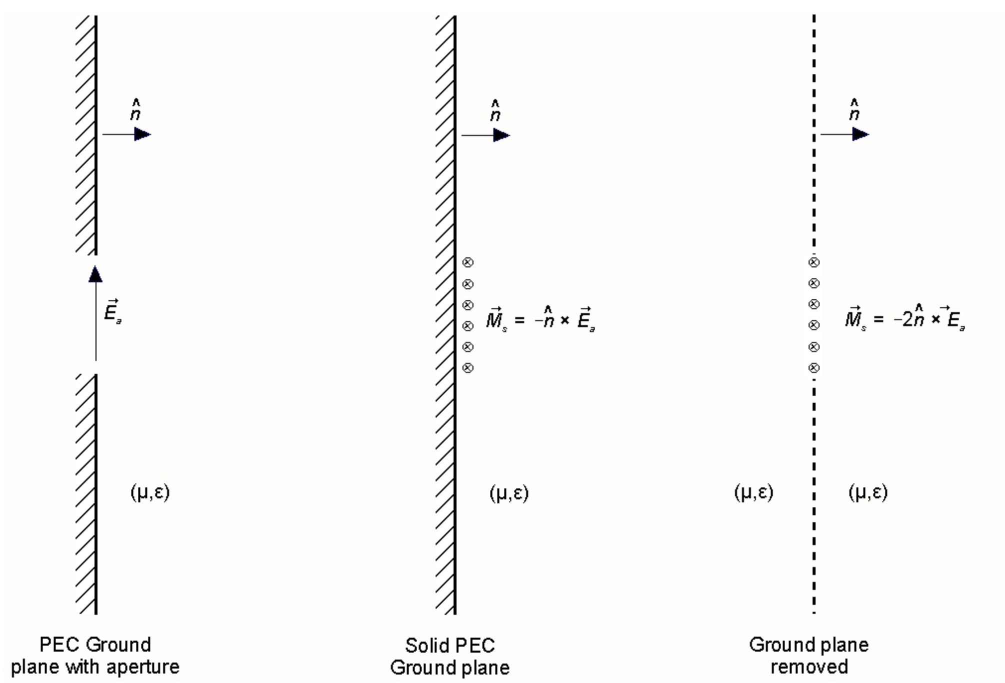
Aperture Antennas

An aperture antenna contains some sort of opening through which electromagnetic waves are transmitted or received. Examples of aperture antennas include slots, waveguides, horns, reflectors and lenses. Aperture antennas are commonly used in aircraft or spacecraft applications. The aperture can be mounted flush with the surface of the vehicle, and the opening can be covered with a dielectric which allows electromagnetic energy to pass through. The fields radiated by aperture antennas are typically determined using a special case of the *field equivalence principle*.

Consider an aperture (opening) in an otherwise closed PEC, through which EM radiation occurs. According to the field equivalence principle, the fields external to the PEC can be determined using an equivalent geometry. The equivalent geometry consists of a magnetic surface current (related to the aperture field) located on the surface of the closed PEC (the aperture is covered by a PEC surface). For an aperture in a ground plane, image theory can then be applied, resulting in a magnetic surface current radiating in a homogeneous medium, which greatly simplifies the analysis.



<u>Open Ended Rectangular Waveguide</u>

Consider an open-ended rectangular waveguide which connects to a conducting ground plane which covers the x-y plane. If we assume that the waveguide carries only the dominant TE_{10} mode, the field distribution in the aperture of the waveguide is

$$\boldsymbol{E}_a = E_o \cos\left(\frac{\pi x}{a}\right) \boldsymbol{a}_y \qquad \left\{ -\frac{a}{2} \leq x \leq \frac{a}{2}, -\frac{b}{2} \leq y \leq \frac{b}{2} \right\}$$

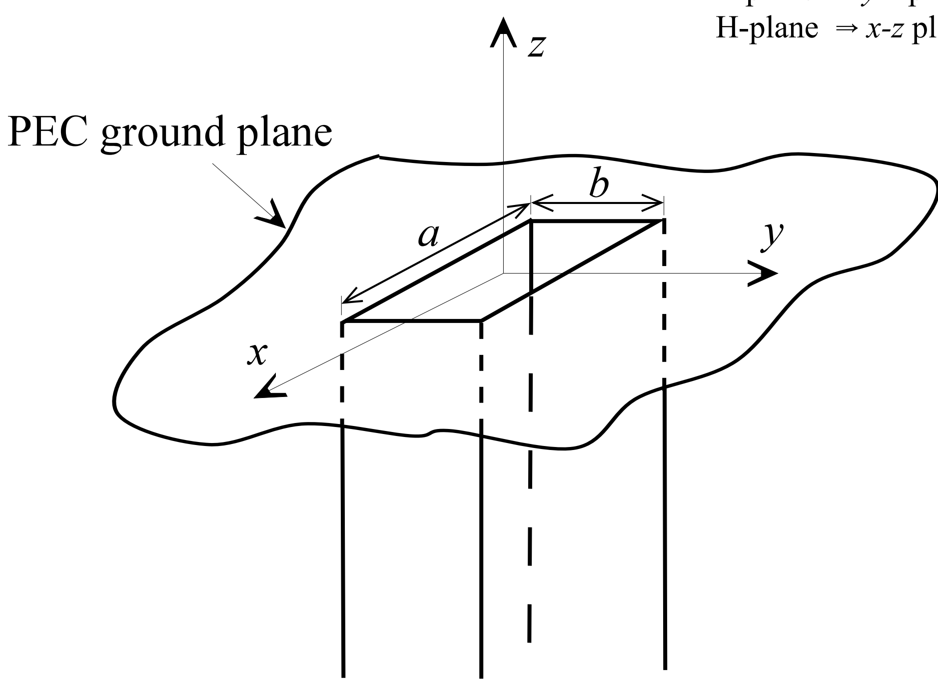
$$\boldsymbol{H}_a = \frac{-\gamma_{10}}{j\omega\mu} E_o \cos\left(\frac{\pi x}{a}\right) \boldsymbol{a}_x$$

where

$$\gamma_{10} = jk\sqrt{1 - \left(\frac{f_{c_{10}}}{f}\right)^2}$$

$$f_{c_{10}} = \frac{1}{2a\sqrt{\mu\varepsilon}}$$

E-plane ⇒ y-z plane
H-plane ⇒ x-z plane



According to the field equivalence principle, the equivalent magnetic surface current radiating in free space is given by

$$\boldsymbol{M}_s = -2\boldsymbol{a}_z \times \boldsymbol{E}_a = 2E_o \cos\left(\frac{\pi x}{a}\right)\boldsymbol{a}_x \qquad \left\{-\frac{a}{2} \le x \le \frac{a}{2}, -\frac{b}{2} \le y \le \frac{b}{2}\right\}$$

The resulting radiated far fields are

$$E_r = H_r = 0$$

$$E_\theta = -j\frac{abkE_o e^{-jkr}}{4r}\sin\varphi \frac{\cos\left(\frac{ka}{2}\sin\theta\cos\varphi\right)}{\left(\frac{ka}{2}\sin\theta\cos\varphi\right)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin\left(\frac{kb}{2}\sin\theta\sin\varphi\right)}{\frac{kb}{2}\sin\theta\sin\varphi}$$

$$E_\varphi = -j\frac{abkE_o e^{-jkr}}{4r}\cos\theta\cos\varphi \frac{\cos\left(\frac{ka}{2}\sin\theta\cos\varphi\right)}{\left(\frac{ka}{2}\sin\theta\cos\varphi\right)^2 - \left(\frac{\pi}{2}\right)^2} \frac{\sin\left(\frac{kb}{2}\sin\theta\sin\varphi\right)}{\frac{kb}{2}\sin\theta\sin\varphi}$$

$$H_\theta = -\frac{E_\varphi}{\eta}$$

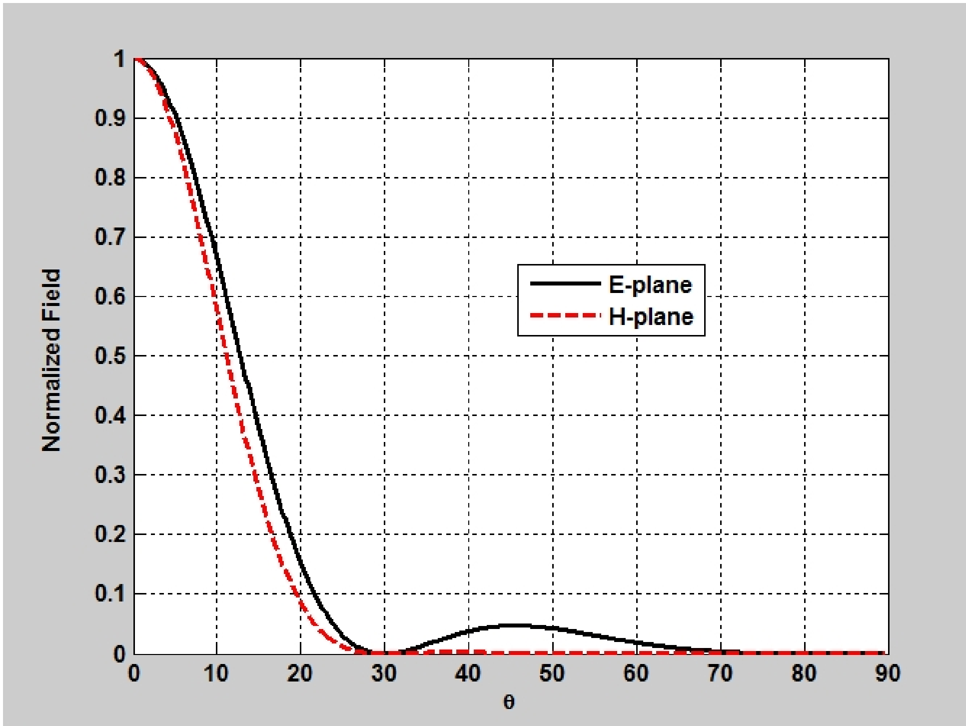
$$H_\varphi = \frac{E_\theta}{\eta}$$

The fields in the *E*-plane ($\varphi = 90^o$) and *H*-plane ($\varphi = 0^o$) reduce to

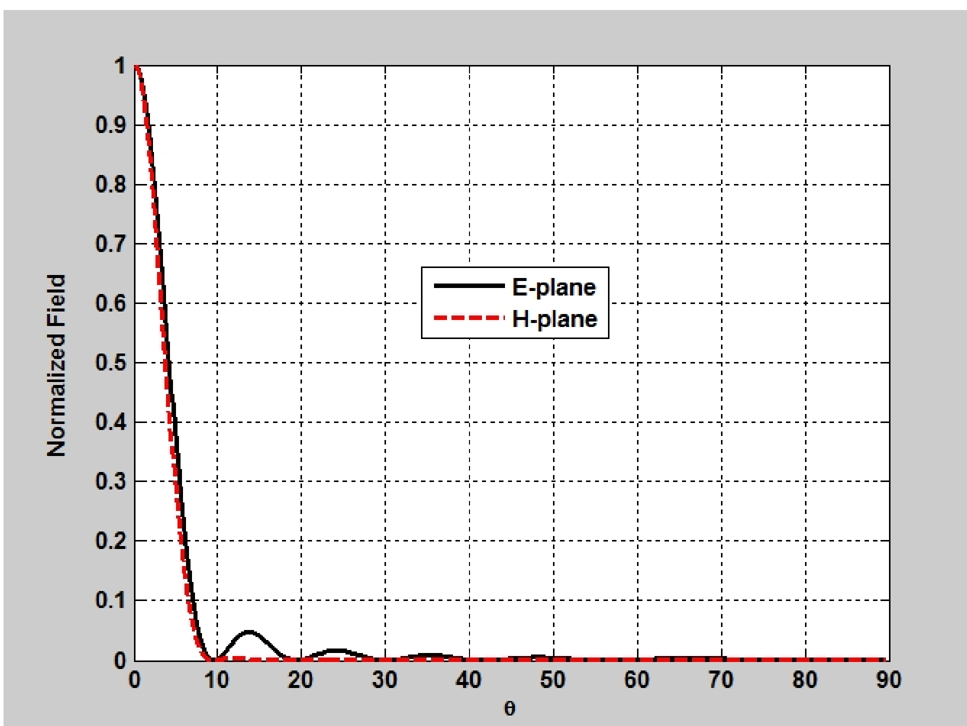
$$E_\theta = j\frac{abkE_o e^{-jkr}}{4r}\frac{4}{\pi^2}\frac{\sin\left(\frac{kb}{2}\sin\theta\right)}{\frac{kb}{2}\sin\theta} \qquad (\varphi = 90^o)$$

$$E_\varphi = -j\frac{abkE_o e^{-jkr}}{4r}\cos\theta \frac{\cos\left(\frac{ka}{2}\sin\theta\right)}{\left(\frac{ka}{2}\sin\theta\right)^2 - \left(\frac{\pi}{2}\right)^2} \qquad (\varphi = 0^o)$$

$a = 3\lambda,\ b = 2\lambda$



$a = 9\lambda,\ b = 6\lambda$

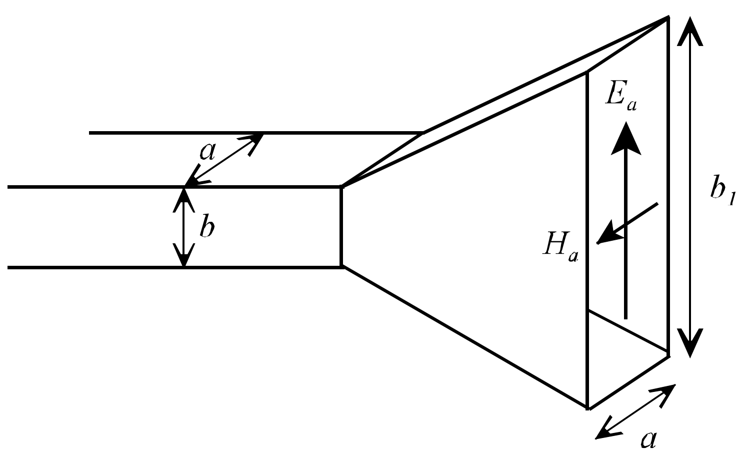


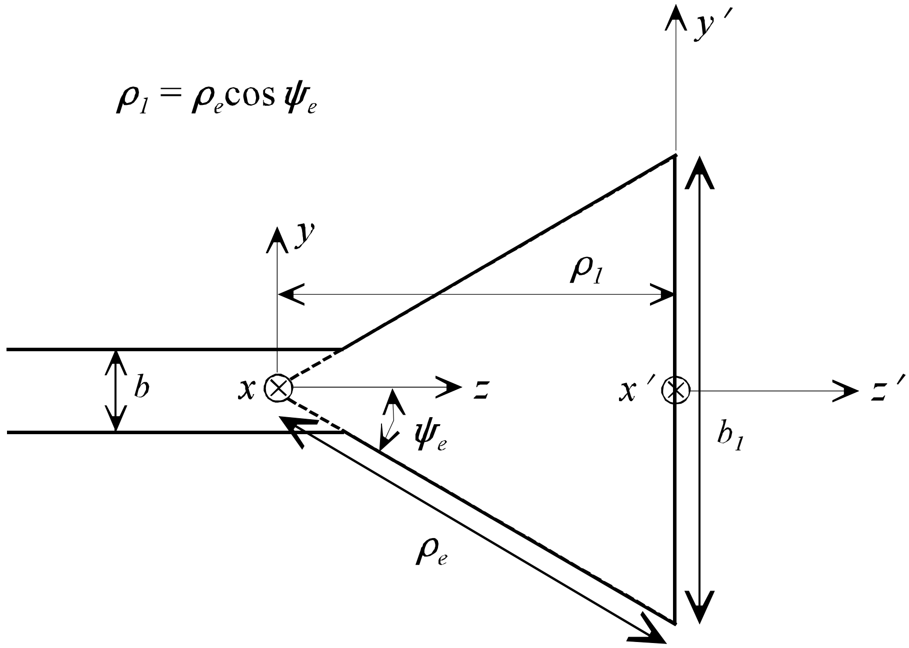
Horn Antennas

The horn antenna represents a transition or matching section from the guided mode inside the waveguide to the unguided (free-space) mode outside the waveguide. The horn antenna, as a matching section, reduces reflections and leads to a lower standing wave ratio. There are three basic types of horn antennas: (a.) the *E*-plane sectoral horn (flared in the direction of the *E*-plane only), (b.) the *H*-plane sectoral horn (flared in the direction of the *H*-plane only), and (c.) the pyramidal horn antenna (flared in both the *E*-plane and *H*-plane). The flare of the horns considered here is assumed to be linear although some horn antennas are formed by other flare types such as an exponential flare.

The horn antenna is mounted on a waveguide that is almost always excited in single-mode operation. That is, the waveguide is operated at a frequency above the cutoff frequency of the TE_{10} mode but below the cutoff frequency of the next highest mode.

E-Plane Sectoral Horn





<u>*E*-plane Sectoral Horn *E*-plane Far Field ($\phi = \pi/2$)</u>

$$E_r = E_\phi = 0$$

$$E_\theta = j\,\frac{a\sqrt{\pi k \rho_1}\,E_1 e^{-jkr}}{8r}\left\{\left(\frac{2}{\pi}\right)^2 e^{j\left(k\rho_1 \sin^2\frac{\theta}{2}\right)}(1+\cos\theta)\,F(t_1',t_2')\right\}$$

$$F(t_1,t_2) = [C(t_2) - C(t_1)] - j[S(t_2) - S(t_1)]$$

$$C(t) = \int_0^t \cos\left(\frac{\pi x^2}{2}\right) dx \qquad \text{(cosine Fresnel integral)}$$

$$S(t) = \int_0^t \sin\left(\frac{\pi x^2}{2}\right) dx \qquad \text{(sine Fresnel integral)}$$

$$t_1' = \sqrt{\frac{k}{\pi\rho_1}}\left(-\frac{b_1}{2} - \rho_1\sin\theta\right) \qquad t_2' = \sqrt{\frac{k}{\pi\rho_1}}\left(\frac{b_1}{2} - \rho_1\sin\theta\right)$$

E-plane Sectoral Horn H-plane Far Field ($\varphi = 0$)

$$E_r = E_\theta = 0$$

$$E_\varphi = -j\frac{a\sqrt{\pi k \rho_1}E_1 e^{-jkr}}{8r}\left\{(1+\cos\theta)\left[\frac{\cos\left(\frac{ka}{2}\sin\theta\right)}{\left(\frac{ka}{2}\sin\theta\right)^2 + \left(\frac{\pi}{2}\right)^2}\right]F(t_1'',t_2'')\right\}$$

$$t_1'' = \sqrt{\frac{k}{\pi\rho_1}}\left(-\frac{b_1}{2}\right)$$

$$t_2'' = \sqrt{\frac{k}{\pi\rho_1}}\left(\frac{b_1}{2}\right)$$

The directivity of the E-plane sectoral horn (D_E) is given by

$$D_E = \frac{64a\rho_1}{\pi\lambda b_1}\left[C^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right) + S^2\left(\frac{b_1}{\sqrt{2\lambda\rho_1}}\right)\right]$$

A plot of the E-plane and H-plane patterns for the E-plane horn shows that the H-plane pattern is much broader than the E-plane pattern. Thus, the E-plane sectoral horn tends to focus the beam of the antenna in the E-plane (see Figures 13.3 and 13.4). Design curves for the E-plane sectoral horn are given in Figure 13.8.

Example (*E*-plane sectoral horn design, Problem 13.6)

An *E*-plane horn is fed by a WR 90 (X-band) rectangular waveguide (a = 2.286 cm, b = 1.016 cm). Design the horn so that its maximum directivity at 11 GHz is 30 (14.77 dB).

$$\lambda = \frac{3 \times 10^8}{11 \times 10^9} = 0.02727\ m = 2.727\ cm$$

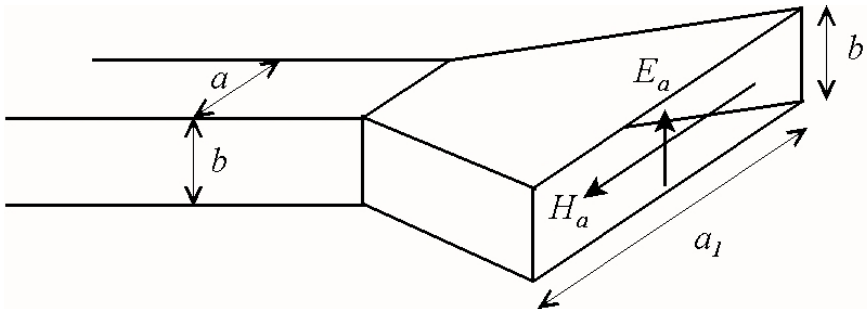
$$\frac{\lambda}{a} = \frac{2.727}{2.286} = 1.19 \qquad\qquad \frac{\lambda}{a} D_E = 1.19 \times 30 = 35.79$$

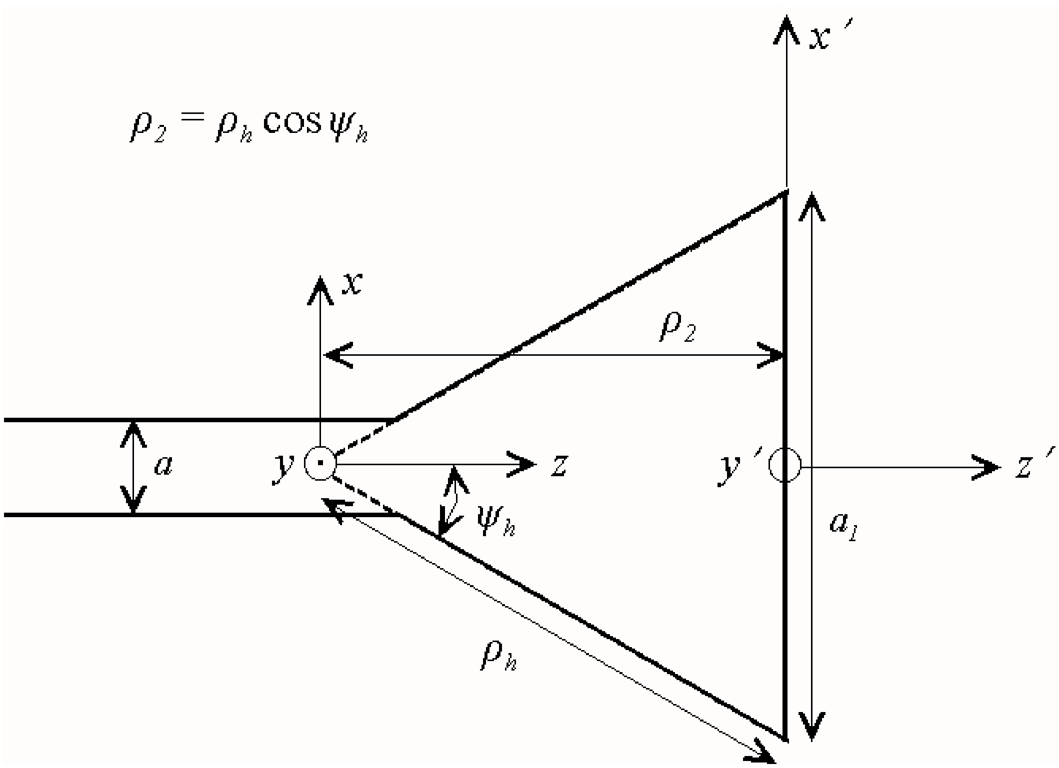
$$\text{From Figure 13.8} \quad \Rightarrow \quad \rho_1 = 10\lambda \quad \text{yields} \quad \frac{\lambda}{a} D_E \approx 36 \quad \text{at} \quad b_1 \approx 4.5\lambda$$

$$\psi_e = \tan^{-1}\left(\frac{b_1/2}{\rho_1}\right) = \tan^{-1}(0.225) = 12.68^o$$

$$\psi_e = 25.36^o$$

H-Plane Sectoral Horn





H-plane Sectoral Horn *E*-plane Far Field ($\varphi = \pi/2$)

$$E_r = E_\varphi = 0$$

$$E_\theta = j\frac{b\sqrt{k\rho_2}\,E_2 e^{-jkr}}{8r\sqrt{\pi}}$$

$$\times\left\{(1+\cos\theta)\,\frac{\sin Y}{Y}\left[e^{jf_1}F(t_1',t_2') + e^{jf_2}F(t_1'',t_2'')\right]\right\}$$

$$Y = \frac{kb}{2}\sin\theta \qquad f_1 = \frac{k_x'^2\rho_2}{2k} \qquad f_2 = \frac{k_x''^2\rho_2}{2k}$$

$$k_x' = \frac{\pi}{a_1} \qquad k_x'' = -\frac{\pi}{a_1}$$

H-plane Sectoral Horn *H*-plane Far Field ($\varphi = 0$)

$$E_r = E_\theta = 0$$

$$E_\varphi = j\frac{b\sqrt{k\rho_2}\,E_2 e^{-jkr}}{8r\sqrt{\pi}}\left\{(1+\cos\theta)\left[e^{jf_1}F(t_1',t_2') + e^{jf_2}F(t_1'',t_2'')\right]\right\}$$

$$k_x' = k\sin\theta + \frac{\pi}{a_1} \qquad k_x'' = k\sin\theta - \frac{\pi}{a_1}$$

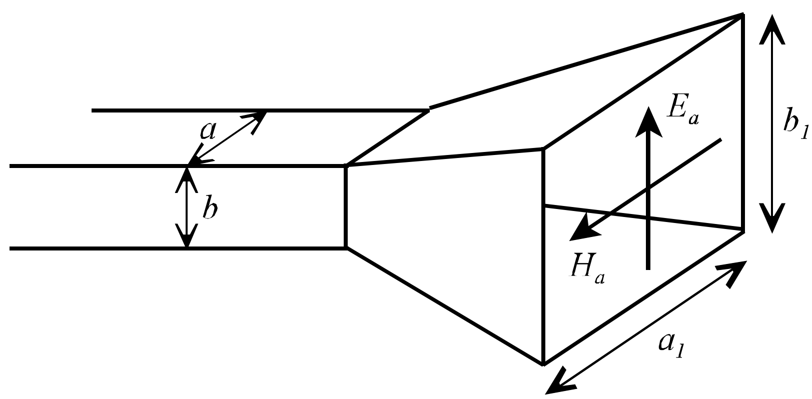
The directivity of the *H*-plane sectoral horn (D_H) is given by

$$D_H = \frac{4\pi b\rho_2}{\lambda a_1}\left\{\left[C(u) - C(v)\right]^2 + \left[S(u) - S(v)\right]^2\right\}$$

$$u = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\rho_2}}{a_1} + \frac{a_1}{\sqrt{\lambda\rho_2}}\right) \qquad v = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\rho_2}}{a_1} - \frac{a_1}{\sqrt{\lambda\rho_2}}\right)$$

A plot of the E-plane and H-plane patterns for the H-plane horn shows that the E-plane pattern is much broader than the H-plane pattern. Thus, the H-plane sectoral horn tends to focus the beam of the antenna in the H-plane (see Figures 13.11 and 13.12). Design curves for the H-plane sectoral horn are given in Figure 13.16.

Pyramidal Horn

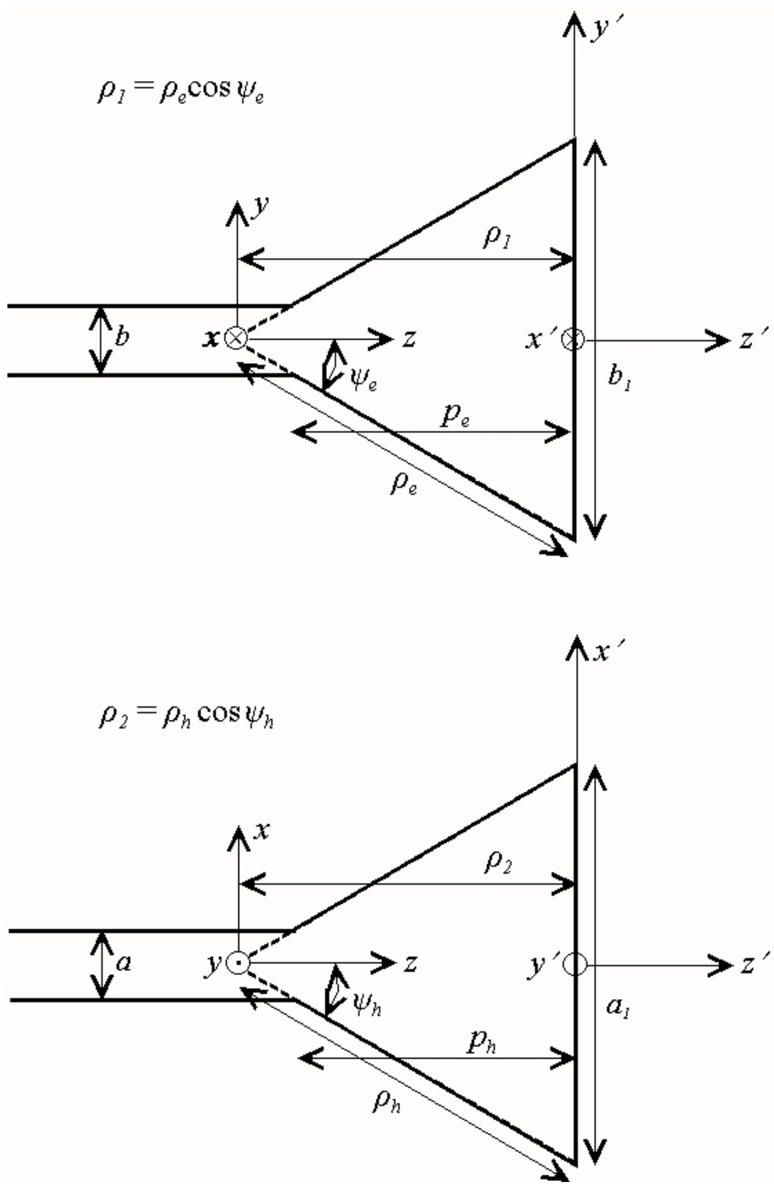


Based on the pattern characteristics of the E-plane and H-plane sectoral horns, the pyramidal horn should focus the beam patterns in both the E-plane and the H-plane. In fact, the E-plane and H-plane patterns of the pyramidal horn are identical to the E-plane pattern of the E-plane sectoral horn and the H-plane pattern of the H-plane sectoral horn, respectively (See Figure 13.19).

The directivity of the pyramidal horn (D_P) can be written in terms of the directivities of the E-plane and H-plane sectoral horns:

$$D_P = \frac{\pi \lambda^2}{32 a b} D_E D_H$$

The length of the pyramidal horn flared region must be equal for the E-plane and H-plane sectoral designs (p_e and p_h, respectively).



$$p_e = (b_1 - b)\left[\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2} = p_h = (a_1 - a)\left[\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

Example (Optimal pyramidal horn design)

A pyramidal horn is fed by a WR 90 (X-band) rectangular waveguide ($a = 2.286$ cm, $b = 1.016$ cm). Design an optimum gain pyramidal horn so that its maximum directivity at 10 GHz is 20 dB. Use the DESIGN.m MATLAB code provided by the author.

```
--------------------------------------
Specify the following input parameters:
--------------------------------------
DESIRED GAIN OF THE HORN IN dB: Go(dB) = 20
FREQUENCY OF OPERATION IN GHz: fo(GHz) = 10
HORN DIMENSION A IN CM: a(cm) = 2.286
HORN DIMENSION B IN CM: b(cm) = 1.016

--------------------------------------------
DESIGNED PARAMETERS FOR THE OPTIMUM GAIN HORN
--------------------------------------------
a1     =   13.426281   cm
b1     =   10.425397   cm
RHOe   =   18.114818   cm
RHOh   =   20.029445   cm
Pe     =   15.657917   cm
Ph     =   15.657917   cm
PSIe   =   16.723827   Deg
PSIh   =   19.582484   Deg
```

The dimensions provided by the DESIGN.m code can be checked according to the pyramidal horn directivity equation based on E-plane and H-plane sectoral horns.

$$D_P = \frac{\pi\lambda^2}{32ab} D_E D_H \qquad \lambda = 3\,\text{cm} \qquad D_P = 0.3804\, D_E D_H$$

E-plane sectoral horn

$$D_E = \frac{64 a \rho_e \cos\psi_e}{\pi\lambda b_1}\left[C^2\left(\frac{b_1}{\sqrt{2\lambda\rho_e\cos\psi_e}}\right) + S^2\left(\frac{b_1}{\sqrt{2\lambda\rho_e\cos\psi_e}}\right)\right]$$

$$D_E = 25.8320\left[C^2(1.0218) + S^2(1.0218)\right]$$
$$= 25.8320\left[0.8187\right]$$
$$= 21.1487$$

H-plane sectoral horn

$$D_H = \frac{4\pi b \rho_h \cos\psi_h}{\lambda a_1}\left\{\left[C(u) - C(v)\right]^2 + \left[S(u) - S(v)\right]^2\right\}$$
$$u = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\rho_h\cos\psi_h}}{a_1} + \frac{a_1}{\sqrt{\lambda\rho_h\cos\psi_h}}\right)$$
$$v = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{\lambda\rho_h\cos\psi_h}}{a_1} - \frac{a_1}{\sqrt{\lambda\rho_h\cos\psi_h}}\right)$$

$$D_H = 5.9816\left\{\left[C(u) - C(v)\right]^2 + \left[S(u) - S(v)\right]^2\right\}$$
$$u = \frac{1}{\sqrt{2}}\left(0.5604 + 1.7844\right) = 1.6580$$
$$v = \frac{1}{\sqrt{2}}\left(0.5604 - 1.7844\right) = -0.8655$$
$$D_H = 5.9816\left\{1.1852 + 0.8042\right\} = 11.8999$$

Pyramidal horn

$$D_P = 0.3804\,(21.1487)(11.8999) = 95.7343 = 19.81 \text{ dB}$$

The patterns of the pyramidal horn can be determined using the ANALYSIS.m MATLAB code provided by the author.

```
E-Plane and H-Plane Horn Specifications
-------------------------------------------
rho1(in wavelengths) = 5.7829
rho2(in wavelengths) = 6.2903

Waveguide Dimensions
-------------------------------------------
a(in wavelengths) = 2.286/3
b(in wavelengths) = 1.016/3

Horn Aperture Dimensions
-------------------------------------------
a1(in wavelengths) = 13.4263/3
b1(in wavelengths) = 10.4254/3

Select output option
-------------------------------------------
1. Screen
2. Output file
->1

please wait
*****************************************************************************
PROGRAM OUTPUT
*****************************************************************************

Pyramidal Horn
--------------
Directivity = 19.81 dB
Directivity = 95.74 dimensionless

E-Plane Sectoral Horn
---------------------
Directivity = 13.253 dB
Directivity = 21.1497 dimensionless

H-Plane Sectoral Horn
---------------------
Directivity = 10.7553 dB
Directivity = 11.8996 dimensionless
```

Pyramidal Horn E-plane and H-Plane Patterns