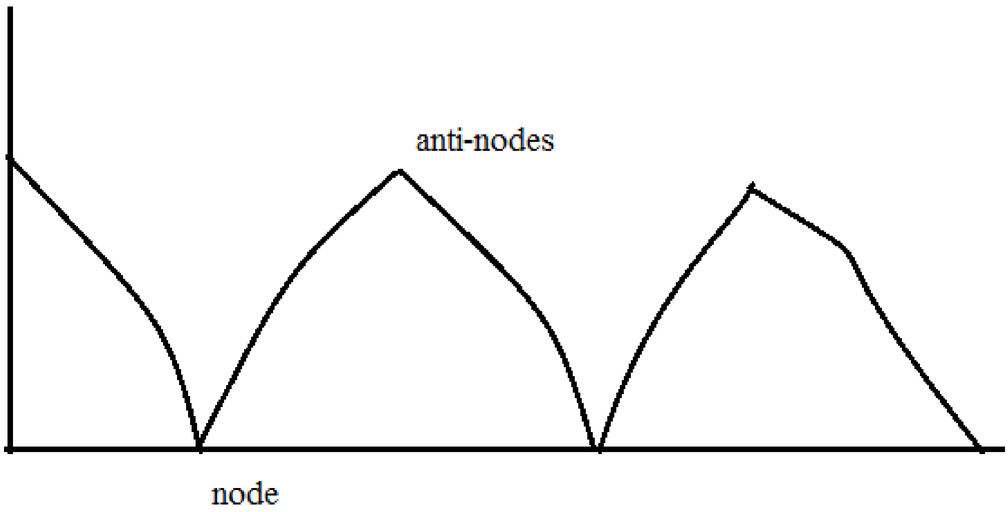
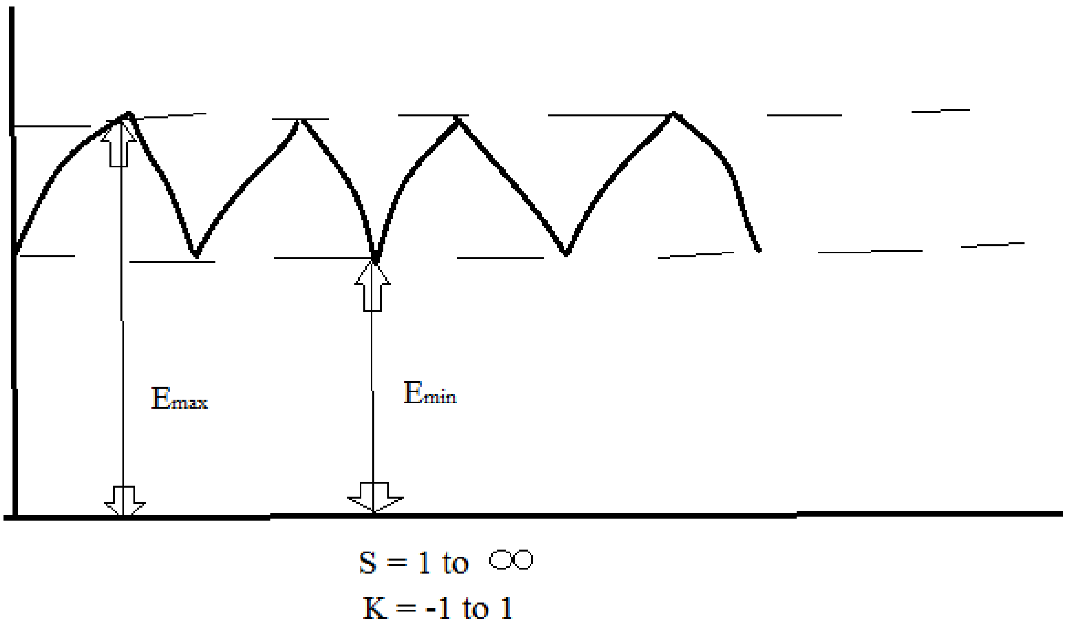
2.3 RELATIONSHIP BETWEEN STANDING WAVE RATIO AND MAGNITUDE OF REFLECTION COEFFICIENT



(a)



(b)

Fig : 2.3.1 (a) Standing waves on a dissipation less line terminated in a load not equal to R_O; (b) Standing waves on a line having open-or short-circuit termination.

If voltage magnitudes are measured long the length of a line terminated in a load other than R_O, the plotted values will appear as in Fig 2.3.1 (a). Fig 2.7 (b) is drawn for a resistive load of value not equal to R_O, and Fig 2.3.1 (b) is the case for either open or short circuit.

When the load is terminated properly with characteristic impedance the distribution of voltage and current along the line consists of maximum and minimum values of voltage and current. These values are called as standing waves.

NODES:

The points along the line were magnitude of current and voltage is zero are called nodes.

ANTINODES:

The points along the line where the magnitude of voltage and current are maximum then it is called anti-modes or Zoops.

STANDING WAVE RATIO:

The ratio of maximum to minimum magnitude of voltage or current on a line having standing waves is called standing wave ratio.

SWR = S = $\frac{|E_{max}|}{|E_{min}|} = \frac{|I_{max}|}{|I_{min}|}$

VSWR = voltage standing wave ratio

ISWR = current standing wave ratio

When the line is not terminated properly standing wave are produced. Then the total power absorption is not possible.

The ratio of E_{max} to E_{min} is referred to as voltage standing wave ratio (VSWR).

The ratio of I_{max} to I_{min} is referred to as current standing wave ratio (ISWR).

But is practice ISWR calculation is not used. Hence, practically VSWR calculation will be done. VSWR is nothing but SWR. Theoretically the value of 's' lies b/w 1 to ∞

RELATION SHIP BETWEEN STANDING WAVE RATIO AND REFLECTION COEFFICIENT:

In the high frequency transmission line at high frequencies reflections takes place.

The incident wave amplitude is E^{+} and reflected wave magnitude is E^{-}

If both of them inphase then it will be added and becomes E_{max}

If it is out off phase then it will be subtracted and becomes E_{min}

$E_{max} = E^{+} + E^{-}$

$E_{min} = E^{+} - E^{-}$

SWR = $\frac{|E_{max}|}{|E_{min}|}$

SWR = $\frac{|E^{+} + E^{-}|}{|E^{+} - E^{-}|}$

SWR = $\frac{E^{+}\left|1 + \frac{E^{-}}{E^{+}}\right|}{E^{+}\left|1 - \frac{E^{-}}{E^{+}}\right|}$

Wkt,

$\frac{E^{-}}{E^{+}}$ = K

S = $\frac{1+ |k|}{1- |k|}$

(1 – K) S= 1 + K

S – KS = 1+K

S = 1+ K + KS

S = 1 + K (1+S)

S – 1 = K (1 + S)

K = $\frac{S-1}{S+1}$

RELATION SHIP BETWEEN STANDING WAVE RATIO AND MAGNITUDE OF REFLECTION COEFFICIENT:

The voltage at a point 's' away from the receiving end is given by,

E = E_R cosh (jβs) + $I_R Z_O$ sinh (jβs) .....(1)

Coshθ = $\frac{e^{\theta} + e^{-\theta}}{2}$

$\text{Sinh}\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

$\text{E} = E_R \left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + I_R Z_O \left(\frac{e^{j\beta s} - e^{-j\beta s}}{2}\right)$ ......(2)

$\text{E} = \frac{E_R}{2} e^{j\beta s} + \frac{E_R}{2} e^{-j\beta s} + \frac{I_R Z_O}{2} e^{j\beta s} - \frac{I_R Z_O}{2} e^{-j\beta s}$

$\text{E} = \frac{e^{j\beta s}}{2} [E_R + I_R Z_O] + \frac{e^{-j\beta s}}{2} [E_R - I_R Z_O]$

$E_R = I_R Z_R$

sub E_R value in above equ....

$\text{E} = \frac{e^{j\beta s}}{2} [I_R Z_R + I_R Z_O] + \frac{e^{-j\beta s}}{2} [I_R Z_R - I_R Z_O]$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} [Z_R + Z_O] + \frac{I_R\, e^{-j\beta s}}{2} [Z_R - Z_O]$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} \left[(Z_R + Z_O) + \frac{e^{-j\beta s}}{e^{j\beta s}} (Z_R - Z_O)\right]$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} \left[(Z_R + Z_O) + e^{-j2\beta s} (Z_R - Z_O)\right]$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} (Z_R + Z_O) \left[1 + e^{-j2\beta s} \left(\frac{Z_R - Z_O}{Z_R + Z_O}\right)\right]$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} (Z_R + Z_O) \left[1 + e^{-j2\beta s} . k\right]$

$$\left(k = \frac{Z_R - Z_O}{Z_R + Z_O}\right)$$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} (Z_R + Z_O) [1 \angle 0° + 1 \angle -2\beta s |\, k| \angle \emptyset]$

$\text{E} = \frac{I_R\, e^{j\beta s}}{2} (Z_R + Z_O) [1 \angle 0° + |\, k| \angle \emptyset - 2\beta s]$

i) <u>At E$_{max}$:</u>

$\emptyset - 2\beta s = 0$

$E_{max} = \frac{I_R\, e^{j\beta s}}{2} (Z_R + Z_O) [1 \angle 0° + |\, k| \angle 0°]$

$E_{max} = \frac{I_R\, e^{j\beta s}}{2} (Z_R + Z_O) [1 + |\, k|]$

ii) <u>At E$_{min}$:</u>

$\emptyset - 2\beta s = \pi = 180°$

$$E_{min} = \frac{I_R\, e^{j\beta s}}{2}(Z_R + Z_O)[1 \angle 0° + \; |k| \; \angle \pi\,]$$

$$E_{min} = \frac{I_R\, e^{j\beta s}}{2}(Z_R + Z_O)[1 - \; |k|\,]$$

We know that,

$$S = \frac{E_{max}}{E_{min}}$$

$$S = \frac{\frac{I_R\, e^{j\beta s}}{2}(Z_R+Z_O)[1+\;|k|\,]}{\frac{I_R\, e^{j\beta s}}{2}(Z_R+Z_O)[1-|k|\,]}$$

$$S = \frac{1+|k|}{1-|k|}$$

S (1-k) = 1+k

S - kS = 1 + k

S - 1 = k + kS

S - 1 = k (1 + S)

$$k = \frac{S-1}{S+1}$$

INPUT IMPEDANCE FOR THE DISSIPATION-LESS LINE:

In Fig 2.3.2 the voltage and current of a transmission line at a distance 's' from the receiving end is given by,

$E_S = E_R\cos\beta s + j\, I_R R_O \sin\beta s$

$I_S = I_R\cos\beta s + j\, \frac{E_R}{R_O}\sin\beta s$

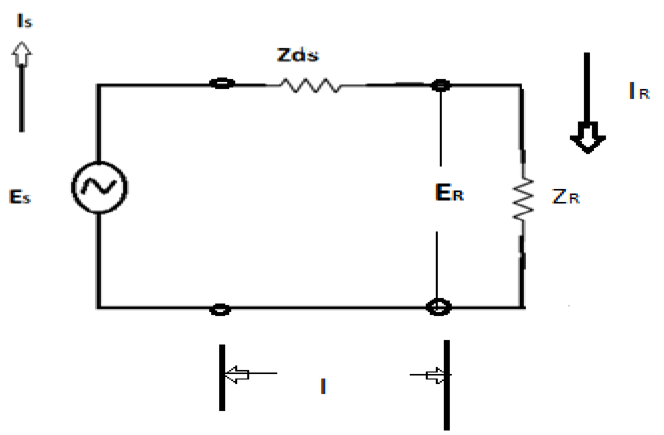


Fig: 2.3.2 Input impedance for the dissipation less line

The input impedance of the transmission line is given by,

$$z_{in} = z_s = \frac{E_s}{I_s} \qquad \text{......(1)}$$

$$z_{in} = z_s = \frac{\mathrm{E_R}\cos\beta s + j\, \mathrm{I_R} R_O \sin\beta s}{\mathrm{I_R}\cos\beta s + j\, \frac{\mathrm{E_R}}{R_O}\sin\beta s}$$

$$z_s = R_O \left[\frac{\mathrm{E_R}\cos\beta s + j\, \mathrm{I_R} R_O \sin\beta s}{\mathrm{I_R} R_O \cos\beta s + j\, \mathrm{E_R}\sin\beta s}\right] \qquad \text{......(2)}$$

we know,

$$E_R = I_R Z_R$$

sub E_R value in above equ,

$$z_s = R_O \left[\frac{I_R Z_R\cos\beta s + j\, \mathrm{I_R} R_O \sin\beta s}{\mathrm{I_R} R_O \cos\beta s + j\, I_R Z_R\sin\beta s}\right]$$

$$z_s = \frac{R_O I_R \cos\beta s}{I_R \cos\beta s}\left[\frac{Z_R + \frac{jR_O \sin\beta s}{\cos\beta s}}{R_O + \frac{jZ_R \sin\beta s}{\cos\beta s}}\right]$$

$$z_s = R_O \left[\frac{Z_R + jR_O \tan\beta s}{R_O + jZ_R \tan\beta s}\right] \qquad \text{.........(3)}$$

The another method to represent input impedance of the transmission line is given by,

$$z_s = R_O \left[\frac{I_R Z_R\cos\beta s + j\, \mathrm{I_R} R_O \sin\beta s}{\mathrm{I_R} R_O \cos\beta s + j\, I_R Z_R\sin\beta s}\right]$$

$$z_s = \frac{\mathrm{I_R}\, R_O}{\mathrm{I_R}}\left[\frac{Z_R\cos\beta s + jR_O \sin\beta s}{R_O\cos\beta s + j\, Z_R\sin\beta s}\right]$$

$$\mathrm{Cos}\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\mathrm{Sin}\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$z_s = R_O \left[\frac{Z_R\left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + jR_O\left(\frac{e^{j\beta s} - e^{-j\beta s}}{2j}\right)}{R_O\left(\frac{e^{j\beta s} + e^{-j\beta s}}{2}\right) + jZ_R\left(\frac{e^{j\beta s} - e^{-j\beta s}}{2j}\right)}\right]$$

$$z_s = \frac{2R_O}{2}\left[\frac{Z_R e^{j\beta s}+Z_R e^{-j\beta s}+R_O e^{j\beta s}-R_O e^{-j\beta s}}{R_O e^{j\beta s}+R_O e^{-j\beta s}+Z_R e^{j\beta s}-Z_R e^{-j\beta s}}\right]$$

$$z_s = R_O\left[\frac{e^{j\beta s}(Z_R+R_O)+e^{-j\beta s}(Z_R-R_O)}{e^{j\beta s}(Z_R+R_O)-e^{-j\beta s}(Z_R-R_O)}\right]$$

$$z_s = R_O\,\frac{e^{j\beta s}(Z_R+R_O)}{e^{j\beta s}(Z_R+R_O)}\left[\frac{1+\frac{e^{-j\beta s}(Z_R-R_O)}{e^{j\beta s}(Z_R+R_O)}}{1-\frac{e^{-j\beta s}(Z_R-R_O)}{e^{j\beta s}(Z_R+R_O)}}\right]$$

$$z_s = R_O\left[\frac{1+K\ e^{-j2\beta s}}{1-K\ e^{-j2\beta s}}\right]$$

$$z_s = R_O\left[\frac{1+|K|\ 1\ \lfloor\emptyset\,.1\ \lfloor -2\beta s}{1-|K|\ 1\ \lfloor\emptyset\,.1\ \lfloor -2\beta s}\right]$$

$$z_s = R_O\left[\frac{1+|K|\ \lfloor\emptyset-2\beta s}{1-|K|\ \lfloor\emptyset-2\beta s}\right]$$

CONDITION FOR z_{max}:

The input impedance will be maximum when both incident and reflected waves are inphase.

$\emptyset - 2\beta s = 0$

$\emptyset = 2\beta s$

$s = \frac{\emptyset}{2\beta}$

$$z_{max} = R_O\left[\frac{1+|K|\ \lfloor 0°}{1-|K|\ \lfloor 0°}\right]$$

$$z_{max} = R_O\left[\frac{1+|K|}{1-|K|}\right]$$

CONDITION FOR z_{min}:

The input impedance will be maximum when both incident and reflected waves are inphase.

$\emptyset - 2\beta s = -\pi$

$\emptyset = -\pi + 2\beta s$

$\emptyset + \pi = 2\beta s$

$s = \frac{\emptyset+\pi}{2\beta}$

$$s = \frac{\emptyset}{2\beta} + \frac{\pi}{2\beta}$$

$$s = \frac{\emptyset}{2\beta} + \frac{\lambda}{4} \qquad (\lambda = \frac{2\pi}{\beta})$$

$$z_{min} = R_O \left[\frac{1+|\ K|\ \llcorner -\pi}{1-|\ K|\ \llcorner -\pi}\right]$$

$$z_{min} = R_O \left[\frac{1-|\ K|}{1+|\ K|}\right]$$

$$z_{min} = \frac{R_O}{s}$$

s = standing wave ratio

$$s = \frac{1+|\ K|}{1-|\ K|}$$