4.2 FORMATION OF FLEXIBILITY MATRIX METHOD

For simple,determinate flexural members,developing the {F} matrix is only a process of finding deflections (and/ or rotations) due to a set of simple forces (including moments).We may have to invoke some of the known methods of comuting deflections (like Mohr´s theorems,conjugate beam method etc).

Again, as in the flexibility method, we see that the greater the degree of indeterminacy (kinematic in this case) the greater the number of equations requiring solution, so that a computer-based approach is necessary when the degree of indeterminacy is high.

Generally this requires that the force–displacement relationships in a structure are expressed in matrix form. We therefore need to establish force–displacement relationships for structural members and to examine the way in which these individual force–displacement relationships are combined to produce a force–displacement relationship for the complete structure. Initially we shall investigate members that are subjected to axial force only.

4.2.1.ANALYSING STRUCTURES BY FLEXIBILITY METHOD

Using Flexibility Method for analyzing simple indeterminate structures is like using a helicoptor to go to the next street.The real value of the method can be realized only when we deal with a large number of redundancies.This method is specifically meant for use with computers.

However, the following examples (of manually solving simple structures) are aimed at reinforcing the fundamental principles and procedures involved.

Example 4.21

For the beam shown in fig.,find the deflection at the mid span.

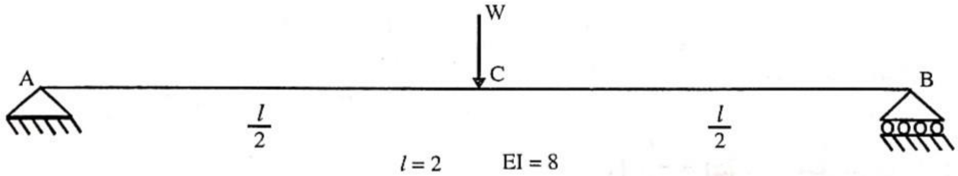
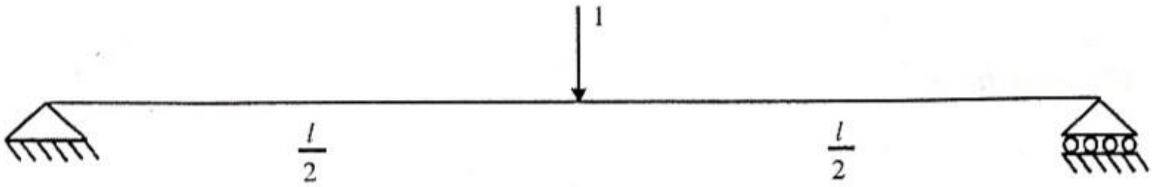


Fig. 4.26

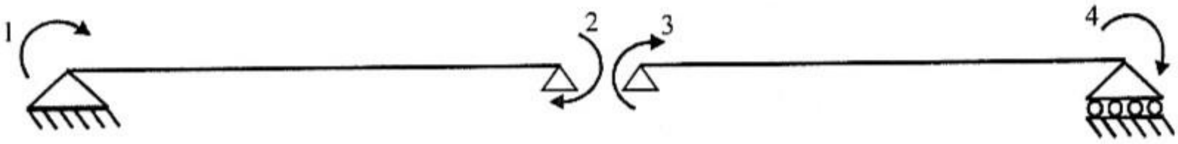
Solution:

Step 1:The given structure, being determinate, is also the primary structure.Hence the system co-ordinate is as under

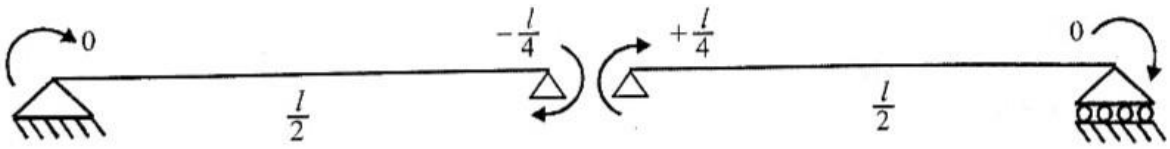


System co-ordinate

Step 2. Let us choose AC and CB as elements and define element co-ordinates as under



Step 3. To generate the $[b]$ matrix relating system force vector, {F} and element force vector {P}, Let us get the equilibrium conditions for the 2 elements, when $F_1 = 1$



$$[b] = [0 \;\; -l/4 \;\; +l/4 \;\; 0]^T$$

$$= [0 \;\; -1/2 \;\; +1/2 \;\; 0]^T$$

This equation simply means that the bending moment at mid span is Wl/4,where W = 1

Step 4:Element flexibility matrix [α] is given by

$$\frac{l/2}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \frac{1}{48}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

Step 5. Compute $[a]$

$$[a] = [b]^T \quad [\alpha] \quad [b]$$

$$= \begin{bmatrix} 0 & -\frac{1}{2} & +\frac{1}{2} & 0 \end{bmatrix} \frac{1}{48}\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ \frac{-1}{2} \\ \frac{+1}{2} \\ 0 \end{bmatrix}$$

$$[a] = \frac{1}{48}\,[1]$$

Step 6:Find the form {u} = [a] {F}

{u} = 1 x W/48 = W/48

The ludicrous fact is that we have used a lio ofelaborate steps to arrive at the conclusion that the mid span deflection is $Wl^3/48EI$.

Example 4.22

A cantilever of length 15 m is subjected to a single concentrated load of 50 kN at the middle of the span. Find the deflection at the free end using flexibility matrix method. EI is uniform throughout.

Solution:

Step1:

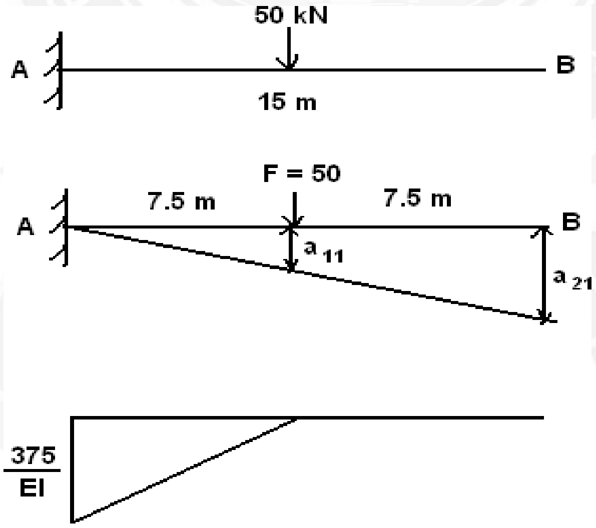
Static Indeterminacy :

Degree of redundancy = 3-3= 0

It is static determinate structures.

Step 2: Deflection at B:

Apply a unit force at given load.



The deflection is calculated by M/EI

Deflection at (a) = {1/2x7.5x375/EI}

Deflection at B = 17578.125/EI

4.2.3.PROCEDURE FOR FLEXIBILITY METHOD:

There are eight steps to find the support moments in continuous beams,frames and trusses;they are

- Decide on the primary structure;indicate redundants $\{F\}^0$
- Select {P} co-ordinates for internal forces.
- Compile external forces $\{F\}^*$,This would include the effects of off node forces.

$$\{F\} = \begin{Bmatrix} \{F\}^* \\ \cdots\cdots \\ \{F\}^0 \end{Bmatrix} \quad \text{and} \quad \{u\} = \begin{bmatrix} \{u\}^* \\ \cdots\cdots \\ \{u\}^0 \end{bmatrix}, \{u\}^0 = \{0\}$$

Then,

Generate [b] matrix such that $\{P\} = [b]\ \{F\}; [b] = [b]^* [b]^0$

- Synthesize element flexibility matrix [α]
- Compute [a] using [a] = [α] [b]; $[a] = \begin{bmatrix} [a]_{11} & [a]_{12} \\ [a]_{21} & [a]_{22} \end{bmatrix}$
- Isolate $\{F\}^0$,the redundant forces, from the condition

  $\{P\}^0 = \{0\}$

  $\{F\}^0 = - [a]_{22}{}^{-1}\ [a]_{21}\ \{F\}^*$
- Isolate $[a]^*$ $\quad [a]^* = [a]_{11} - [a]_{22}{}^{-1}\ [a]_{21}$
- Get {P} from $\{P\} = [b]\ \{F\}$ and $\{P\} - \{P\}^e$

  $\{P\}^f$ are the final member forces.

  $\{P\}^f$ are also the support moments at all joints.