UNIT I

STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

CHAPTER 3

Working Stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

Factor of Safety

It is defined, in general, as the ratio of the maximum stress to the working stress. Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum Stress}}{\text{Working or Design Stress}}$$

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{Yield point Stress}}{\text{Working or Design Stress}}$$

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials.

Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\therefore \text{Factor of safety} = \frac{\text{Ultimate Stress}}{\text{Working or Design Stress}}$$

This relation may also be used for ductile materials.

Note: The above relations for factor of safety are for static loading.

Selection of Factor of Safety

The selection of a proper factor of safety to be used in designing any machine component depends upon a number of considerations, such as the material, mode of manufacture, type of stress, general service conditions and shape of the parts. Before selecting a proper factor of safety, a design engineer should consider the following points:

1. The reliability of the properties of the material and change of these properties during service.

2. The reliability of test results and accuracy of application of these results to actual machine parts.

3. The reliability of applied load.

4. The certainty as to exact mode of failure.

5. The extent of simplifying assumptions.

6. The extent of localised stresses.

7. The extent of initial stresses set up during manufacture.

8. The extent of loss of life if failure occurs and

9. The extent of loss of property if failure occurs.

Each of the above factors must be carefully considered and evaluated. The high factor of safety results in unnecessary risk of failure. The values of factor of safety based on ultimate strength for different materials and type of load are given in the following table:

Table 3.1 Values of factor of safety.

Material	Steady load	Live load	Shock load
Cast iron	5 to 6	8 to 12	16 to 20
Wrought iron	4	7	10 to 15
Steel	4	8	12 to 16
Soft materials and alloys	6	9	15
Leather	9	12	15
Timber	7	10 to 15	20

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 102]

Linear and Lateral Strain

Consider a circular bar of diameter d and length l, subjected to a tensile force P as shown in Fig 3.1

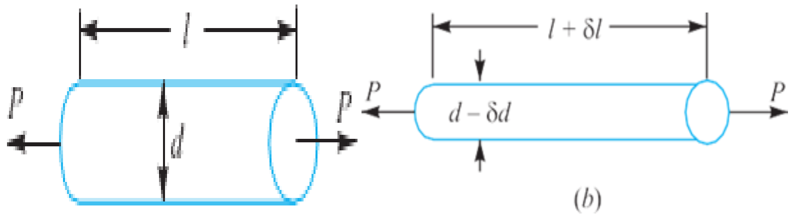

Fig 3.1 Linear and lateral strain

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 111]

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in Fig. 3.1 (b). Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as linear strain and an opposite kind of strain in every direction, at right angles to it, is known as lateral strain.

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{Lateral\ strain}{Linear\ strain} = \text{Constant}$$

This constant is known as Poisson's ratio and is denoted by 1/m or μ.

Following are the values of Poisson's ratio for some of the materials commonly used in engineering practice

Table 3.2 Values of Poisson's ratio for commonly used materials.

S.No.	Material	Poisson's ratio (1/m or μ)
1	Steel	0.25 to 0.33
2	Cast iron	0.23 to 0.27
3	Copper	0.31 to 0.34
4	Brass	0.32 to 0.42
5	Aluminium	0.32 to 0.36
6	Concrete	0.08 to 0.18
7	Rubber	0.45 to 0.50

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 111]

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as volumetric strain. Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

where δV = Change in volume, and V = Original volume.

Notes: 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta V}{V}$$

$$\varepsilon_v = \varepsilon \left(1 - \frac{2}{m}\right) \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions x-axis, y-axis and z-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as bulk modulus. It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{\text{Direct Stress}}{\text{Volumetric Strain}} = \frac{\sigma}{\delta V / V}$$

Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3(m-2)} = \frac{E}{3(1-2\mu)}$$

Relation Between Young's Modulus and Modulus of Rigidity

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2(m+1)} = \frac{E}{2(1+\mu)}$$

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as impact stress.

Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig. 3.2

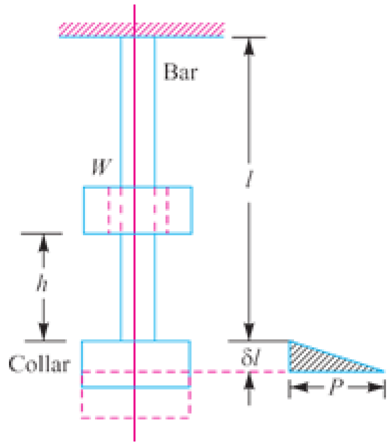

Fig. 3.2 Impact Stress

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 114]

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy = $\frac{1}{2} \times P \times \delta l$

and potential energy lost by the weight = W (h + δl)

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W (h + \delta l)$$

$$\frac{1}{2} \times \sigma_i \times A \times \frac{l \times \sigma_i}{E} = W \left(h + \frac{l \times \sigma_i}{E}\right)$$

$$\frac{Al}{2E}(\alpha_i)^2 - \frac{Wl}{E}(\alpha_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1+\sqrt{1+\frac{2hEA}{Wl}}\right) \qquad \text{... [Taking +ve sign for maximum value]}$$

Note: When h = 0, then σi = 2W/A. This means that the stress in the bar when the load in applied suddenly is double of the stress induced due to gradually applied load.

Problem 3.1

An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm^2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm^2.

Given Data:

h = 10 mm

l = 3 m = 3000 mm

A = 600 mm^2

δl = 2 mm

E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

$$\sigma = \frac{E\delta l}{l} = \frac{2\times200\times10^3}{3000}$$

$$= \frac{400}{3}$$

$$\sigma = 133.33 \text{ N/mm}^2$$

Value of the unknown weight

Let W = Value of the unknown weight.

$$\sigma = \frac{W}{A}\left(1+\sqrt{1+\frac{2hEA}{Wl}}\right)$$

$$\frac{400}{3} = \frac{W}{600}\left(1+\sqrt{1+\frac{2\times10\times600\times200\times10^3}{W\times3000}}\right)$$

$$\frac{400\times600}{3\times W} = 1+\sqrt{1+\frac{800000}{W}}$$

$$\frac{80000}{3\times W} - 1 = \sqrt{1+\frac{800000}{W}}$$

Squaring both sides,

$$\frac{6400\times10^6}{W^2} + 1 - \frac{16000}{W} = 1 + \frac{80000}{W}$$

$$\frac{6400\times10^6}{W^2} - 16 = 80\ \frac{6400\times10^6}{W^2} = 96$$

$$W = \frac{6400\times10^6}{96}$$

W = 6666.7 N

Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as strain energy. The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as resilience and the maximum energy which can be stored in a body up to the elastic limit is called proof resilience. The proof resilience per unit volume of a material is known as modulus of resilience. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,

$$U = \frac{V \times \sigma^2}{2E}$$

and Modulus of resilience = $\frac{\sigma^2}{2E}$

where σ = Tensile or compressive stress,

V = Volume of the body, and

E = Young's modulus of the material of the body.

Notes: 1. When a body is subjected to a shear load, then modulus of resilience (shear)

$$= \frac{\tau^2}{2C}$$

τ = Shear stress, and

C = Modulus of rigidity.

2. When the body is subjected to torsion, then modulus of resilience

$$= \frac{\tau^2}{4C}$$

Torsional and Bending Stresses in Machine Parts

Introduction

Sometimes machine parts are subjected to pure torsion or bending or combination of both torsion and bending stresses. We shall now discuss these stresses in detail in the following pages.

Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to torsion. The stress set up by torsion is known as torsional shear stress. It is zero at the centroidal axis and maximum at the outer surface.

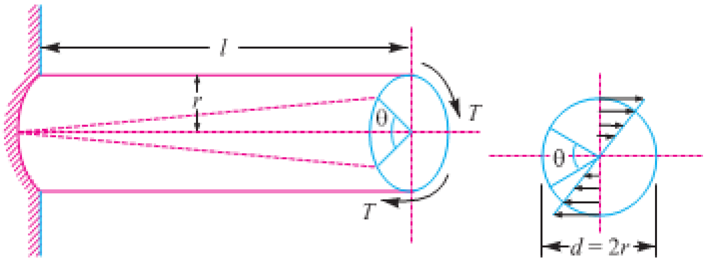

Fig 3.3 Torsional shear stress.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 121]

Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Fig. 4.1. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C.\theta}{l}$$

where

τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l.

The equation (i) is known as torsion equation. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.

2. The twist along the length of the shaft is uniform.

3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Notes:

1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2.From equation (i), we know that

$$\frac{\tau}{r} = \frac{T}{J}$$

$$T = \frac{\tau}{r} \times J$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY}$$

$$= \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4$$

$$= \frac{\pi}{32} \times d^4$$

$$T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d}$$

$$T = \frac{\pi}{16} \times d^3$$

In case of a hollow shaft with external diameter (do) and internal diameter (di), the polar moment of inertia,

$$J = \frac{\pi}{32} \times ((d_o)^2 - (d_i)^2) \qquad \text{and } r = d_o/2$$

$$T = \tau \times \frac{\pi}{32} \times ((d_o)^2 - (d_i)^2) \times \frac{2}{d_o}$$

$$T = \tau \times \frac{\pi}{16} \times \frac{(d_o)^2 - (d_i)^2}{d_o}$$

$$T = \frac{\pi}{16} \times \tau \times (d_o)^3(1-k^4) \qquad \text{... Substituting, } k = \frac{d_i}{d_o}$$

3.The expression (C × J) is called torsional rigidity of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\pi NT}{60} = T.\ \omega \qquad \text{...........} \qquad (\omega = \frac{2\pi N}{60})$$

where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Problem 3.2

A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Given Data:

$P = 100\ kW = 100 \times 10^3\ W$

$N = 160$ r.p.m

$T_{max} = 1.25\ T_{mean}$

$\tau = 70\ MPa = 70\ N/mm^2$

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$P = \frac{2\pi N T_{mean}}{60}$$

$$100 \times 10^3 = \frac{2\pi \times 160 \times T_{mean}}{60}$$

$$100 \times 10^3 = 16.76 T_{mean}$$

$$T_{mean} = \frac{100 \times 10^3}{16.67}$$

$$T_{mean} = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6$$

$$= 7458 \text{ N-m}$$

$$= 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3$$

$$= \frac{\pi}{16} \times 70 \times d^3$$

$$= 13.75\, d^3$$

$$d^3 = 7458 \times 10^3 / 13.75$$

$$= 524.3 \times 10^3$$

$$d = 81.5 \text{ mm}$$