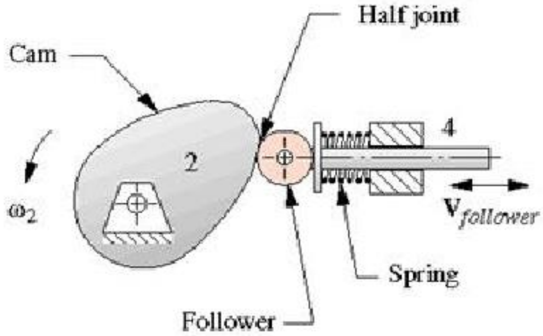
# UNIT V

# DESIGN OF CAM, CLUTCHES AND BRAKES

## 5.1 Introduction

C**AMS** in some form or other are essential to the operation of many kinds of mechanical devices. Their best known application is in the valve operating gear of internal combustion engines, but they play an equally important part in industrial machinery, from printing presses to reaping machines. In general, a cam can be defined as a projection on the face of a disc or the surface of a cylinder for the purpose of producing intermittent reciprocating motion of a contacting member or follower. Most cams operate by rotary motion, but this is not an essential condition and in special cases the motion may be semi-rotary Oscillatory or swinging. Even straight-line motion of the operating member is possible, though the term cam may not be considered properly applicable in such circumstances. Most text books on mechanics give some information on the design of cams and show examples of cam forms plotted to produce various orders of motion. Where neither the operating speed nor the mechanical duty is very high, there is a good deal of latitude in the permissible design of the cam and it is only necessary to avoid excessively steep contours or abrupt changes which would result in noise, impact shock, and side pressure on the follower.



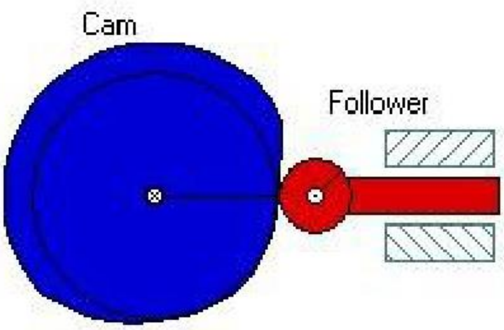



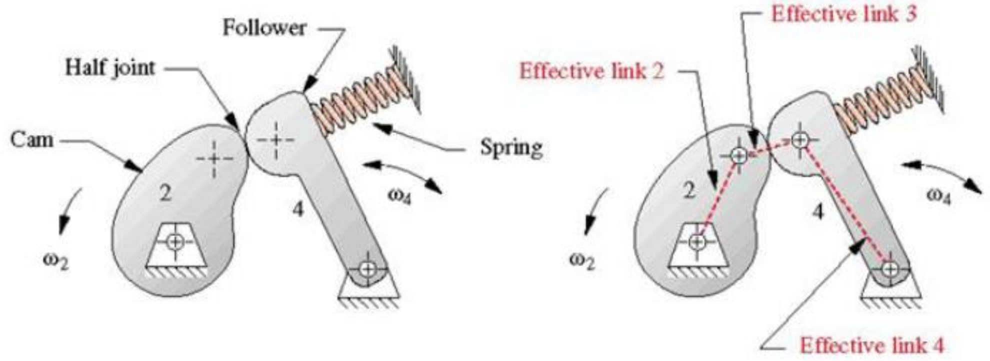

Figure : Cam and follower

But, with increase of either speed or load, much more exacting demands are made on the cam, calling for the most careful design and, at very high speed, the effect of inertia on the moving parts is most pronounced, so that the further factors of acceleration and rate of lift have to be taken into account and these are rarely dealt with in any detail in the standard text books. The design of the cam follower is also of great importance and bears a definite relation to the shape of the cam itself. This is because the cam cannot make contact with the follower at a single fixed point. Surface contact is necessary to distribute load and avoid excess wear, thus the cam transmits its motion through various points of location on the follower, depending on the shape of the two complementary members. The cams for operating i.c. engine valves present specially difficult problems in design. In the case of racing engines, both the load and speed may be regarded as extreme, because in many engines the rate at which the valves can be effectively controlled is the limiting factor in engine performance. In some respects, cam design of miniature engines is simplified by reason of their lighter working parts (and consequent less inertia) but on the other hand, working friction is usually greater and rotational speeds are generally considerably higher than in full-size practice.

## 5.2 Terminology:

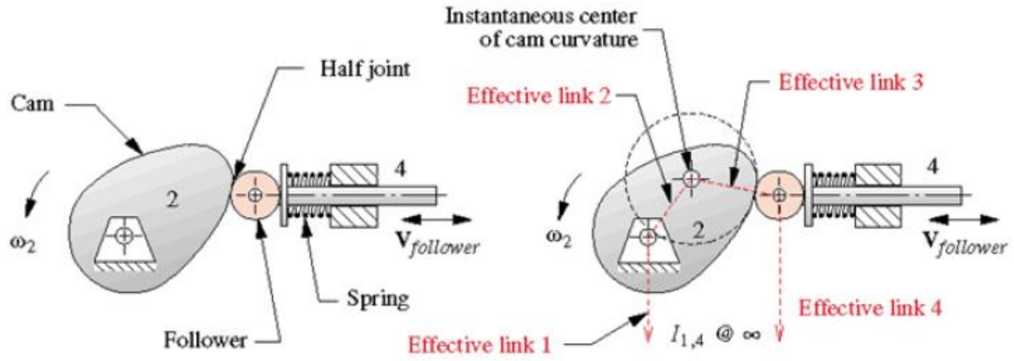
### 5.2.1 Type of Follower Motion

**Rotating follower**

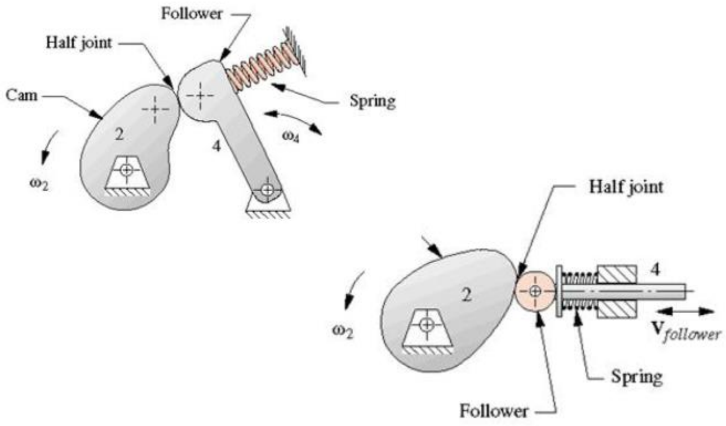



### 5.2.2 Type of Follower Motion

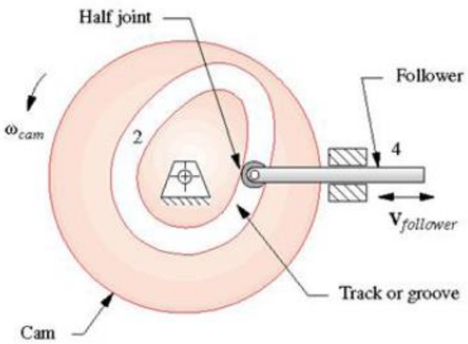
**Translating follower**

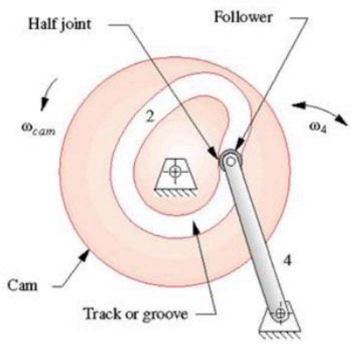



### 5.2.3 Type of Joint Closure –Force



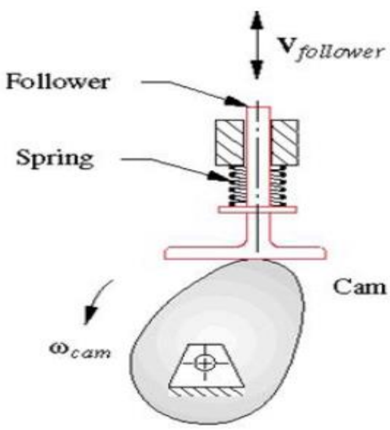

## 5.2.4 Type of Joint Closure – Form



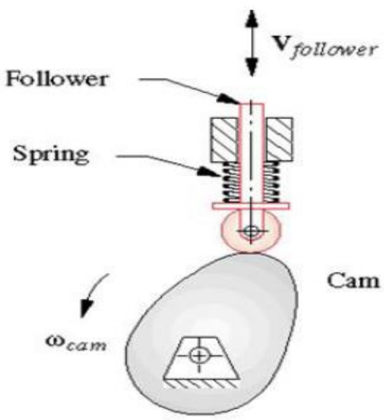




## 5.2.5 Type of Follower

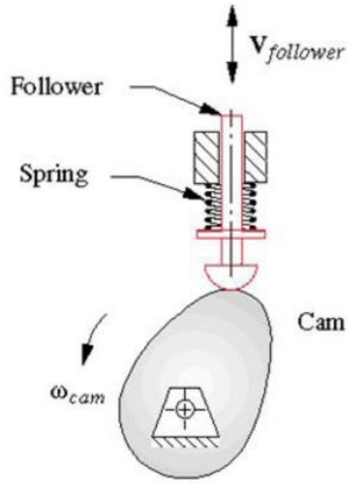
* Flat-faced type follower



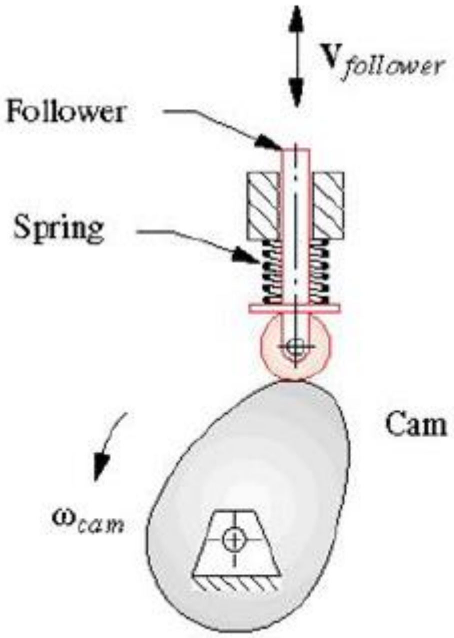

*Roller type follower



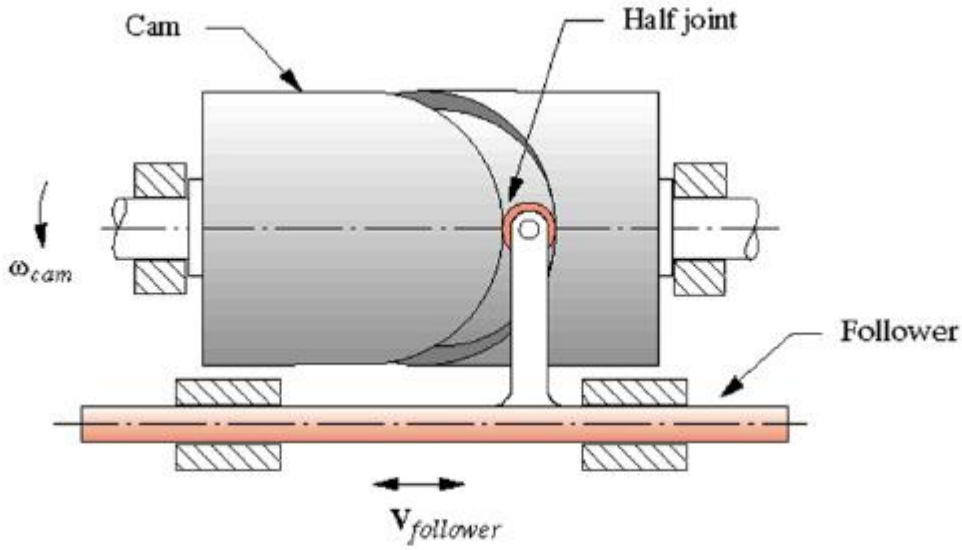

## 5.2.6 Mushroom type follower




## 5.3 Types of CAM




**Radial Type CAM**




**Axial Type CAM**

## 5.4 Sizing of CAM

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Major factor that affect cam size

– Pressure angle

– Radius of curvature

– Base circle radius (flat)

• The smallest circle that can be drawn tangent to the physical cam surface

– Prime circle radius (roller or curved)

• The smallest circle that can be drawn tangent to the locus of the centerline of the follower

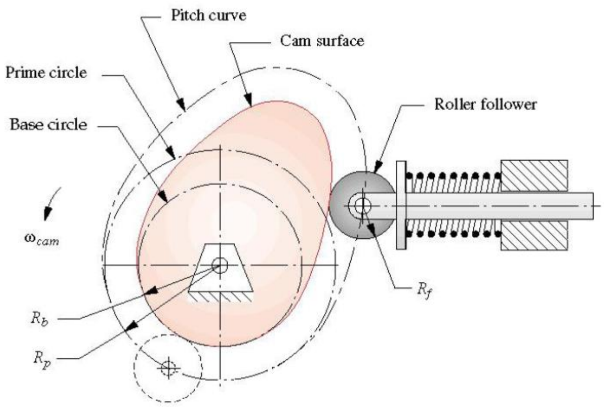
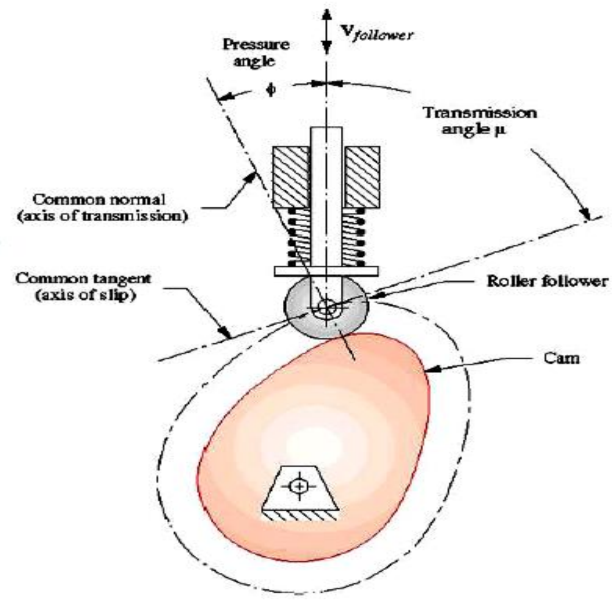
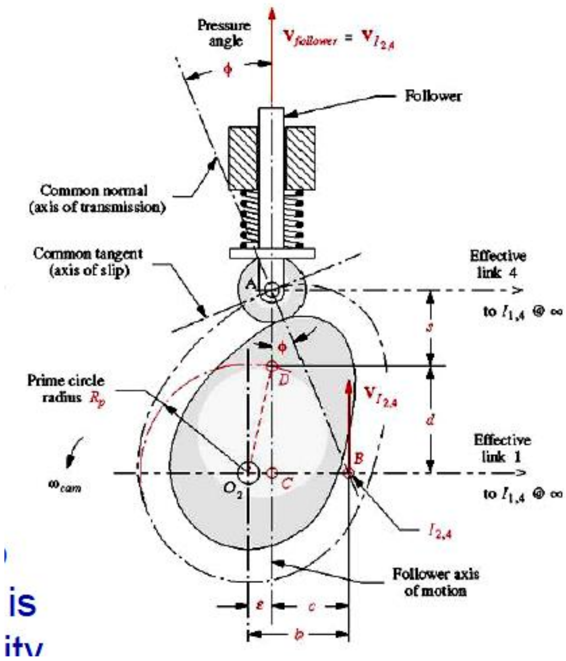



**Figure:** Base circle Rb, Prime circle Rp, and pitch circle of a radial CAM with roller follower

## 5.5 Pressure angle

The angle between the direction of motion (velocity) of the follower and the direction of the axis of transmission between 0°and 30°.




## 5.6 Eccentricity

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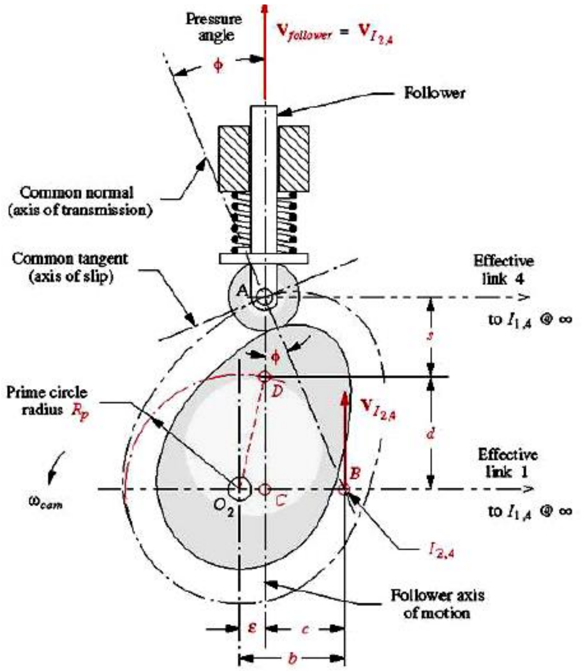
- Perpendicular distance between the follower's axis of motion and the center of the cam

$$V_{I_{2,4}} = b\omega = \dot{s}$$

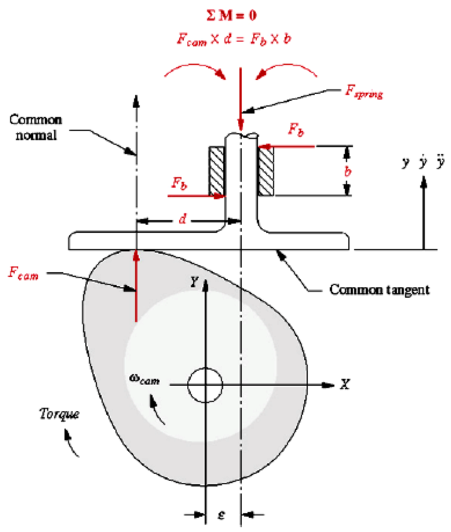
$$b = v$$

- The distance b to the instant center is equal to the velocity of the follower

$$\phi = \arctan \frac{v - \varepsilon}{s + \sqrt{R_P^2 - \varepsilon^2}}$$




### 5.7 Overturning –Translating Flat-Faced Follower

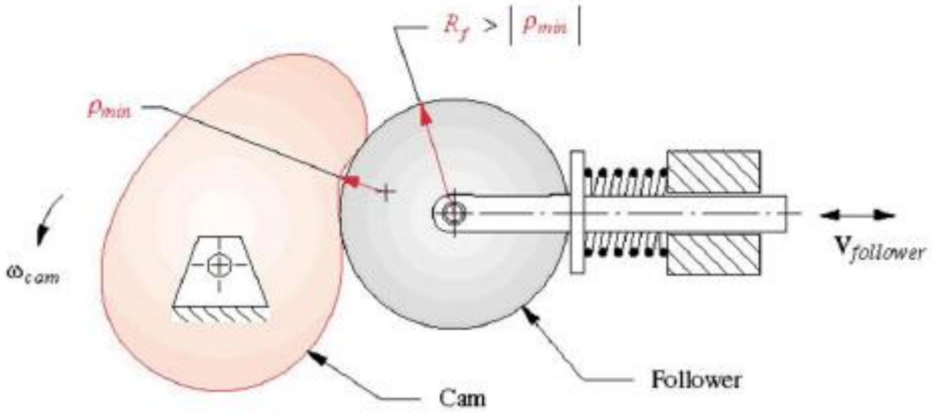


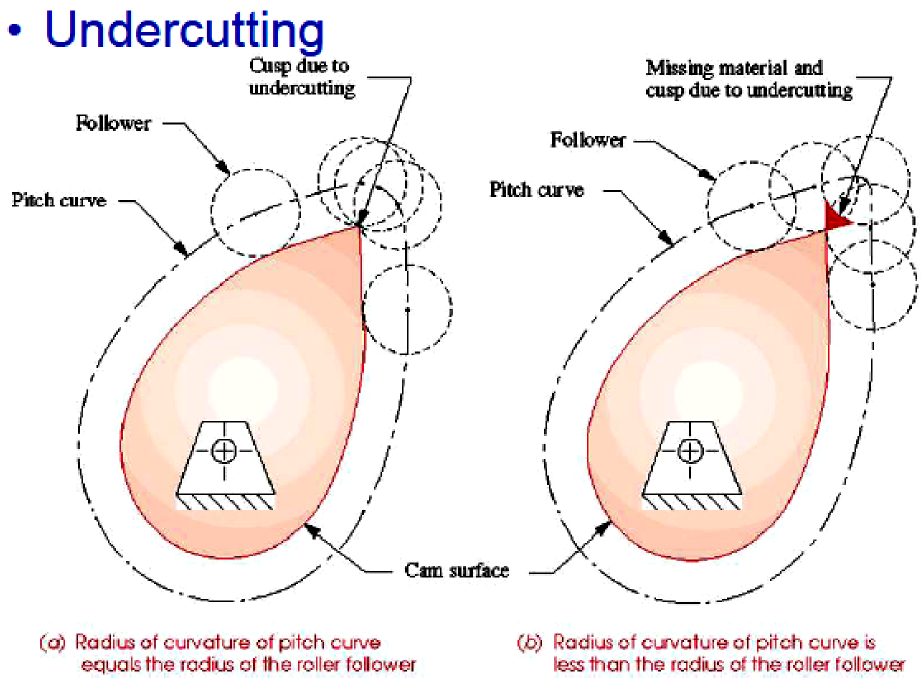

## 5.8 Radius of Curvature (Roller)

- No matter how complicated a curve's shape may be, nor how high the degree of the describing function, it will have a instantaneous radius of curvature

- Concerns

a) Large radius




The rule of thumb is to keep the absolute value of the minimum radius of curvature of the cam pitch curve 2 to 3 times as large as the radius of the follower

$$|\rho_{\min}| >> R_f$$

$$\rho_{pitch} = \frac{\left[(R_p + s)^2 + v^2\right]^{3/2}}{(R_p + s)^2 + 2v^2 - a(R_p + s)}$$

## 5.9 Radius of curvature for Flat

$$R_A = x + j(R_b + s)$$

$$x = v$$

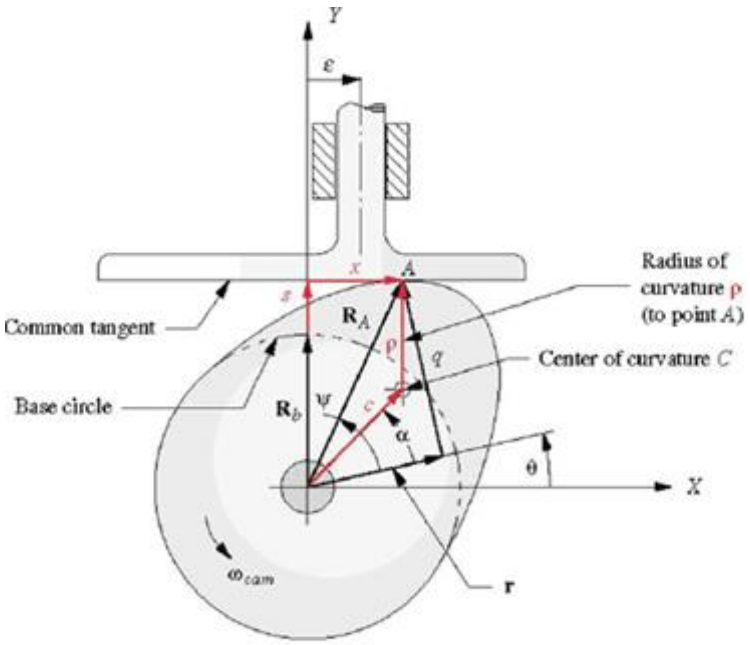
$$facewidth > v_{\max} - v_{\min}$$

$$\rho_{\min} = R_b + (s + a)_{\min}$$

Cam Contour;

$$r = (R_b + s)\sin\theta + v\cos\theta$$

$$q = (R_b + s)\cos\theta - v\sin\theta$$




**5.10 CAM FORCES**

At any instant both inertia force and spring force are acting on the cam. During the phase of cam rotation when $\frac{\theta}{\beta}$. varies from 0.5 to 1, the total force on the follower, F, is the difference between the inertia force (upward)and the spring force (downward). Hence the maximum force during this phase is due to spring force and this occurs, when the follower reaches the highest position i.e., at $\frac{\theta}{\beta}$.=1.

$F_{n\ max}$ =$F_s$=h x k = 25 x 633.7=15842.5N

(At $\frac{\theta}{\beta}$.=1, inertia force, $F_I$=0 and $\alpha$ =0$^o$)

When $\frac{\theta}{\beta}$. varies from 0 to 0.5, inertia force as well as spring force are downward. These two forces do not reach their maximum at the same time. Therefore it is necessary to calculate both the forces and obtain their sum for different values of $\frac{\theta}{\beta}$..

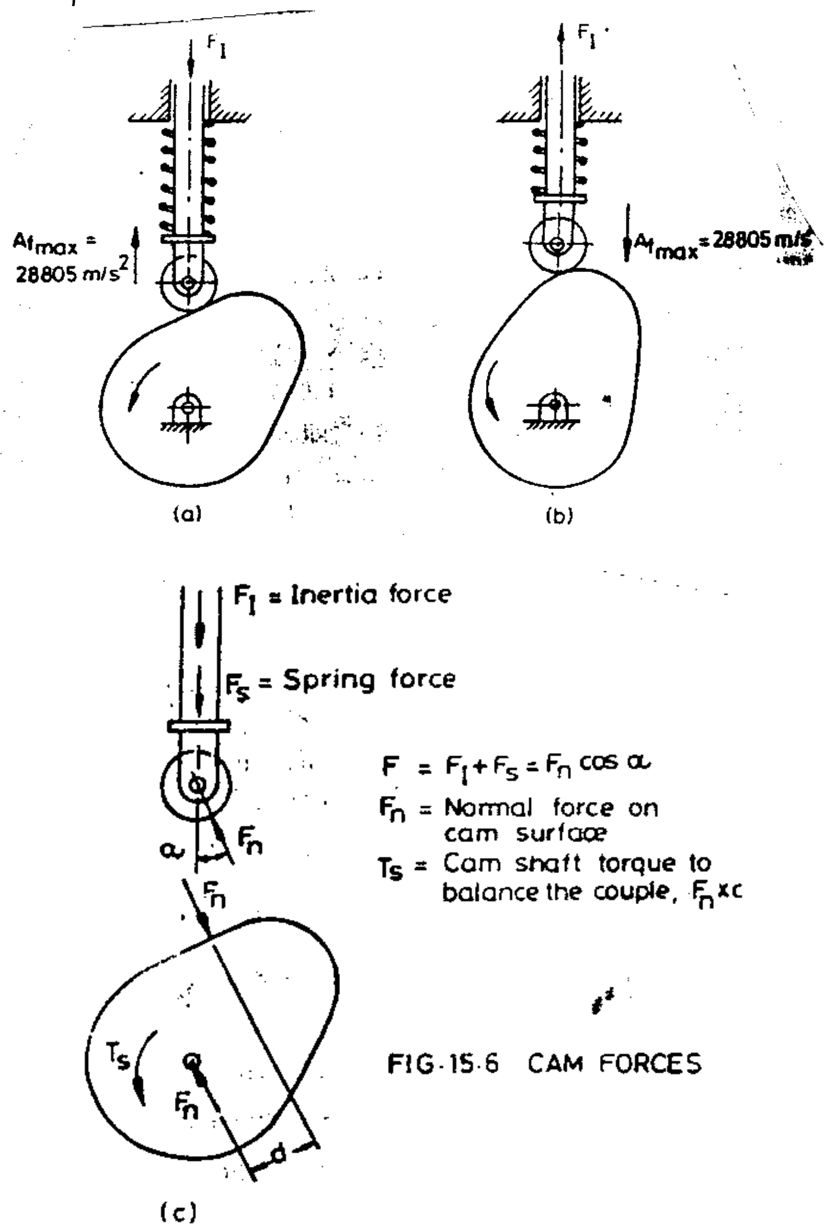


FIG 15.6 CAM FORCES

1. $\frac{\theta}{\beta}$.=0;y=0;$F_s$=$F_1$=0;α=0

2. $\frac{\theta}{\beta}$.=0.25;y=2.269mm;$F_s$=633.7 x 2.269 ≈ 1438N

$F_1$=14,403N;α=11.3°

F=$F_s$+$F_1$=1438+14403=15841 N

3. $\frac{\theta}{\beta}$.=0375;y=25[0.375-$\frac{1}{2\pi}$. sin (2π x 0.375)]=6.56mm

$F_S$=633.7 x 6.56=4157N

$$F_I = m_f \times \frac{d^2}{dt^2} = m_f \times 2\pi h \frac{\omega^2}{\beta^2} \sin 2\pi \frac{\theta}{\beta}.$$

$$= \frac{0.5 \times 2\pi \times 25 \times 10^{-3} \times 523.3^2}{1.22117^2} \sin(2\pi \times 0.375) = 10{,}184\text{N}$$

$$F = 4157 + 10{,}184 = 14{,}341\text{N}$$

$$\frac{dy}{d\theta} = \frac{h}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right)$$

$$= \frac{25}{1.2217}\left[1 - \cos(2\pi \times 0.375)\right] = 34.9\text{mm/rad}$$

$$r = R_P + y = 100$$

$$6.56 = 106.56\text{mm}$$

$$\alpha = \tan^{-1}\left(\frac{1}{r}, \frac{dy}{d\theta}\right) = \tan^{-1}\frac{34.9}{106.56} = 18..15^o$$

$$\frac{\theta}{\beta} = 0.5; y = 12.5\text{mm}; \frac{d^2y}{dt^2} = 0$$

$F_S$=12.5 x 633.7=7921N;$F_I$=0

F=$F_S$=7921N

$$\frac{dy}{d\theta} = \frac{25}{1.2217}\left[1 - \cos(2\pi x 0.5)\right] = 40.93\text{mm/rad}$$

$$r = 100 + 12.5 = 112.5\text{mm}$$

$$\alpha = \tan^{-1}\left(\frac{1}{112.5} \times 40.93\right) \approx 20^o$$

The normal force acting on the cam surface is

$F_n$= $\frac{F}{\text{Cos}\alpha}$ (Fig) ignoring the friction between follower and guide

$F_n = 0$ $\left(\frac{\theta}{\beta} = 0\right)$

$F_n = \frac{15841}{\cos 113^o} = 16164\text{N}$ $\left(\frac{\theta}{\beta} = 0.25\right)$

$F_n = \frac{14341}{\cos 18.15^o} = 15128\text{N}$ $\left(\frac{\theta}{\beta} = 0.375\right)$

$F_n = \frac{7921}{\cos 20^o} = 8427\text{N}$ $\left(\frac{\theta}{\beta} = 0.5\right)$

Thus we find $F_{n\,max}$ =16,164N

**Example 1:** The displacement function of a cam-follower mechanism is given by y $(\theta) = 100(1 - \cos\ \theta)\text{mm},\ 0 \le \theta \le 2\pi$. Where y is the follower displacement and $\theta$ is the cam rotation. The cam speed is 1000 rpm. The spring constant is 20 N/mm and the spring has an initial compression of 10 mm, when the roller follower is 10 N, length of the follower outside the guide A = 40 mm, length of the guide B = 100 mm, $R_b$ = 50 mm, $R_r$ = 10 mm and the coefficient of friction between the guide and the follower $\mu = 0.05$. Compute $F_n$ and the cam shaft torque when the cam has rotated 60 degrees.

**Solution:**



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$y = 100(1-\cos\theta)$

$\frac{dy}{dt} = \frac{dt}{d\theta}\frac{d\theta}{dt}\omega(100\sin\theta)$

$\frac{d^2y}{dt^2} = \frac{d}{d\theta}(\omega(100\sin\theta)).\frac{d\theta}{dt}\omega^2(100\cos\theta)$

$\frac{d^2y}{dt^2} = \omega^2 100x10^{-3}\cos 60^0 = 104.7^2 x0.05 \quad \omega=\frac{1000x2\pi}{60}$

$(\theta = 60^0)$ $=547.8$ m/s$^2$ $= 104.7$ rad/s

Acceleration is positive upward.

Inertia force is downward.

$F_I = \frac{W}{g}\frac{d^2y}{dt^2} = \frac{10}{9.81}x547.8 = 558.4N$

Displacement at $\theta = 60^0$ is y=100 $(1\text{-}\cos 60^0) = 50mm$

Spring Force = ( Initial compression + y) x spring stiffness $F_s$= ( 10 + 50 ) 20 = 1200 N

This force is also downward. $p_k = \frac{\left[97.73^2 + 23.87^2\right]^{3/2}}{(97.73)^2 + 2x23.87^2 - 97.73(= -143.24)}mm$

Total downward force $F=F_I+F_s$=558.4+1200=1758.4 N

Pressure angle $\alpha=\tan^{-1}\frac{1}{r}\frac{dy}{d\theta}$ $r=R_b + R_r + y =$

$50 + 10 + 50 = 110$ mm

$=\tan^{-1}\left[\frac{1}{110}100\sin\theta\right] = \tan^{-1}\left[\frac{100\sin 60^0}{110}\right] = 38.2^0$

$F_n = \frac{F}{\cos\alpha - \mu\left(\frac{2A+B}{B}\right)\sin\alpha}$

$= \frac{1758.4}{\cos 38.2^0 - 0.05\left(\frac{2x40+100}{100}\right)\sin 38.2^0}$

$= \frac{1758.4}{0.7857 - 0.0556} = 2408.6N$

$F_n$ can be used to check the induced contact stress.

Cam shaft torque = $F.\frac{dy}{dt}/\omega$

$\frac{1758.4x9.067}{104.7}$ $\frac{dy}{dt} = 100\omega\sin 60^0$

$\theta=60^0 = 100x104.7x0.866$

= 9067.3 mm/s

**Example 2:** Given h = 25 mm, $R_p$=mm, $\beta = 60^0$ Find $\rho_{kmin}$for parabolic cyclodial and harmonic motions.

**Solution:**

1. **Parabolic motion**

At $\frac{\theta}{\beta} = 1$, **maximum negative acceleration occurs and velocity is zero.**

Y = h = 25 mm

$R_p$+y=75+25= 100 mm

$$\frac{1}{\omega}\frac{dy}{dt}=0$$

$$\frac{1}{\omega^2}\frac{d^2y}{\beta^2}=-\frac{4h}{\beta^2}=-\frac{4x25}{\left(\frac{\pi}{180}x60\right)^2}=-91.18\ mm/rad^2$$

$$P_k=\frac{\left[\left(R_p+y\right)^2+\left(\frac{1}{\omega}\frac{dy}{dt}\right)^2\right]^{3/2}}{\left(R_p+y\right)^2+2\left(\frac{1}{\omega}\frac{dy}{dt}\right)^2-\left(R_p+y\right)\left(\frac{1}{\omega^2}\frac{d^2y}{dt^2}\right)}$$

$$=\frac{100^2+0}{100^2+\left(2x0\right)-100\left(-91.18\right)}=52.3mm$$

$$\frac{1}{\omega^2}.\frac{d^2y}{dt^2}=\frac{2h\pi}{\beta^2}\sin\frac{2\pi\theta}{\beta}=-\frac{2x25x\pi}{\left(\pi/3\right)^2}=-143.24mm/rad^2$$

**2. Cycloidal motion**

At $\frac{\theta}{\beta}=0.75,$ Maximum negative acceleration occurs. $\frac{1}{\omega}.\frac{dy}{dt}=\frac{h}{\beta}\left(1-\cos\frac{2\pi\theta}{\beta}\right)=\frac{25}{\pi/3}\left(1-\cos\left(2\pi x0.75\right)\right)$

$$=23.87\ mm/rad$$

$$y=\frac{h}{\pi}\left(\frac{\pi\theta}{\beta}-\frac{1}{2}\sin\frac{2\pi\theta}{\beta}\right)$$

$$=\frac{25}{\pi}\left(\pi x0.75-\frac{1}{2}\sin\left(2\pi x0.75\right)\right)=22.73mm$$

$$R_p+y=75+22.73=97.73mm$$

$$\frac{1}{\omega^2}.\frac{d^2y}{dt^2}=\frac{2h\pi}{\beta^2}\sin\frac{2\pi\theta}{\beta}=-\frac{2x25x\pi}{\left(\pi/3\right)^2}=143.24mm/rad^2$$

$$p_k=\frac{\left[97.73^2+23.872^2\right]}{\left(97.73\right)^2+2x23.87^2-97.73\left(-143.24\right)}=41.24mm$$

3. **Simple Harmonic Motion**

At $\frac{\theta}{\beta}=1,$ maximum negative acceleration occurs.

$$\frac{1}{\omega}\frac{dy}{dt}=0, y=h=25mm$$

$$R_p+y=75+25=100\ mm$$

$$\frac{1}{\omega^2}\frac{d^2y}{dt^2}=\frac{\pi^2h}{2\beta^2}\cos\frac{\pi\theta}{\beta}=-\frac{\pi^2x25}{2x\left(\pi/3\right)^2}=-112.5mm/rad^2$$

$$\rho_k=\frac{\left(100^2+0\right)}{100^2+\left(2x0\right)-100\left(-112.5\right)}=47.05mm$$

**Example 3:** A cycloidal motion cam has the following data; h=25 mm, $\beta=60^0$, $\alpha_{max}=38^0$. Calculate the minimum radius of the basic circle if the radius of the roller is 6 mm. Also calculate the amount of offset that is necessary to reduce the maximum pressure angle to $25^0$.

**Solution:**

**Determination of $R_b$**

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$$\frac{1}{\omega}\left(\frac{dt}{dy}\right)=\frac{h}{\beta}(1-\cos\pi)=\frac{2h}{\beta}\text{at } \theta=\beta/2$$

$$y=\frac{h}{2}\text{at } \theta=\frac{\beta}{2}$$

$$\tan\ \alpha_{max}=\frac{\left(\frac{dy}{dt}\right)max}{\left(R_p+y\right)\omega}=\frac{2h}{\left(R_p+\frac{h}{2}\right)\beta}=\frac{2x25}{\left(R_p+\frac{h}{2}\right)\beta}=\frac{2x25}{\left(R_p+\frac{25}{2}\right)\frac{\pi}{3}}$$

$$\tan 38^0=\frac{150}{\left(R_p+12.5\right)\pi},\qquad R_p=48.6mm$$

$$R_b=R_p-R_r=48.6-6=42.6\text{ mm}$$

**Offset required reducing the pressure angle**

$$\tan\ \alpha=\frac{dy/d\theta\text{-e}}{\sqrt{R_p^2-e^2+y}}=\frac{\frac{2h}{\beta}-e}{\sqrt{R_p^2-e^2+h/2}}$$

$$\tan\ 25^0=\frac{\frac{2x25}{\pi/3}-e}{\sqrt{48.6^2-e^2+\frac{25}{2}}}$$

$$0.4663=\frac{47.75-e}{\sqrt{2361.9-e^2+12.5}}$$

Solving by trial and error we get e = 22 mm that gives $\alpha=24.75^0$

The following data refer to a cam operating the suction valve of a 4-stroke petrol engine: Least radius = 18 mm, Lift = 10 mm, Nose radius = 3 mm, Crank angle when the suction valve opens before TDC = $4^0$, Crank angle when the suction valve closes after BDC = $50^0$, Cam shaft speed = 80 rpm. The cam is of circular type with circular nose and flanks. It is integral with the cam shaft and operates a flat faced follower.

Calculate (i) the maximum acceleration and retardation of the valve (ii) the minimum force to be exerted by the spring to overcome inertia of valve parts whose mass is 250 gram (M.U. Oct'96).

Also calculate the angles of cam rotation during the acceleration and retardation of follower.

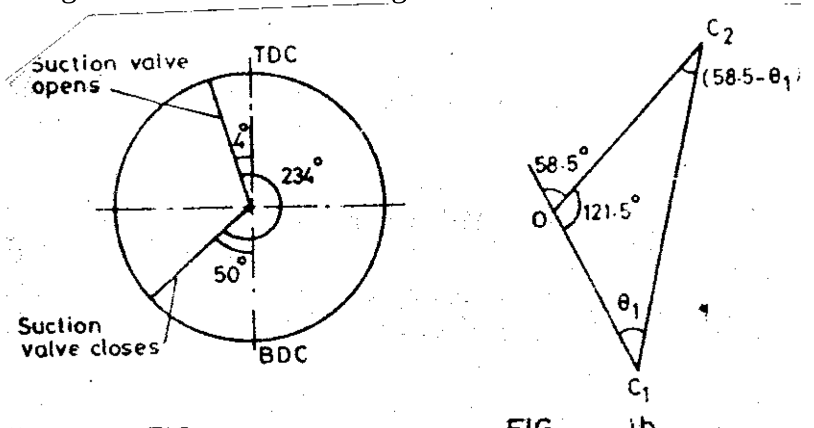


FIG. a FIG. 1b

1. Refer to figure (a)
   Angle of crank rotation during suction
   = 4+180+50 = $234^0$
2. Corresponding angle of cam shaft rotation

$2\theta_2 = 234/2 = 117^0, \theta_2 = 58.5^0$

(Speed of cam shaft = $\frac{1}{2}$xspeed of crankshaft

for four stroke cycle)

3. Acceleration of the follower is maximum force to be exert by the spring; maximum retardataion when the follower is on the nose, $a_{min}$ should be evaluated. Hence we use equation

$a = -\omega^2 (r_3 - r_1) = OC_1$

and put $\theta = \theta_2$ to obtain $\alpha_{min.}$

Now the unknown is ($r_5$-$r_4$) i.e., $OC_2$ in figure

4. $C_1C_2 = r_3 - r_4 = r_1 + OC_1 - r_4$

$= 18 + OC_1 - 3 = 15 + OC_1$ $\quad r_1 = 18$ mm

$OC_2 = r_1 +$ lift $- r_4 = 18+10-3 = 25$mm $\quad r_4 = 3$ mm

Lift = 10 mm

5. Refer to figure 15.10b. Consider triangle $OC_1C_2$, Using cosine law, we write

$C_1C_2^2 = OC_1^2 + OC_2^2 - 2.\ OC_2.\cos\ 121.5^0$

$(15 + OC_1)^2 = OC_1^2 + 25^2 - 2.)OC_1.25.COS\ 121.5^0$

$OC_1 = 103.36$ mm=118.36=118.36 mm

i.e., $C_1C_2 = 15 + OC_1 = 15 + 103.36 = 118.36$ mm

6. Now use sine rule

$\frac{OC_2}{\sin\theta_1} = \frac{C_1C_2}{\sin\ 121.5^0}$

$\frac{25}{\sin\theta_1} = \frac{118.36}{\sin\ 121.5^0} \theta_1 = 10.4^0$

Angle of cam rotation during acceleration $\theta_1 = 10.4^0$

Angle of cam rotation during retardation

$\theta_2 - \theta_1 = 58.5 - 10.4 = 48.1^0$

7. Acceleration is maximum when $\theta = 0$.

$\alpha_{max} = \omega^2 (r_3 - r_1)$

$= 8.4^2 xOC_1 = 8.4^2 x0.1034 \quad \omega =: \frac{80x2\pi}{60} 8.4$ rad/s

$=7.3$ m/s$^2$

$\alpha_{min}(\text{retardation}) = -\omega^2 (r_5 - r_4)$

$= -\omega^2 OC_2 = -8.4^2 x0.025$

$= -1.76$ m/s

8. When the follower reaches the topmost position (D) maximum retardation occurs (figure 15.8.b), i.e., the acceleration is downward. Hence the inertia force acting o the follower is upward (similar to the case shown in the figure) and to to prevent the loss of contact between the follower and cam the spring has to exert a downward force. Minimum value of this force is the spring has to exert a downward force. Minimum value of this force is

$F_s =$ mass of the follower x $\alpha_{min}$

$= 0.25$ x 1. 76 $\quad$ m = 250 gm

0.44N

## 5.11 CLUTCHES

### 5.11.1 Introduction

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A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles. A little consideration will show that in order to change gears or to stop the vehicle, it is required that the driven shaft should stop, but the engine should continue to run. It is, therefore, necessary that the driven shaft should be disengaged from the driving shaft. The engagement and disengagement of the shafts is obtained by means of a clutch which is operated by a lever.

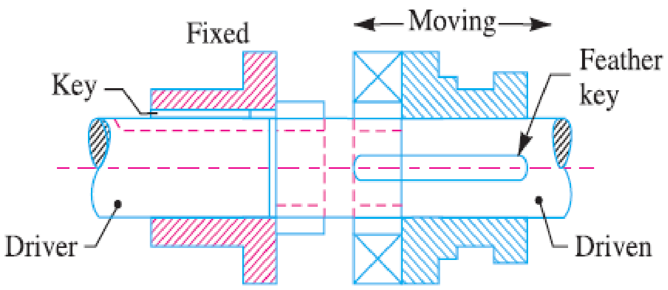
### 5.12 Types of Clutches

Following are the two main types of clutches commonly used in engineering practice :

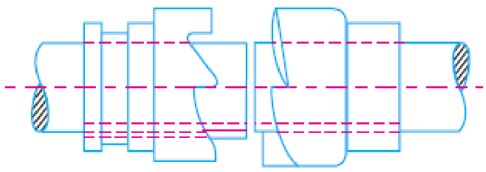
1. Positive clutches.
2. Friction clutches

### 5.12.1 Positive Clutches

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a ***jaw*** or ***claw clutch***. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. It consists of two halves, one of which is permanently fastened to the driving shaft by a sunk key. The other half of the clutch is movable and it is free to slide axially on the driven shaft, but it is prevented from turning relatively to its shaft by means of feather key. The jaws of the clutch may be of square type as shown in Fig. (*a*) or of spiral type as shown in Fig. (*b*).




(*a*) Square jaw clutch.



(*b*) Spiral jaw clutch.

A square jaw type is used where engagement and disengagement in motion and under load is not necessary. This type of clutch will transmit power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only. This type of clutch is occasionally used where the clutch must be engaged and disengaged while in motion. The use of jaw clutches are frequently applied to sprocket wheels, gears and pulleys. In such a case, the non-sliding part is made integral with the hub.

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. Its application is also found in cases in which power is to be delivered to machines partially or fully loaded. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces. In automobiles, friction clutch is used to connect the engine to the drive shaft. In operating such a clutch, care should be taken so that the friction surfaces engage easily and gradually bring the driven shaft up to proper speed. The proper alignment of the bearing must be maintained and it should be located as close to the clutch as possible. It may be noted that :

**1.** The contact surfaces should develop a frictional force that may pick up and hold the load with reasonably low pressure between the contact surfaces.
**2.** The heat of friction should be rapidly *dissipated and tendency to grab should be at aminimum.
**3.** The surfaces should be backed by a material stiff enough to ensure a reasonably Uniform distribution of pressure.

### 5.13 Material for Friction Surfaces

The material used for lining of friction surfaces of a clutch should have the following characteristics :
During operation of a clutch, most of the work done against frictional forces opposing the motion is liberated as heat at the interface. It has been found that at the actual point of contact, the temperature as high as 1000°C is reached for a very short duration (*i.e.* for 0.0001 second). Due to this, the temperature of the contact surfaces will increase and may destroy the clutch.

**1.** It should have a high and uniform coefficient of friction.
**2.** It should not be affected by moisture and oil.
**3**. It should have the ability to withstand high temperatures caused by slippage.
**4.** It should have high heat conductivity.
**5.** It should have high resistance to wear and scoring.
The materials commonly used for lining of friction surfaces and their important properties are shown in the following table.

**Table 24.1. Properties of materials commonly used for lining of friction surfaces.**

| *Material of friction surfaces* | *Operating condition* | *Coefficient of friction* | *Maximum operating temperature (°C)* | *Maximum pressure (N/mm²)* |
|---|---|---|---|---|
| Cast iron on cast iron or steel | dry | 0.15 – 0.20 | 250 – 300 | 0.25– 0.4 |
| Cast iron on cast iron or steel | In oil | 0.06 | 250 – 300 | 0.6 – 0.8 |
| Hardened steel on Hardened steel | In oil | 0.08 | 250 | 0.8 – 0.8 |
| Bronze on cast iron or steel | In oil | 0.05 | 150 | 0.4 |
| Pressed asbestos on cast iron or steel | dry | 0.3 | 150 – 250 | 0.2 – 0.3 |
| Powder metal on cast iron or steel | dry | 0.4 | 550 | 0.3 |
| Powder metal on cast iron or steel | In oil | 0.1 | 550 | 0.8 |

### 5.14 Considerations in Designing a Friction Clutch

The following considerations must be kept in mind while designing a friction clutch.
**1.** The suitable material forming the contact surfaces should be selected.
**2.** The moving parts of the clutch should have low weight in order to minimise the inertia load, especially in high speed service.
**3.** The clutch should not require any external force to maintain contact of the friction surfaces.
**4.** The provision for taking up wear of the contact surfaces must be provided.
**5.** The clutch should have provision for facilitating repairs.

**6.** The clutch should have provision for carrying away the heat generated at the contact surfaces.

**7.** The projecting parts of the clutch should be covered by guard.

**5.15 Types of Friction Clutches**

Though there are many types of friction clutches, yet the following are important from the subject point of view :

**1.** Disc or plate clutches (single disc or multiple disc clutch),

**2.** Cone clutches, and

**3.** Centrifugal clutches.

We shall now discuss these clutches, in detail, in the following pages.

**Note :** The disc and cone clutches are known as ***axial friction clutches,*** while the centrifugal clutch is called ***radial friction clutch.***

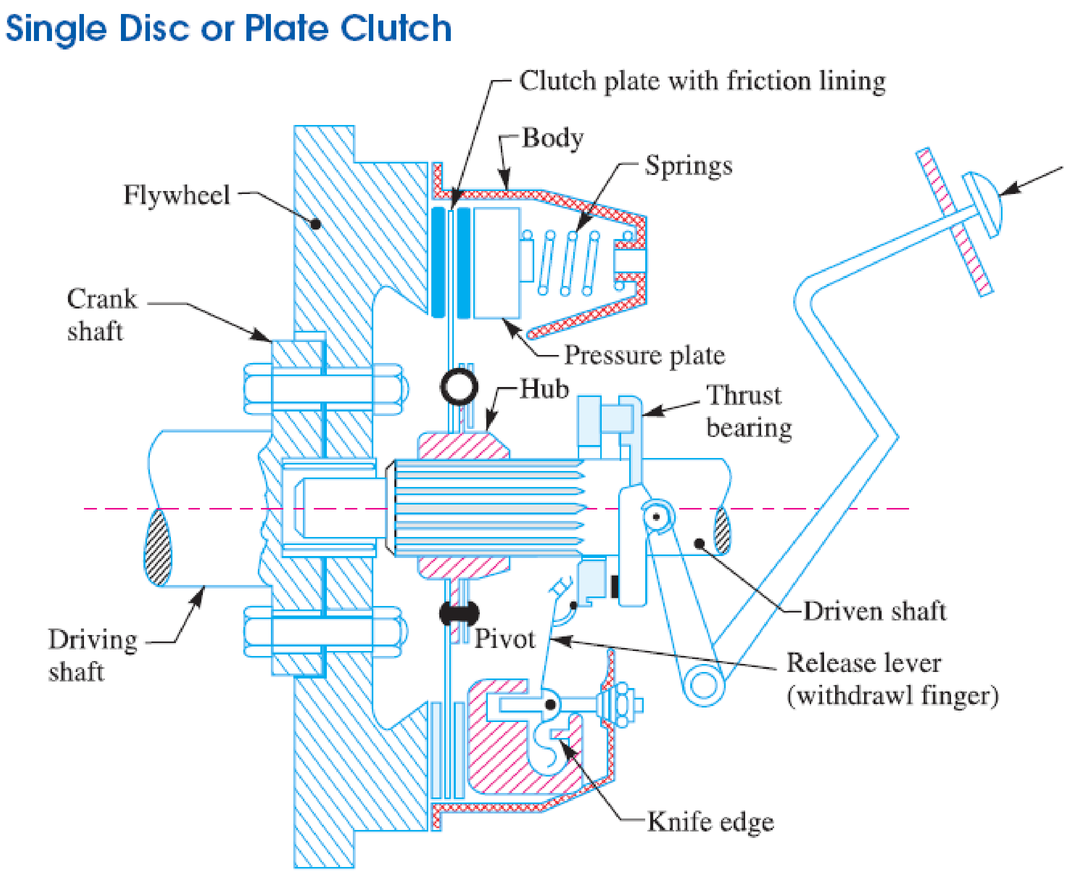



**Fig. 24.2.** Single disc or plate clutch.

A single disc or plate clutch, as shown in Fig 24.2, consists of a clutch plate whose both sides are faced with a frictional material (usually of Ferrodo). It is mounted on the hub which is free to move axially along the splines of the driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a

set of strong springs which are arranged radially inside the body. The three levers (also known as release levers or fingers) are carried on pivots suspended from the case of the body. These are arranged in such a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing.

The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed. When the clutch pedal is pressed down, its linkage forces the thrust release bearing to move in towards the flywheel and pressing the longer ends of the levers inward. The levers are forced to turn on their suspended pivot and the pressure plate moves away from the flywheel by the knife edges, thereby compressing the clutch springs. This action removes the pressure from the clutch plate and thus moves back from the flywheel and the driven shaft becomes stationary. On the other hand, when the foot is taken off from the clutch pedal, the thrust bearing moves back by the levers. This allows the springs to extend and thus the pressure plate pushes the clutch plate back towards the flywheel. The axial pressure exerted by the spring provides a frictional force in the circumferential direction when the relative motion between the driving and driven members tends to take place. If the torque due to this frictional force exceeds the torque to be transmitted, then no slipping takes place and the power is transmitted from the driving shaft to the driven shaft.

## Design of a Disc or Plate Clutch

:onsider two friction surfaces maintained in contact by an axial thrust ($W$) as shown in 4.3 ($a$).

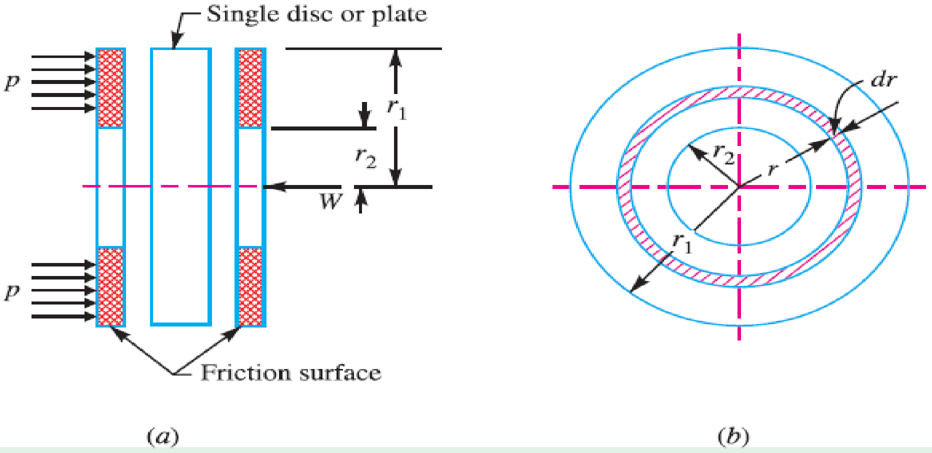



**Fig. 24.3.** Forces on a disc clutch.

Let $T$ = Torque transmitted by the clutch,

$p$ = Intensity of axial pressure with which the contact surfaces are held together,

$r_1$ and $r_2$ = External and internal radii of friction faces,

$r$ = Mean radius of the friction face, and

$\mu$ = Coefficient of friction.

Consider an elementary ring of radius $r$ and thickness $dr$ as shown in Fig. 24.3 (*b*).

We know that area of the contact surface or friction surface

$$= 2\pi\, r.dr$$

∴ Normal or axial force on the ring,

$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi\, r.dr$$

and the frictional force on the ring acting tangentially at radius $r$,

$$F_r = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi\, r.dr \times r = 2\,\pi\,\mu\, p.\, r^2.dr$$

We shall now consider the following two cases :

1. When there is a uniform pressure, and
2. When there is a uniform axial wear.

**1. *Considering uniform pressure.*** When the pressure is uniformly distributed over the entire area of the friction face as shown in Fig. 24.3 (*a*), then the intensity of pressure,

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

where $W$ = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius $r$ and thickness $dr$ is

$$T_r = 2\pi\,\mu.p.r^2.dr$$

Integrating this equation within the limits from $r_2$ to $r_1$ for the total friction torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\,\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\,\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right] = 2\pi\,\mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

... (Substituting the value of $p$)

$$= \frac{2}{3}\,\mu.\,W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu\,.W.R$$

where $R = \dfrac{2}{3}\left[\dfrac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$ = Mean radius of the friction surface.

**2. *Considering uniform axial wear.*** The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction.

The work of friction is proportional to the product of normal pressure ( $p$) and the sliding velocity ($V$). Therefore,

$$\text{Normal wear} \propto \text{Work of friction} \propto p.V$$

or $$p.V = K \text{ (a constant) or } p = K/V \qquad \ldots(i)$$

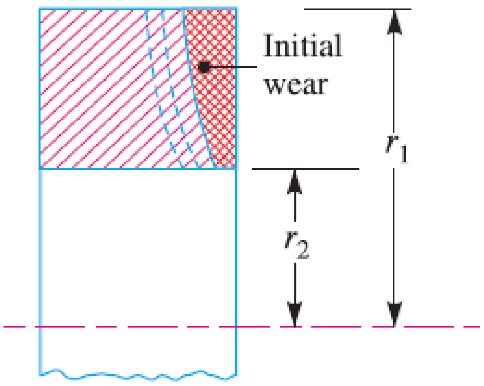



**Fig. 24.4.** Uniform axial wear.

It may be noted that when the friction surface is new, there is a uniform pressure distribution over the entire contact surface. This pressure will wear most rapidly where the sliding velocity is maximum and this will reduce the pressure between the friction surfaces. This wearing-in process continues until the product *p.V* is constant over the entire surface. After this, the wear will be uniform as shown in Fig. 24.4. Let *p* be the normal intensity of pressure at a distance *r* from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r = C \text{ (a constant) or } p = C/r$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\, C\, dr = 2\pi\, C\, [r]_{r_2}^{r_1} = 2\pi\, C\, (r_1 - r_2)$$

or $$C = \frac{W}{2\pi\,(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\,\mu.p.r^2.dr = 2\pi\,\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr \qquad ...(\because p = C/r)$$

∴ Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\,\mu\, C.r.dr = 2\pi\,\mu\, C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\,\mu.C\,[(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of the friction surface.

**Notes : 1.** In general, total frictional torque acting on the friction surfaces (or on the clutch) is given by

$$T = n.\mu.W.R$$

where $n$ = Number of pairs of friction (or contact) surfaces, and

$R$ = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \qquad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \qquad \text{... (For uniform wear)}$$

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**2.** For a single disc or plate clutch, normally both sides of the disc are effective. Therefore a single disc clutch has two pairs of surfaces in contact (*i.e.* $n = 2$).

**3.** Since the intensity of pressure is maximum at the inner radius ($r_2$) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C / r_2$$

**4.** Since the intensity of pressure is minimum at the outer radius ($r_1$) of the friction or contact surface, therefore equation (*ii*) may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C / r_1$$

**5.** The average pressure ($p_{av}$) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

**6.** In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch, the uniform wear theory is more approximate.

**7.** The uniform pressure theory gives a higher friction torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

## 5.16 Multiple Disc Clutch

A multiple disc clutch, as shown in Fig. 24.5, may be used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft. The multiple disc clutches are extensively used in motor cars, machine tools etc.

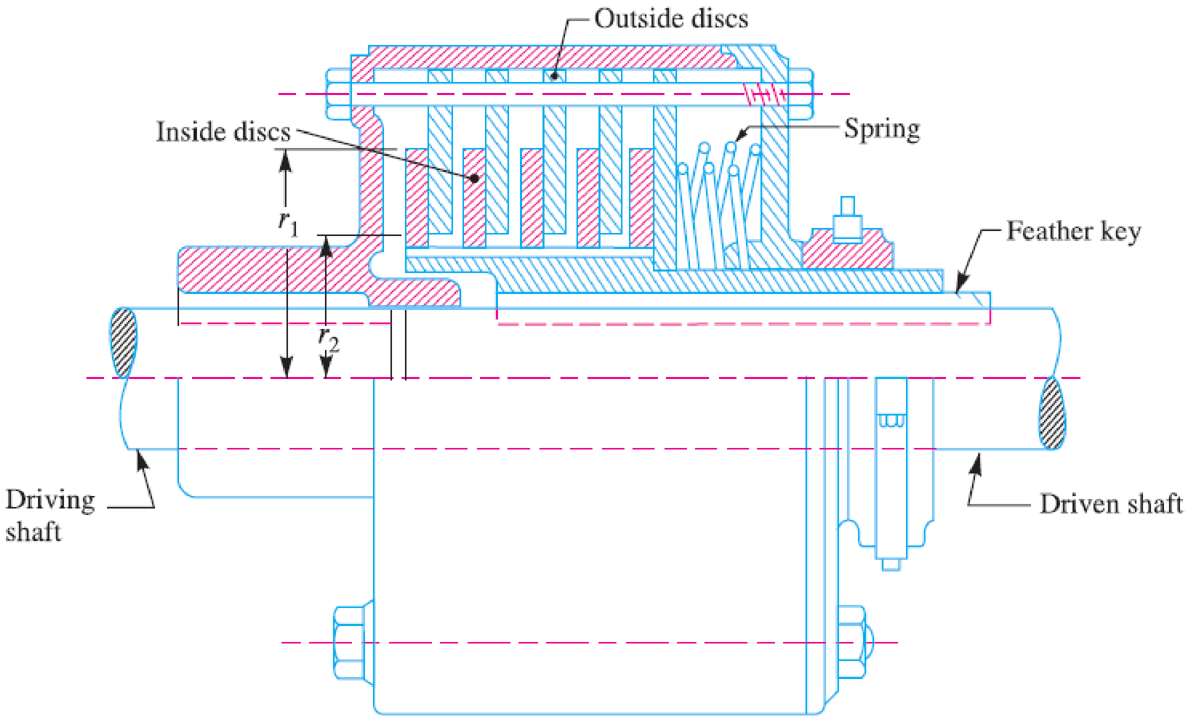


**Fig. 24.5.** Multiple disc clutch.

Let $n_1$ = Number of discs on the driving shaft, and

$n_2$ = Number of discs on the driven shaft.

∴ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

$$T = n.\mu.W.R$$

where $R$ = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

## 5.17 Cone Clutch

A cone clutch, as shown in Fig. 24.6, was extensively used in automobiles, but now-a-days it has been replaced completely by the disc clutch. It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

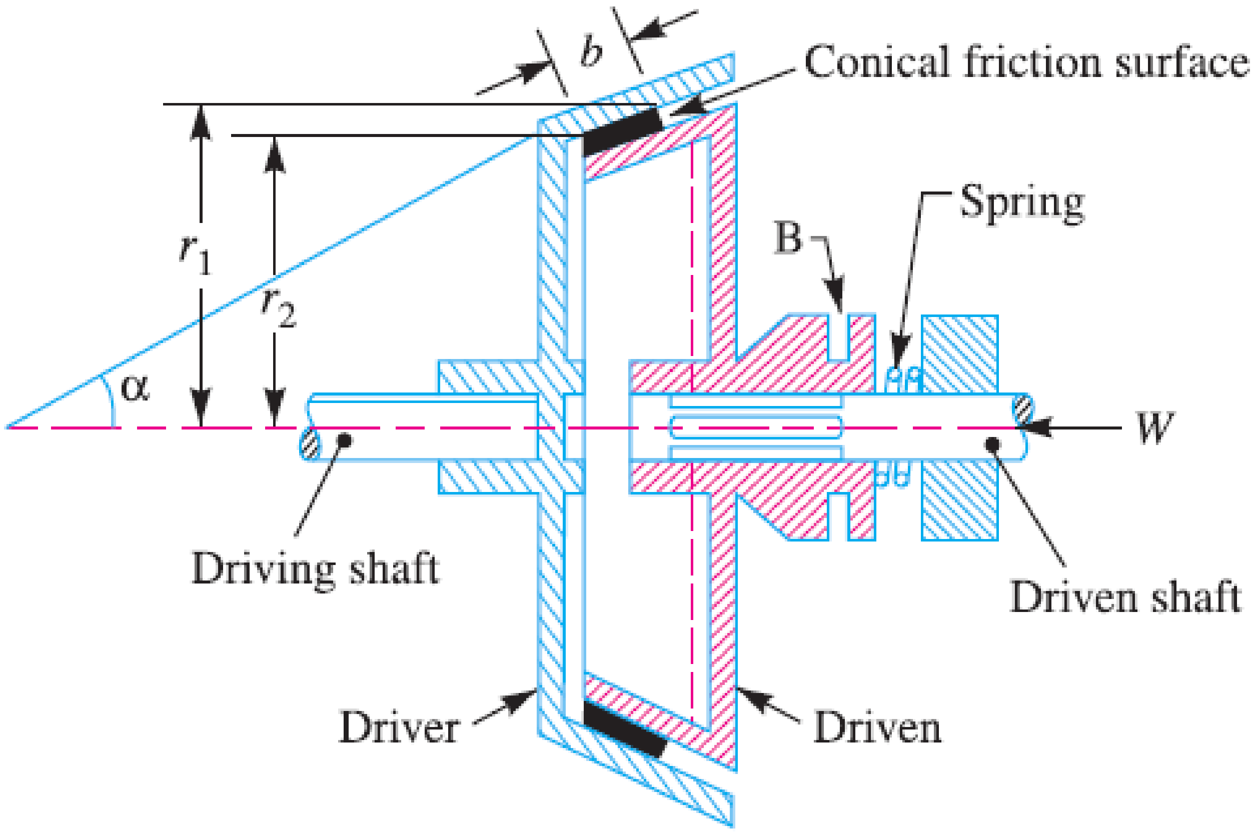


**Fig. 24.6.** Cone clutch.

### 5.18 Design of a Cone Clutch

Consider a pair of friction surfaces of a cone clutch as shown in Fig. 24.7. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone.

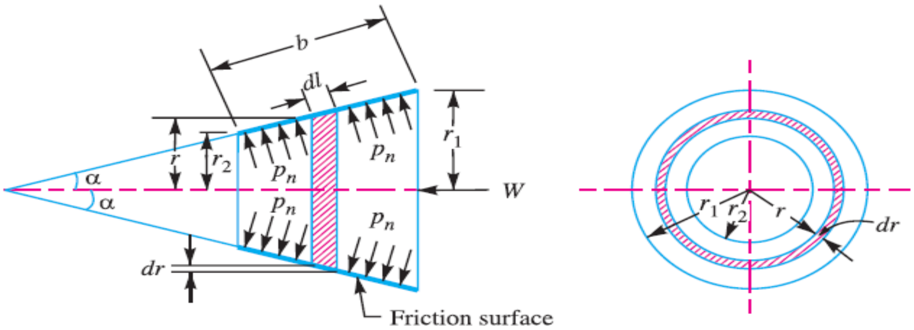


**Fig. 24.7.** Friction surfaces as a frustrum of a cone.

Let $p_n$ = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between the contact surfaces),

$r_1$ = Outer radius of friction surface,

$r_2$ = Inner radius of friction surface,

$R$ = Mean radius of friction surface = $\frac{r_1 + r_2}{2}$,

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between the contact surfaces, and

$b$ = Width of the friction surfaces (also known as face width or cone face).

Consider a small ring of radius *r* and thickness *dr* as shown in Fig.. Let *dl* is the length of ring of the friction surface, such that,

$$dl = dr \operatorname{cosec} \alpha$$

$$\therefore \quad \text{Area of ring} = 2\pi r.\, dl = 2\pi r.dr \operatorname{cosec} \alpha$$

We shall now consider the following two cases :

**1.** When there is a uniform pressure, and
**2.** When there is a uniform wear.

**1. *Considering uniform pressure***

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi\, r.dr \text{ cosec } \alpha$$

and the axial force acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$

$$= \delta W_n \times \sin\alpha = p_n \times 2\pi\, r.dr \text{ cosec } \alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

$\therefore$ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi \times p_n.r.dr = 2\pi\ p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\, p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\, p_n\, [(r_1)^2 - (r_2)^2]$$

and $$p_n = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]} \qquad \ldots(i)$$

We know that frictional force on the ring acting tangentially at radius $r$,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr \text{ cosec } \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr \text{ cosec } \alpha \times r$$

$$= 2\pi\, \mu.p_n \text{ cosec } \alpha.r^2\, dr$$

Integrating this expression within the limits from $r_2$ to $r_1$ for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{ cosec } \alpha.r^2\ dr = 2\pi\ \mu.p_n\ \text{cosec } \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\ \mu.p_n \text{ cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

**2. *Considering uniform wear***

In Fig. 24.7, let $p_r$ be the normal intensity of pressure at a distance $r$ from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$\therefore$ $$p_r.r = C \text{ (a constant)} \quad \text{or} \quad p_r = C / r$$

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr \text{ cosec } \alpha$$

and the axial force acting on the ring,

$$\delta W = \delta W_n \times \sin\alpha = p_r \times 2\pi\, r.dr \text{ cosec } \alpha \times \sin\alpha$$

$$= 2\pi \times p_r.r\ dr$$

$$= 2\pi \times \frac{C}{r} \times r.\ dr = 2\pi\ C.dr \qquad \ldots\left(\because p_r = \frac{C}{r}\right)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\ C.dr = 2\pi\ C\ [r]_{r_2}^{r_1} = 2\pi\ C\ (r_1 - r_2)$$

or $$C = \frac{W}{2\pi\,(r_1 - r_2)} \qquad \ldots (iii)$$

We know that frictional force on the ring acting tangentially at radius $r$,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi\, r.dr \text{ cosec } \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\pi\, r.dr \operatorname{cosec} \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi\, r.dr \operatorname{cosec} \alpha \times r = 2\pi\, \mu.C \operatorname{cosec} \alpha \times r\, dr$$

Integrating this expression within the limits from $r_2$ to $r_1$ for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu.C \operatorname{cosec} \alpha \times r\, dr = 2\pi\, \mu.C \operatorname{cosec} \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\, \mu.C \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of $C$ from equation ***(iii)***, we have

$$T = 2\pi\mu \times \frac{W}{2\pi\,(r_1 - r_2)} \times \operatorname{cosec} \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W \operatorname{cosec} \alpha \left[\frac{r_1 + r_2}{2}\right] = \mu\, WR \operatorname{cosec} \alpha \qquad \text{...(iv)}$$

where $R = \dfrac{r_1 + r_2}{2}$ = Mean radius of friction surface.

Since the normal force acting on the friction surface, $W_n = W \operatorname{cosec} \alpha$, therefore the equation ***(iv)*** may be written as

$$T = \mu\, W_n\, R \qquad \text{...(v)}$$

The forces on a friction surface, for steady operation of the clutch and after the clutch is engaged, is shown in Fig. 24.8 (*a*) and (*b*) respectively.

From Fig. 24.8 (*a*), we find that

$$r_1 - r_2 = b \sin \alpha \quad \text{and} \quad R = \frac{r_1 + r_2}{2} \quad \text{or} \quad r_1 + r_2 = 2R$$

$\therefore$ From equation ***(i)***, normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]} = \frac{W}{\pi\,(r_1 + r_2)\,(r_1 - r_2)} = \frac{W}{2\pi\, R.b \sin \alpha}$$

or $W = p_n \times 2\pi\, R.b \sin \alpha = W_n \sin \alpha$

where $W_n$ = Normal load acting on the friction surface = $p_n \times 2\pi R.b$

Now the equation ***(iv)*** may be written as

$$T = \mu\,(p_n \times 2\pi\, R.\, b \sin \alpha)\, R \operatorname{cosec} \alpha = 2\pi\, \mu.p_n\, R^2.b$$

The following points may be noted for a cone clutch :

**1.** The above equations are valid for steady operation of the clutch and after the clutch is engaged.

**2.** If the clutch is engaged when one member is stationary and the other rotating (*i.e.* during engagement of the clutch) as shown in Fig. 24.8 (*b*), then the cone faces will tend to slide on each other due to the presence of relative motion. Thus an additional force (of magnitude $\mu . W_n \cos \alpha$) acts on the clutch which resists the engagement, and the axial force required for engaging the clutch increases.

∴ Axial force required for engaging the clutch,

$$W_e = W + \mu . W_n \cos \alpha = W_n . \sin \alpha + \mu W_n \cos \alpha$$
$$= W_n (\sin \alpha + \mu \cos \alpha)$$

It has been found experimentally that the term ($\mu W_n . \cos \alpha$) is only 25 percent effective.

$$\therefore \quad W_e = W_n \sin \alpha + 0.25 \mu W_n \cos \alpha = W_n (\sin \alpha + 0.25 \mu \cos \alpha)$$

**3.** Under steady operation of the clutch, a decrease in the semi-cone angle ($\alpha$) increases the torque produced by the clutch ($T$) and reduces the axial force ($W$). During engaging period, the axial force required for engaging the clutch ($W_e$) increases under the influence of friction as the angle $\alpha$ decreases. The value of $\alpha$ can not be decreased much because smaller semi-cone angle ($\alpha$) requires larger axial force for its disengagement.

If the clutch is to be designed for free disengagement, the value of $\tan \alpha$ must be greater than $\mu$. In case the value of $\tan \alpha$ is less than $\mu$, the clutch will not disengage itself and axial force required to disengage the clutch is given by

$$W_d = W_n ( \mu \cos \alpha - \sin \alpha)$$

## 5.19 Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 24.10. The outer surface of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

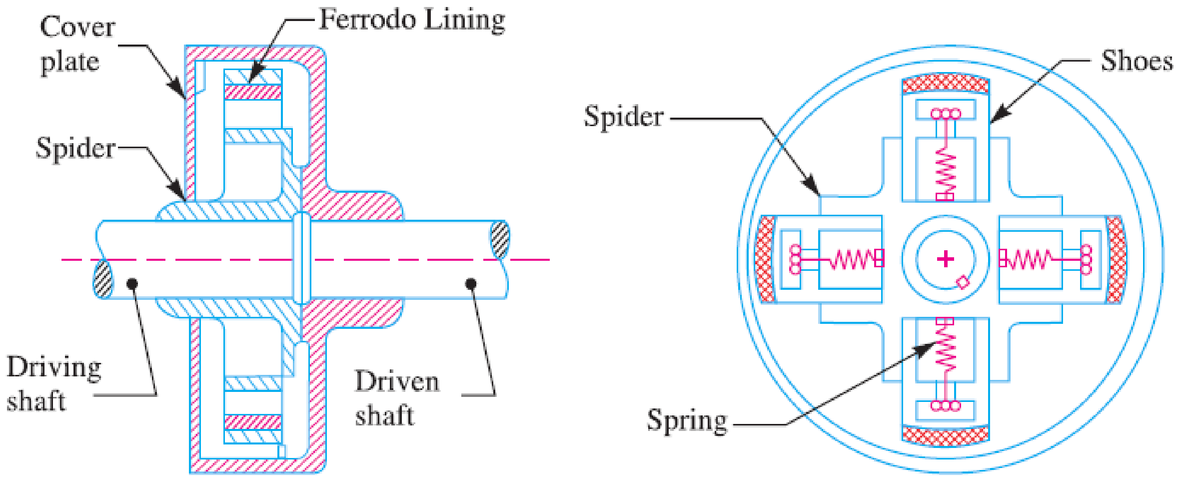


**Fig. 24.10.** Centrifugal clutch.

### 5.20 Design of a Centrifugal Clutch

In designing a centrifugal clutch, it is required to determine the weight of the shoe, size of the shoe and dimensions of the spring. The following procedure may be adopted for the design of a centrifugal clutch.

**1. *Mass of the shoes***

Consider one shoe of a centrifugal clutch as shown in Fig. 24.11.

Let $m$ = Mass of each shoe,

$n$ = Number of shoes,

$r$ = Distance of centre of gravity of the shoe from the centre of the spider,

$R$ = Inside radius of the pulley rim,

$N$ = Running speed of the pulley in r.p.m.,

$\omega$ = Angular running speed of the pulley in rad / s = $2\pi N / 60$ rad/s,

$\omega_1$ = Angular speed at which the engagement begins to take place, and

$\mu$ = Coefficient of friction between the shoe and rim.

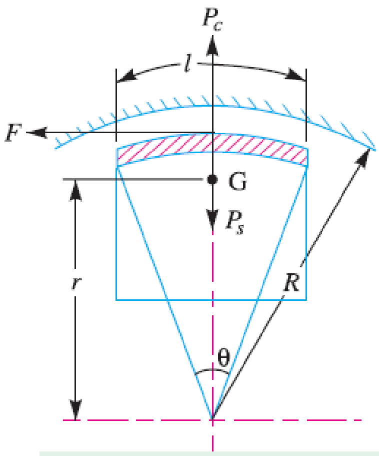


**Fig. 24.11.** Forces on a shoe of a centrifugal clucth.

We know that the centrifugal force acting on each shoe at the running speed,

$$*P_c = m.\omega^2.r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m\,(\omega_1)^2\, r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16}\, m.\omega^2.r$$

∴ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16}\, m.\omega^2.r = \frac{7}{16}\, m.\omega^2.r$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

∴ Frictional torque acting on each shoe

$$= F \times R = \mu\,(P_c - P_s)\, R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\, R \times n = n.F.R$$

From this expression, the mass of the shoes ($m$) may be evaluated.

### 3. *Size of the shoes*

Let $l$ = Contact length of the shoes,

$b$ = Width of the shoes,

$R$ = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley,

θ□= Angle subtended by the shoes at the centre of the spider in radians, and

$p$ = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, it may be taken as 0.1 N/mm2

We know that $\theta = \dfrac{l}{R}$ or $l = \theta.R = \dfrac{\pi}{3} R$ ...(Assuming $\theta = 60° = \pi / 3$ rad)

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•

∴ Area of contact of the shoe

$$= l \, . \, b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$, therefore

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe ($b$) may be obtained.

**3. *Dimensions of the spring***

We have discussed above that the load on the spring is given by

$$P_s = \frac{9}{16} \times m.\omega^2.r$$

The dimensions of the spring may be obtained as usual.