4.3 COMPOSITE SHAFT

A shaft made up of two or more different materials and behaving as a single shaft is known as composite shaft. Hence in a composite shaft one type of is rigidly sleeved over another type of shaft. The total torque transmitted by a composite shaft is the sum of the torques transmitted by each individual shaft. But the angle of twist in each shaft will be equal.

4.3.1 PARALLEL SHAFT

It is also same as the composite shaft, here the shaft are arranged in parallel to one thick cylinder covered with another sleeve shaft and covered with number of sleeve of different material is named as parallel shaft.

The shaft are said to be in parallel when the driving torque is applied at the junction of the shaft and the resisting torque is at the other ends of the shafts. Here the angle of twist is same for each shaft, but the applied torque is divided between the two shafts. Angle of twist are same $\theta = \theta_1 = \theta_2$

From the torsion equation, $\theta = \frac{T \times l}{C \times J}$

Then $$\frac{T_1 \times l_1}{C_1 \times J_1} = \frac{T_2 \times l_2}{C_2 \times J_2}$$

And $$T = T_1 + T_2$$

If the shaft are made of same material then $C_1 = C_2$

Then $$\frac{T_1 \times l_1}{J_1} = \frac{T_2 \times l_2}{J_2} \text{ or } \frac{T_1}{T_2} = \frac{J_1 \times l_2}{J_2 \times l_1}$$

When torque is shared equally by both the shafts then

$$T_1 = T_2 \text{ then } J_1 \times l_2 = J_2 \times l_1$$

Problem:4.3.1: A composite shaft consists of a steel rod 60mm diameter surrounded by a closely fitting tube of brass. Find the outer diameter of the tube so that when a torque of 1000Nm is applied to the composite shaft, it will be shared equally by the two materials. Take C for steel 8.4× 10^4 N/mm^2 and C for brass 4.2× 10^4 N/mm^2. Find also the maximum shear stress in each material and common angle of twist in a length of 4 m.

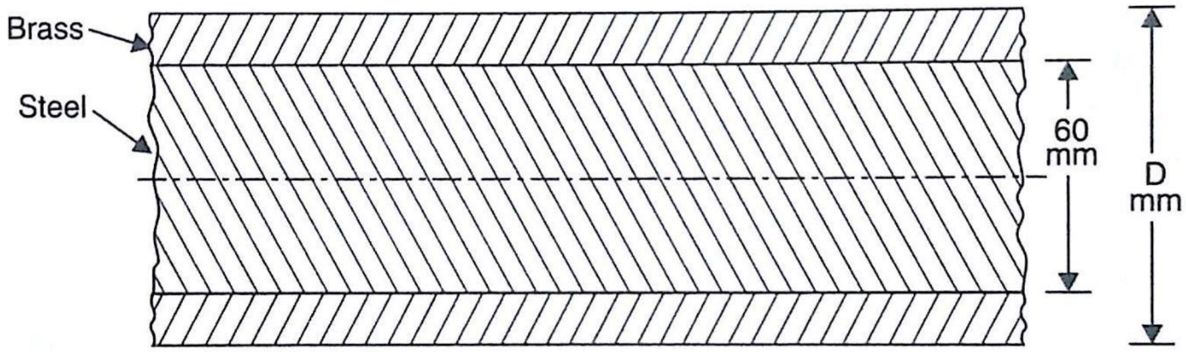
Given data:

Diameter of steel rod d = 80mm

Total torque T = 1000 Nm = 1000× 10^3 Nmm

C for steel C_s = 8.4× 10^4 N/mm^2

C for brass C_b = 4.2× 10^4 N/mm^2



To find out:

Outside diameter, maximum shear stress on steel and brass, twist angle.

Solution:

Based on the condition of parallel shaftT = $T_S + T_b$ and $\theta_1 = \theta_2$

Based on given data $T_S = T_b$

∴ Total torque T = $T_S + T_b = T_S + T_s = 2T_S$

Then $T_S = \frac{1000 \times 10^3}{2} = 500 \times 10^3$ Nmm

∴ $T_S = T_b = 500 \times 10^3$ Nmm

From the torsion equation $\frac{T}{J} = \frac{C\theta}{l}$, which gives T = $\frac{C \times \theta \times J}{l}$

For steel rod $T_S = \frac{C_S \times \theta_S \times J_S}{l_S}$ and for brass rod $T_b = \frac{C_b \times \theta_b \times J_b}{l_b}$

But $T_S = T_b$ then $\frac{C_S \times \theta_S \times J_S}{l_S} = \frac{C_b \times \theta_b \times J_b}{l_b}$ for compound shaft $l_s = l_s$ and $\theta_s = \theta_b$

Now the above equation become

$$C_s \times J_s = C_b \times J_b$$

Wkt polar moment of inertia for steel $J_s = \frac{\pi \times d^4}{32} = \frac{\pi \times 60^4}{32} = 1.27 \times 10^6$ mm^4

And polar moment of inertia for brass $J_b = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(D^4 - 60^4)$

Substitute the value in the above equation

$$8.4 \times 10^4 \times 1.27 \times 10^6 = 4.2 \times 10^4 \times \frac{\pi}{32}(D^4 - 60^4)$$

D = 78.98mm

To find the Shear stress on each section

Wkt $\frac{T}{J} = \frac{\tau}{R}$ then $\tau = \frac{T \times R}{J}$

For steel rod $\tau_s = \frac{T_s x R_s}{J_s} = \frac{500 \times 10^3 x\ 30}{1.27 \times 10^6} =$ **11.79 N/mm²**

For brass sleeve $\tau_b = \frac{T_b x R_b}{J_b} = \frac{500 \times 10^3 x\ 78.98}{\frac{\pi}{32}(78.98^4 - 60^4)} = 7.76$ **N/mm²**

Common angle of twist

From the equation $T_s = \frac{C_s \times \theta_s \times J_s}{l_s}$ $\theta_s = \frac{T_s \times l_s}{C_s \times J_s}$

$$= \frac{500 \times 10^3 \times 4000}{8.4 \times 10^4 \times 1.27 \times 10^6} = \textbf{0.0181 rad}$$

$$= 0.0181 \times \frac{180}{\pi} = \textbf{1.072°}$$

Result:

Outside diameter = **78.98mm**

Maximum shear stress on steel = **11.79 N/mm²**

Maximum shear stress on brass = 7.**76 N/mm²**

Common twist angle = **1.072°**

4.3.2 COMBINED BENDING AND TORSION

When a shaft is transmitting torque, it is due to shear stresses. At the same time the shaft is also subjected to bending moment due to the inertia loads. Due to bending moment, bending stresses are also set up in the shaft. Hence each particle in a shaft is subjected to shear stress and bending moment.

Consider any point on the cross-section of a shaft.

Let T = Torque at the section

D = Diameter of the shaft

M = B.M. at the section

The torque T will produce shear stress at the point whereas the B.M. will produce bending stress.

Let q = shear stress at the point produced by torque Tand

σ = Bending stress at the point produced by B.M.

The shear stress at any point due to torque (T) is given by

$$\frac{q}{r} = \frac{T}{J}$$

or $$q = \frac{T}{J} r$$

The bending stress at any point due to bending moment (M) is given by

$$\frac{M}{I} = \frac{\sigma}{y} \quad \text{or} \quad \sigma = \frac{M \times y}{I} =$$

When two mutual perpendicular force and shear force act on a shaft, we know that the angle θ made by the plane of maximum shear with the normal cross-section is given by,

$$\tan 2\theta = \frac{2\,\tau}{\sigma}$$

The bending stress and shear stress is maximum at a point on the surface of the shaft. where $r = R = \frac{D}{2}$ and $y = \frac{D}{2}$

Let σ = Maximum bending stress i.e., on the surface of the shaft.

$$= \frac{M \times (\frac{D}{2})}{\frac{\pi}{64} D^4} = \frac{32\ M}{\pi D^3}$$

$$\tau = \frac{T}{J} \times\ R = \frac{T}{\frac{\pi}{32} D^4} \times \frac{D}{2} = \frac{16\,T}{\pi D^3}$$

$$\therefore \quad \tan 2\theta = \frac{2\,\tau}{\sigma} = \frac{2 \times \frac{16\,T}{\pi D^3}}{\frac{32\ M}{\pi D^3}} = \frac{T}{M}$$

Major Principal stress

$$= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{32\ M}{2 \times \pi D^3} + \sqrt{\left(\frac{32\ M}{2 \times \pi D^3}\right)^2 + \left(\frac{16\ T}{\pi D^3}\right)^2}$$

$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$ and

Minor Principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress = $\frac{\text{Major Principal Stress} - \text{Minor Principal Stress}}{2}$

$$= \frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)$$

For hollow shaft

Major Principal stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(M + \sqrt{M^2 + T^2}\right)$

Minor Principal stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(\sqrt{M^2 + T^2}\right)$

Problem:4.3.2: A solid shaft of diameter 80mm is subjected to bending and twisting moment of 5MNmm and 8MNmm respectively at a point. Determine the Principal stresses, Maximum shear stress and position of plane on which they act.

Given data:

Diameter of shaft D = 80mm

Bending moment M =5 MNmm = 5 × 10^6 Nmm Twisting moment T = 8 MNmm = 8 × 10^6 Nmm

To find out:

Principal stresses, Maximum shear stress and position of plane on which they act.

Solution:

Major Principal stress $= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$

$$= \frac{16}{\pi D^3}\left(5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right)$$

= **143.57 N/mm^2**

Minor Principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

$$= \frac{16}{\pi D^3}\left(5 \times 10^6 - \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right)$$

= - 44.1 = **44.1N/mm^2**

Maximum shear stress = $\frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)$

$$= \frac{16}{\pi D^3}\left(\sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right)$$

=

position of plane = tan 2θ $= \frac{T}{M} = \frac{8 \times 10^6}{5 \times 10^6} = 1.6$

$2\theta = \tan^{-1} 1.6$ = 57°59' or 237°59'

∴ θ = 28°59' or **118°59'**

Result:

Major Principal stress = **143.57 N/mm^2**

Minor Principal stress = **44.1N/mm^2**

Max. shear stress =

Position of plane = **118°59'**

Problem:4.3.3: The maximum allowable shear stress in a hollow shaft of external diameter equal to twice the internal diameter is 80N/mm^2. Determine the diameter of the shaft if it is subjected to a torque of 4× 10^6 Nmm and a bending moment of 3× 10^6 Nmm.

Given data:

Maximum shear stress = 80N/mm^2

Diameter of shaft $D_o = 2\ D_i$

Bending moment M =3× 10^6 Nmm

Twisting moment T = 4× 10^6 Nmm

To find out:

Diameter of the shaft **Solution:**

Wkt, Maximum shear stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(\sqrt{M^2 + T^2}\right)$

$$80 = \frac{16D_o}{\pi[D_o{}^4 - (0.5\,D_o)^4]}\left(\sqrt{(3\times10^6)^2 + (4\times10^6)^2}\right)$$

$$80 = \frac{16D_o}{\pi D_o{}^4[1-(0.5)^4]}\left(5\times10^6\right)$$

$$D_o{}^3 = \frac{16}{\pi\times80[1-(0.5)^4]}\left(5\times10^6\right) = 0.3395\times10^6$$

$$D_o = \sqrt[3]{0.3395\times10^6} \qquad = \mathbf{69.78\ mm}$$

Then $D_i = \frac{69.78}{2}$ $= \mathbf{34.89\ mm}$

Result:

Outer diameter D_o = **69.78 mm**

Inner diameter D_i = **34.89 mm**

4.3.3 STRAIN ENERGY STORED IN A SHAFT DUE TO TORSION:

Consider a solid circular shaft of length l and radius R, subjected to a torque T producing a twist θin the length of the shaft.

Strain energy stored U = Average torque x angle of twist

$$U = \frac{1}{2} \times T \times \theta$$

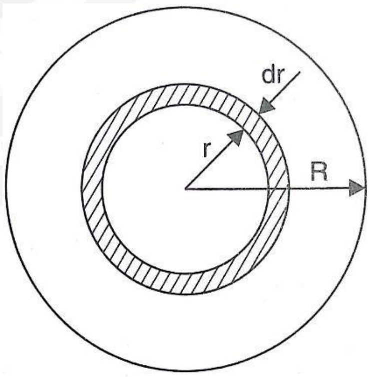
Wkt Torsion equation $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$

Where T = Torque

J = polar moment of inertia

C = Modulus of rigidity

τ = Maximum shear stress



From the torsion equation $\frac{T}{J} = \frac{J\tau}{R}$ and $\theta = \frac{l\tau}{CR}$

Substitute the T andθ value n the energy stored equation

Then, Strain energy stored U = $\frac{1}{2} \times \frac{J\tau}{R} \times \frac{l\tau}{CR}$

Where polar moment of inertia J = $\frac{\pi}{32}D^4 = \frac{\pi}{2}R^4$ (∵ D = 2R)

∴ Strain energy stored U = $\frac{1}{2} \times \frac{\frac{\pi}{2}R^4\,\tau}{R} \times \frac{l\tau}{CR}$

$$= \frac{\pi R^2 \tau^2 l}{4C}$$

$$= \left(\frac{\tau^2}{4C}\right) \text{x } \pi R^2 l$$

$\therefore$ Strain energy stored (U) $= \left(\frac{\tau^2}{4C}\right)$ x V ($\because \pi R^2 x\ l = Volume = V$)

Strain Energy Stored in hollow shaft due to torsion

Strain energy stored (U) $= \left(\frac{\tau^2}{4CD^2}\right)$ x $(D^2 + d^2)$V

Where D = Outer diameter of hollow shaft

d = Inner diameter of hollow shaft

Problem 4.3.4. Determine the maximum strain energy stored in a solid shaft of diameter 10cm and of length 1.25 m, if the maximum allowable shear stress is 50 N/mm^2. Take C 8×10^4 N/mm^2.

Given data:

Length of shaft l = 1.25 m = 1250 mm

Diameter of shaft D = 10 cm = 100mm

Maximum shear stress τ = 50N/mm^2

Modulus of rigidity C = 8×10^4 N/mm^2 **To find out:**

Maximum strain energy stored **Solution:**

Wkt, Strain energy stored for solid shaft (U) $= \left(\frac{\tau^2}{4C}\right)$ x V

Where Volume V = $\pi \frac{D^2}{4} x l = \pi \frac{100^2}{4}$ x $1250 = 9.8175 \times 10^6$ mm^3

Now, Strain energy stored (U) $= \left(\frac{50^2}{4\times 8 \times 10^4}\right)$ x $9.8175 \times 10^6 = \mathbf{7.669 \times 10^4}$ **Nmm**

Result:

Strain energy stored (U) = $\mathbf{7.669 \times 10^4}$ **Nmm**

Problem 4.3.5. A solid circular shaft of 4m length and 10cm diameter is to transmit 112.5 kW power at 150 rpm. Determine the maximum shear stress and Strain energy stored in the shaft. Take C = 8×10^4 N/mm^2.

Given data:

Length of shaft l = 4 m = 4×10^3 mm

Diameter of shaft D = 10 cm = 100 mm

Power transmit P = 112.5 kW = 112.5 × 10^3 W

Speed N = 150 rpm

Modulus of rigidity C = 8 × 10^4 N/mm^2 **To find out:**

Maximum shear stress, strain energy stored **Solution:**

Wkt, power transmitted P = $\frac{2\pi NT}{60}$

$112.5 \times 10^3 = \frac{2 \times \pi \times 150 \times T}{60}$

T = $\frac{60 \times 112.5 \times 10^3}{2 \times \pi \times 150}$ = 7159 Nm = 7.159 × 10^6 Nmm

But mean torque T = $\frac{\pi}{16} \tau D^3$

Then, shear stress $\tau = \frac{T \times 16}{\pi D^3} = \frac{7.159 \times 10^6 \times 16}{\pi\, 100^3}$ = 36.5 N/mm^2

Strain energy stored for solid shaft (U) $= \left(\frac{\tau^2}{4C}\right) \times$ V

Where Volume V = $\pi \frac{D^2}{4} \times l = \pi \frac{100^2}{4} \times 4 \times 10^3 = 2.27 \times 10^8$ mm^3

Now, Strain energy stored (U) $= \left(\frac{36.5^2}{4 \times 8 \times 10^4}\right) \times 2.27 \times 10^8 =$ **1.308×10^5 Nmm**

Result:

Strain energy stored (U) = **1.308×10^5 Nmm**

Problem 4.3.5. A hollow shaft of internal diameter 10cm is subjected to pure torque and attains a maximum shear stress q on the outer surface of the shaft. If the Strain energy stored in the hollow shaft is given by $\frac{\tau^2}{3C} V$, determine the external diameter of the shaft.

Given data:

Internal diameter d = 10 cm = 100 mm 4 m = 4× 10^3 mm

Strain energy stored U = $\frac{\tau^2}{3C} V$

To find out:

External diameter of hollow shaft D **Solution:**

Wkt, Strain energy stored in hollow shaft (U) $= \left(\frac{\tau^2}{4CD^2}\right)$ x $(D^2 + d^2)$V

Equating the two values of strain energy, we get

$$\left(\frac{\tau^2}{4CD^2}\right) \text{x } (D^2 + d^2)\text{V} = \frac{\tau^2}{3C}\ V$$

$$\frac{D^2+d^2}{4D^2} = \frac{1}{3} \qquad \left(\text{Cancelling } \frac{\tau^2}{3C} \text{ V on both sides}\right)$$

$$3D^2 + 3d^2 = 4D^2$$

$$3d^2 = 4D^2 - 3D^2$$

$$\frac{D^2}{d^2} = 3 \text{ then } \frac{D}{d} = \sqrt{3} = 1.732$$

Then D = 1.732 d D = 1.732 x 10 = **17.32 cm**

Result: External Diameter (D) = **17.32 cm**