

## UNIT II - KINEMATICS OF LINKAGE MECHANISMS

### 2.1 Displacement, velocity and acceleration analysis in simple mechanisms:

#### Important Concepts in Velocity Analysis

1. The absolute velocity of any point on a mechanism is the velocity of that point with reference to ground.
2. Relative velocity describes how one point on a mechanism moves relative to another point on the mechanism.
3. The velocity of a point on a moving link relative to the pivot of the link is given by the equation:  $V = \omega r$ , where  $\omega$  = angular velocity of the link and  $r$  = distance from pivot.

#### Acceleration Components

- | **Normal Acceleration:**  $A^n$  = Points toward the centre of rotation
- | **Tangential Acceleration:**  $A^t$  = In a direction perpendicular to the link
- | **Coriolis Acceleration:**  $A^c$  = In a direction perpendicular to the link
- | **Sliding Acceleration:**  $A^s$  = In the direction of sliding.

A rotating link will produce normal and tangential acceleration components at any point a distance,  $r$ , from the rotational pivot of the link. The total acceleration of that point is the vector sum of the components. A slider attached to ground experiences only sliding acceleration.

The total acceleration of a point is the vector sum of all applicable acceleration components:

$$\mathbf{A} = \mathbf{A}^n + \mathbf{A}^t + \mathbf{A}^c + \mathbf{A}^s$$

These vectors and the above equation can be broken into x and y components by applying sines and cosines to the vector diagrams to determine the x and y components of each vector. In this way, the x and y components of the total acceleration can be found.

### 2.2 Graphical Method, Velocity and Acceleration polygons:

#### \* Graphical velocity analysis:

It is a very short step (using basic trigonometry with sines and cosines) to convert the graphical results into numerical results. The basic steps are these:

1. Set up a velocity reference plane with a point of zero velocity designated.
2. Use the equation,  $V = \omega r$ , to calculate any known linkage velocities.
3. Plot your known linkage velocities on the velocity plot. A linkage that is rotating about ground gives an absolute velocity. This is a vector that originates at the zero-velocity point and runs perpendicular to the link to show the direction of motion. The vector,  $\mathbf{V}_A$ , gives the velocity of point A.
4. Plot all other velocity vector directions. A point on a grounded link (such as point B) will produce an absolute velocity vector passing through the zero-velocity point and perpendicular to the link. A point on a floating link (such as B relative to point A) will produce a relative velocity vector. This vector will be perpendicular to the link AB and pass through the reference point (A) on the velocity diagram.
5. One should be able to form a closed triangle (for a 4-bar) that shows the vector equation:  $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{B/A}$  where  $\mathbf{V}_B$  = absolute velocity of point B,  $\mathbf{V}_A$  = absolute velocity of point A, and  $\mathbf{V}_{B/A}$  is the velocity of point B relative to point A.

### 2.3 Velocity and Acceleration analysis of mechanisms (Graphical Methods):

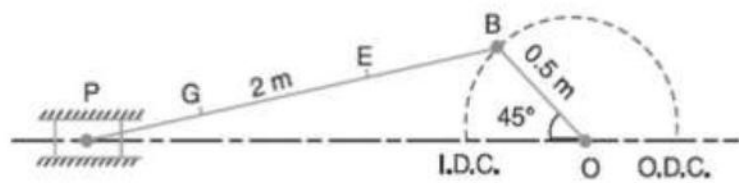
Velocity and acceleration analysis by vector polygons: Relative velocity and accelerations of particles in a common link, relative velocity and accelerations of coincident particles on separate link, Coriolis component of acceleration.

Velocity and acceleration analysis by complex numbers: Analysis of single slider crank mechanism and four bar mechanism by loop closure equations and complex numbers.

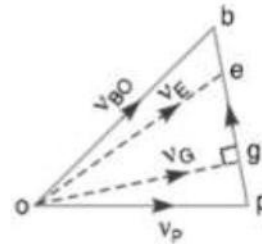
### 2.4 Coincident points, Coriolis Acceleration:

Γ **Coriolis Acceleration:**  $A^c = 2 (dr/dt)$ . In a direction perpendicular to the link. A slider attached to ground experiences only sliding acceleration.

**Example: 1** The crank and connecting rod of a steam engine are 0.5m and 2 m long. The crank makes 180 rpm in CW direction. When it turned 45° from IDC determine 1. Velocity of piston. 2. Angular velocity of connecting rod, 3. Velocity of point E on connecting rod, 1.5 m from gudgeon pin 4. Velocities of rubbing at the pins of crank shaft, crank and crosshead when the diameters of the pins are 50mm, 60mm, 30mm respectively



(a) Space diagram.



(b) Velocity diagram.

$$\text{vector } oa = v_{AO} = v_A = 1.76 \text{ m/s}$$

$$v_D = \text{vector } od = 1.6 \text{ m/s}$$

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

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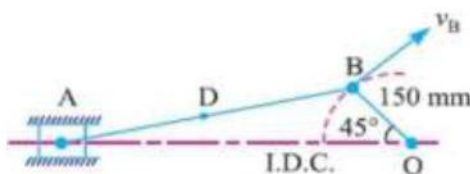
$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about B)}$$

**Example** The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

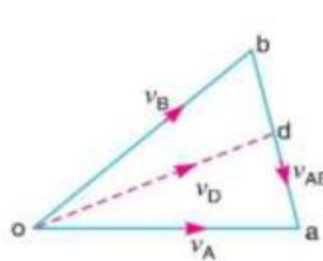
**Solution.** Given :  $N_{BO} = 300 \text{ r.p.m.}$  or  $\omega_{BO} = 2\pi \times 300/60 = 31.42 \text{ rad/s}$ ;  $OB = 150 \text{ mm}$   
 $0.15 \text{ m}$ ;  $BA = 600 \text{ mm} = 0.6 \text{ m}$

We know that linear velocity of B with respect to O or velocity of B,

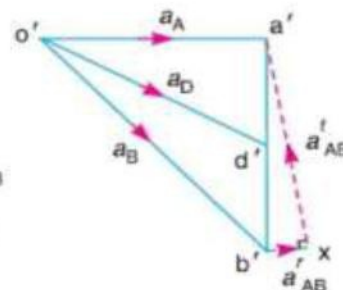
$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$



(a) Space diagram.



(b) Velocity diagram.



(c) Acceleration diagram.

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

$$\text{Velocity of } A, v_A = \text{vector } oa = 4 \text{ m/s}$$

$$v_D = \text{vector } od = 4.1 \text{ m/s}$$

$$a_{BO}^r = a_B = \frac{v_{BO}^2}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

$$a_{AB}^r = \frac{v_{AB}^2}{BA} = \frac{(3.4)^2}{0.6} = 19.3 \text{ m/s}^2$$

$$\text{vector } o'b' = a_{BO}^r = a_B = 148.1 \text{ m/s}^2$$

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2$$

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B)$$

$$a_{AB}^t = 103 \text{ m/s}^2$$

$$\alpha_{AB} = \frac{a_{AB}^t}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B)$$

An engine mechanism is shown in Fig. 8.5. The crank  $CB = 100 \text{ mm}$  and the connecting rod  $BA = 300 \text{ mm}$  with centre of gravity  $G$ ,  $100 \text{ mm}$  from  $B$ . In the position shown, the crankshaft has a speed of  $75 \text{ rad/s}$  and an angular acceleration of  $1200 \text{ rad/s}^2$ . Find: 1. velocity of  $G$  and angular velocity of  $AB$ , and 2. acceleration of  $G$  and angular acceleration of  $AB$ .

**Solution.** Given :  $\omega_{BC} = 75 \text{ rad/s}$  ;  $\alpha_{BC} = 1200 \text{ rad/s}^2$ ,  $CB = 100 \text{ mm} = 0.1 \text{ m}$ ;  $BA = 300 \text{ mm} = 0.3 \text{ m}$

We know that velocity of  $B$  with respect to  $C$  or velocity of  $B$ ,

$$v_{BC} = v_B = \omega_{BC} \times CB = 75 \times 0.1 = 7.5 \text{ m/s} \quad \dots(\text{Perpendicular to } BC)$$

Since the angular acceleration of the crankshaft,  $\alpha_{BC} = 1200 \text{ rad/s}^2$ , therefore tangential component of the acceleration of  $B$  with respect to  $C$ ,

$$a_{BC}^t = \alpha_{BC} \times CB = 1200 \times 0.1 = 120 \text{ m/s}^2$$

Note: When the angular acceleration is not given, then there will be no tangential component of the acceleration.

#### 1. Velocity of $G$ and angular velocity of $AB$

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.6 (a). Now the velocity diagram, as shown in Fig. 8.6 (b), is drawn as discussed below:

1. Draw vector  $cb$  perpendicular to  $CB$ , to some suitable scale, to represent the velocity of  $B$  with respect to  $C$  or velocity of  $B$  (i.e.  $v_{BC}$  or  $v_B$ ), such that

$$\text{vector } cb = v_{BC} = v_B = 7.5 \text{ m/s}$$

2. From point  $b$ , draw vector  $ba$  perpendicular to  $BA$  to represent the velocity of  $A$  with respect to  $B$  i.e.  $v_{AB}$ , and from point  $c$ , draw vector  $ca$  parallel to the path of motion of  $A$  (which is along  $AC$ ) to represent the velocity of  $A$  i.e.  $v_A$ . The vectors  $ba$  and  $ca$  intersect at  $a$ .

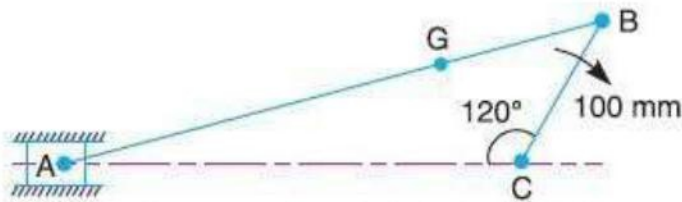
3. Since the point  $G$  lies on  $AB$ , therefore divide vector  $ab$  at  $g$  in the same ratio as  $G$  divides  $AB$  in the space diagram. In other words,

$$ag / ab = AG / AB$$

The vector  $cg$  represents the velocity of  $G$ .

By measurement, we find that velocity of  $G$ ,

$$v_G = \text{vector } cg = 6.8 \text{ m/s} \quad \text{Ans.}$$

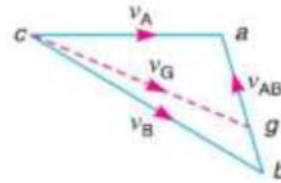


From velocity diagram, we find that velocity of  $A$  with respect to  $B$ ,

$$v_{AB} = \text{vector } ba = 321 \text{ m/s}$$

We know that angular velocity of  $AB$ ,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{4}{0.3} = 13.3 \text{ rad/s (Clockwise) Ans.}$$



## 2. Acceleration of $G$ and angular acceleration of $AB$

We know that radial component of the acceleration of  $B$  with respect to  $C$ ,

$$a_{BC}^r = \frac{v_{BC}^2}{CB} = \frac{(7.5)^2}{0.1} = 562.5 \text{ m/s}^2$$

and radial component of the acceleration of  $A$  with respect to  $B$ ,

$$a_{AB}^r = \frac{v_{AB}^2}{BA} = \frac{4^2}{0.3} = 53.3 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.6 (c), is drawn as discussed below:

1. Draw vector  $c'b''$  parallel to  $CB$ , to some suitable scale, to represent the radial component of the acceleration of  $B$  with respect to  $C$ , i.e.  $a_{BC}^r$ , such that

$$\text{vector } c'b'' = a_{BC}^r = 562.5 \text{ m/s}^2$$

2. From point  $b''$ , draw vector  $b''b'$  perpendicular to vector  $c'b''$  or  $CB$  to represent the tangential component of the acceleration of  $B$  with respect to  $C$  i.e.  $a_{BC}^t$ , such that

$$\text{vector } b''b' = a_{BC}^t = 120 \text{ m/s}^2 \quad \dots \text{(Given)}$$

3. Join  $c'b'$ . The vector  $c'b'$  represents the total acceleration of  $B$  with respect to  $C$  i.e.  $a_{BC}$ .

4. From point  $b'$ , draw vector  $b'x$  parallel to  $BA$  to represent radial component of the acceleration of  $A$  with respect to  $B$  i.e.  $a_{AB}^r$  such that

$$\text{vector } b'x = a_{AB}^r = 53.3 \text{ m/s}^2$$

5. From point  $x$ , draw vector  $xa'$  perpendicular to vector  $b'x$  or  $BA$  to represent tangential component of the acceleration of  $A$  with respect to  $B$  i.e.  $a_{AB}^t$ , whose magnitude is not yet known.

6. Now draw vector  $c'a'$  parallel to the path of motion of  $A$  (which is along  $AC$ ) to represent the acceleration of  $A$  i.e.  $a_A$ . The vectors  $xa'$  and  $c'a'$  intersect at  $a'$ . Join  $b'a'$ . The vector  $b'a'$  represents the acceleration of  $A$  with respect to  $B$  i.e.  $a_{AB}$ .

7. In order to find the acceleration of  $G$ , divide vector  $a'b'$  in  $g'$  in the same ratio as  $G$  divides  $BA$  in Fig. 8.6 (a). Join  $c'g'$ . The vector  $c'g'$  represents the acceleration of  $G$ .

By measurement, we find that acceleration of  $G$ ,

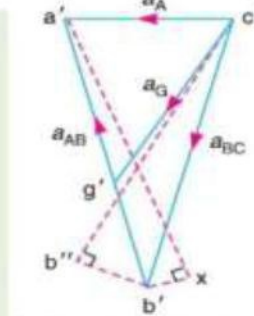
$$a_G = \text{vector } c'g' = 414 \text{ m/s}^2 \text{ Ans.}$$

From acceleration diagram, we find that tangential component of the acceleration of  $A$  with respect to  $B$ ,

$$a_{AB}^t = \text{vector } xa' = 546 \text{ m/s}^2 \quad \dots \text{(By measurement)}$$

$\therefore$  Angular acceleration of  $AB$ ,

$$\alpha_{AB} = \frac{a_{AB}^t}{BA} = \frac{546}{0.3} = 1820 \text{ rad/s}^2 \text{ (clockwise) Ans.}$$



(c) Acceleration diagram.

Fig. 8.6

**Example 8.3.** In the mechanism shown in Fig. 8.7, the slider  $C$  is moving to the right with a velocity of  $1 \text{ m/s}$  and an acceleration of  $2.5 \text{ m/s}^2$ .

The dimensions of various links are  $AB = 3 \text{ m}$  inclined at  $45^\circ$  with the vertical and  $BC = 1.5 \text{ m}$  inclined at  $45^\circ$  with the horizontal. Determine: 1. the magnitude of vertical and horizontal component of the acceleration of the point  $B$ , and 2. the angular acceleration of the links  $AB$  and  $BC$ .

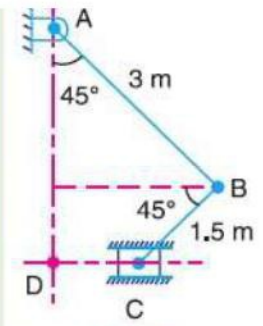


Fig. 8.7

**Solution.** Given :  $v_C = 1 \text{ m/s}$  ;  $a_C = 2.5 \text{ m/s}^2$  ;  $AB = 3 \text{ m}$  ;  $BC = 1.5 \text{ m}$

First of all, draw the space diagram, as shown in Fig. 8.8 (a), to some suitable scale. Now the velocity diagram, as shown in Fig. 8.8 (b), is drawn as discussed below:

1. Since the points  $A$  and  $D$  are fixed points, therefore they lie at one place in the velocity diagram. Draw vector  $dc$  parallel to  $DC$ , to some suitable scale, which represents the velocity of slider  $C$  with respect to  $D$  or simply velocity of  $C$ , such that

$$\text{vector } dc = v_{CD} = v_C = 1 \text{ m/s}$$

2. Since point  $B$  has two motions, one with respect to  $A$  and the other with respect to  $C$ , therefore from point  $a$ , draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$ , i.e.  $v_{BA}$  and from point  $c$  draw vector  $cb$  perpendicular to  $CB$  to represent the velocity of  $B$  with respect to  $C$  i.e.  $v_{BC}$ . The vectors  $ab$  and  $cb$  intersect at  $b$ .

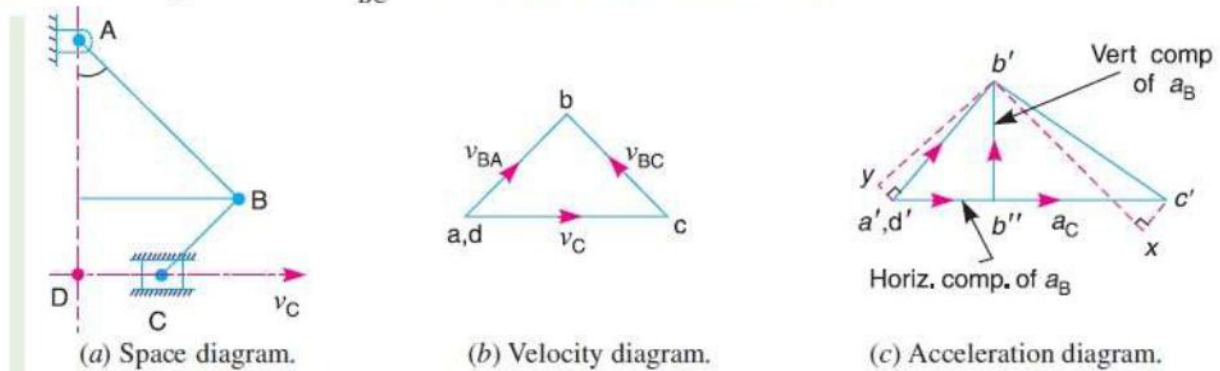


Fig. 8.8

By measurement, we find that velocity of  $B$  with respect to  $A$ ,

By measurement, we find that velocity of  $B$  with respect to  $A$ ,

$$v_{BA} = \text{vector } ab = 0.72 \text{ m/s}$$

and velocity of  $B$  with respect to  $C$ ,

$$v_{BC} = \text{vector } cb = 0.72 \text{ m/s}$$

**Example 8.4.** PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5 mm ; QR = 175 mm ; RS = 112.5 mm ; and PS = 200 mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.

**Solution.** Given :  $\omega_{QP} = 10 \text{ rad/s}$  ;  $PQ = 62.5 \text{ mm} = 0.0625 \text{ m}$  ;  $QR = 175 \text{ mm} = 0.175 \text{ m}$  ;  $RS = 112.5 \text{ mm} = 0.1125 \text{ m}$  ;  $PS = 200 \text{ mm} = 0.2 \text{ m}$

We know that velocity of Q with respect to P or velocity of Q,

$$v_{QP} = v_Q = \omega_{QP} \times PQ = 10 \times 0.0625 = 0.625 \text{ m/s}$$

...(Perpendicular to PQ)

### Angular velocity of links QR and RS

First of all, draw the space diagram of a four bar chain, to some suitable scale, as shown in Fig. 8.9 (a). Now the velocity diagram as shown in Fig. 8.9 (b), is drawn as discussed below:

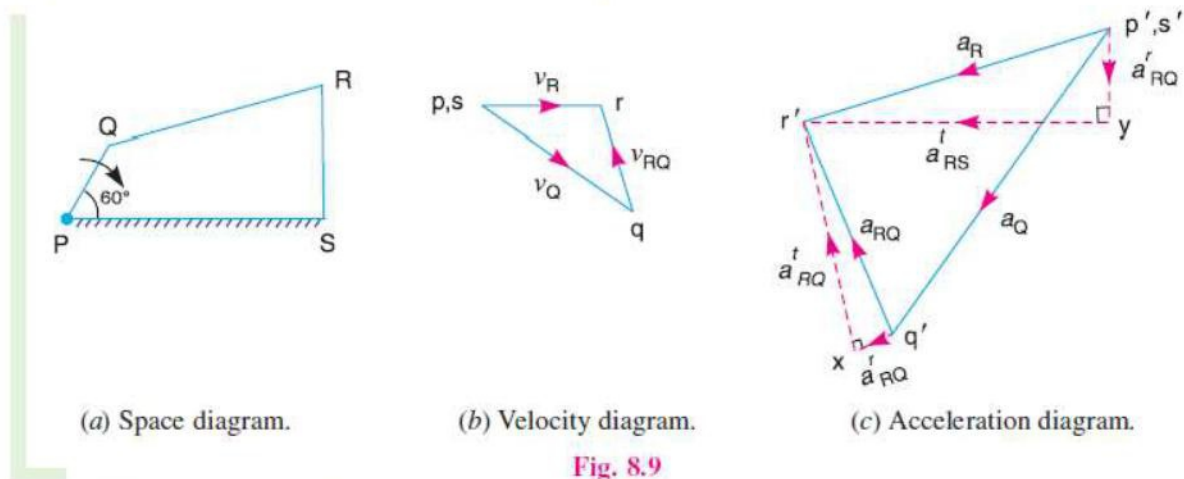


Fig. 8.9

1. Since P and S are fixed points, therefore these points lie at one place in velocity diagram. Draw vector pq perpendicular to PQ, to some suitable scale, to represent the velocity of Q with respect to P or velocity of Q i.e.  $v_{QP}$  or  $v_Q$  such that

$$\text{vector } pq = v_{QP} = v_Q = 0.625 \text{ m/s}$$

2. From point q, draw vector qr perpendicular to QR to represent the velocity of R with respect to Q (i.e.  $v_{RQ}$ ) and from point s, draw vector sr perpendicular to SR to represent the velocity of R with respect to S or velocity of R (i.e.  $v_{RS}$  or  $v_R$ ). The vectors qr and sr intersect at r. By measurement, we find that

$$v_{RQ} = \text{vector } qr = 0.333 \text{ m/s, and } v_{RS} = v_R = \text{vector } sr = 0.426 \text{ m/s}$$

We know that angular velocity of link QR,

$$\omega_{QR} = \frac{v_{RQ}}{RQ} = \frac{0.333}{0.175} = 1.9 \text{ rad/s (Anticlockwise) Ans.}$$

### Angular acceleration of links QR and RS

Since the angular acceleration of the crank  $PQ$  is not given, therefore there will be no tangential component of the acceleration of  $Q$  with respect to  $P$ .

We know that radial component of the acceleration of  $Q$  with respect to  $P$  (or the acceleration of  $Q$ ),

$$a_{QP}^r = a_{QP} = a_Q = \frac{v_{QP}^2}{PQ} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Radial component of the acceleration of  $R$  with respect to  $Q$ ,

$$a_{RQ}^r = \frac{v_{RQ}^2}{QR} = \frac{(0.333)^2}{0.175} = 0.634 \text{ m/s}^2$$

and radial component of the acceleration of  $R$  with respect to  $S$  (or the acceleration of  $R$ ),

$$a_{RS}^r = a_{RS} = a_R = \frac{v_{RS}^2}{SR} = \frac{(0.426)^2}{0.1125} = 1.613 \text{ m/s}^2$$

The acceleration diagram, as shown in Fig. 8.9 (c) is drawn as follows :

1. Since  $P$  and  $S$  are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector  $p'q'$  parallel to  $PQ$ , to some suitable scale, to represent the radial component of acceleration of  $Q$  with respect to  $P$  or acceleration of  $Q$  i.e.  $a_{QP}^r$  or  $a_Q$  such that

$$\text{vector } p'q' = a_{QP}^r = a_Q = 6.25 \text{ m/s}^2$$

2. From point  $q'$ , draw vector  $q'x$  parallel to  $QR$  to represent the radial component of acceleration of  $R$  with respect to  $Q$  i.e.  $a_{RQ}^r$  such that

$$\text{vector } q'x = a_{RQ}^r = 0.634 \text{ m/s}^2$$

3. From point  $x$ , draw vector  $xr'$  perpendicular to  $QR$  to represent the tangential component of acceleration of  $R$  with respect to  $Q$  i.e.  $a_{RQ}^t$  whose magnitude is not yet known.

4. Now from point  $s'$ , draw vector  $s'y$  parallel to  $SR$  to represent the radial component of the acceleration of  $R$  with respect to  $S$  i.e.  $a_{RS}^r$  such that

$$\text{vector } s'y = a_{RS}^r = 1.613 \text{ m/s}^2$$

5. From point  $y$ , draw vector  $yr'$  perpendicular to  $SR$  to represent the tangential component of acceleration of  $R$  with respect to  $S$  i.e.  $a_{RS}^t$ .

6. The vectors  $xr'$  and  $yr'$  intersect at  $r'$ . Join  $p'r$  and  $q'r'$ . By measurement, we find that

$$a_{RQ}^t = \text{vector } xr' = 4.1 \text{ m/s}^2 \text{ and } a_{RS}^t = \text{vector } yr' = 5.3 \text{ m/s}^2$$

We know that angular acceleration of link  $QR$ ,

$$\alpha_{QR} = \frac{a_{RQ}^t}{QR} = \frac{4.1}{0.175} = 23.43 \text{ rad/s}^2 \text{ (Anticlockwise) Ans.}$$

and angular acceleration of link  $RS$ ,

$$\alpha_{RS} = \frac{a_{RS}^t}{SR} = \frac{5.3}{0.1125} = 47.1 \text{ rad/s}^2 \text{ (Anticlockwise) Ans.}$$

**Example 8.6.** In the mechanism, as shown in Fig. 8.12, the crank  $OA$  rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks  $B$  and  $D$ . The dimensions of the various links are  $OA = 300$  mm;  $AB = 1200$  mm;  $BC = 450$  mm and  $CD = 450$  mm.

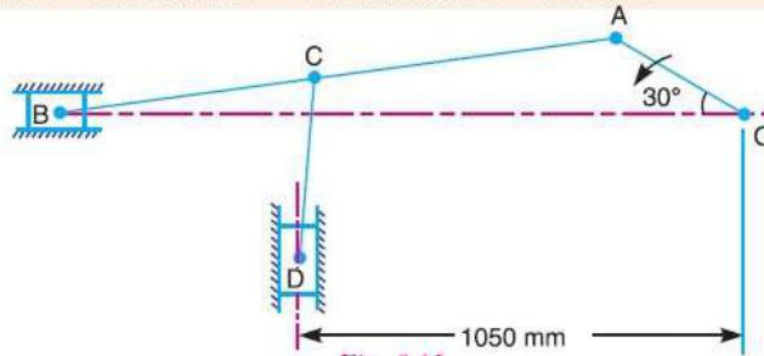


Fig. 8.12

For the given configuration, determine : 1. velocities of sliding at  $B$  and  $D$ , 2. angular velocity of  $CD$ , 3. linear acceleration of  $D$ , and 4. angular acceleration of  $CD$ .

**Solution.** Given :  $N_{AO} = 20$  r.p.m. or  $\omega_{AO} = 2\pi \times 20/60 = 2.1$  rad/s ;  $OA = 300$  mm = 0.3 m ;  $AB = 1200$  mm = 1.2 m ;  $BC = CD = 450$  mm = 0.45 m

### 1. Velocities of sliding at $B$ and $D$

First of all, draw the space diagram, to some suitable scale, as shown in Fig. 8.13 (a). Now the velocity diagram, as shown in Fig. 8.13 (b), is drawn as discussed below:

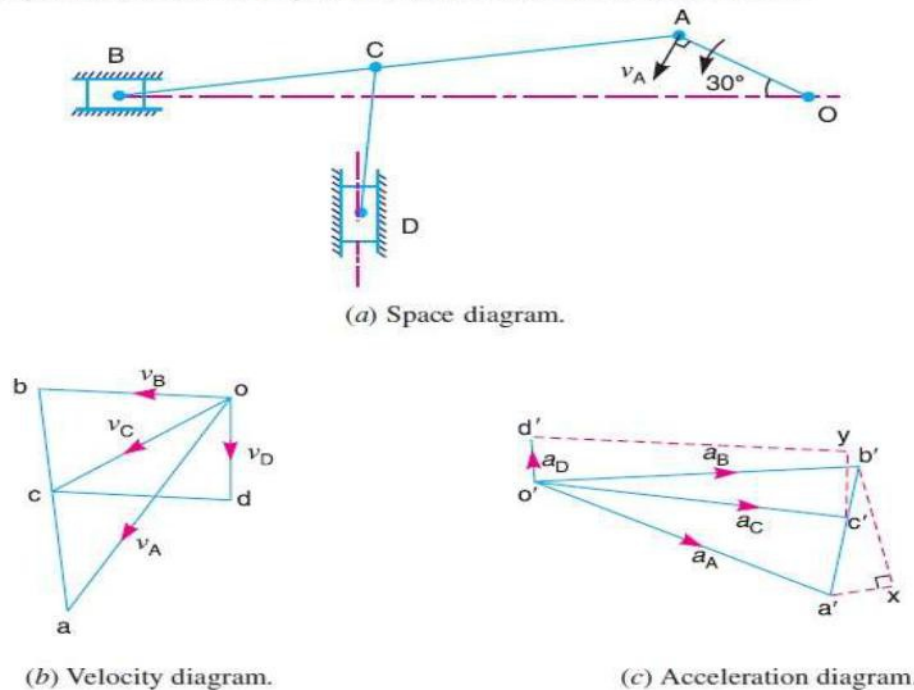


Fig. 8.13

1. Draw vector  $oa$  perpendicular to  $OA$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $O$  (or simply velocity of  $A$ ), such that

$$\text{vector } oa = v_{AO} = v_A = 0.63 \text{ m/s}$$

2. From point  $a$ , draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  (i.e.  $v_{BA}$ ) and from point  $o$  draw vector  $ob$  parallel to path of motion  $B$  (which is along  $BO$ ) to represent the velocity of  $B$  with respect to  $O$  (or simply velocity of  $B$ ). The vectors  $ab$  and  $ob$  intersect at  $b$ .

3. Divide vector  $ab$  at  $c$  in the same ratio as  $C$  divides  $AB$  in the space diagram. In other



4. Now from point  $c$ , draw vector  $cd$  perpendicular to  $CD$  to represent the velocity of  $D$  with respect to  $C$  (i.e.  $v_{DC}$ ) and from point  $o$  draw vector  $od$  parallel to the path of motion of  $D$  (which along the vertical direction) to represent the velocity of  $D$ .

By measurement, we find that velocity of sliding at  $B$ ,

$$v_B = \text{vector } ob = 0.4 \text{ m/s Ans.}$$

and velocity of sliding at  $D$ ,

$$v_D = \text{vector } od = 0.24 \text{ m/s Ans.}$$

### 2. Angular velocity of $CD$

By measurement from velocity diagram, we find that velocity of  $D$  with respect to  $C$ ,

$$v_{DC} = \text{vector } cd = 0.37 \text{ m/s}$$

$\therefore$  Angular velocity of  $CD$ ,

$$\omega_{CD} = \frac{v_{DC}}{CD} = \frac{0.37}{0.45} = 0.82 \text{ rad/s (Anticlockwise). Ans.}$$

### 3. Linear acceleration of $D$

We know that the radial component of the acceleration of  $A$  with respect to  $O$  or acceleration of  $A$ ,

$$a_{AO}^r = a_A = \frac{v_{AO}^2}{OA} = \omega_{AO}^2 \times OA = (2.1)^2 \times 0.3 = 1.323 \text{ m/s}^2$$

Radial component of the acceleration of  $B$  with respect to  $A$ ,

$$a_{BA}^r = \frac{v_{BA}^2}{AB} = \frac{(0.54)^2}{1.2} = 0.243 \text{ m/s}^2$$

...(By measurement,  $v_{BA} = 0.54 \text{ m/s}$ )

Radial component of the acceleration of  $D$  with respect to  $C$ ,

$$a_{DC}^r = \frac{v_{DC}^2}{CD} = \frac{(0.37)^2}{0.45} = 0.304 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.13 (c), is drawn as discussed below:

1. Draw vector  $o'a'$  parallel to  $OA$ , to some suitable scale, to represent the radial component of the acceleration of  $A$  with respect to  $O$  or simply the acceleration of  $A$ , such that

$$\text{vector } o'a' = a_{AO}^r = a_A = 1.323 \text{ m/s}^2$$

2. From point  $a'$ , draw vector  $a'x$  parallel to  $AB$  to represent the radial component of the acceleration of  $B$  with respect to  $A$ , such that

$$\text{vector } a'x = a_{BA}^r = 0.243 \text{ m/s}^2$$

3. From point  $x$ , draw vector  $xb'$  perpendicular to  $AB$  to represent the tangential component of the acceleration of  $B$  with respect to  $A$  (i.e.  $a_{BA}^t$ ) whose magnitude is not yet known.

4. From point  $o'$ , draw vector  $o'b'$  parallel to the path of motion of  $B$  (which is along  $BO$ ) to represent the acceleration of  $B$  ( $a_B$ ). The vectors  $xb'$  and  $o'b'$  intersect at  $b'$ . Join  $a'b'$ . The vector  $a'b'$  represents the acceleration of  $B$  with respect to  $A$ .

5. Divide vector  $a'b'$  at  $c'$  in the same ratio as  $C$  divides  $AB$  in the space diagram. In other words,

$$BC / BA = b'c' / b'a'$$

6. From point  $c'$ , draw vector  $c'y$  parallel to  $CD$  to represent the radial component of the acceleration of  $D$  with respect to  $C$ , such that

### 4. Angular acceleration of $CD$

From the acceleration diagram, we find that the tangential component of the acceleration of  $D$  with respect to  $C$ ,

$$a_{DC}^t = \text{vector } yd' = 1.28 \text{ m/s}^2 \quad \dots(\text{By measurement})$$

∴ Angular acceleration of  $CD$ ,

$$\alpha_{CD} = \frac{a_{DC}^t}{CD} = \frac{1.28}{0.45} = 2.84 \text{ rad/s}^2 \text{ (Clockwise) Ans.}$$

**Example 8.7.** Find out the acceleration of the slider  $D$  and the angular acceleration of link  $CD$  for the engine mechanism shown in Fig. 8.14.

The crank  $OA$  rotates uniformly at 180 r.p.m. in clockwise direction. The various lengths are:  $OA = 150 \text{ mm}$ ;  $AB = 450 \text{ mm}$ ;  $PB = 240 \text{ mm}$ ;  $BC = 210 \text{ mm}$ ;  $CD = 660 \text{ mm}$ .

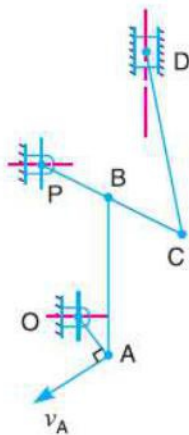
**Solution.** Given:  $N_{AO} = 180 \text{ r.p.m.}$ , or  $\omega_{AO} = 2\pi \times 180/60 = 18.85 \text{ rad/s}$ ;  $OA = 150 \text{ mm} = 0.15 \text{ m}$ ;  $AB = 450 \text{ mm} = 0.45 \text{ m}$ ;  $PB = 240 \text{ mm} = 0.24 \text{ m}$ ;  $CD = 660 \text{ mm} = 0.66 \text{ m}$

We know that velocity of  $A$  with respect to  $O$  or velocity of  $A$ ,

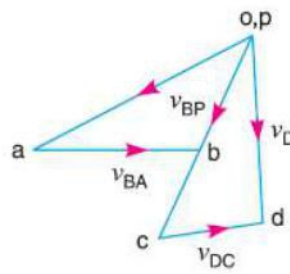
$$\begin{aligned} v_{AO} = v_A &= \omega_{AO} \times OA \\ &= 18.85 \times 0.15 = 2.83 \text{ m/s} \end{aligned}$$

...(Perpendicular to  $OA$ )

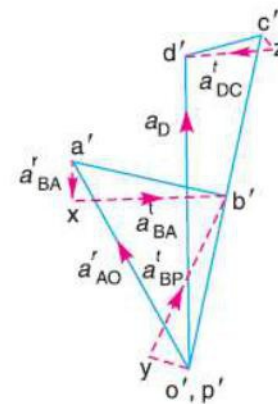
First of all draw the space diagram, to some suitable scale, as shown in Fig. 8.15 (a). Now the velocity diagram, as shown in Fig. 8.15 (b), is drawn as discussed below:



(a) Space diagram.



(b) Velocity diagram.



(c) Acceleration diagram.

Fig. 8.15

1. Since  $O$  and  $P$  are fixed points, therefore these points lie at one place in the velocity diagram. Draw vector  $oa$  perpendicular to  $OA$ , to some suitable scale, to represent the velocity of  $A$  with respect to  $O$  or velocity of  $A$  (i.e.  $v_{AO}$  or  $v_A$ ), such that

$$\text{vector } oa = v_{AO} = v_A = 2.83 \text{ m/s}$$

2. Since the point  $B$  moves with respect to  $A$  and also with respect to  $P$ , therefore draw vector  $ab$  perpendicular to  $AB$  to represent the velocity of  $B$  with respect to  $A$  i.e.  $v_{BA}$ , and from point  $p$  draw vector  $pb$  perpendicular to  $PB$  to represent the velocity of  $B$  with respect to  $P$  or velocity of  $B$  (i.e.  $v_{BP}$  or  $v_B$ ). The vectors  $ab$  and  $pb$  intersect at  $b$ .

3. Since the point  $C$  lies on  $PB$  produced, therefore divide vector  $pb$  at  $c$  in the same ratio as  $C$  divides  $PB$  in the space diagram. In other words,  $pb/pc = PB/PC$ .

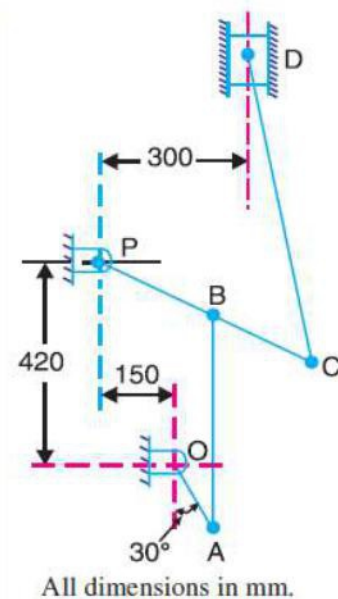


Fig. 8.14

4. From point  $c$ , draw vector  $cd$  perpendicular to  $CD$  to represent the velocity of  $D$  with respect to  $C$  and from point  $o$  draw vector  $od$  parallel to the path of motion of the slider  $D$  (which is vertical), to represent the velocity of  $D$ , i.e.  $v_D$ .

By measurement, we find that velocity of the slider  $D$ ,

$$v_D = \text{vector } od = 2.36 \text{ m/s}$$

Velocity of  $D$  with respect to  $C$ ,

$$v_{DC} = \text{vector } cd = 1.2 \text{ m/s}$$

Velocity of  $B$  with respect to  $A$ ,

$$v_{BA} = \text{vector } ab = 1.8 \text{ m/s}$$

and velocity of  $B$  with respect to  $P$ ,  $v_{BP} = \text{vector } pb = 1.5 \text{ m/s}$

### Acceleration of the slider $D$

We know that radial component of the acceleration of  $A$  with respect to  $O$  or acceleration of  $A$ ,

$$a_{AO}^r = a_A = \omega_{AO}^2 \times AO = (18.85)^2 \times 0.15 = 53.3 \text{ m/s}^2$$

Radial component of the acceleration of  $B$  with respect to  $A$ ,

$$a_{BA}^r = \frac{v_{BA}^2}{AB} = \frac{(1.8)^2}{0.45} = 7.2 \text{ m/s}^2$$

Radial component of the acceleration of  $B$  with respect to  $P$ ,

$$a_{BP}^r = \frac{v_{BP}^2}{PB} = \frac{(1.5)^2}{0.24} = 9.4 \text{ m/s}^2$$

Radial component of the acceleration of  $D$  with respect to  $C$ ,

$$a_{DC}^r = \frac{v_{DC}^2}{CD} = \frac{(1.2)^2}{0.66} = 2.2 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.15 (c), is drawn as discussed below:

1. Since  $O$  and  $P$  are fixed points, therefore these points lie at one place in the acceleration diagram. Draw vector  $o'a'$  parallel to  $OA$ , to some suitable scale, to represent the radial component of the acceleration of  $A$  with respect to  $O$  or the acceleration of  $A$  (i.e.  $a_{AO}^r$  or  $a_A$ ), such that

$$\text{vector } o'a' = a_{AO}^r = a_A = 53.3 \text{ m/s}^2$$

2. From point  $a'$ , draw vector  $a'x$  parallel to  $AB$  to represent the radial component of the acceleration of  $B$  with respect to  $A$  (i.e.  $a_{BA}^r$ ), such that

$$\text{vector } a'x = a_{BA}^r = 7.2 \text{ m/s}^2$$

3. From point  $x$ , draw vector  $xb'$  perpendicular to the vector  $a'x$  or  $AB$  to represent the tangential component of the acceleration of  $B$  with respect to  $A$  i.e.  $a_{BA}^t$  whose magnitude is yet unknown.

4. Now from point  $p'$ , draw vector  $p'y$  parallel to  $PB$  to represent the radial component of the acceleration of  $B$  with respect to  $P$  (i.e.  $a_{BP}^r$ ), such that

$$\text{vector } p'y = a_{BP}^r = 9.4 \text{ m/s}^2$$

5. From point  $y$ , draw vector  $yb'$  perpendicular to vector  $b'y$  or  $PB$  to represent the tangential component of the acceleration of  $B$ , i.e.  $a_{BP}^t$ . The vectors  $xb'$  and  $yb'$  intersect at  $b'$ . Join  $p'b'$ . The vector  $p'b'$  represents the acceleration of  $B$ , i.e.  $a_B$ .

6. Since the point  $C$  lies on  $PB$  produced, therefore divide vector  $p'b'$  at  $c'$  in the same ratio as  $C$  divides  $PB$  in the space diagram. In other words,  $p'b'/p'c' = PB/PC$

7. From point  $c'$ , draw vector  $c'z$  parallel to  $CD$  to represent the radial component of the acceleration of  $D$  with respect to  $C$  i.e.  $a_{DC}^r$ , such that

$$\text{vector } c'z = a_{DC}^r = 2.2 \text{ m/s}^2$$

8. From point  $z$ , draw vector  $zd'$  perpendicular to vector  $c'z$  or  $CD$  to represent the tangential component of the acceleration of  $D$  with respect to  $C$  i.e.  $a_{DC}^t$ , whose magnitude is yet unknown.

9. From point  $o'$ , draw vector  $o'd'$  parallel to the path of motion of  $D$  (which is vertical) to represent the acceleration of  $D$ , i.e.  $a_D$ . The vectors  $zd'$  and  $o'd'$  intersect at  $d'$ . Join  $c'd'$ .

By measurement, we find that acceleration of  $D$ ,

$$a_D = \text{vector } o'd' = 69.6 \text{ m/s}^2 \text{ Ans.}$$

### Angular acceleration of $CD$

From acceleration diagram, we find that tangential component of the acceleration of  $D$  with respect to  $C$ ,

$$a_{DC}^t = \text{vector } zd' = 17.4 \text{ m/s}^2 \quad \dots(\text{By measurement})$$

We know that angular acceleration of  $CD$ ,

$$\alpha_{CD} = \frac{a_{DC}^t}{CD} = \frac{17.4}{0.66} = 26.3 \text{ rad/s}^2 \text{ (Anticlockwise) Ans.}$$