







Low Frequency circuit can be described by two port networks and their parameters such as Z,Y,H,ABCD etc. as per network theory. Here network parameters relate the total voltages and total currents as shown in fig. 1.3.In similar way at microwave frequencies, we talk of travelling waves with associated powers instead of voltages and current and the microwave junction can be defined by what are called as S-parameters or scattering parameters(similar to H, Y, Z parameter).

Referring to fig.1.4, it can be seen that for an input at one port, we have four outputs. Similarly if we apply inputs to all the ports, we have 16 combinations, which are represented in matrix form and that matrix is called as SCATTERING MATRIX. It is a square matrix which gives all the combinations of power relationships between the various input and output port of a microwave junction. The elements of this matrix are called scattering coefficients or Scattering (S) parameters. To obtain the relationship between the scattering matrix and the input/output powers at different ports, Consider a junction of "n" number is terminated in a source as shown in fig. 1.5.



Figure 1.5 Four port waveguide with matched termination

• S-parameters, scattering refers to the way in which the traveling currents and voltages in a transmission line are affected when they meet a discontinuity caused by the insertion of a network into the transmission line. This is equivalent to the wave meeting an impedance differing from the line's characteristic impedance



For s-parameters, the definition is:

$$S_{11} = \frac{b_1}{a_1} = \frac{V_1^-}{V_1^+}$$

$$S_{21} = \frac{b_2}{a_1} = \frac{V_2^-}{V_1^+}$$

$$S_{12} = \frac{b_1}{a_2} = \frac{V_1^-}{V_2^+}$$

$$S_{22} = \frac{b_2}{a_2} = \frac{V_2^-}{V_2^+}$$

# **Scattering Matrix Formulation**

To obtain the relationship between the scattering matrix and the input/output powers at different ports consider a junction of n number of transmission lines wherein the ith line (i can be any line from 1 to n) is terminated in a source as shown in fig 1.7

Case 1: Let the first line be terminated in an impedance other than the characteristic impedance(i.e  $Z_L Z_0$ )





Figure 1.7 Microwave Junction of n-number of lines

- If ai is the incident wave, it divides among the n-1 number of lines as a1,a2 .....an
- No reflections from 2<sup>nd</sup> to n<sup>th</sup> line
- The incident waves are absorbed since their impedance is equal to the characteristic impedance
- 1<sup>st</sup> line mismatch- wave reflected back to b1
  - b1 related to a1 by
  - b1= (reflection co -efficient) a1

=Si1. al.....(1)

Where Si1 = reflection co efficient of 1 <sup>st</sup> line

1<sup>^</sup> reflection from 1<sup>st</sup> line

source connected at ith line

Hence the contribution to the outward travelling wave in the i<sup>th</sup> line is given by

[ b2 =b3 = .....=bn=0]

bi = Sil. a1 .....(2)

### Case 2: Let all the (n-1) lines be terminated in an impedance other than $Z_0$ (i.e. $Z_L Z_0$ )

Then there will be reflections into the junction from every line and hence the total contribution to the outward travelling wave in the i<sup>th</sup> line is given by

bi = 1 to n since I can be any line from 1 to n

Therefore, we have

$$bl -$$
•S'llCti + «S'12-1-....+-S'lTi<2.7i

$$b-2 = S'ai^{TZ}l + S-2-2.CL-2. + \langle S'23 \langle 3 \rangle + \dots \wedge InO-n$$

 $b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \dots + S_{nn}a_n$ 



When this whole thing is kept in a matrix form,

The column matrix [b] corresponds to the reflected waves or the output, while the matrix [a] corresponds to the incident waves or the input. The scattering column matrix [s] which is of the order of n n contains the reflection coefficients and transmission coefficients. Therefore,

## [b]=[S][a]

## **Properties of [S] Matrix**

The scattering matrix is indicated as [S] matrix. There are few standard properties for [S] matrix. They are -

• [S] is always a square matrix of order nxn

[S]nxn

• [S] is a symmetric matrix

i.e., Sij=Sji

• [S] is a unitary matrix

i.e., [S][S]=I

• The sum of the products of each term of any row or column multiplied by the complex

conjugate of the corresponding terms of any other row or column is zero. i.e.,

$$\sum_{i=j}^n S_{ik}S^*_{ik} = 0 \ for \ k 
eq j$$

$$(k = 1, 2, 3, \dots n)$$
 and  $(j = 1, 2, 3, \dots n)$ 

If the electrical distance between some k<sup>th</sup> port and the junction is pklk, then the coefficients of Sij involving k, will be multiplied by the factor e-jpkIk

#### **1.4 MICROWAVE TEE JUNCTIONS**

### **E-plane** Tee

An E-Plane Tee junction is formed by attaching a simple waveguide to the broader dimension of a rectangular waveguide, which already has two ports. The arms of rectangular waveguides make two ports called **collinear ports** i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or **E-arm**. T his E-plane Tee is also called as **Series Tee**. As the axis of the side arm is parallel to the electric field, this junction is called E-Plane Tee junction. This is also called as **Voltage** or **Series junction**. The ports 1 and 2 are 180° out of phase with each other. The cross-sectional details of E-plane tee can be understood by the following fig 1.8





Figure 1.8 Microwave E plane Tee junctions

The connection made by the sidearm to the bi-directional waveguide to form the parallel port is shown in fig 1.9



Figure 1.9 E plane bidirectional waveguide

# **Properties of E-Plane Tee**

The properties of E-Plane Tee can be defined by its [S]3x3 matrix.

It is a 3 \*3 matrix as there are 3 possible inputs and 3 possible outputs.

$$\begin{bmatrix} S \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 513 \\ S_{21} & S_{22} & 523 \\ S_{31} & S_{32} & 5s3 \end{bmatrix}$$

Scattering coefficients S13 and S23 are out of phase by 180° with an input at port 3.

$$S_{23} = -S_{13}$$

The port is perfectly matched to the junction.

$$S_{33} = 0$$

From the symmetric property,

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21} \ S_{23} = S_{32} \ S_{13} = S_{31}$$

Considering equations 3 & 4, the [S] matrix can be written

We can say that we have four unknowns, considering the symmetry property. From the Unitary property

 $R_SC_3$ :  $|S_{13}|^2 + IS13I^2 = 1$ 

Multiplying the rows and columns we get,

# $R_1C_1:S_{11}S_{11}^*+S_{12}S_{12}^*+S_{13}S_{13}^*=1$

Equating the equations 6 & 7, we get

$$S_{13} \left(S_{11}^* - S_{12}^*\right)$$

$$S_{11} = S_{22}$$
 Or  $S_{11} = S_{12} = S_{22}$ 

Using the equations 10, 11, and 12 in the equation 6,

$$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$$

Substituting the values from the above equations in [S] matrix,

$$[S] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

We know that [b]= [S][a]



This is the scattering matrix for E-Plane Tee, which explains its scattering properties.

## **H PLANE TEE**

An H-Plane Tee junction is formed by attaching a simple waveguide to a rectangular waveguide which already has two ports. The arms of rectangular waveguides make two ports called **collinear ports** i.e., Port1 and Port2, while the new one, Port3 is called as Side arm or **Harm**. This H-plane Tee is also called as **Shunt Tee**.

As the axis of the side arm is parallel to the magnetic field, this junction is called H- Plane Tee junction. This is also called as **Current junction**, as the magnetic field divides itself into arms. The cross-sectional details of H-plane tee can be understood by the figure 1.20



The following figure 1.21 shows the connection made by the sidearm to the bi-directional waveguide to form the serial port.





# **Properties of H-Plane Tee**

The properties of H-Plane Tee can be defined by its [S]3\*3 matrix. It is a 3\*3 matrix as there are 3 possible inputs and 3 possible outputs.

$$\begin{bmatrix} S \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & ^{23} \\ S_{31} & S_{32} & ^{33} \end{bmatrix}$$

Scattering coefficients S13 and S23 are equal here as the junction is symmetrical in plane. From the symmetric property,

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21} \ S_{23} = S_{32} = S_{13} \ S_{13} = S_{31}$$

Now, the [5] matrix can be written as,

$$[S] = \begin{bmatrix} 3 \\ 512 \\ 512 \\ 512 \\ 513 \\ 513 \\ 0 \end{bmatrix}$$

We can say that we have four unknowns, considering the symmetry property. From the Unitary property

$$\begin{bmatrix} 5 \end{bmatrix} \begin{bmatrix} 5 \end{bmatrix}^* = \begin{bmatrix} J \end{bmatrix}$$

$$\begin{vmatrix} 5_n & 512 & 513' \\ 512 & 522 & 513 \\ .5_{15} & 513 & 0 \end{vmatrix} \begin{vmatrix} Q_{11}^* & Q_{12}^* & Q_{13}^* \\ C_{11}^* & C_{12}^* & C_{13}^* \\ C_{12}^* & Q_{13}^* & Q_{13}^* \\ C_{13}^* & C_{13}^* & C_{13}^* \\ C_{13}^* & C$$

Multiplying we get,

$$RIC$$
, : SHSJ'J +  $S_{12}S^*u$  + SI<sub>3</sub>5;<sub>3</sub> = 1

 $|Sn|^2 + |5i21^2 + |S_{13}|^2 = 1$ 

$$R_2C_2: |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$$

$$\begin{split} R_{3}C_{3}: |S_{13}|^{2} + |S_{13}|^{2} &= 1 \\ R_{3}C_{1}: S_{13}S_{11}^{*} - S_{13}S_{12}^{*} &= 0 \\ 2|S_{13}|^{2} &= 1 \quad or \quad S_{13} = \frac{1}{\sqrt{2}} \\ |S11|^{2} &= 1^{\wedge}22 \\ 5n &= S22 \\ S_{13}(S_{11}^{*} + S_{12}^{*}) &= 0 \\ S_{13} &\neq 0, S_{11}^{*} + S_{12}^{*} &= 0, \text{ or } S_{11}^{*} &= -S_{12}^{*} \\ Sn &= -\$12 \text{ or } S12 &= -s_{n} \\ \\ |S_{11}|^{2} + |S_{11}|^{2} + \frac{1}{2} &= 1 \quad or \quad 2|S_{11}|^{2} &= \frac{1}{2} \quad or \quad S_{11} &= \frac{1}{2} \\ S_{12} &= -\frac{1}{2} \\ S_{22} &= \frac{1}{2} \end{split}$$

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Substituting the coefficients we get

$$[S] = egin{bmatrix} rac{1}{2} & -rac{1}{2} & rac{1}{\sqrt{2}} \ -rac{1}{2} & rac{1}{2} & rac{1}{\sqrt{2}} \ rac{1}{\sqrt{2}} & rac{1}{\sqrt{2}} & 0 \ rac{1}{\sqrt{2}} & rac{1}{\sqrt{2}} & 0 \ \end{bmatrix}$$

We know that [6] = [s] [a]

This is the scattering matrix for H-Plane Tee, which explains its scattering properties.

## MAGIC TEE

An E-H Plane Tee junction is formed by attaching two simple waveguides one parallel and the other series, to a rectangular waveguide which already has two ports. This is also called as **Magic Tee**, or **Hybrid** or **3dB coupler**.

The arms of rectangular waveguides make two ports called **collinear ports** i.e., Port 1 and Port 2, while the Port 3 is called as **H-Arm** or **Sum port** or **Parallel port**. Port 4 is called as **EArm** or **Difference port** or **Series port**.

The cross-sectional details of Magic Tee can be understood by the following figure 1.22



Figure 1.22 Magic Tee

The following figure 1.23 shows the connection made by the side arms to the bi-directional waveguide to form both parallel and serial ports.





Figure 1.23 Bi-directional Magic Tee waveguide

# **Characteristics of E-H Plane Tee**

- If a signal of equal phase and magnitude is sent to port 1 and port 2, then the output at port 4 is zero and the output at port 3 will be the additive of both the ports 1 and 2.
- If a signal is sent to port 4, E-arm then the power is divided between port 1 and 2 equally but in opposite phase, while there would be no output at port 3. Hence, S34 = 0.
- If a signal is fed at port 3, then the power is divided between port 1 and 2 equally, while there would be no output at port 4. Hence, S43 = 0.
- If a signal is fed at one of the collinear ports, then there appears no output at the other collinear port, as the E-arm produces a phase delay and the H-arm produces a phase advance. So, S12 = S21 = 0.

#### **Properties of E-H Plane Tee**

The properties of E-H Plane Tee can be defined by its [S]4x4 matrix.

It is a 4x4 matrix as there are 4 possible inputs and 4 possible outputs.

[S] =	Sil	S12	S13	S14
	Sai	S22	S23	S24
	S₃i	S32	S33	S34
	S₄i	S42	S43	S44

As it has H-Plane Tee section

$$S_{23} = S_{13}$$

As it has E-Plane Tee section

$$S_{24} = -S_{14}$$

The E-Arm port and H-Arm port are so isolated that the other won't deliver an output, if an input is applied at one of them. Hence, this can be noted as

$$S_{34} = S_{43} = 0$$

From the symmetry property, we have

$$S_{ij} = S_{ji}$$

$$5_{12} = S_{21J}S_{13} = S_{31T}S_{14} = S_{41}$$

 $^{23} = <532, S24 = S_{42?}S_{34} = S_{43}$ 

If the ports 3 and 4 are perfectly matched to the junction, then

 $S_{33} = S_{44} = 0$ 

Substituting all the above equations in equation 1, to obtain the [S] matrix,



$$\begin{split} |S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} &= 1 \\ |S_{11}|^2 + |S_{12}|^2 &= 0 \\ S_{11} &= S_{22} &= 0 \end{split}$$

Now we understand that ports 1 and 2 are perfectly matched to the junction. As this is a 4 port junction, whenever two ports are perfectly matched, the other two ports are also perfectly matched to the junction.

The junction where all the four ports are perfectly matched is called as Magic Tee Junction.

By substituting the equations we obtain the scattering matrix of Magic Tee as

We already know that, [6] = [SI a]

Applications of E-H Plane Tee

Some of the most common applications of E-H Plane Tee are as follows -

- E-H Plane junction is used to measure the impedance A null detector is connected to E-Arm port while the Microwave source is connected to H-Arm port. The collinear ports together with these ports make a bridge and the impedance measurement is done by balancing the bridge.
- E-H Plane Tee is used as a duplexer A duplexer is a circuit which works as both the transmitter and the receiver, using a single antenna for both purposes. Port 1 and 2 are used as receiver and transmitter where they are isolated and hence will not interfere. Antenna is connected to E-Arm port. A matched load is connected to H-Arm port, which provides no reflections. Now, there exists transmission or reception without any problem.
- E-H Plane Tee is used as a mixer E-Arm port is connected with antenna and the HArm port is connected with local oscillator. Port 2 has a matched load which has no reflections and port 1 has the mixer circuit, which gets half of the signal power and half of the oscillator power to produce IF frequency.

In addition to the above applications, an E-H Plane Tee junction is also used as Microwave bridge, Microwave discriminator, etc.

## **Directional Coupler**

A Directional coupler is a device that samples a small amount of Microwave power for measurement purposes. The power measurements include incident power, reflected power, VSWR values, etc.

Directional Coupler is a 4-port waveguide junction consisting of a primary main waveguide and a secondary auxiliary waveguide. The following figure 1.25 shows the image of a directional coupler.



Directional coupler is used to couple the Microwave power which may be unidirectional or bidirectional.

## **Properties of Directional Couplers**

The properties of an ideal directional coupler are as follows.

- All the terminations are matched to the ports.
- When the power travels from Port 1 to Port 2, some portion of it gets coupled to Port 4 but not to Port 3.
- As it is also a bi-directional coupler, when the power travels from Port 2 to Port 1, some portion of it gets coupled to Port 3 but not to Port 4.
- If the power is incident through Port 3, a portion of it is coupled to Port 2, but not to Port 1.
- If the power is incident through Port 4, a portion of it is coupled to Port 1, but not to Port 2.
- Port 1 and 3 are decoupled as are Port 2 and Port 4.

Ideally, the output of Port 3 should be zero. However, practically, a small amount of power called back power is observed at Port 3. The following figure 1.26 indicates the power flow in a directional coupler

Incident power 1	Main waveguide	2 Received power
$P_t$		Pr
$P_b$		Pf
Back power 3	Auxiliary waveguide	4 Forward power

**Directional Coupler indicating powers** 

## **Figure 1.26 Directional Coupler indicating powers**

## Where

- Pi = Incident power at Port 1
- Pr = Received power at Port 2
- Pf= Forward coupled power at Port 4
- Pb= Back power at Port 3

Following are the parameters used to define the performance of a directional coupler.

#### **Coupling Factor C**

The Coupling factor of a directional coupler is the ratio of incident power to the forward power, measured in dB.

$$C = 10 \log_{10} \frac{P_i}{P_f} dB$$

## **Directivity D**

The Directivity of a directional coupler is the ratio of forward power to the back power, measured in dB.

$$D = 10 \log_{10} \frac{P_f}{P_h} dB$$

#### Isolation

It defines the directive properties of a directional coupler. It is the ratio of incident power to the back power, measured in dB.

$$I = 10 \log_{10} \frac{P_i}{P_b} dB$$

Isolation in dB = Coupling factor + Directivity

## **Two-Hole Directional Coupler**

This is a directional coupler with same main and auxiliary waveguides, but with two small holes that are common between them. These holes are Ag/4 distance apart where Ag is the guide wavelength. The following figure 1.27 shows the image of a two-hole directional coupler.



A two-hole directional coupler is designed to meet the ideal requirement of directional coupler, which is to avoid back power. Some of the power while travelling between Port 1 and Port 2, escapes through the holes 1 and 2.

The magnitude of the power depends upon the dimensions of the holes. This leakage power at both the holes are in phase at hole 2, adding up the power contributing to the forward power Pf. However, it is out of phase at hole 1, cancelling each other and preventing the back power to occur.

Hence, the directivity of a directional coupler improves.

# 2.1 Four Port Networks (Directional Coupler)

It has two inputs and two outputs. After considering using the features of matrix for [S] reciprocal, matched and lossless Network, the possible solutions S14=S23=0 are means Directional Coupler . Using different phase references, Symmetrical or Anti - symmetrical Directional Coupler may be defined. The design parameters of directional coupler are

Coupling - C - 20/cr#

 $Directivity = D = 20log \frac{|S_{13}|}{|S_{14}|}$ 

#### Waveguide Directional Coupler

**Bethe Hole Coupler** : Couple one waveguide to another through a single small hole in the common wall. Types of the parallel guides and skewed guides work properly only at the design frequency (narrow bandwidth in terms of its directivity).

**Multi Hole Coupler :** Series of coupling holes are used to increase bandwidth as similar design to multi section transformer. Making coupling coefficients proportional to binominal coefficients, maximally flat response can be obtained. Using Chebysev polynomial, different responses are possible.



Wilkinson Power Divider The lossless T-junction divider suffers from the disadvantage of not being matched at all ports, and it does not have isolation between output ports. The resistive divider can be matched at all ports, but even though it is not lossless, isolation is still not achieved • The Wilkinson power divider is such a network, with the useful property of appearing lossless when the output ports are matched; that is, only reflected power from the output ports is dissipated. • The Wilkinson power divider can be made with arbitrary power division, but we will first consider the equalsplit (3 dB) case • This divider is often made in microstrip line or strip line form Matched to all its ports • Reciprocal • Lossy



The Wilkinson power divider, (a) An equal-split Wilkinson power divider in mi crostrip line form, (b) Equivalent transmission line circuit.

## Figure 2.6 Wilkinson Power Divider

It is a network with the useful property of being lossless when the output ports are matched, that is, only reflected power is dissipated. It is known that a lossy three port network can be made having all ports are matched with isolation between the output ports. Wilkinson Power Divider can be made in microstrip or stripline form with arbitrary power division of way Divider or Combiner. The even -odd mode technique is used for analysis.

There are 2- X/4 sections with the characteristic impedance • All the input side characteristic impedance is z0 • When power is given input at port 1, divide into two parts port 2 and 3, if there is any reflected power from the o/p port due to mismatch will couple into the resistor 2zo and dissipated. • Using this lossy property to dissipate the reflected power so that it the conditions are satisfied



# **Hybrid Coupler**

It has C= 3dB having types of the following

# Quadrature Hybrid (Hybrid)

This is a 3dB directional coupler (knows as Branch Line Hybrid ) with a phase difference in outputs (2 ^3). Even -odd mode technique can be applied for analysis. [S] matrix has a high degree of symmetry means any port can be used for input as given below



$$[S] = \begin{bmatrix} 0 & J & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & J & 0. \end{bmatrix}$$

## 180 Hybrid

It is a four port network with a phase shift (2^3) between two outputs (also may be in phase). It can be used as a combiner and has unitary symmetric scattering matrix as It may be produces as the form of ring hybrid (rate race), tapered matching lines and hybrid waveguide junction (Magic T, (Rate Race)) in which symmetrically (or antisymmetrical ) placed tuning ports (or irises) can be used for matching.

$[S] = \frac{-j}{\sqrt{2}}$	ГО	1	1	0'-
	1	0	0	1
	1	0	0	1
	Lo	-1	-1	0.

**Coupled Line Directional Coupler** Coupled lines of two (or more) transmission lines are closed together, power can be coupled between the lines. Generally TEM mode is assumed rigorously valid for striplines, but approximately valid for microstrips. Coupled Line Theory is based on types of excitations as even mode (strip currents are equal in amplitude with same directions) and odd mode (strip currents are equal in amplitude with same directions) and odd mode (strip currents are equal in amplitude with opposite directions). Arbitrary excitation can be treated as a superposition of appropriate even and odd modes amplitudes. Moreover design graphs are present for coupled lines.



Figure 2.8 Couple line coupler Design Considerations:

Although a single section coupled line has limited bandwidth due to requirement, the bandwidth can be increased using multiple sections coupled line having close relations to multisection QWT.

The assumption of the same velocity of propagation for even and odd modes in design, generally not satisfied for a coupled microstrip or non TEM lines. This gives poor directivity. By using more effective dielectric constant (smaller phase velocity) for even mode, phase differences should be minimized. This also produces problems as the mismatching phase velocities for multisection case and degrades coupler directivity. Increasing bandwidth can be obtained with low coupling limits.

#### Lange Coupler

To increase coupling factor, Lange Coupler (several lines) with phase difference between outputs is used as a 3 dB coupling ratio in an octave or more bandwidth can be achieved. The main disadvantage of it (a type of quadrature hybrid) is difficult to fabricate due to very narrow lines.



## Figure 2.9 Lange Coupler

Folded Lange coupler is also used for more easily analysis to model equivalent circuit.

Other Couplers Moreno Crossed Guide Coupler Schwinger Reversed Phase Coupler Riblet Short Slot Coupler Symmetric Tapered Coupled Line Coupler Coupler with Apertures in Planar Lines

As an example of a device uses a directional coupler is Reflectometer isolate and sample the incident and reflected powers from a mismatch load as a heart of a scalar (or vectorial) network analyzer.

