UNIT I

STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

CHAPTER 5

Eccentric Loading - Direct and Bending Stresses Combined

An external load, whose line of action is parallel but does not coincide with the centroidal axis of the machine component, is known as an eccentric load. The distance between the centroidal axis of the machine component and the eccentric load is called eccentricity and is generally denoted by e. The examples of eccentric loading, from the subject point of view, are C-clamps, punching machines, brackets, offset connecting links etc.

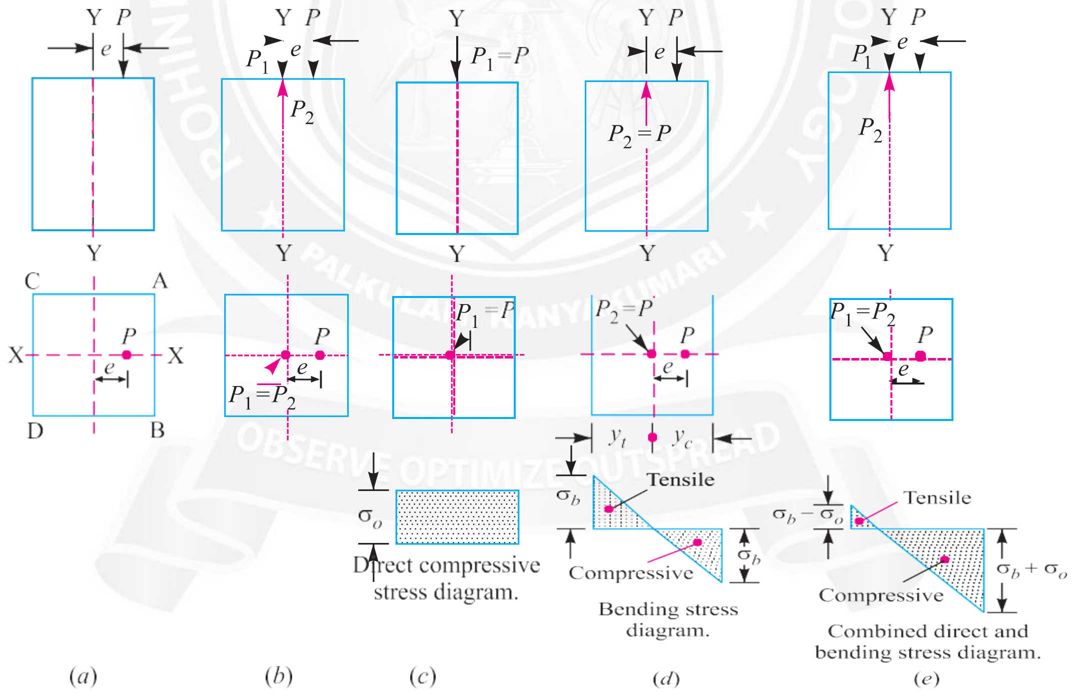


Fig 5.1 Eccentric concentration.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 160]

Consider a short prismatic bar subjected to a compressive load P acting at an eccentricity of e as shown in Fig. 5.1 (a). Let us introduce two forces P_1 and P_2 along the centre line or neutral axis equal in magnitude to P, without altering the equilibrium of the bar as shown in Fig. 5.1 (b). A little consideration will show that the force P1 will induce a direct compressive stress over the entire cross-section of the bar, as shown in Fig. 5.1 (c). The magnitude of this direct compressive stress is given by

$$\sigma_o = \frac{P_1}{A} = \frac{P}{A}$$ …... where A is the cross-sectional area of the bar.

The forces P_1 and P_2 will form a couple equal to P × e which will cause bending stress. This bending stress is compressive at the edge AB and tensile at the edge CD, as shown in Fig. 5.1 (d). The magnitude of bending stress at the edge AB is given by

$$\sigma_b = \frac{P.e.y_c}{I} \text{ (compressive)}$$

and bending stress at the edge CD,

$$\sigma_b = \frac{P.e.y_t}{I} \text{ (tensile)}$$

where,

y_c and y_t = Distances of the extreme fibres on the compressive and tensile sides, from the neutral axis respectively, and

I = Second moment of area of the section about the neutral axis i.e. Y-axis.

According to the principle of superposition, the maximum or the resultant compressive stress at the edge AB,

$$\sigma_c = \frac{P.e.y_c}{I} + \frac{P}{A} = \frac{M}{Z} + \frac{P}{A} = \sigma_b + \sigma_o$$

and the maximum or resultant tensile stress at the edge CD,

$$\sigma_c = \frac{P.e.y_c}{I} - \frac{P}{A} = \frac{M}{Z} - \frac{P}{A} = \sigma_b - \sigma_o$$

The resultant compressive and tensile stress diagram is shown in Fig. 5.1 (e).

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhoods of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration. It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a tensile load as shown in Fig. 5.2.

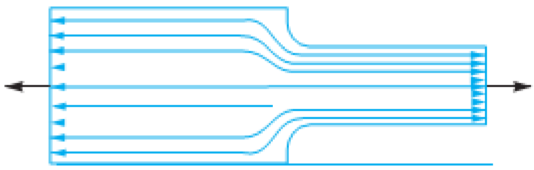


Fig 5.2 Stress Concentration

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 187]

A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area.

Mathematically, theoretical or form stress concentration factor

$$K = \frac{\text{Maximum Stress}}{\text{Nominal Stress}}$$

The value of K_t depends upon the material and geometry of the part.

Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig 5.3 (a). We see from the stress-distribution that the stress at the point away from the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\alpha_{max} = \alpha \left(1 + \frac{2a}{b}\right)$$

and the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} \left(1 + \frac{2a}{b}\right)$$

When a/b is large, the ellipse approaches a crack transverse to the load and the value of K_t becomes very large. When a/b is small, the ellipse approaches a longitudinal slit [as shown in Fig. 5.3 (b)] and the increase in stress is small. When the hole is circular as shown in Fig. 5.3 (c), then a/b = 1 and the maximum stress is three times the nominal value.

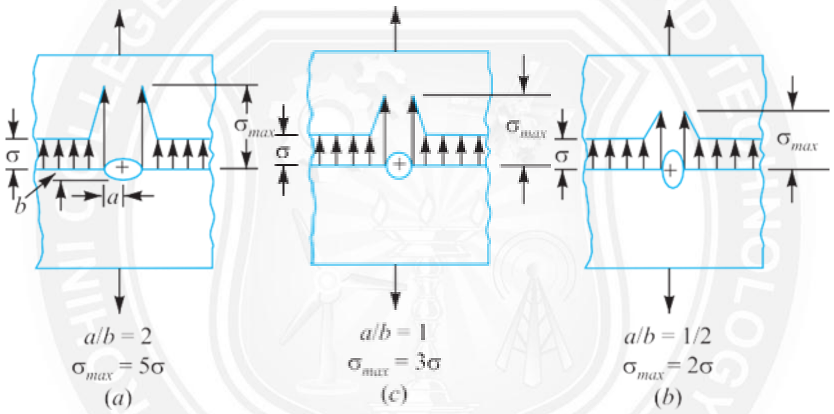


Fig 5.3 Stress concentration due to holes.

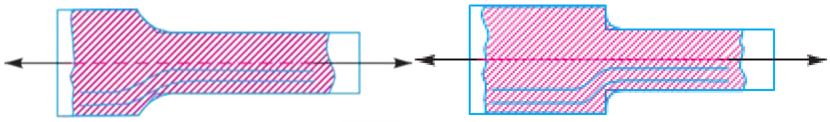
[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 188]

The stress concentration in the notched tension member, as shown in Fig. 5.3, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the width of the plate, may be obtained by the following equation,

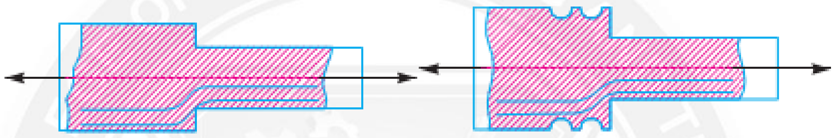
$$\alpha_{max} = \alpha \left(1 + \frac{2a}{r}\right)$$

Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration cannot be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig. 5.4 The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.



(a) Good (b) Poor



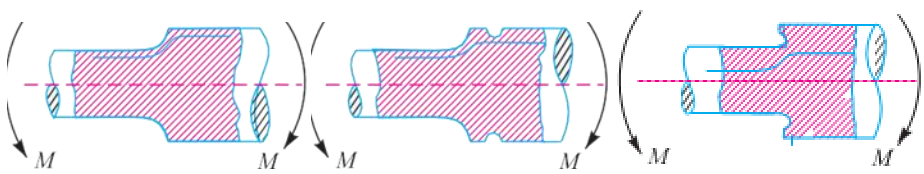
(c) Preferred (d) Preferred

Fig 5.4

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 189]

In Fig. 5.4 (a) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 5.4 (b) and (c) to give more equally spaced flow lines.

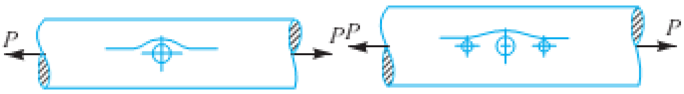
Figs. 5.5 to 5.7 show the several ways of reducing the stress concentration in shafts and other cylindrical members with shoulders, holes and threads respectively. It may be noted that it is not practicable to use large radius fillets as in case of ball and roller bearing mountings. In such cases, notches may be cut as shown in Fig. 5.4 (d) and Fig. 5.5 (b) and (c).



(a) Poor (b) Good (c) Preferred

Fig 5.5 Methods of reducing stress concentration in cylindrical members with shoulders.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 189]



(a) Poor (b) Preferred

Fig 5.6 Methods of reducing stress concentration in cylindrical members with holes.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 189]

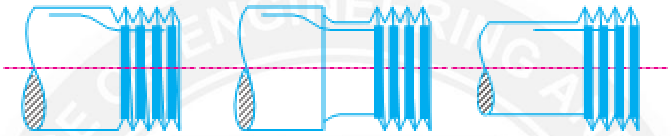


Fig 5.7 Methods of reducing stress concentration in cylindrical members with holes.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 189]

Factors to be Considered while Designing Machine Parts to Avoid Fatigue Failure

The following factors should be considered while designing machine parts to avoid fatigue failure:

1. The variation in the size of the component should be as gradual as possible.

2. The holes, notches and other stress raisers should be avoided.

3. The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.

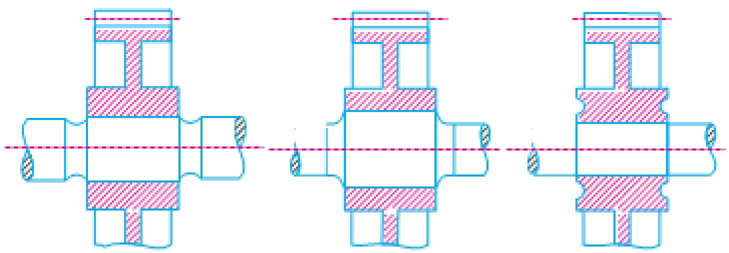


Fig 5.8 Methods of reducing stress concentration of a press fit.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 190]

4. The parts should be protected from corrosive atmosphere.

5. A smooth finish of outer surface of the component increases the fatigue life.

6. The material with high fatigue strength should be selected.

7. The residual compressive stresses over the parts surface increases its fatigue strength.

Problem 5.1

Find the maximum stress induced in the following cases taking stress concentration into account: 1. A rectangular plate 60 mm × 10 mm with a hole 12 diameter as shown in Fig. 5.9 (a) and subjected to a tensile load of 12 kN. 2. A stepped shaft as shown in Fig. 5.9 (b) and carrying a tensile load of 12 kN.

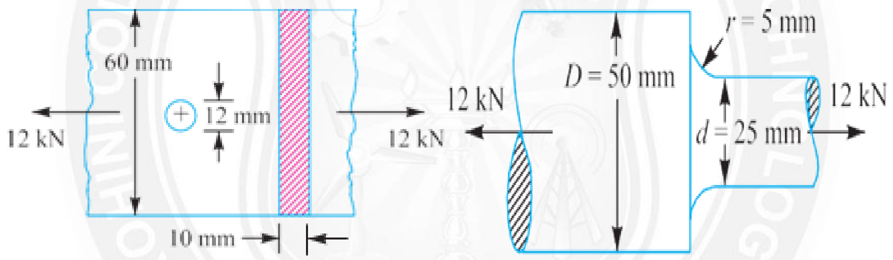


Fig 5.9

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 194]

Given Data:

Case 1:

$b = 60$ mm

$t = 10$ mm

$d = 12$ mm

$W = 12$ kN $= 12 \times 10^3$ N

We know that cross-sectional area of the plate,

$A = (b - d)\, t = (60 - 12)\, 10 = 480$ mm^2

$$\text{Nominal Stress} = \frac{W}{A} = \frac{12 \times 10^3}{480}$$

$$= 25 \text{ N/mm}^2$$

Nominal Stress = 25 MPa

Ratio of diameter of hole to width of plate,

$$\frac{d}{b} = \frac{12}{60} = 0.2$$

From Table

we find that for d / b = 0.2, theoretical stress concentration factor,

$K_t = 2.5$

∴ Maximum stress = K_t × Nominal stress

= 2.5 × 25

∴ Maximum stress = 62.5 MPa

Case 2:

D = 50 mm

d = 25 mm

r = 5 mm

W = 12 kN = 12×10^3 N

We know that cross-sectional area for the stepped shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 25^2$$

$A = 491\ mm^2$

$$\text{Nominal Stress} = \frac{W}{A} = \frac{12\times10^3}{491}$$

$= 24.4\ N/mm^2$

Nominal Stress = 24.4 MPa

Ratio of maximum diameter to minimum diameter,

D/d = 50/25 = 2

Ratio of radius of fillet to minimum diameter, r/d = 5/25 = 0.2

From Table

we find that for D/d = 2 and r/d = 0.2, theoretical stress concentration factor,

$K_t = 1.64.$

∴ Maximum stress = Kt × Nominal stress

= 1.64 × 24.4

∴ Maximum stress = 40 MPa

Completely Reversed or Cyclic Stresses

Consider a rotating beam of circular cross-section and carrying a load W, as shown in Fig. 5.10. This load induces stresses in the beam which are cyclic in nature. A little consideration will show that the upper fibres of the beam (i.e. at point A) are under compressive stress and the lower fibres (i.e. at point B) are under tensile stress. After half a revolution, the point B occupies the position of point A and the point A occupies the position of point B. Thus the point B is now under compressive stress and the point A under tensile stress. The speed of variation of these stresses depends upon the speed of the beam. From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or vice versa, are known as completely reversed or cyclic stresses.

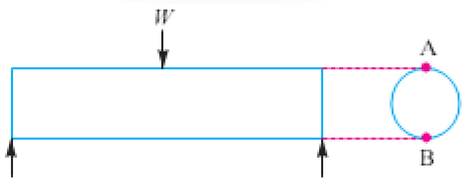
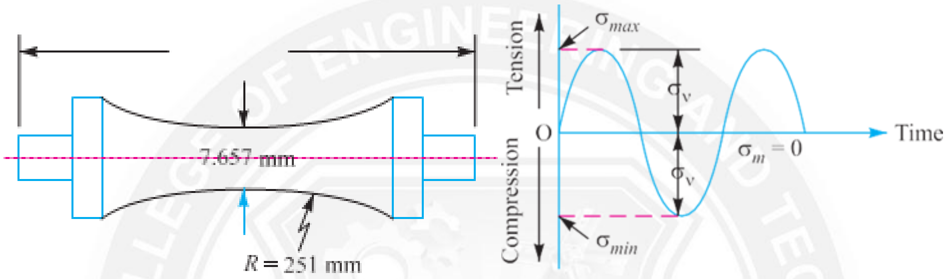


Fig 5.10 Reversed or cyclic stresses.

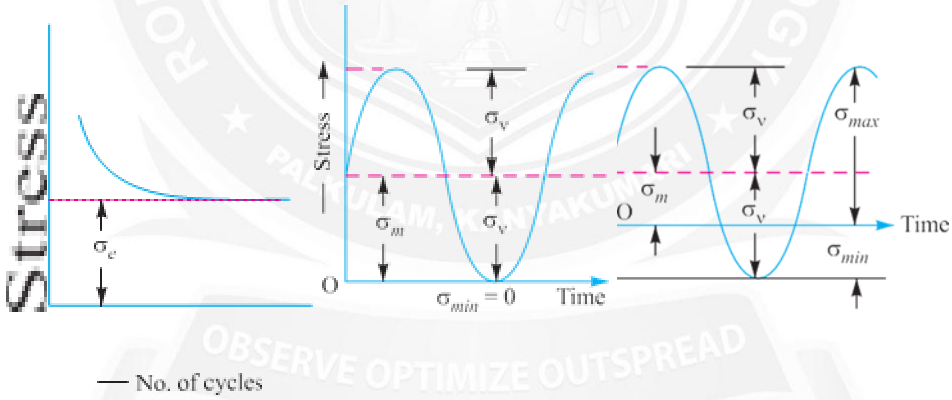
[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 182]

<u>Fatigue and Endurance Limit</u>

It has been found experimentally that when a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any prior indication. The fatigue of material is effected by the size of the component, relative magnitude of static and fluctuating loads and the number of load reversals.



(a) Standard Specimen (b) Completely Revered Stress



(c) Endurance or Fatigue limit (d) Repeated Stress (e) Fluctuating Stresses

Fig 5.11 Time-stress diagrams

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 182]

In order to study the effect of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig. 5.11 (a), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the specimen rotates, the bending stress at the upper fibres varies from maximum compressive to maximum tensile while the bending stress at the lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig. 5.11 (b). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in Fig. 5.11 (c). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig. 6.2 (c), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line, is known as endurance or fatigue limit (σ_e). It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can withstand without failure, for infinite number of cycles (usually 107 cycles). It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term endurance strength may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig. 5.11 (b). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress verses time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig. 5.11 (e). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ_v. The following relations are derived from Fig. 5.11 (e):

1. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

2. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

where

σ'_e = Endurance limit for any stress range represented by R.

σ_e = Endurance limit for completely reversed stresses, and

R = Stress ratio.

Effect of Loading on Endurance Limit—Load Factor

The endurance limit (σ_e) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_s = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

$\therefore$ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e . Kb = \sigma_e$ ... $(Kb = 1)$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e . K_a$

and

endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e . Ks$

Effect of Surface Finish on Endurance Limit—Surface Finish Factor

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. Fig. 5.12 shows the values of surface finish factor for the various surface conditions and ultimate tensile strength.

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition.

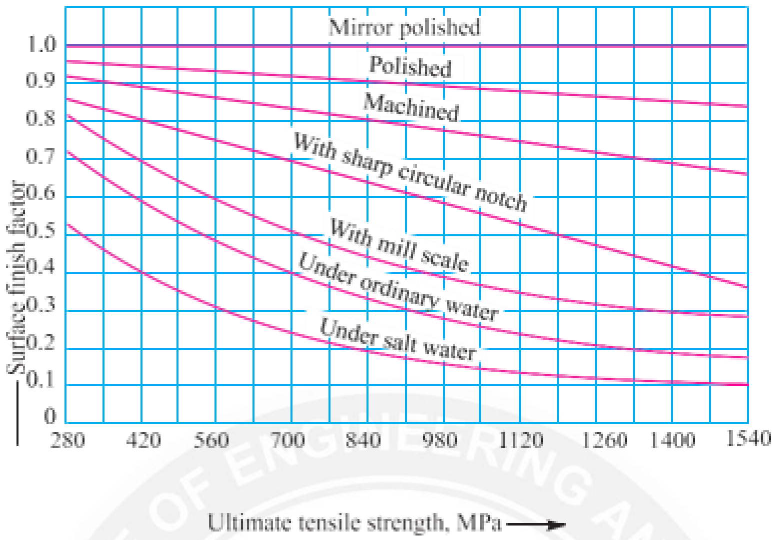


Fig 5.12 Surface finish factor for various surface conditions.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 184]

Let K_{sur} = Surface finish factor.

∴ Endurance limit,

$\sigma_{e1} = \sigma_{eb}.K_{sur}$

$= \sigma_e.K_b.K_{sur}$

$= \sigma_e.K_{sur}$ ...($K_b = 1$)

... (For reversed bending load)

$= \sigma_{ea}.K_{sur}$

$= \sigma_e.K_a.K_{sur}$ ...(For reversed axial load)

$= \tau_e.K_{sur}$

$= \sigma_e.K_s.K_{sur}$ ...(For reversed torsional or shear load)

The surface finish factor for non-ferrous metals may be taken as unity.

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig. 5.11 (a) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one.

Let K_{sz} = Size factor.

∴ Endurance limit,

$\sigma_{e2} = \sigma_{e1} \times K_{sz}$ ... (Considering surface finish factor also)

$= \sigma_{eb}.K_{sur}.K_{sz}$

$= \sigma_{e}.K_{b}.K_{sur}.K_{sz}$

$= \sigma_{e}.K_{sur}.K_{sz}$ $(K_b = 1)$

$= \sigma_{ea}.K_{sur}.K_{sz}$

$= \sigma_{e}.K_{a}.K_{sur}.K_{sz}$ ...(For reversed axial load)

$= \tau_{e}.K_{sur}.K_{sz}$

$= \sigma_{e}.K_{s}.K_{sur}.K_{sz}$ ... (For reversed torsional or shear load)

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions:

1. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

2. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

3. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation Between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u). Fig. 5.13 shows the endurance limit of steel corresponding to ultimate tensile strength for different surface conditions. Following are some empirical relations commonly used in practice:

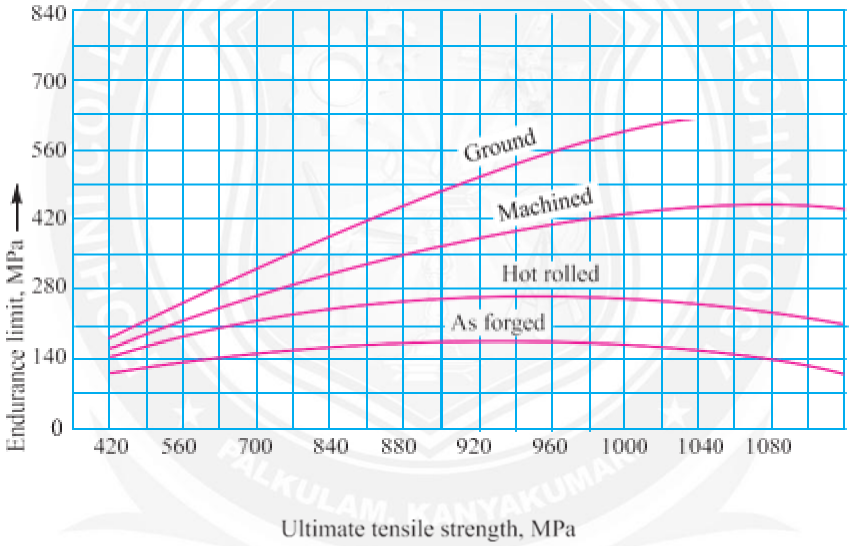


Fig 5.13 Endurance limit of steel corresponding to ultimate tensile strength.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 186]

For steel, $\sigma_e = 0.5\ \sigma_u$

For cast steel, $\sigma_e = 0.4\ \sigma_u$

For cast iron, $\sigma_e = 0.35\ \sigma_u$

For non-ferrous metals and alloys, $\sigma_e = 0.3\ \sigma_u$

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety (F.S.)} = \frac{\text{Endurance Limit Stress}}{\text{Design or Working Stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8$ to $0.9\ \sigma_y$

where σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term notch sensitivity is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (q) is not available, therefore the curves, as shown in Fig. 5.14, may be used for determining the values of q for two steels.

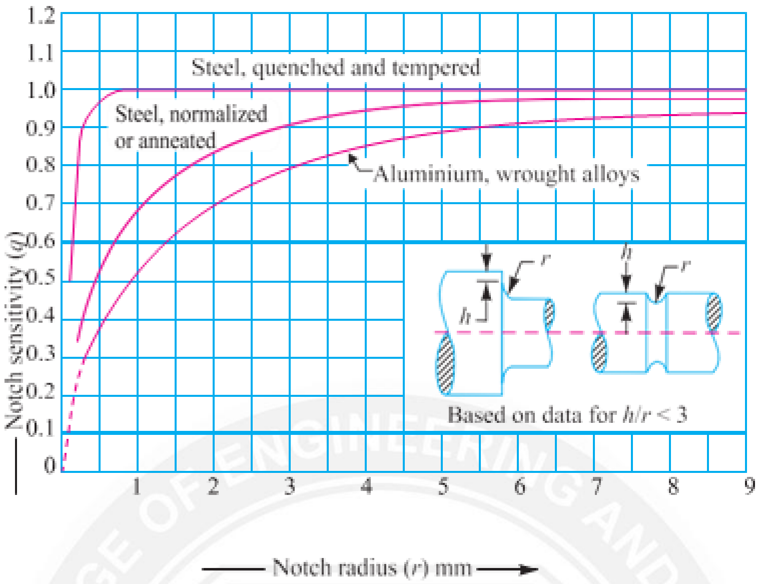


Fig 5.14 Notch sensitivity.

[Source: "A Textbook of Machine Design" by R.S. Khurmi & J.K. Gupta, Page: 195]

When the notch sensitivity factor q is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

$K_f = 1 + q\,(K_t - 1)$ ...[For tensile or bending stress]

and $K_{fs} = 1 + q\,(K_{ts} - 1)$ ...[For shear stress]

where K_t = Theoretical stress concentration factor for axial or bending loading, and

K_{ts} = Theoretical stress concentration factor for torsional or shear loading.