4.2.1 PROPERTIES OF CROSS CORRELATION FUNCTION

Property 1: If {X(t)} and {Y(t)} are two WSS processes, then $R_{XY}(-\tau) = R_{YX}(\tau)$.

Proof:

$R_{XY}(\tau) = E[X(t+\tau)\ Y((t)]$

Replace τ by $-\tau$

$R_{XY}(-\tau) = E[X(t-\tau)\ Y((t)]$

$= E[Y(t)\ X(t-\tau)]$

$= R_{YX}(\tau)$

$R_{XY}(-\tau) = R_{YX}(\tau)$

Property 2: Prove that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0) R_{YY}(0)}$

$R_{XY}(\tau) = E[X(t_1)\ Y(t_2)]$

$[R_{XY}(\tau)]^2 = [\ E[X(t_1)\ Y(t_2)]]^2 \leq \quad R_{XY}(\tau) = E[X^2(t_1)]\ E[Y^2(t_2)]$ [by Scwartz inequality]

$= R_{XX}(0) R_{YY}(0)$

$|\ R_{XY}(\tau)|^2 \leq R_{XX}(0) R_{YY}(0)$

$|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0) R_{YY}(0)}$

Property 3: Prove that | $R_{XY}(\tau)$ |≤ $\frac{R_{XX}(0)R_{YY}(0)}{2}$

Proof: We know that

$|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$ …………………(1)

Also we know that geometric mean ≤ arithmetic mean

i.e $\sqrt{ab} \leq \frac{a+b}{2}$ ……………(2)

Take a= $R_{XX}(0)$, b= $R_{YY}(0)$ in (1), we get

$\sqrt{R_{XX}(0)R_{YY}(0)} \leq \frac{R_{XX}(0)R_{YY}(0)}{2}$ …………(3)

From (1) & (3) , we get

$| R_{XY}(\tau) | \leq \frac{R_{XX}(0)R_{YY}(0)}{2}$

Property 4: If {X(t)} and {Y(t)} are two random processes and

E [(X (0) – Y(0))^2] = 0 then prove that $R_{XY}(\tau) = R_{YY}(\tau)$.

Proof: Given E [(X (0) – Y(0))^2] = 0 …………(1)

Let Z(t) = X (t+τ) and W(t)= X(t) – Y (t) …………….(2)

By Cauchy Schwartz inequality,

$$[E(Z(t)\,W(t))]^2 \leq E[Z^2(t)]\,E[W^2(t)]$$

i.e, $[E[X(t+\tau)(X(t)-Y(t)]]^2 \leq E[X^2(t+\tau)]\, E[(X(t)-Y(t))^2]$ from (2)

$[E[X(t+\tau)(X(t)] - [E[X(t+\tau)Y(t)]]^2 \leq R_{XX}(0)\, E[(X(t)-Y(t))^2]$

$$[R_{XX}(\tau) - R_{XY}(\tau)]^2 \leq R_{XX}(0)\, E[(X(t)-Y(t))^2]$$

By putting t=0 we get,

$[R_{XX}(\tau) - R_{XY}(\tau)]^2 \leq R_{XX}(0)\, E[(X(0)-Y(0))^2]$

$[R_{XX}(\tau) - R_{XY}(\tau)]^2 = 0$ from (1)

$R_{XX}(\tau) - R_{XY}(\tau) = 0$

$R_{XX}(\tau) = R_{XY}(\tau)$ …………(3)

Similarly if we take $Z(t) = X(t) - Y(t)$ and $W(t) = Y(t-\tau)$,

We can prove that

$R_{YY}(\tau) = R_{XY}(\tau)$ …………(4)

From (3) and (4), we get $R_{XX}(\tau) = R_{XY}(\tau) = R_{YY}(\tau)$