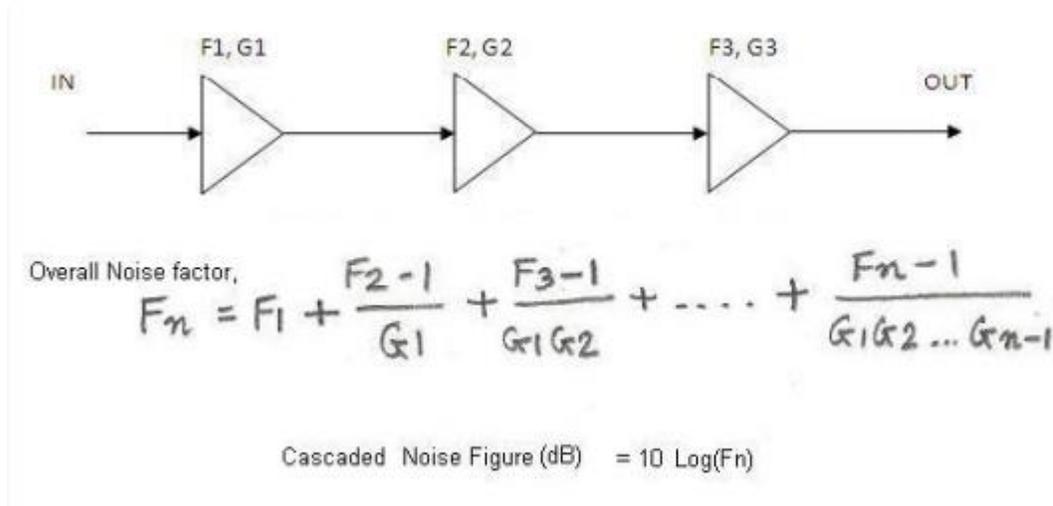


### Noise In Cascade Systems

Cascade noise figure calculation is carried out by dealing with gain and noise figure as a ratio rather than decibels, and then converting back to decibels at the end. As the following equation shows, cascaded noise figure is affected most profoundly by the noise figure of components closest to the input of the system as long as some positive gain exists in the cascade.



**Figure 4.4.1 Block Diagram of Cascaded Systems**

*Diagram Source Brain kart*

If only loss exists in the cascade, then the cascaded noise figure equals the magnitude of the total loss. The figure 4.4.1 is used to calculate cascaded noise figure as a ratio based on ratio values for gain and noise figure (do not use decibel values).

#### Cascaded Network:

A receiver systems usually consists of a number of passive or active elements connected in series, each element is defined separately in terms of the gain (greater than 1 or less than 1 as the case may be), noise figure or noise temperature and bandwidth (usually the 3 dB bandwidth). These elements are assumed to be matched.

A typical receiver block diagram is shown below, with example

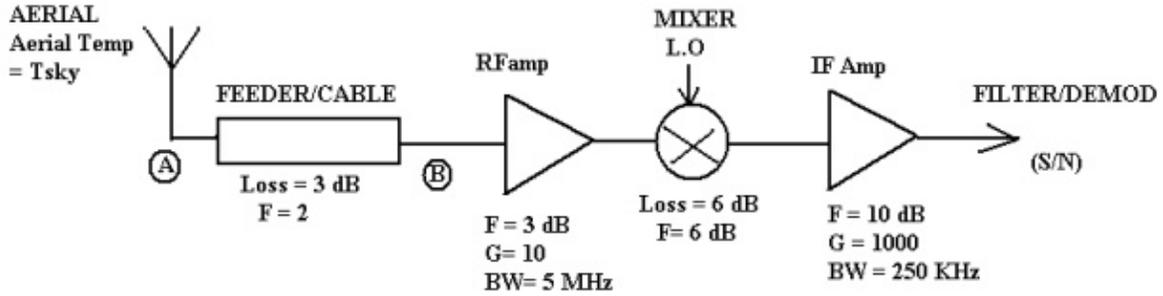


Figure 4.4.2 Block diagram for Cascade System,

Diagram Source Brain Kart

In order to determine the (S/N) at the input, the overall receiver noise figure or noise temperature must be determined in the figure 4.4.2. In order to do this all the noise must be referred to the same point in the receiver, for example to A, the feeder input or B, the input to the first amplifier. The equations so far discussed refer the noise to the input of that specific element i.e.

$T_e$  or  $N_e$  is the noise referred to the input.

To refer the noise to the output we must multiply the input noise by the gain G. For example, for a lossy feeder, loss L, we had

$N_e = (L-1) N_{IN}$ , noise referred to input Or  $T_e = (L-1) T_S$  - referred to the input. Noise referred to output is gain x noise referred to input, hence

$$N_e \text{ referred to output} = G N_e = \frac{1}{L} (L-1) N_{IN}$$

$$= (1 - \frac{1}{L}) N_{IN}$$

Similarly, the equivalent noise temperature referred to the output is

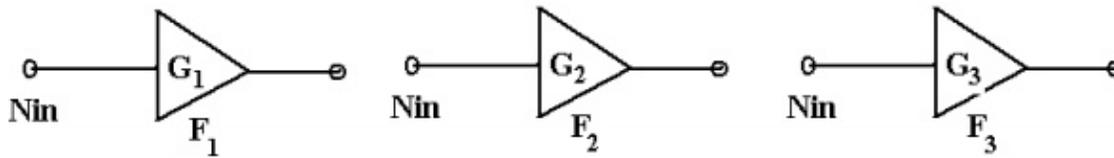
$$T_e \text{ referred to output} = (1 - \frac{1}{L}) T_S$$

These points will be clarified later; first the system noise figure will be considered.

**System Noise Figure:**

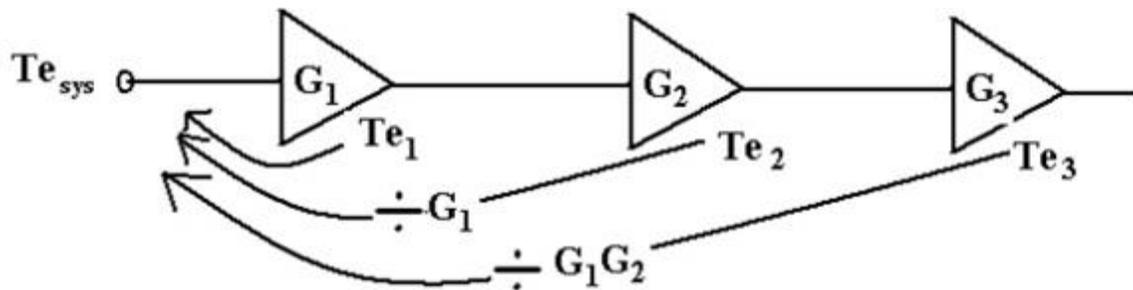
Assume that a system comprises the elements shown below, each element defined and specified separately shown in the figure 4.4.3 & 4.4.4.

The gains may be greater or less than 1, symbols F denote noise factor (not noise figure, i.e. not in dB). Assume that these are now cascaded and connected to an aerial at the input, with



**Figure 4.4.3 Circuit Diagram of cascaded Systems**

*Diagram Source Brain Kart*



**Figure 4.4.4 Cascaded Systems**

*Diagram Source Brain Kart*

$$\begin{aligned} \text{Now, } N_{OUT} &= G_3 (N_{IN3} + N_{e3}) \\ &= G_3 (N_{IN3} + (F_3 - 1)N_{IN}) \end{aligned}$$

$$\text{Since } N_{IN3} = G_2 (N_{IN2} + N_{e2}) = G_2 (N_{IN2} + (F_2 - 1)N_{IN})$$

$$\text{and similarly } N_{IN2} = G_1 (N_{ae} + (F_1 - 1)N_{IN})$$

then

$$N_{OUT} = G_3 [G_2 [G_1 N_{ae} + G_1 (F_1 - 1)N_{IN}] + G_2 (F_2 - 1)N_{IN}] + G_3 (F_3 - 1)N_{IN}$$

The overall system Noise Factor is

$$\begin{aligned} F_{sys} &= \frac{N_{OUT}}{GN_{IN}} = \frac{N_{OUT}}{G_1 G_2 G_3 N_{ae}} \\ &= 1 + (F_1 - 1) \frac{N_{IN}}{N_{ae}} + \frac{(F_2 - 1) N_{IN}}{G_1 N_{ae}} + \frac{(F_3 - 1) N_{IN}}{G_1 G_2 N_{ae}} \end{aligned}$$

If we assume  $N_{ae}$  is  $\approx N_{IN}$ , i.e. we would measure and specify  $F_{sys}$  under similar conditions as  $F_1, F_2$  etc (i.e. at 290 K), then for n elements in cascade.

### FRIIS Formula

$$F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \frac{(F_4 - 1)}{G_1 G_2 G_3} + \dots + \frac{(F_n - 1)}{G_1 G_2 \dots G_{n-1}}$$

The equation is called FRIIS Formula. This equation indicates that the system noise factor depends largely on the noise factor of the first stage if the gain of the first stage is reasonably large. This explains the desire for —low noise front ends‖ or low noise most head preamplifiers for domestic TV reception. There is a danger however; if the gain of the first stage is too large, large and unwanted signals are applied to the mixer which may produce intermodulation distortion. Some receivers apply signals from the aerial directly to the mixer to avoid this problem. Generally a first stage amplifier is designed to have a good noise factor and some gain to give an acceptable overall noise figure.

**System Noise Temperature:**

Since  $T_e = (L-1)T_s$ , i.e.  $F = 1 + \frac{T_e}{T_s}$

Then  $F_{sys} = 1 + \frac{T_{e,sys}}{T_s}$  } *where  $T_{e,sys}$  is the equivalent Noise temperature of the system  
and  $T_s$  is the noise temperature of the source*

and

$$\left(1 + \frac{T_{e,sys}}{T_s}\right) = \left(1 + \frac{T_{e1}}{T_s}\right) + \frac{\left(1 + \frac{T_{e2}}{T_s} - 1\right)}{G_1} + \dots etc$$

i.e. from  $F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \dots etc$

which gives

$$T_{e,sys} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \dots$$

Overall Noise factor,

$$F_n = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}}$$

Cascaded Noise Figure (dB) = 10 Log(Fn)

**Application:**

It is important to realize that the previous sections present a technique to enable a receiver performance to be calculated. The essence of the approach is to refer all the noise contributed at various stages in the receiver to the input and thus contrive to make all the stages ideal, noise free.