated above.

$$Q_1 = \frac{0.66 \times 0.05745}{0.52274} = 0.07254\ m^3/s,$$

$$Q_2 = \frac{0.66 \times 0.13558}{0.52274} = 0.17117\ m^3/s$$

$$Q_3 = \frac{0.66 \times 0.32971}{0.52274} = 0.41629\ m^3/s$$

Calculation for frictional loss.

Pipe 1 $$h_f = \frac{8 \times 0.022 \times 800 \times 0.07254^2}{\pi^2 \times 9.81 \times 0.2^5} = 23.91\ m$$

Pipe 2 $$h_f = \frac{8 \times 0.02 \times 1200 \times 0.17117^2}{\pi^2 \times 9.81 \times 0.3^5} = 23.91\ m$$

Pipe 3 $$h_f = \frac{8 \times 0.019 \times 900 \times 0.41629^2}{\pi^2 \times 9.81 \times 0.4^5} = 23.91 \text{ m}$$

Electrical analogy : For parallel pipe network also electrical analogy can be used. In the case of parallel flow as the pressure drop is the same

$$h_f = R_1 Q_1^2 = R_2 Q_2^2 = R_3 Q_3^2 \text{ .... or}$$

$$Q_1 = \sqrt{h_f / R_1} \text{, Total flow equals } Q_1 + Q_2 + Q_3 \text{ ....}$$

$$Q = \sqrt{h_f}\left[\frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \text{.........}\right]$$

An equivalent resistance R can be obtained by

$$\frac{1}{\sqrt{R}} = \frac{1}{\sqrt{R_1}} + \frac{1}{\sqrt{R_2}} + \text{......... and } Q^2 = h_f/R$$

Example 3.14. *Work out problem 7.13 by analogy method.*

Case 1. $$\boldsymbol{R_1} = \frac{8 \times 0.022 \times 800}{\pi^2 \times 9.81 \times 0.2^5} = \mathbf{4544.48},\ \boldsymbol{R_2} = \frac{8 \times 0.02 \times 1200}{\pi^2 \times 9.81 \times 0.3^5} = \mathbf{816.07}$$

$$\boldsymbol{R_3} = \frac{8 \times 0.019 \times 900}{\pi^2 \times 9.81 \times 0.4^5} = \mathbf{137.98},\ \frac{1}{\sqrt{R}} = \frac{1}{\sqrt{4544.48}} + \frac{1}{\sqrt{816.07}} + \frac{1}{\sqrt{137.98}}$$

$\therefore$ $$\mathbf{R} = \left(\frac{1}{0.13497}\right)^2 = \mathbf{54.893} \quad \therefore \quad Q^2 = 15/54.893 = 0.2733$$

$\therefore$ Q = **0.52274** (checks with the previous case)

Case 2. $Q = 0.66$, $R = 54.893$

$\therefore$ $h_f = R\,Q^2 = 54.893 \times 0.52274^2 = 23.91$ m

$Q_1^2 = h_f/R_1 = 23.91/4544.48$, $Q_1 = 0.07254$ m³/s

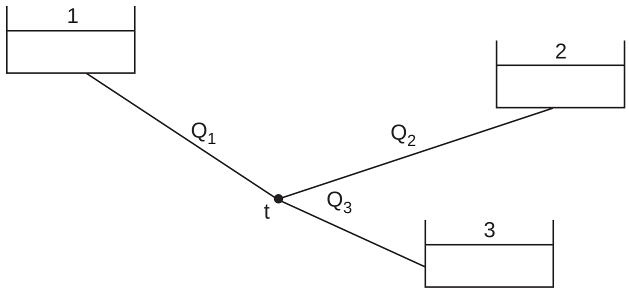
$Q_2^2 = h_f/R_2 = 23.91/816.07$, $Q_2 = 0.17117$ m³/s

$Q_3^2 = h_f/R_3 = 23.91/137.98$, $Q_3 = 0.41629$ m³/s

Note: Checks in all cases.

3.19.3 Branching Pipes

The simplest case is a three reservoir system interconnected by three pipes (Ref. Fig. 3.19.3). The conditions to be satisfied are (*i*) The net flow at any junction should be zero due to continuity principle. (*ii*) The Darcy-Weisbach equation should be satisfied for each pipe. If flows are Q_1, Q_2, Q_3 , then the algebraic sum of $Q_1 + Q_2 + Q_3 = 0$. **If one of the flow rate is specified the solution is direct. If none are specified, trial solution becomes necessary.** The flow may be from the higher reservoir to the others or it may be from both high level reservoirs to the low level one. The hydraulic grade line controls the situation. If the head at the junction is above both the lower reservoirs, both of these will receive the flow. If the head at the junction is below the middle one, the total flow will be received by the lowest level reservoir. This is shown in Fig 3.19.3.



The method of solution requires iteration.

(*i*) **A value for the head at the junction is assumed and the flow rates are calculated from pipe details.**

(*ii*) **The sum of these (algebraic) should be zero. But at the first attempt, the sum may have a positive value or negative value.**

(*iii*) **If it is positive, inflow to the junction is more. So increase the value of head assumed at the junction.**

(*iv*) **If it is negative, the outflow is more. So reduce the value of head assumed. Such iteration can be also programmed for P.C.**

Example 3.15 *Three reservoirs A, B and C at water levels of 25 m, 12 m and 8 m are connected by a pipe network. 1200 m length pipe of diameter 0.5 m and f = 0.013 draws water from A. 1000 m length pipe of diameter 0.4 m and f = 0.015 draws water from B and joins the pipe end from A. The reservoir C is connected to this junction by 900 m length of pipe 0.6 m diameter with f = 0.011. Determine the flow from/to each reservoir.*

Using the equation 3.11.15

$$h_f = \frac{8\,fLQ^2}{\pi^2\,g\,D^5}, \text{ and writing this } h_f = R\,Q^2$$

$$\mathbf{R_A} = \frac{8 \times 0.013 \times 1200}{\pi^2 \times 9.81 \times 0.5^5} = \mathbf{41.2473},$$

$$\mathbf{R_B} = \frac{8 \times 0.015 \times 1000}{\pi^2 \times 9.81 \times 0.4^5} = \mathbf{121.0354}$$

$$\mathbf{R_C} = \frac{8 \times 0.011 \times 900}{\pi^2 \times 9.81 \times 0.6^5} = \mathbf{10.5196}$$

Considering flow from A, $(25 - Z_J)/41.2473 = Q_A^2$,

where Z_J level at junction J. Similarly for flow from B and C,

$\mathbf{(12 - Z_J)/121.0354 = Q_B^2,\ (8 - Z_J)/10.5196 = Q_C^2}$

Assumed value of Z_J	Q_A	Q_B	Q_C	$Q_A + Q_B + Q_C$
10	0.6030	0.1286	– 0.4360	0.2556
13.0	0.5394	– 0.0909	– 0.6894	– 0.2409
11.5	0.5721	0.06423	– 0.5768	0.0596
11.8	**0.5657**	**0.0406**	**– 0.6010**	**0.0053**

This is sufficient for the trial. The flow rates in the last column can be used.

Z_J = 11.825 m gives a residue of 0.00019.

Considering the value, the flows are

Q_A = 0.56517 m³/s, Q_B = 0.03802 m³/s and Q_C = – 0.603 m³/s

3.19.4 Pipe Network

More complex network of pipes exist in practice. A sample is shown in Fig. 3.12

For analysis of the system the following conditions are used.

1. **The algebraic sum of the pressure drop around each circuit must be zero.**
2. **The flow into the junction should equal the flow out of the junction.**
3. **For each pipe the proper relation between head loss and discharge** should be **maintained.**

Analytical solution to such a problem is more involved. Methods of successive approximation are used. With the use of computers, it is now possible to solve any number of simultaneous equations rather easily. Use of the above conditions leads to a set of simultaneous equations. This set can be solved using computers.

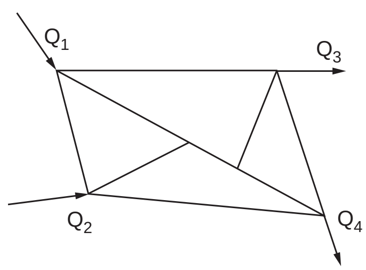


Figure 3.19.4 Pipe network

SOLVED PROBLEMS

Problem. 3.1. *An oil of specific gravity 0.82 and kinematic viscosity 16 × 10⁻⁶ m²/s flows in a smooth pipe of 8 cm diameter at a rate of 2l/s. Determine whether the flow is laminar or turbulent. Also calculate the velocity at the centre line and the velocity at a radius of 2.5 cm. What is head loss for a length of 10 m.* ***What will be the entry length****? Also determine the* ***wall shear****.*

Average flow velocity = volume flow/area = 4 × 0.002/π × 0.08² = 0.4 m/s

$$Re = \frac{uD}{v} = \frac{0.4 \times 0.08}{16 \times 10^{-6}} = 2000$$

This value is very close to transition value. However for smooth pipes the flow may be taken as laminar.

Centre line velocity = 2 × average velocity = 0.8 m/s

For velocity at 2.5 cm radius

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2 \quad \therefore \quad \mathbf{u} = 0.8\left[1-\left(\frac{2.5}{4}\right)^2\right] = \mathbf{0.4875\ m/s}$$

$$\mathbf{f} = 64/\text{Re} = 64/2000 = \mathbf{0.032}$$

$$\mathbf{h_f} = fLu^2/2gd = (0.032 \times 10 \times 0.4^4)/(2 \times 9.81 \times 0.08)$$

$$= \mathbf{0.03262\ m} \text{ of oil}$$

as $$\mathbf{\Delta p} = h_f\gamma = 0.03262 \times 9810 \times 0.82 = \mathbf{262.4}\ \text{N/m}^2$$

$$\textbf{Entry length} = 0.058\ \text{Re}.D. = 0.058 \times 2000 \times 0.08 = \mathbf{9.28\ m}$$

For highly viscous fluid entry length will be long. Wall shear is found from the definition of f.

$$\tau_o = \frac{f}{4}\frac{\rho}{g_o}\frac{u_m{}^2}{2} = \frac{0.032}{4} \times \frac{820}{1} \times \frac{0.4^2}{2} = \mathbf{0.5248\ N/m^2}$$

Wall shear can also be found using, $\tau_o = -\rho\, v\dfrac{du}{dr}$

$$u = u_{max}\left[1-\frac{r^2}{R^2}\right], \frac{du}{dr} = -\frac{U_{max}\ 2r}{R^2}, \text{ at } r = R, \frac{du}{dr} = -u_{max}\frac{2}{R}$$

Substituting, $\tau_o = 820 \times 16 \times 10^{-6} \times 0.8 \times 2/0.04 = 0.5248$ N/m^2.

Problem 3.2. *A circular and a square pipe are of equal sectional area. For the same flow rate, determine which section will lead to a higher value of Reynolds number.*

$\text{Re} = uD_h/v$, For the same section and same flow rate of a specified fluid, $R_e \propto D_h$ hydraulic Diameter.

Circular Pipe : $D_h = D$

Square Pipe of side a : $D_h = 4a^2/4a = a$

as areas are equal, $a^2 = \pi D^2/4, \quad \therefore \quad a = 0.886\ D$

The hydraulic diameter of a square section of the same area is lower by about 11.4%. So the Reynolds number in this case will be lower by about 11.4% and hence for the same flow rate f will be higher for the square section.

Problem 3.3. *The kinematic viscosity of water at 30°C is 0.832 × 10^{-6} m^2/s.* ***Determine the maximum flow rate*** *through a 10 cm dia pipe for the flow to be laminar. Assume smooth pipe. Also determine the head loss/m at this flow condition.*

The condition is that Reynolds number should be about 2000.

$$2000 = (0.1 \times u)/(0.832 \times 10^{-6})$$

$$\therefore \quad \mathbf{u = 0.01664\ m/s}.$$

The flow rate will be = $(\pi \times 0.1^2/4) \times 0.01664$ = **1.307×10^{-4} m³/s = 0.1307 l/s.**

$f = 64/2000 = 0.032$

Head of water, $h_f = 0.032 \times 1 \times 0.01664^2/(2 \times 9.81 \times 0.1) = 4.516 \times 10^{-6}$ **m/m.**

Note: The flow turns turbulent even at a low flow velocity as the kinematic viscosity is low.

Problem 3.4. *Air at 1 atm and 30 °C flows through a pipe of 30 cm dia. The kinematic viscosity at this condition is 16×10^{-6} m²/s. The density is 1.165 kg/m³. Determine the maximum average velocity for the flow to remain laminar. What will be the* ***volume and mass flow rates*** *at this condition? Also determine the head loss/m due to friction.*

The condition is that Reynolds number should equal 2000.

∴ $2000 = (u_m \times 0.3)/16 \times 10^{-6}$ ∴ $u_m = 0.107$ m/s

Volume flow rate = $u A = 0.107 \times \pi \times 0.3^2/4$ = **7.54×10^{-3} m³/s** or **7.54 l/s**

Mass flow = volume flow × density = $7.54 \times 10^{-3} \times 1.165$

= **8.784×10^{-3} kg/s**

$f = 64/\text{Re} = 64/2000 = 0.032$

$h_f = (0.032 \times 0.107^2 \times 1)/(2 \times 9.81 \times 0.3)$

$= 62.2 \times 10^{-6}$ m/m (head of air)

Problem 3.5. *Oil with a kinematic viscosity of 241×10^{-6} m²/s and density of 945 kg/m³ flows through a pipe of 5 cm dia. and 300 m length with a velocity of 2 m/s. Determine* **the *pump power***, *assuming an overall pump efficiency of 45%, to overcome friction.*

$\text{Re} = uD/v = 2 \times 0.05/241 \times 10^{-6} = 415$. So the flow is laminar.

$h_f = (64/415) \times [(2^2 \times 300)/(2 \times 9.81 \times 0.05)] = 188.67$ m head of oil

Mass flow = $(\pi \times 0.05^2/4)\ 2 \times 945$ kg/s = 3.711 kg/s

Power required = $mg\,H/\eta = 3.711 \times 9.81 \times 188.67/0.45\ W$

= 15,263 W or **15.263 kW.**

Problem 3.6. *If, in problem P.7.5. the power available was 10 kW, what will be the pumping rate?*

Power available to overcome friction P = power × pump efficiency

= 10 × 0.45 = 4.5 kW or 4500 W

Mass flow = $(\pi D^2/4)\ u\rho$

Frictional loss in head of fluid = $(64/\text{Re}) \times (u^2L/2gD)$

$= (64v/uD) \times (u^2L/2gD)$

∴ Power = mass flow × g × frictional head

$= (\pi D^2/4)\ u\ \rho\ g\ (64vu^2\ L/u\ D\ 2g\ D) = 8\ \pi\ \rho\ v\ Lu^2$

$4500 = 8 \times \pi \times 945 \times 241 \times 10^{-6} \times 300\ u^2$

∴ **u = 1.619 m/s**

Flow rate = $(\pi \times 0.05^2/4) \times 1.619 \times 945$ = **3kg/s**

Note: Check for the flow to be laminar. $\text{Re} = 1.619 \times 0.05/241 \times 10^{-6} = 336$

Problem 3.7. *Oil of specific gravity 0.92 flows at a rate of 4.5 litres/s through a pipe of 5 cm dia, the pressure drop over 100 m horizontal length being 15 N/cm². **Determine the dynamic viscosity of the oil**.*

Using the equation 7.9.2 – Hagen-Poiseuille eqn. $\Delta p = 128\ \mu LQ/\pi D^4$

$$\mu = \Delta p\ .\ \pi . D^4/128LQ$$

$$= 15 \times 10^4 \times \pi \times 0.05^4/128 \times 100 \times 0.0045 = \mathbf{0.05113\ Ns/m^2}\ \text{(Pa.s)}$$

(**Note:** $N/cm^2 \rightarrow 10^4\ N/m^2$, litre = 0.001 m^3)

Reynolds number $= uD\ \rho/\mu,\ u = Q \times 4/\pi D^2$

$\therefore$ $$\text{Re} = (4Q/\pi\ D^2) \times (D\ \rho/\mu) = (0.0045 \times 920 \times 4)/(\pi \times 0.05 \times 0.05113)$$

$$= 2061.6$$

$\therefore$ Flow is laminar but just on the verge of turning turbulent

(Note: Re = 4Q/π Dv)

Problem 3.8. *In a capillary viscometer the tube is of 2 mm dia and 0.5 m length. If 60 cm³ of liquid is collected during 10 min with a constant pressure difference of 5000 N/m², **determine the viscosity of the oil.***

Using $\Delta p = 128\ \mu L\ Q/\pi D^4$ (Hagen Poiseuille equation 7.9.2)

$\mu = \Delta p . \pi D^4/128\ L\ Q$ where Q is the discharge in m^3 per second.

Discharge $= 60 \times 10^{-6}\ m^3/600$ sec $= 10^{-7}\ m^3/s$

$\therefore$ $$\mu = 5000 \times \pi \times 0.002^4/128 \times 0.5 \times 10^{-7} = \mathbf{0.0393\ Ns/m^2}\ \ \textbf{(or Pa.s)}$$

Problem 3.9. *If an oil of viscosity of 0.05 Ns/m² is used in the experiment of problem P.7.8 **calculate how long it will take to collect 60 cc**. Assume that the other conditions remain unaltered.*

$$\Delta p = 128\ \mu L\ Q/\pi D^4$$

$\therefore$ $Q = \Delta p \times \pi D^4/128\ \mu L$ where Q is in m^3/s

$$Q = 5000 \times \pi \times 0.002^4/128 \times 0.05 \times 0.5 = 7.854 \times 10^{-8}\ m^3/s$$

or 7.854×10^{-2} cc/s

$\therefore$ Time for 60 cc $= 60/7.854 \times 10^{-2}$ s = 763.94 s or **12.73 min**

Problem 3.10. *Oil of viscosity 0.1 Ns/m² is to flow through an inclined pipe by gravity. The pipe diameter is 25 mm and the density of the oil is 930 kg/m³. If the flow rate is to be 0.25 l/s **determine the pipe inclination** with horizontal.*

The inclination of the pipe should be such that the drop in head should equal the friction drop along the length or

$$h_f = L \sin\theta,\ \Delta h = fLu^2/2gD,\ f = 64/\text{Re},\ h = \Delta p/\gamma$$

Using Darcy–Weisbach equation and substituting for f in terms of Re

$$h_f = \frac{64}{\text{Re}}\frac{u^2 L}{2gD} = L \sin\theta \text{ or } \sin\theta = \frac{64u^2}{2\,\text{Re}\,gD}$$

$$u = 4Q/\pi D^2 = 4 \times 0.25/(\pi \times 0.025^2)\ 1000 = 0.5093\ m/s$$

$\therefore$ $$\text{Re} = \frac{uD\rho}{\mu} = 0.5093 \times 0.025 \times 930/0.1 = 118.41$$

∴ Flow is laminar

$$\sin\theta = (64 \times 0.5093^2)/(2 \times 118.41 \times 9.81 \times 0.025) = 0.28582$$

∴ **θ = 16.6° with horizontal**

Problem 3.11. *In a double pipe heat exchanger (to obtain chilled water) water at 10° C flows in the annular area between 30 mm OD inside pipe and the 50 mm ID outer pipe. The kinematic viscosity at this temperature is 1.4 × 10^{-6} m^2/s.* ***Determine the maximum flow rate if the flow should be laminar***.

The sectional area for flow = (π/4) $(D^2 - d^2)$ where D = out side dia, d = inside dia. of the annular area. Wetted perimeter = π $(D + d)$

∴ $\mathbf{D_h} = 4 \times (\pi/4)(D^2 - d^2)/\pi(D + d) = D - d = 0.05 - 0.03 =$ **0.02 m**

For laminar conditions Re should be less than 2000.

$$\text{Re} = 2000 = (0.02 \times u)/(1.4 \times 10^{-6}) \quad \therefore \quad u = 0.14 \text{ m/s}$$

∴ **flow rate** = $(\pi/4)(0.05^2 - 0.03^2)\ 0.14$

= **1.76 × 10^{-4} m^3/s or 0.176 l/s or 633.3 l/hr.**

The friction factor and friction drop in head and power required for a flow rate etc can be determined as in problem P. 7.5. taking care to use D_h in place of D.

Problem 3.12. *Water flows in an experimental 50 mm square pipe at a temperature of 10°C. The flow velocity is 0.012 m/s.* ***Determine the head drop over a length of 10 m.*** *Compare the same with circular section of the same area, v = 1.4 × 10^{-6} m^2/s.*

As the **section is square**, the hydraulic diameter is to be used.

$$D_h = 4 \text{ area/perimeter} = 4a^2/4a = a = 0.05 \text{ m}$$

$$\text{Re} = D_h\, u/v = 0.05 \times 0.012/1.4 \times 10^{-6} = 428.6$$

∴ The flow is laminar.

$$f = 64/\text{Re} = 64/428.6$$

$$h_f = fLu^2/2g\, D_h = (64/428.6) \times 10 \times 0.012^2/(2 \times 9.81 \times 0.05)$$

= **2.19 × 10^{-4} m head of water.**

Circular section: $\pi D^2/4 = 0.05^2 \quad \therefore \quad D = 0.05642$ m

$$Re = 0.05642 \times 0.012/1.4 \times 10^{-6} = 483.6, \text{ laminar,}$$

$$f = 64/483.6 = 0.1323$$

$$\mathbf{h_f} = 0.1323 \times 10 \times 0.012^2/2 \times 9.81 \times 0.05642$$

= **1.722 × 10^{-4} m** ∴ Lower by: 21.4%

Problem 3.13. *If in the place of square a rectangular section of 100 mm × 25 mm is used for the data of P. 7.12 determine the head drop over a length of 10 m.*

Hydraulic diameter $= 4A/P = 4 \times 0.025 \times 0.1/2(0.1 + 0.025) = 0.04$ m

$$\text{Re} = D_h\, u/v = 0.4 \times 0.012/1.4 \times 10^{-6} = 342.86$$

Frictional drop in head = $f.L.u^2/2g\, D_h = (64/342.86) \times 10 \times 0.012^2/2 \times 9.81 \times 0.04$

= **3.425 × 10^{-4} m head of water**

For the same flow area as compared to 1.722 × 10^{-4} m head of water for the circular section there is an **increase of 100% in friction drop** for the rectangular section.

Problem 3.14. *Water flows out from a storage tank through a pipe of 50 mm dia. at a rate of 9.82 l/s.* ***Determine the loss of head at entrance*** *if it is (i)* ***bell mounted*** *(ii)* ***square edged*** *and (iii)* ***reentrant****.*

Refer section 3.12

(*i*) In this case the loss coefficient is 0.04

$$u = 4Q/\pi D^2 = 4 \times 9.82/(1000 \times \pi \times 0.05^2) = 5 \text{ m/s}$$

$\therefore$ $h_f = 0.04 \times 5^2/2 \times 9.81 = \mathbf{0.051\ m.}$

(*ii*) In this case loss coeffcient is 0.5. $\therefore$ $\mathbf{h_f} = 0.5 \times 5^2/2 \times 9.81 = \mathbf{0.637\ m}$

(*iii*) The loss coefficient in this case = 0.8 $\therefore$ $\mathbf{h_f} = 0.8 \times 5^2/2 \times 9.81 = \mathbf{1.019\ m}$

Problem 7.15. *Water flowing in a pipe of 500 mm dia suddenly passes into a pipe of 750 mm dia. Determine the loss of head if the initial velocity was 2 m/s.*

Ref. eqn 7.12.1. In this case,

$$h_f = (u_2 - u_1)^2/2g,\ u_1 = 2 \text{ m/s},\ u_2 = 2 \times (0.5/0.75)^2 = 0.889 \text{ m/s.}$$

$\therefore$ $\mathbf{h_f} = (2 - 0.889)^2/2 \times 9.81 = \mathbf{0.0629\ m.}$

Problem 3.16. *A 30 cm pipe with friction factor f = 0.024 carries water to a turbine at the rate of 0.25 m³/s over a distance of 160 m. The difference in levels between the water inlet and turbine inlet is 36 m.* ***Determine the efficiency of transmission****. The turbine outlet delivery is submerged into the tailrace and the velocity at the exit is 0.4 times the velocity in the pipe.*

$$\text{The efficiency of transmission} = \frac{\text{Available head for conversion to work}}{\text{Difference in datum}}$$

The losses in this case are the friction head and the dynamic head at exit.

Flow rate = 0.25 m³/s,

$\therefore$ $u_m = 0.25 \times 4/\pi \times 0.3^2 = 3.54$ m/s.

Friction head $= fLu^2/2gD = [0.024 \times 160 \times 3.54^2/(2 \times 9.81 \times 0.3)] = 8.176$ m

Dynamic head: Exit velocity $= 0.4 \times 3.54$ *m/s*.

$\therefore$ Dynamic head $= (0.4 \times 3.54)^2/2 \times 9.81 = 0.102$ m

Total losses $= 8.176 + 0.102 = 8.28$ m

Efficiency is high but the power delivered is not maximum.

$\therefore$ **Efficiency of transmission** = (36 – 8.28)/36 = **0.77 or 77%**

Problem 3.17. *The flow in a pipe of 100 mm dia with Reynolds number value of 10^5 is found to have a friction factor f = 0.032.* ***Determine the thickness of laminar sublayer****. Also indicate whether the pipe is* ***hyraulicaly smooth or not if*** *the roughness height is 0.4 mm.*

Ref. section 3.5.

$$\delta_l = 32.8\ v/u_m \sqrt{f},\ Re = u_m D/v = 10^5$$

$\therefore$ $v/u_m = D/10^5$, substituting for v/u_m

$\therefore$ $\delta_l = (32.8 \times D)/(10^5 \times \sqrt{f}\,) = 32.8 \times 0.1/10^5 \sqrt{0.032}$

$= 1.83 \times 10^{-4}$ m = 0.183 mm

$\varepsilon =$ 0.4 mm 0.3 ε = 0.12 mm, 6ε = 1.098 mm

The sublayer thickness is larger than 0.3ε but less than 6ε.

The pipe cannot be classified definitely as smooth or rough.

Problem 3.18. *Petrol of sp. gravity 0.7 and kinematic viscosity of 0.417 × 10^{-6} m^2/s flows through a smooth pipe of 250 mm ID. The pipe is 800 m long. The pressure difference between the ends is 0.95 bar.* ***Determine the flow rate***.

In this case the determination off involves the velocity as the Reynolds number depends on velocity. The flow rate depends on velocity. A trial solution is necessary. So a value of f = 0.02 is first assumed.

Pressure difference = 0.95 bar or 0.95 × 10^5 N/m^2. Converting the same to head of fluid, 0.95 × 10^5/700 × 9.81 = 13.834 m of petrol column.

$$13.834 = (fLu^2/2gD) + (u^2/2g)$$

$$= [(0.02 \times 800 \times u^2)/(2 \times 9.81 \times 0.25)] + u^2/2 \times 9.81$$

$$= (3.26 + 0.051)u^2$$

$\therefore$ u = 2.045 m/s.

Now $\text{Re} = uD/v = 2.045 \times 0.25/0.417 \times 10^{-6} = 1.226 \times 10^6$

Ref. section 3.11, eqn 7.11.12, $f = 0.0032 + (0.221/\text{Re}^{0.237}) = 0.01117$

or $1/\sqrt{f} = 1.8.\log \text{Re} - 1.5186$ $\therefore$ $f = 0.01122$

so the value 0.02 is on the higher side. Now using the value 0.01117,

$$13.834 = [0.01117 \times 800 \times u^2)/(2 \times 9.81 \times 0.25)] + [u^2/(2 \times 9.81)]$$

$$= 1.8727\ u^2$$

$\therefore$ u = 2.7185 m/s, Re = 2.7185 × 0.25/0.417 × 10^{-6} = 1.63 × 10^6

f = 0.1065.

This is nearer the assumed value and further refinements can be made by repeating the procedure.

Flow rate = 2.7185 × π × 0.25^2/4 = 0.1334 m^3/s = 93.4 kg/s

Problem 3.19. ***Determine the diameter of the pipe*** *(smooth) required to convey 150 l of kerosene over a length 1000 m with the loss of head by friction limited to 10 m of kerosene. Density = 810 kg/m^3, kinematic viscosity = 2.37 × 10^{-6} m^2/s*

In this problem also as in P. 7.18, trial is necessary. Assume f = 0.012

Neglecting dynamic head, As $u = Q/A$,

$$10 = [(0.012 \times 1000)/(2 \times 9.81 \times D)] \times [(0.15 \times 4)/(\pi D^2)]^2$$

$\therefore$ $u = (4 \times 0.15)/\pi D^2$, Simplifying

$$D^5 = [(0.012 \times 1000)/(2 \times 9.81 \times 10)] \times [(0.15^2 \times 4^2)/\pi^2]$$

$$= 2.231 \times 10^{-3}$$

$\therefore$ **D = 0.295 m** and **u = 2.195 m/s**

$$\text{Re} = 0.295 \times 2.195/2.37 \times 10^{-6} = 0.273 \times 10^6$$

Refer eqn. 7.11.11, $f = 0.0032 + (0.221/\text{Re}^{0.0237}) = 0.0146$

Assuming **f = 0.014,** to repeat the procedure

$$10 = [(0.014 \times 10000)/(2 \times 9.81 \times D)] \times [(0.15 \times 4)/\pi D^2]^2$$

$$\therefore \quad D^5 = [(0.014 \times 1000)/(2 \times 9.82 \times 10)]\ [(0.15^2 \times 4^2)/\pi^2] = 2.6 \times 10^{-3}$$

$$\mathbf{D = 0.304\ m,} \quad u = 0.15 \times 4/\pi \times 0.304^2 = 2.065 \text{ m/s}$$

$$\text{Re} = 2.065 \times 0.304/2.37 \times 10^{-6} = 0.265 \times 10^6$$

$$f = 0.0032 + 0.221/(0.265 \times 10^6)^{0.237} = 0.01466$$

The answer can be refined further using this value of f and reworking on the same lines.

Problem 3.20. *Two pipes of 0.35 m and 0.25 m dia and length 2000 m and 1500 m with f values 0.021 and 0.018 connected is series carry water from a reservoir to a supply system, the head available being 8 m.* ***Determine the flow quantity neglecting minor losses.***

The head available should be equal to the sum of the frictional losses in the two pipes. Neglecting loss at sudden contraction

$$\delta = [(0.021 \times 2000 \times u_1^2)/(2 \times 9.81 \times 0.35)] + [(0.018 \times 1500 \times u_2^2)/(2 \times 9.81 \times 0.25)]$$

From continuity equation, we get

$$[(\pi \times 0.35^2)/4] \times u_1 = [(\pi \times 0.25^2)/4]u_2$$

$$\therefore \quad u_2 = (0.35/0.25)^2\, u_1 \quad \text{or} \quad u_2^2 = (0.35/0.25)^4\, u_1^2$$

Substituting, and simplifying and solving,

$$u_1 = 0.542 \text{ m/s}, \ u_2 = 1.062 \text{ m/s}$$

flow rate $= (0.542 \times \pi \times 0.35^2)/4 =$ **0.0521 m³/s or 187.7 m³/hr**

check the frictional drop:

$$h_f = [(0.021 \times 2000 \times 0.542^2)/(2 \times 9.81 \times 0.35)] + [(0.018 \times 1500 \times 1.062^2)/(2 \times 9.81 \times 0.25)]$$

$$h_f = 1.8 + 6.2 = 8 \text{ m.}$$

Problem 3.21. *A 300 mm dia pipe carries kerosene at a rate of 200 l/s. The roughness is 0.2 mm.* ***Determine the frictional drop*** *over 100 m length of pipe.*

using equation (7.11.13). $\dfrac{1}{\sqrt{f}} = 2 \log \dfrac{R}{\varepsilon} + 1.74 = 2 \log \dfrac{0.15}{0.2 \times 10^{-3}} + 1.74 = 7.49$

$$\therefore \quad f = 0.01782 \text{ m} \qquad u = 4 \times 0.2/\pi \times 0.3^2 = 2.829 \text{ m/s}$$

$$h_f = \frac{0.01782 \times 1000 \times u^2}{2 \times 9.81 \times 0.3} + \frac{u^2}{2 \times 9.81} = 3.0785u^2$$

substituting the value of u, $\mathbf{h_f}$ **= 24.65 m (head of kerosene)**

Problem 3.22. *Water is drawn from a reservoir through a pipe of diameter D and a constant friction factor f. Along the length water is drawn off at the rate of Km³/s per unit length and the length is L. There is no flow at the end. Derive an expression for the loss of head.*

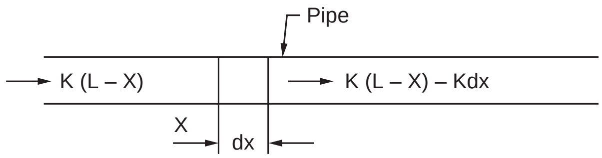


Figure P. 3.22

Consider a length dx at location x, using the equation, the drop dh over length dx is

$$h_f = \frac{fLu^2}{2gD} \quad \therefore \quad dh = \frac{fdx}{D}\frac{u^2}{2g} \tag{1}$$

At this location, the flow rate Q can be obtained as

$Q = K(L - x)$, as total flow is KL and draw off upto x is Kx.

$$u = \frac{4Q}{\pi D^2},\ u^2 = \frac{16Q^2}{\pi^2 D^4} = \frac{16K^2 (L-x)^2}{\pi^2 D^4}$$

Substituting in eqn. (1)

$$dh = \frac{fdx}{2g}\frac{16K^2 (L-x)^2}{\pi^2 D^5} = \frac{8fK^2}{g\pi^2 D^5}(L-x)^2\, dx$$

Integrating from $x = 0$ to L

$$h_2 - h_1 = h_f = \frac{8\, fK^2}{g\pi^2 D^5}\int_0^L [L^2 - 2Lx + x^2]dx = \frac{8\, fK^2}{g\pi^2 D^5}\left[L^3 - L^3 + \frac{L^3}{3}\right]$$

$$\therefore \qquad h_f = \frac{8\, fL^3\, K^2}{3g\pi^2\, D^5} \quad \text{(Note: } K \text{ has a unit m}^3\text{/sm)}$$

for the following data,

$f = 0.024$, $K = 7.5\ l/hr/m = 2.085 \times 10^{-6}$ m^3/s/m,

$D = 0.1$ m, $L = 4.8 \times 10^3$ m,

$$\mathbf{h_f} = \frac{8 \times 0.024 \times (4.8 \times 10^3)^3\ (2.085 \times 10^{-6})^2}{3 \times 9.81 \times \pi^2 \times 0.1^5} = \mathbf{31.73\ m}$$

The head drop between lengths L_1 and L_2 can be determined by difference *i.e.,* $(h_{f2} - h_{f1})$

Problem 3.23. *A pipe line 200 mm dia. and 4000 m long connects two reservoirs with a difference in level of 60 m. Water is drawn at 1500 m point at a rate of 50 l/s. Friction coefficient f = 0.024.* ***Determine the flow rates*** *in the two sections. Neglect minor losses*

$$h_f = \frac{fLu^2}{2gD},\ u = \frac{4Q}{\pi D^2},\ u^2 = \frac{16Q^2}{\pi^2 D^2},\ h_f = \frac{8\, fLQ^2}{\pi^2\, gD^5}$$

Considering the two sections, (total drop)

$$60 = \frac{8 \times 0.024 \times 1500 \times Q_1^{\,2}}{\pi^2 g \times 0.2^5} + \frac{8 \times 0.024 \times 2500 \times Q_2^{\,2}}{\pi^2 g \times 0.25}$$

$$= 9295.5\ Q_1^{\,2} + 15492.54\ Q_2^{\,2}$$

but $Q_2^2 = (Q_1 - 0.05)^2$, Substituting and simplifying

$$60 = 9295.52\, Q_1^2 + 1549.25(Q_1^2 + 0.05^2 - 2Q_1 \times 0.05)$$

or

$$24788.05\, Q_1^2 - 1549.25\, Q_1 - 21.268 = 0$$

$$Q_1 = \frac{1549.25 \pm [(-1549.25)^2 + 4 \times 21.268 \times 24788.05]^{0.5}}{2 \times 24788.05}$$

$$= \frac{1549.25 \pm 2123.45}{2 \times 24788.05} = \mathbf{0.074082\ m^3/s,}$$

$\therefore$ $\mathbf{Q_2 = 0.024082\ m^3/s}$

The other solution is negative.

Check: For the first section

$$h_f = \frac{8 \times 0.024 \times 1500 \times 0.074082^2}{\pi^2 \times 9.81 \times 0.2^5} = 51.015\ \text{m}$$

For the second section

$$h_f = \frac{8 \times 0.024 \times 2500 \times 0.024082^2}{\pi^2 \times 9.81 \times 0.2^5} = 8.985\ \text{m}, \quad \text{Total head} = 60\ \text{m}$$

Problem 3.24. *Two adjacent city centres B and D receive water from separate sources A and C. The water level in A is 4 m above that in C. Reservoir A supplies city centre B by 0.4 m diameter pipe of 3000 m length with a level difference of 10 m. City centre D's is supplied by reservoir C through a 4000 m long pipe of 0.45 m diameter, with a level difference of 15 m. After sometime it is found that centre B has excess water while centre D is staraved. So it is proposed to interconnect these lines and draw 100 l/s from the line A to B. The junction on AB is at a distance of 2000 m from A. The junction CD is at 3000 m from C.* ***Determine the original supply rates and supply rates with interconnection*** *to centres B and D. Also determine the diameter of the interconnecting pipe, if the length is 1500 m Friction factor, f = 0.01 in all cases.*

The arrangement is shown in Fig. P. 3.24

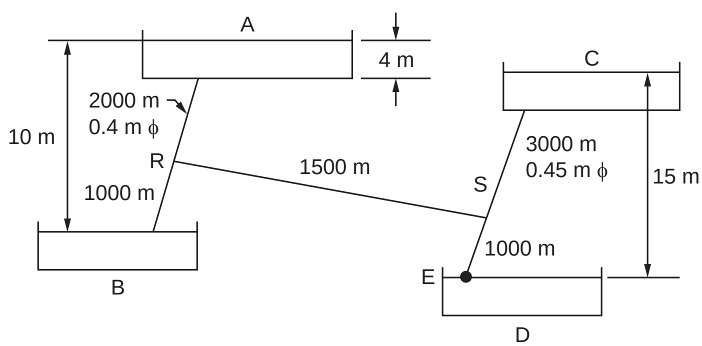


Figure P. 3.24

(*i*) Without interconnection : using equation 3.11.15

$$h_f = \frac{8\, fLQ^2}{\pi^2\, gD^5}$$

Drop in Line AB is 10 m

$$10 = \frac{8 \times 0.01 \times 3000 \times Q^2}{\pi^2 \times 9.81 \times 0.4^5}$$

∴ $\mathbf{Q_{AB} = 0.20325\ m^3/s}$ or **203.25 l/s**

Drop in line CD is 15 m

$$15 = \frac{8 \times 0.01 \times 4000 \times Q^2}{\pi^2 \times 9.81 \times 0.45^5}$$

∴ $\mathbf{Q_{CD} = 0.2894\ m^3/s}$ **or 289.4 l/s**

After interconnection: line AB:

Let the flow up to R be Q and then in RB $(Q - 0.1)$

Total frictional loss $$= 10 = \frac{8 \times 0.01 \times 2000 \times Q^2}{\pi^2 \times 9.81 \times 0.4^{0.5}} + \frac{8 \times 0.01 \times 1000(Q - 0.1)^2}{\pi^2 \times 9.81 \times 0.4^5}$$

This reduces to $3Q^2 - 0.2Q - 0.11393 = 0$. **Solving Q = 0.231 m³/s or 231 l/s**

Now the centre B will receive 131 *l/s* (previous 203 *l/s*)

Line CD: Let the flow upto S be Q and then $(Q + 0.1)$ upto D

Total head loss $$= 15 = \frac{8 \times 0.01 \times 3000 \times Q^2}{\pi^2 \times 9.81 \times 0.45^{0.5}} + \frac{8 \times 0.01 \times 1000(Q + 0.1)^2}{\pi^2 \times 9.81 \times 0.45^5}$$

This reduces to $4Q^2 + 0.2Q - 0.32499 = 0$

Solving $\mathbf{Q = 0.261\ m^3/s}$ **or 261 l/s**

Now the city center C will receive 361 *l/s* (previous 289.4 *l/s*)

To determine the diameter of the connecting pipe RS:

Head drop from A to R

$$h_{f1} = \frac{8 \times 0.01 \times 2000 \times 0.231^2}{\pi^2 \times 9.81 \times 0.4^5} = 8.61 \text{ m}$$

Head drop from S to E

$$h_{f2} = \frac{8 \times 0.01 \times 1000 \times 0.361^2}{\pi^2 \times 9.81 \times 0.45^5} = 5.84 \text{ m}$$

Head drop from A to E = 4 + 15 = 19 m

∴ Head available between

$$RS = 19 - 8.61 - 5.84 = 4.55 \text{ m}$$

Considering Pipe RS

$$4.55 = \frac{8 \times 0.01 \times 1500 \times 0.1^2}{\pi^2 \times 9.81 \times D^5}$$ **Solving D = 0.307 m.**