4.11 TUTORIAL PROBLEMS:

1. A machine of mass 100 kg is supported on openings of total stiffness 800 kN/m and has a rotating unbalanced element which results in a disturbing force of 400 N at a speed of 3000 r.p.m. Assum-ing the damping ratio as 0.25, determine : 1. the amplitude of vibrations due to unbalance ; and 2. the transmitted force. **[Ans. 0.04 mm ; 35.2 N]**

2. A mass of 500 kg is mounted on supports having a total stiffness of 100 kN/m and which provides viscous damping, the damping ratio being 0.4. The mass is constrained to move vertically and is subjected to a vertical disturbing force of the type $F \cos \omega t$. Determine the frequency at which resonance will occur and the maximum allowable value of F if the amplitude at resonance is to be restricted to 5 mm. **[Ans. 2.25 Hz ; 400 N]**

3. A machine of mass 75 kg is mounted on springs of stiffness 1200 kN/m and with an assumed damping factor of 0.2. A piston within the machine of mass 2 kg has a reciprocating motion with a stroke of 80 mm and a speed of 3000 cycles/min. Assuming the motion to be simple harmonic, find : 1. the amplitude of motion of the machine, 2. its phase angle with respect to the exciting force, 3. the force transmitted to the foundation, and 4. the phase angle of transmitted force with respect to the exciting force. **[Ans. 1.254 mm ; 169.05° ; 2132 N ; 44.8°]**

UNIT-V MECHANISMS FOR CONTROL

5.1 INTRODUCTION TO GOVERNOR:

A **centrifugal governor** is a specific type of governor that controls the speed of an engine by regulating the amount of fuel (or working fluid) admitted, so as to maintain a near constant speed whatever the load or fuel supply conditions. It uses the principle of proportional control.

It is most obviously seen on steam engines where it regulates the admission of steam into the cylinder(s). It is also found on internal combustion engines and variously fuelled turbines, and in some modern striking clocks.

5.2 PRINCIPLE OF WORKING:

Power is supplied to the governor from the engine's output shaft by (in this instance) a belt or chain (not shown) connected to the lower belt wheel. The governor is connected to a throttle valve that regulates the flow of working fluid (steam) supplying the prime mover (prime mover not shown). As the speed of the prime mover increases, the central spindle of the governor rotates at a faster rate and the kinetic energy of the balls increases. This allows the two masses on lever arms to move outwards and upwards against gravity. If the motion goes far enough, this motion causes the lever arms to pull down on a thrust_bearing, which moves a beam linkage, which reduces the aperture of a throttle valve. The rate of working-fluid entering the cylinder is thus reduced and the speed of the prime mover is controlled, preventing over speeding.

Mechanical stops may be used to limit the range of throttle motion, as seen near the masses in the image at right.

The direction of the lever arm holding the mass will be along the vector _ sum of the reactive centrifugal force vector and the gravitational force.

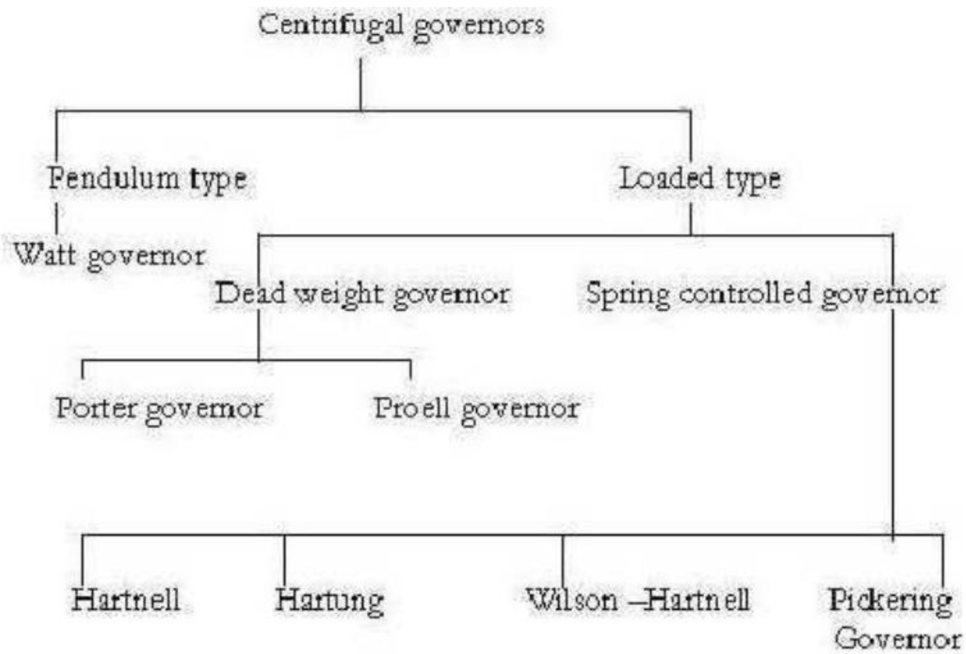
5.3 CLASSIFICATION OF GOVERNORS:

Governors are classified based upon two different principles.

These are:

1. Centrifugal governors
2. Inertia governors

Centrifugal governors are further classified as –



(4) Height of governor

It is the vertical distance between the centre of the governor halls and the point of intersection between the upper arms on the axis of spindle is known as governor height. It is generally denoted by h.

(5) Sleeve lift

The vertical distance the sleeve travels due to change in the equilibrium Speed is called the sleeve lift. The vertical downward travel may be termed as Negative lift

(6) Isochronism

This is an extreme case of sensitiveness. When the equilibrium speed is constant for all radii of rotation of the balls within the working range, the governor is said to be in isochronism. This means that the difference between the maximum and minimum equilibrium speeds is zero and the sensitiveness shall be ininite.

(7) Stability

Stability is the ability to maintain a desired engine speed without Fluctuating. Instability results in hunting or oscillating due to over correction. Excessive stability results in a dead-beat governor or one that does not correct sufficiently for load changes

(8) Hunting

The phenomenon of continuous fluctuation of the engine speed above and below the mean speed is termed as hunting. This occurs in over- sensitive or isochronous governors. Suppose an isochronous governor is fitted to an engine running at a steady load. With a slight increase of load, the speed will fall and the sleeve will immediately fall to its lowest position. This shall open the control valve wide and excess supply of energy will be-given, with the result that the speed will rapidly increase and the sleeve will rise to its higher position. As a result of this movement of the sleeve, the control valve will be cut off; the supply to the engine and the speed will again fall, the cycle being repeated indefinitely. Such a governor would admit either more or less amount of fuel and so effect would be that the engine would hunt.

5.4 SENSITIVENESS

A governor is said to be sensitive, if its change of speed s from no Load to full load may be as small a fraction of the mean equilibrium speed as possible and the corresponding sleeve lift may be as large as possible. Suppose $\omega 1$ = max. Equilibrium speed $\omega 2$ = min. equilibrium speed ω = mean equilibrium speed = $(\omega 1+ \omega 2)/2$ Therefore sensitiveness = $(\omega 1- \omega 2)/2$

5.5 CHARACTERISTICS AND QUALITIES OF CENTRIFUGAL GOVERNOR:

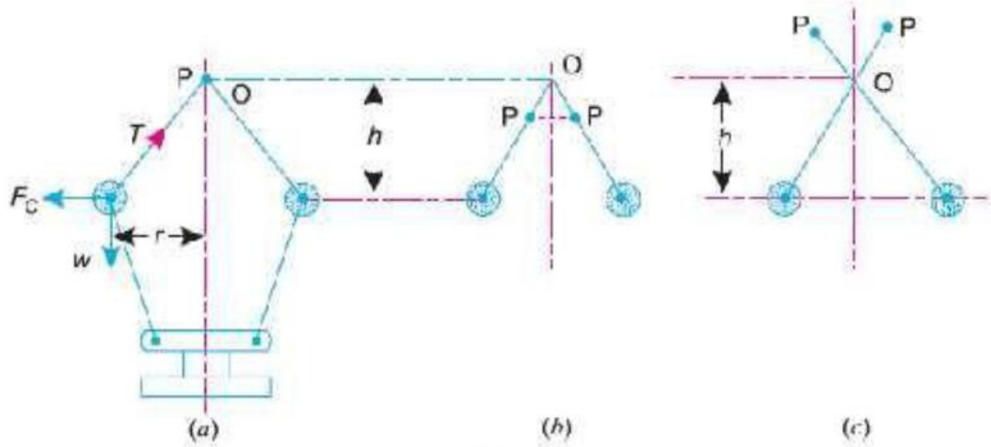
For satisfactory performance and working a centrifugal governor should possess The following qualities.

a. On the sudden removal of load its sleeve should reach at the top most position at Once.
b. Its response to the change of speed should be fast.
c. Its sleeve should float at some intermediate position under normal operating Conditions.
d. At the lowest position of sleeve the engine should develop maximum power.
e. It should have sufficient power, so that it may be able to exert the required force At the sleeve to operate the control & mechanism

5.6 WATT GOVERNOR:

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways :

1. The pivot *P*, may be on the spindle axis as shown in Fig. (*a*).
2. The pivot *P*, may be offset from the spindle axis and the arms when produced intersect at *O*, as shown in Fig (*b*).
3. The pivot *P*, may be offset, but the arms cross the axis at *O*, as shown in Fig (*c*).



Watt governor.

Let m = Mass of the ball in kg.

w = Weight of the ball in newtons = $m.g$,

T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (F_C) acting on the ball, **2.** the tension (T) in the arm, and **3.** the weight (w) of the ball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

or

$$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g / \omega^2 \qquad \ldots (i)$$

When g is expressed in m/s² and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

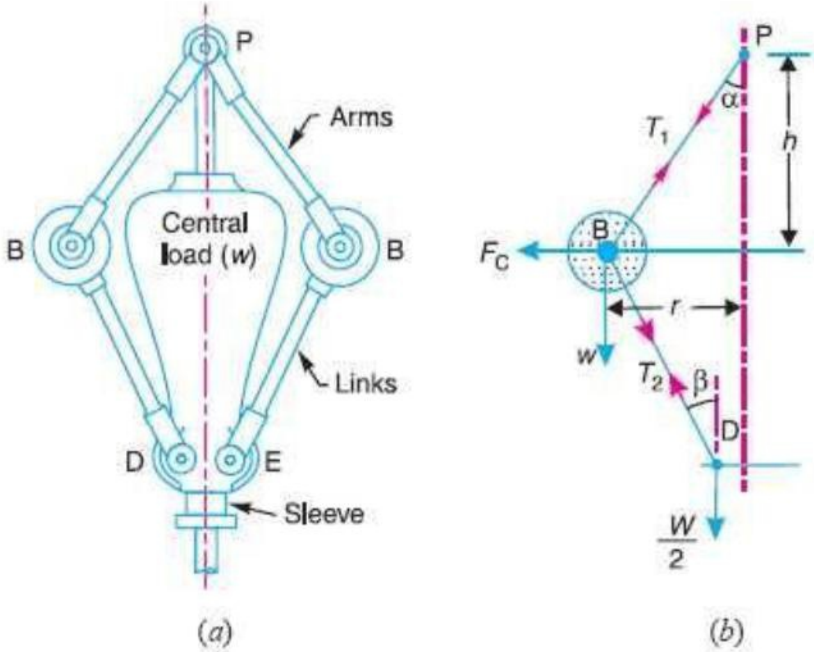
$$\therefore \quad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots (\because g = 9.81 \text{ m/s}^2) \ldots (ii)$$

Note : We see from the above expression that the height of a governor h, is inversely proportional to N^2. Therefore at high speeds, the value of h is small. At such speeds, the change in the value of h corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds *i.e.* from 60 to 80 r.p.m.

5.7 PORTER GOVERNOR:

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. (*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Consider the forces acting on one-half of the governor as shown in Fig. (*b*).



Porter governor.

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

---

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T_1 = Force in the arm in newtons,

T_2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

5.8 PROELL GOVERNOR:

The Proell governor has the balls fixed at B and C to the extension of the links DF and EG, as shown in Fig. (a). The arms FP and GQ are pivoted at P and Q respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig (b). The instantaneous centre (I) lies on the intersection of the line PF produced and the line from D drawn perpendicualr to the spindle axis. The prependicular BM is drawn on ID.

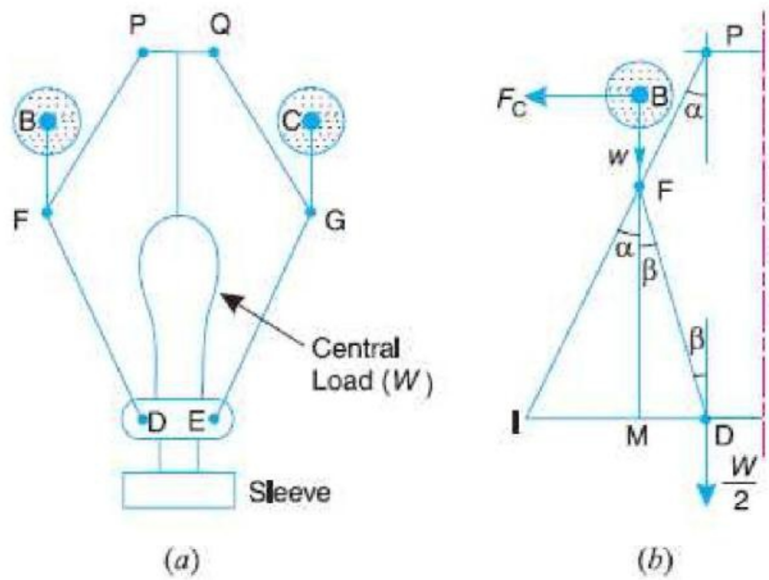
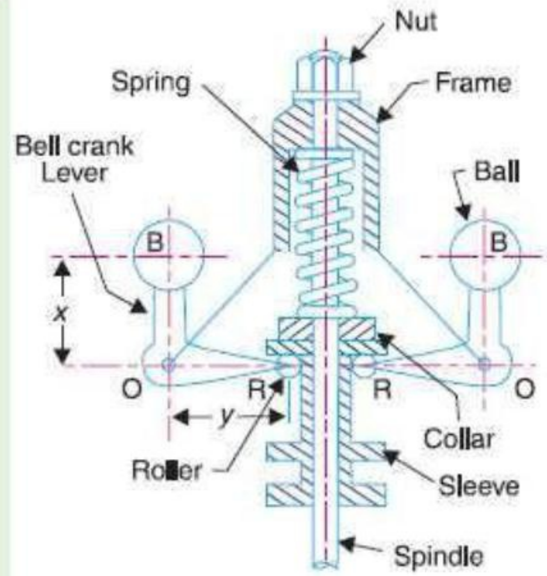


Fig. Proell governor.

5.9 HARTNELL GOVERNOR:

A Hartnell governor is a spring loaded governor as shown in Fig. It consists of two bell crank levers pivoted at the points *O*,*O* to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm *OB* and a roller at the end of the horizontal arm *OR*. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



Hartnell governor.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

r_2 = Maximum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve at ω_2 in newtons,

F_{C1} = Centrifugal force at ω_1 in newtons = $m\,(\omega_1)^2\,r_1$,

F_{C2} = Centrifugal force at ω_2 in newtons = $m\,(\omega_2)^2\,r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum *O* from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

5.10 HARTUNG GOVERNOR:

A spring controlled governor of the Hartung type is shown in Fig. (*a*). In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

Let S = Spring force,

F_C = Centrifugal force,

M = Mass on the sleeve, and

x and y = Lengths of the vertical and horizontal arm of the bell crank lever respectively.

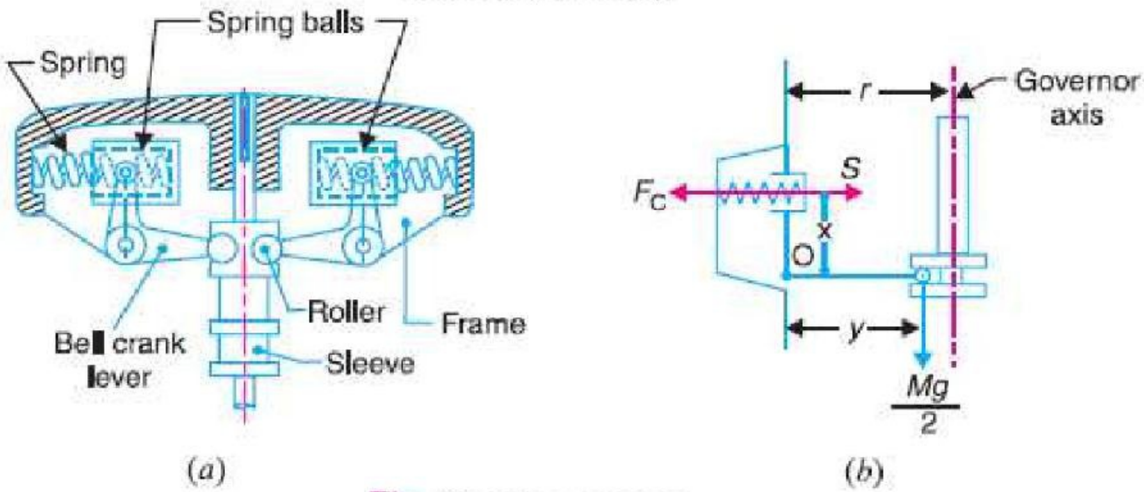


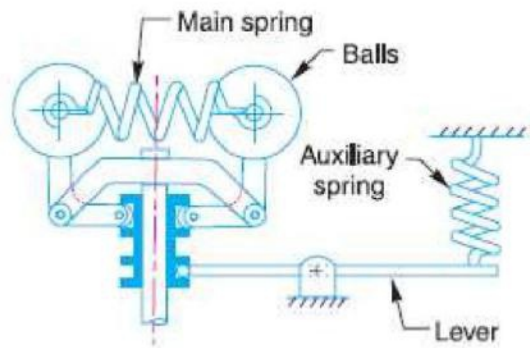
Fig. Hartung governor.

Fig. (*a*) and (*b*) show the governor in mid-position. Neglecting the effect of obliquity of the arms, taking moments about the fulcrum O,

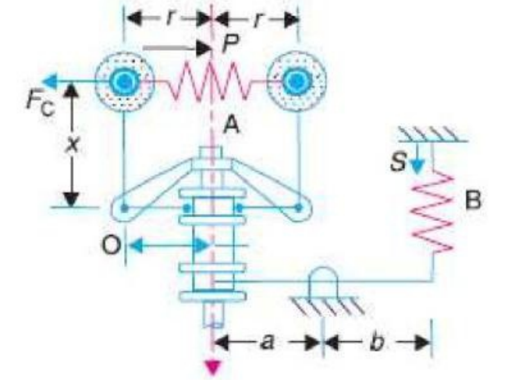
$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

5.11 WILSON HARTNELL GOVERNOR:

A Wilson-Hartnell governor is a governor in which the balls are connected by a spring in tension as shown in Fig. An auxiliary spring is attached to the sleeve mechanism through a lever by means of which the equilibrium speed for a given radius may be adjusted. The main spring may be considered of two equal parts each belonging to both the balls. The line diagram of a Wilson-Hartnell governor is shown in Fig.



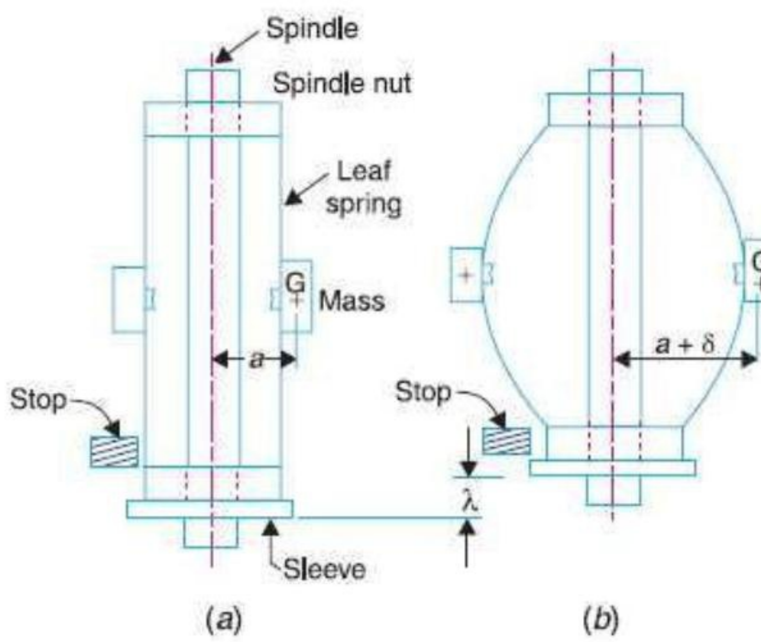
Wilson-Hartnell governor.



Line diagram of Wilson-Hartnell governor.

5.12 PICKERING GOVERNOR:

A Pickering governor is mostly used for driving gramophone. It consists of *three straight leaf springs arranged at equal angular intervals round the spindle. Each spring carries a weight at the centre. The weights move outwards and the springs bend as they rotate about the spindle axis with increasing speed.



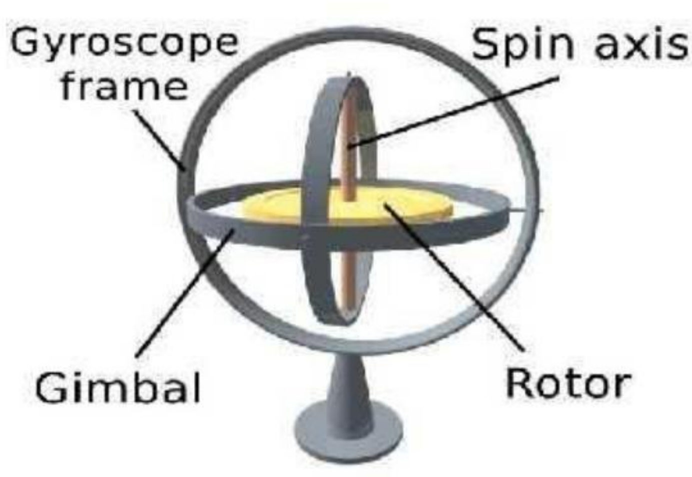
5.13 DIFFERENCE BETWEEN A FLYWHEEL AND A GOVERNOR:

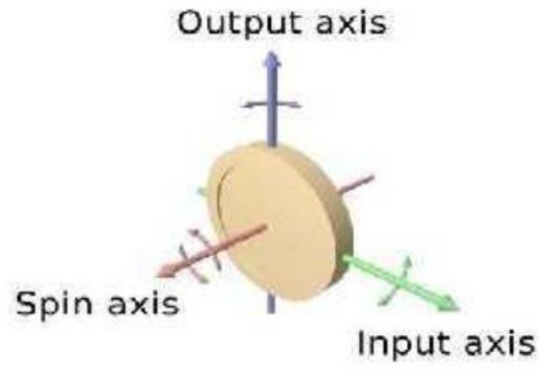
S.no	Flywheel	Governor
1	It is provided on the engine and fabricating machines viz., rolling mills; punching machines; shear machines, presses etc.	It is provided on prime movers such as engines and turbines
2	Its function is to store available mechanical energy when it is in excess of the load requirements and to part with the same when the available energy is less than that required by the load.	Its function is to regulate the supply of driving fluid producing energy, according to the load requirements so that at different loads almost a constant speed is maintained.
3.	In engines it takes care of fluctuations of speed during thermodynamic cycle.	It take care of fluctuation of speed due to variation of load over range of working of engines and turbines.
4.	It works continuously from cycle to cycle.	It works intermittently, i.e. only when there is change in the load.
5.	In fabrication machines it is very economical to use it as its use reduces capital investment on prime movers and their running expenses.	But for governor, there would have been unnecessarily more consumption of driving fluid thus it economizes its consumption

5.14 GYROSCOPE AND ITS APPLICATIONS

(19) Gyroscope

A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum. A mechanical gyroscope is essentially a spinning wheel or disk whose axle is free to take any orientation. This orientation changes much less in response to a given external torque than it would without the large angular momentum associated with the gyroscope's high rate of spin. Since external torque is minimized by mounting the device in gimbals, its orientation remains nearly fixed, regardless of any motion of the platform on which it is mounted. Gyroscopes based on other operating principles also Exit, such as the electronic, microchip-packaged MEMS gyroscope devices found in consumer electronic devices, solid state ring lasers, fiber optic gyroscopes and the extremely sensitive quantum gyroscope. Applications of gyroscopes include navigation (INS) when magnetic compasses do not work (as in the Hubble telescope) or are not precise enough (as in ICBMs) or for the stabilization of flying vehicles like radio-controlled helicopters or UAVs. Due to higher precision, gyroscopes are also used to maintain direction in tunnel mining.



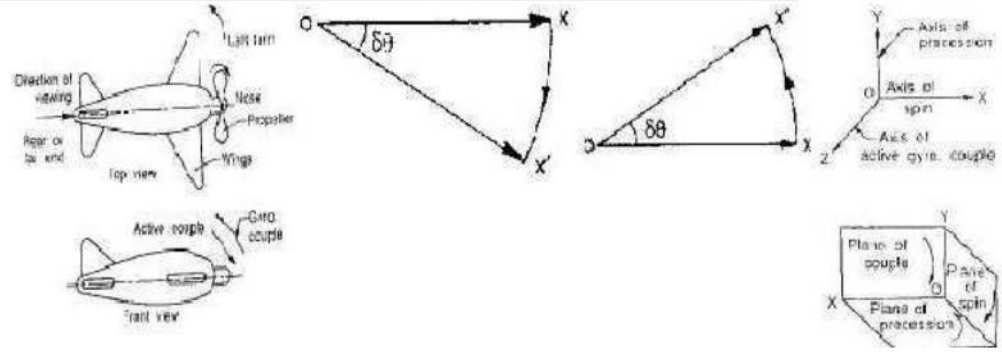


<u>(20) Description and diagram:</u>

Diagram of a gyro wheel. Reaction arrows about the output axis (blue) correspond to forces applied about the input axis (green), and vice versa. Within mechanical systems or devices, a conventional gyroscope is a mechanism comprising a rotor journal led to spin about one axis, the journals of the rotor being mounted in an inner gimbal or ring, the inner gimbal is journal led for oscillation in an outer gimbal which is journal led in another gimbal. So basically there are three gimbals. The **outer gimbal** or ring which is the gyroscope frame is mounted so as to pivot about an axis in its own plane determined by the support. This outer gimbal possesses one degree of rotational freedom and its axis possesses none. The next **inner gimbal** is mounted in the gyroscope frame (outer gimbal) so as to pivot about an axis in its own plane that is always perpendicular to the pivotal axis of the gyroscope frame (outer gimbal). This inner gimbal has two degrees of rotational freedom. Similarly, next **innermost gimbal** is attached to the inner gimbal which has three degree of rotational freedom and its axis posses two. The axle of the spinning wheel defines the spin axis. The rotor is journaled to spin about an axis which is always perpendicular to the axis of the innermost gimbal. So, the rotor possesses four degrees of rotational freedom and its axis possesses three. The wheel responds to a force applied about the input axis by a reaction force about the output axis.

The behavior of a gyroscope can be most easily appreciated by consideration of the front wheel of a bicycle. If the wheel is leaned away from the vertical so that the top of the wheel moves to the left, the forward rim of the wheel also turns to the left. In other words, rotation on one axis of the turning wheel produces rotation of the third axis.

5.15 EFFECT OF THE GYROSCOPIC COUPLE ON AN AERO PLANE



5.16 EFFECT OF GYROSCOPIC COUPLE

This couple is, therefore, to raise the nose and dip the tail of the aero plane.

Notes

1.When the aero plane takes a right turn under similarConditions as discussed above, the effect of the reactiveCouple will be to dip the nose and raise the tail of theaero plane.

2. When the engine or propeller rotates in anticlockwise direction when viewed from the rear or tail end and the aero plane takes a left turn, then the effect of reactive gyroscopic couple will be to dip the nose and raise the tail ofthe aero plane.

3. When the aero plane takes a right turn under similar Conditions as mentioned in note 2 above, the effect of Reactive gyroscopic couple will be to raise the nose and dip the of the aero plane.

4. When the engine or propeller rotates in clockwise direction when viewed from the front and the aero plane takes a left turn, then the effect of reactive gyroscopic couple will be to raise the tail and dip the nose of the aero plane.

5. When the aero plane takes a right turn under similar conditions as mentioned in note4 above, the effect ofreactive gyroscopic couple will be to raise the nose and dip the tail of the aero plane.

5.17 EFFECT OF GYROSCOPIC COUPLE ON SHIP

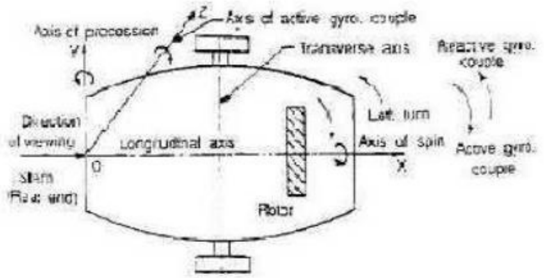
The top and front views of a naval ship are shown in fig. The fore end of the ship is called bow and the rear end is known as stern or aft. The left hand and the right hand sides of the ship, when viewed from the stern are called port and star board respectively. We shall now discuss the effect of gyroscopic couple in the naval ship in the following three cases:

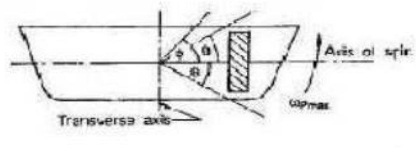
1. Steering

2. Pitching, and

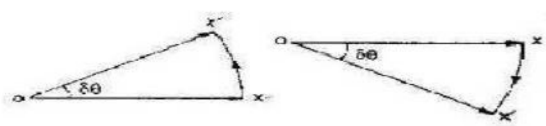
3. Rolling

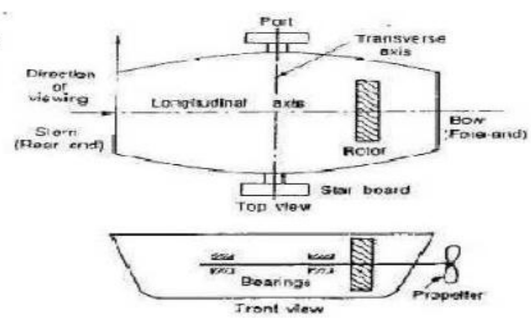
5.17.1 EFFECT OF GYROSCOPIC COUPLE ON A NAVAL SHIP DURING PITCHING& STEERING





(a) Pitching of a naval ship.





Steering is the turning of a complete ship in a curve towards left or right, while it moves forward, considers the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. below. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aero plane as discussed in Art.

When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown in Fig. A1. As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from ox to ox'. The vector xx' now represents the active gyroscopic couple and is perpendicular to ox. Thus the plane of active gyroscopic couple is perpendicular to xx' and its direction in the axis OZ for left hand turn is clockwise as shown in Fig below. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (i.e in anticlockwise direction). The effect of this reactive gyroscopic couple is to raise the bow and lower the stern.

Notes

1. When the ship steers to the right under similar condition as discussed above, the effect of the reactive gyroscopic couple, as shown in Fig. B1, will be to raise the stern and lower the bow.
2. When the rotor rotates in the anticlockwise direction, when viewed from the stern and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to lower the bow and raise the stern.
3. When the ship is steering to the right under similar conditions as discussed in note 2 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.
4. When the rotor rotates in the clockwise direction when viewed from the bow or fore end and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to raise the stern and lower the bow.
5. When the ship is steering to the right under similar conditions as discussed in note 4 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.
6. The effect of the reactive gyroscopic couple on a boat propelled by a turbine taking left or right turn.

5.17.2 Effect of Gyroscopic couple on a Naval Ship during Rolling:

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship. In case of rolling of a ship, the axis of precession (i.e. longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

5.18 EFFECT OF GYROSCOPIC COUPLE ON A 4-WHEEL DRIVE:

Consider the four wheels A, B, C and D of an automobile locomotive taking a turn towards left as shown in Fig. 14.11. The wheels A and C are inner wheels, whereas B and D are outer wheels. The centre of gravity (C.G.) of the vehicle lies vertically above the road surface.

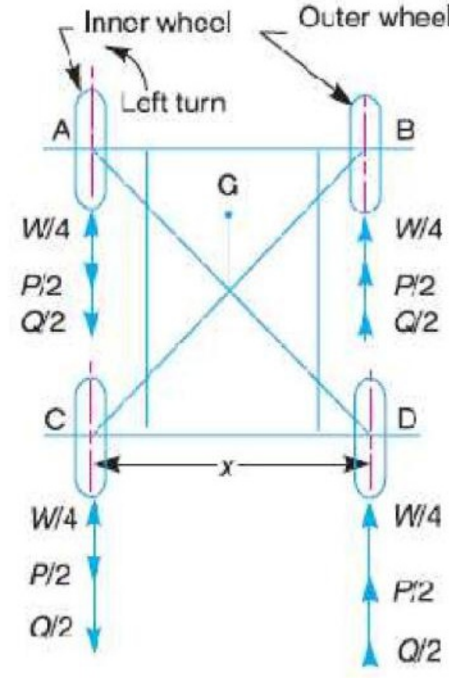


Fig. 14.11. Four wheel drive moving in a curved path.

Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons = $m.g$,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_W$),

h = Distance of centre of gravity, vertically above the road surface in metres,

x = Width of track in metres,

I_W = Mass moment of inertia of one of the wheels in kg-m^2,

ω_W = Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m^2,

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle in m/s = $\omega_W . r_W$

A little consideration will show, that the weight of the vehicle (W) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards. Therefore

Road reaction over each wheel

$$= W/4 = m.g/4 \text{ newtons}$$

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

1. *Effect of the gyroscopic couple*

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.

We know that velocity of precession,

$$\omega_P = v/R$$

∴ Gyroscopic couple due to 4 wheels,

$$C_W = 4\,I_W.\omega_W.\omega_P$$

and gyroscopic couple due to the rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P \qquad \ldots (\because G = \omega_E/\omega_W)$$

∴ Net gyroscopic couple,

$$C = C_W \pm C_E = 4\,I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P$$

$$= \omega_W.\omega_P\,(4\,I_W \pm G.I_E)$$

The *positive* sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then *negative* sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be P newtons. Then

$$P \times x = C \quad \text{or} \quad P = C/x$$

∴ Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

Note: We have discussed above that when rotating parts of the engine rotate in opposite directions, then –ve sign is used, *i.e.* net gyroscopic couple,

$$C = C_W - C_E$$

When $C_E > C_W$, then C will be –ve. Thus the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels.

2. *Effect of the centrifugal couple*

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle. We know that centrifugal force,

$$F_C = \frac{m \times v^2}{R}$$

∴ The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be Q. Then

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$

∴ Vertical reaction at each of the outer or inner wheels,

$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

∴ Total vertical reaction at each of the outer wheel,

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

and total vertical reaction at each of the inner wheel,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

A little consideration will show that when the vehicle is running at high speeds, P_I may be zero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the automobile. In order to have the contact between the inner wheels and the ground, the sum of $P/2$ and $Q/2$ must be less than $W/4$.

19. SOLVED PROBLEMS

1.(i) Explain the function of the proell governor with the help of a neat sketch. Derive that relationship among the various forces acting on the link. (12)

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig. 18.12 (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig. 18.12 (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The perpendicular *B M* is drawn on *ID*.

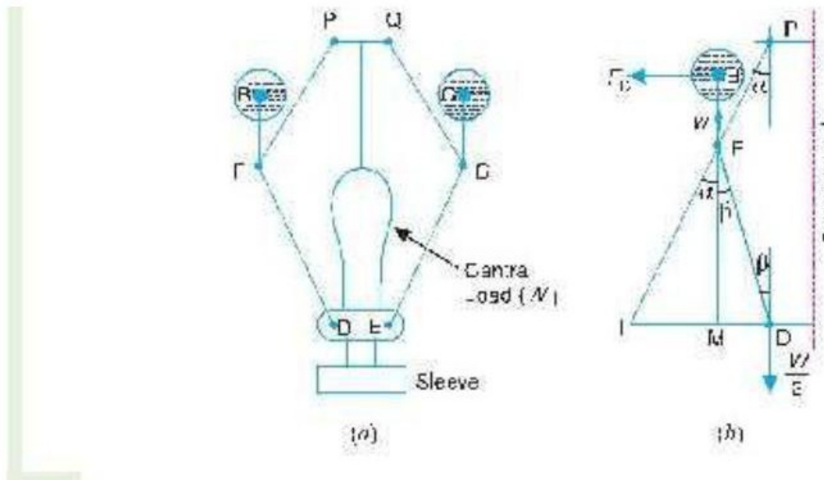


Fig. 18.12. Proell governor.

Taking moments about *I*,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \quad \ldots (i)$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \quad \ldots (\because ID = IM + MD)$$

Multiplying and dividing by *FM*, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2 .r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \quad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s², we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \quad \ldots (iii)$$

1 (ii). What are centrifugal governors? how do they differ from inertia governor? (4)

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force***

In inertia governors the positions of the balls are affected by the rate of change of speed. i.e., angular acceleration or retardation of the governor shaft. The amount of displacement of governor balls is controlled by suitable springs and the fuel supply to the engine is controlled by governor mechanism.

Though the sensitiveness of the inertia governors is more, there is a practical difficulty of balancing the inertia forces caused by the revolving parts of the governor to the controlling force. Hence these governors are not preferred when compared with the centifugal governors.

2. . A Proell governor has equal arms of length 300 m m. The upper and

arms of the lower links are each 80 mm long and parallel to the axis when th ra of rotation of the balls are 150 mm and 200 m m. The mass of each ball is 10 and the mass of the central load is 100 kg. Determine the range of speed of t governor.

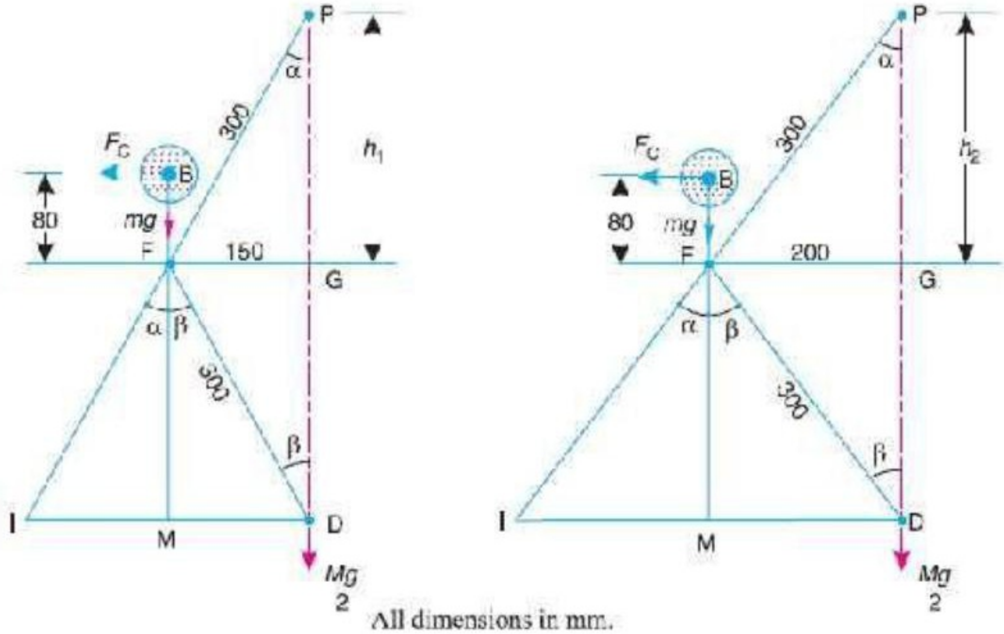
lower ends of the arms are pivoted on the axis of the governor. The extension e

Solution. Given : $PF = DF = 300$ mm ; $BF = 80$ mm ; $m = 10$ kg ; $M = 100$ kg ; $r_1 = 150$ mm; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.13.

Let N_1 = Minimum speed when radius of rotation, $r_1 = FG = 150$ mm ; and

N_2 = Maximum speed when radius of rotation , $r_2 = FG = 200$ mm. From Fig. (*a*), we find that height of the governor,



(*a*) Minimum position. (*a*) Maximum position.

From Fig. (*a*), we find that

$$\sin \alpha = \sin \beta = 150 / 300 = 0.5 \qquad \text{or} \quad \alpha = \beta = 30°$$

and $$MD = FG = 150 \text{ mm} = 0.15 \text{ m}$$

$$FM = FD \cos \beta = 300 \cos 30° = 260 \text{ mm} = 0.26 \text{ m}$$

$$IM = FM \tan \alpha = 0.26 \tan 30° = 0.15 \text{ m}$$

$$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$$

$$ID = IM + MD = 0.15 + 0.15 = 0.3 \text{ m}$$

We know that centrifugal force,

$$F_C = m(\omega_1)^2 r_1 = 10\left(\frac{2\pi N_1}{60}\right)^2 0.15 = 0.0165\,(N_1)^2$$

Now taking moments about point *I*,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

or $$0.0165\,(N_1)^2\, 0.34 = 10 \times 9.81 \times 0.15 + \frac{100 \times 9.81}{2} \times 0.3$$

$$0.0056\,(N_1)^2 = 14.715 + 147.15 = 161.865$$

$$\therefore \quad (N_1)^2 = \frac{161.865}{0.0056} = 28\,904 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

$$h = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$$

$$FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$$

$$BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$$

We know that $(N_2)^2 = \frac{}{BM}\left(\frac{}{m}\right)\frac{}{h_2}$ ... ($\because$ $\alpha = \beta$ or $q = 1$)

$$= \frac{0.224}{0.304}\left(\frac{10 + 100}{10}\right)\frac{895}{0.224} = 32\,385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

We know that range of speed

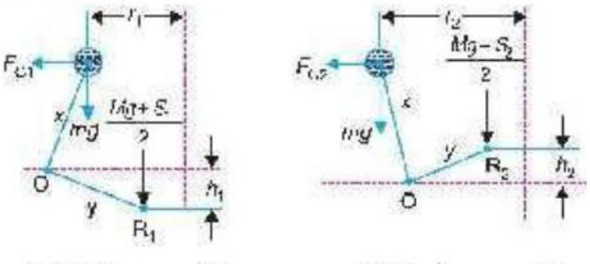
$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m. } \textbf{Ans.}$$

3. The radius of rotation of the balls of a Hartnell governor is 80 mm at the minimum speed of 300 r.p.m. Neglecting gravity effect, determine the speed after the sleeve has lifted by 60 m m. Also determine the initial compression of the spring, the governor effort and the power.

The particulars of the governor are given below:

Length of ball arm = 150 mm ; length of sleeve arm = 100 mm ; mass of each ball = 4 kg ; and stiffness of the spring = 25 N/m m.

Solution. Given : $r_1 = 80$ mm = 0.08 m ; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times 300/60 = 31.42$ rad/s ; $h = 60$ mm = 0.06 m ; $x = 150$ mm = 0.15 m ; $y = 100$ mm = 0.1 m ; $m = 4$ kg ; $s = 25$ N/mm



(a) Minimum position. (b) Maximum position.

The minimum and maximum position of the governor is shown in Fig. 18.36 (*a*) and (*b*) respectively. First of all, let us find the maximum radius of rotation (r_2). We know that lift of the sleeve,

$$h = (r_2 - r_1)\frac{y}{x}$$

or

$$r_2 = r_1 + h \times \frac{x}{y} = 0.08 + 0.06 \times \frac{0.15}{0.1} = 0.17 \text{ m} \quad \ldots (\because h = h_1 + h_2)$$

N

S_1 and S_2= Spring force at the minimum and maximum speed respectively, in newtons

We know centrifugal force at the minimum speed,

$$F_{C1} = m(\omega_1)^2 r_1 = 4\,(31.42)^2\, 0.08 = 316 \text{ N}$$

Now taking moments about the fulcrum *O* of the bell crank lever when in minimum position as shown in Fig 18.36 (*a*). The gravity effect is neglected, *i.e.* the moment due to the weight of balls, sleeve and the bell crank lever arms is neglected.

$$\therefore \quad F_{C1} \times x = \frac{M.g + S_1}{2} \times y \quad \text{or} \quad S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 316 \times \frac{0.15}{0.1} = 948 \text{ N}$$

$$\ldots (\because M = 0)$$

We know that $S_2 - S_1 = h.s$ or $S_2 = S_1 + h.s = 948 + 60 \times 25 = 2448$ N

We know that centrifugal force at the maximum speed,

$$F_{C1} = m(\omega_2)^2 r_2 = \left(\frac{2\pi N_2}{60}\right)^2 r_2 = m\left(\frac{2\pi N_2}{60}\right)^2 0.17 = 0.007\,46\,(N_2)^2$$

Initial compression of the spring

We know that initial compression of the spring $= \frac{S_1}{s} = \frac{918}{25} = 37.92$ mm **Ans.**

Governor effort

We know that the governor effort,

$$P = \frac{S_2 - S_1}{2} = \frac{2448 - 918}{2} = 750 \text{ N} \textbf{ Ans.}$$

Now taking moments about the fulcrum *O* when in maximum position, as shown in Fig. 18.36 (*b*),

$$F_{C2} \times x = \frac{M.g + S_2}{2} \times y$$

$$0.00746\,(N_2)^2\,0.15 = \frac{2448}{2} \times 0.1 \quad \text{or} \quad 0.00112\,(N_2)^2 = 122.4 \qquad \ldots (\because M = 0)$$

$$(N_2)^2 = \frac{122.4}{0.00112} = 109\,286 \quad \text{or} \quad N_2 = 331 \text{ r.p.m.} \textbf{ Ans.}$$

Governor power

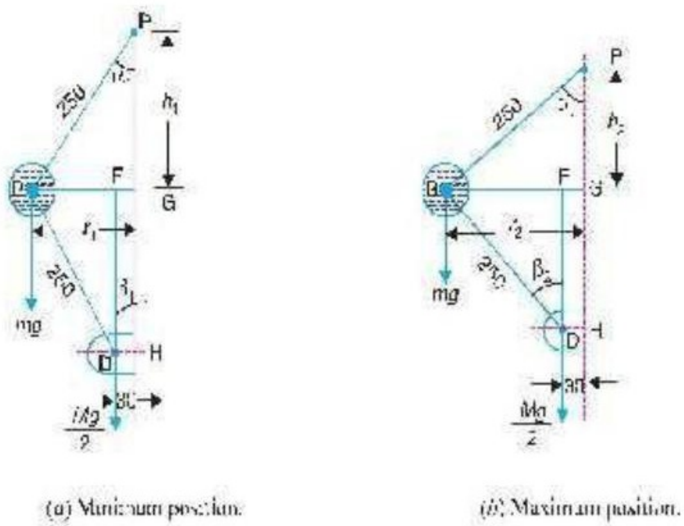
We know that the governor power

$= P \times h = 750 \times 0.06 = 45$ N-m **Ans.**

4. A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 m m from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 m m. Determine the range of speed of the governor.

Solution. Given : $BP = BD = 250$ mm ; $DH = 30$ mm ; $m = 5$ kg ; $M = 50$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.8 (*a*) and (*b*) respectively.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.8

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm ; and N_2 = Maximum speed when $r_2 = BG = 200$ mm.

From Fig. 18.8 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$BF = BG - FG = 150 - 30 = 120 \text{ mm} \qquad \ldots (\because FG = DH)$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (120)^2} = 219 \text{ mm}$$

$\therefore$ $\tan \alpha_1 = BG/PG = 150 / 200 = 0.75$ and $\tan \beta_1 = BF/DF = 120/219 = 0.548$

$$\therefore \quad q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

We know that $$(N_1)^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{5 + \frac{50}{2}(1 + 0.731)}{5} \times \frac{895}{0.2} = 43206$$

$$\therefore \quad N_1 = 208 \text{ r.p.m.}$$

From Fig. 18.8(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$BF = BG - FG = 200 - 30 = 170 \text{ mm}$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (170)^2} = 183 \text{ mm}$$

$\therefore$ $\tan \alpha_2 = BG/PG = 200/150 = 1.333$

and $\tan \beta_2 = BF/DF = 170/183 = 0.93$

$$\therefore \quad q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.93}{1.333} = 0.7$$

We know that

$$(N_2)^2 = \frac{m + \frac{M}{2}(1 + q_2)}{m} \times \frac{895}{h_2} = \frac{5 + \frac{50}{2}(1 - 0.7)}{5} \times \frac{895}{0.15} = 56\,683$$

$$\therefore \quad N_2 = 238 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 238 - 208 = 30 \text{ r.p.m. Ans.}$$

5. A spring loaded governor of the Hartnell type has arms of equal length. The masses rotate in a circle of 130 mm diameter when the sleeve is in the mid position and the ball arms are vertical. The equilibrium speed for this position is 450 r.p.m., neglecting friction. The maximum sleeve movement is to be 25 mm and the maximum variation of speed taking in account the friction to be 5

;

Solution. Given : $x = y$; $d = 130$ mm or $r = 65$ mm $= 0.065$ m ; $N = 450$ r.p.m. or $\omega = 2\pi \times 450/60 = 47.23$ rad/s ; $h = 25$ mm $= 0.025$ m ; $M = 4$ kg ; $F = 30$ N

1. ***Value of each rotating mass***

per cent of the mid position speed. The mass of the sleeve is 4 kg and the fricti may be considered equivalent to 30 N at the sleeve. The power of the govern must be sufficient to over-come the friction by one per cent change of spe either way at mid-position. Determine, neglecting obliquity effect of arms ; The value of each rotating mass : 2. The spring stiffness in N/mm and 3. T initial compression of spring.

Let m = Value of each rotating mass in kg, and

S = Spring force on the sleeve at mid position in newtons.

Since the change of speed at mid position to overcome friction is 1 per cent either way (*i.e.* ± 1%), therefore

Minimum speed at mid position,

$$\omega = \omega - 0.01\omega = 0.99\omega = 0.99 \times 47.13 = 46.66 \text{ rad/s and}$$

maximum speed at mid-position,

$$\omega_2 = \omega + 0.01\omega = 1.01\omega = 1.01 \times 47.13 = 47.6 \text{ rad/s}$$

∴ Centrifugal force at the minimum speed,

$$FC1 = m(\omega_1)^2 r = m(46.66)^2\, 0.065 = 141.5\, m \text{ N}$$

and centrifugal force at the maximum speed,

$$F_{C2} = m(\omega_2)^2 r = m(47.6)^2\, 0.0065 = 147.3$$

m N We know that for minimum speed at mid-position,

$$S + (M.g + F) - 2\, F_{C1} \times \frac{x}{y}$$

or $$S + (4 \times 9.81 - 30) = 2 \times 141.5\, m \times 1 \quad \ldots (\because x = y)$$

∴ $$S + 9.24 = 283\, m \quad \ldots (i)$$

and for maximum speed at mid-position,

$$S + (M.g + F) = 2\, F_{C2} \times \frac{x}{y}$$

$$S + (4 \times 9.81 + 30) = 2 \times 147.3\, m \times 1 \quad \ldots (\because x = y)$$

∴ $$S + 69.24 = 294.6\, m \quad \ldots (ii)$$

From equations (*i*) and (*ii*),

$$m = 5.2 \text{ kg } \textbf{Ans.}$$

2. *Spring stiffness in N/mm*

Let s = Spring stiffness in N/mm.

Since the maximum variation of speed, considering friction is ± 5% of the mid-position speed, therefore,

Minimum speed considering friction,

$$\omega_1' = \omega - 0.05\omega = 0.95\omega = 0.95 \times 47.13 = 44.8 \text{ rad/s}$$

and maximum speed considering friction,

$$\omega_2' = \omega + 0.05\omega = 1.05\omega = 1.05 \times 47.13 = 49.5 \text{ rad/s}$$

We know that minimum radius of rotation considering friction,

$$r_1 = r - h_1 \times \frac{x}{y} = 0.065 - \frac{0.025}{2} = 0.0525 \text{ m}$$

$$\ldots \left(\because x = y \text{ and } h_1 = \frac{h}{2}\right)$$

And maximum radius of rotation considering friction,

We know that for minimum speed considering friction,

$$S_1 + (M.g - F) = 2F_{C1}' \times \frac{x}{y}$$

$$S_1 + (4 \times 9.81 - 30) = 2 \times 548 \times 1 \qquad \ldots (\because x = y)$$

$$\therefore \quad S_1 + 9.24 = 1096 \quad \text{or} \quad S_1 = 1096 - 9.24 = 1086.76 \text{ N}$$

and for maximum speed considering friction,

$$S_2 + (M.g + F) = 2F_{C2}' \times \frac{x}{y} \qquad \ldots (\because x = y)$$

3. Initial compression of the spring

We know that initial compression of the spring

$$= \frac{S_1}{s} = \frac{1086.76}{32.72} = 33.2 \text{ mm} \textbf{ Ans.}$$

6. In an engine governor of the Porter type, the upper and lower arms are 200 m m and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, find, taking friction into account, range of speed of the governor.

Solution . Given : BP = 200 mm = 0.2 m ; BD = 250 mm = 0.25 m ; M = 15 kg ; m = 2 kg ; F = 25 N ; $\alpha_1 = 30°$; $\alpha_2 = 40°$

First of all, let us find the minimum and maximum speed of the governor.

The minimum and maximum position of the governor is shown Fig. 18.7 (*a*) and (*b*) respectively.

Let N_1 = Minimum speed, and N_2 = Maximum speed.

From Fig. 18.7 (*a*), we find that minimum radius of rotation, $r_1 = BG = BP \sin 30° = 0.2 \times 0.5 = 0.1$ m Height of the governor,

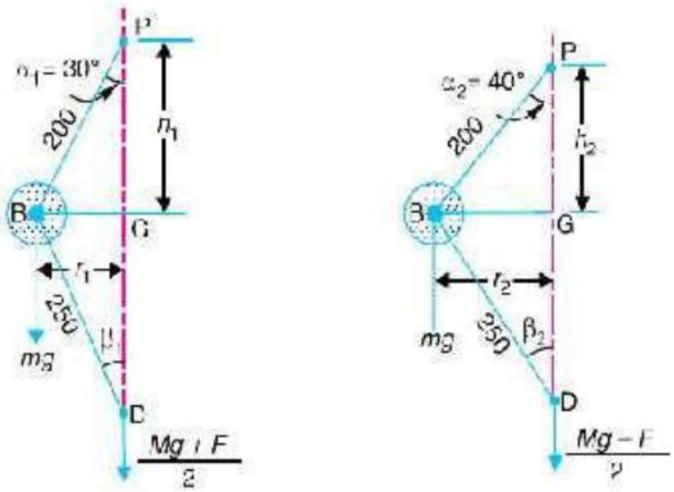
$h_1 = PG = BP \cos 30° = 0.2 \times 0.866 = 0.1732$ m

and $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1)^2} = 0.23 \text{ m}$$

$$\therefore \quad \tan \beta_1 = BG/DG = 0.1/0.23 = 0.4348$$

and $$\tan \alpha_1 = \tan 30° = 0.5774$$

$$\therefore \quad q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.4348}{0.5774} = 0.753$$



All dimensions in mm.

(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.7

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum speed is given by,

$$(N_1)^2 = \frac{m.g - \left(\dfrac{M.g - F}{2}\right)(1 + q_1)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{2 \times 9.81 + \left(\dfrac{15 \times 9.81 - 24}{2}\right)(1 + 0.753)}{2 \times 9.81} \times \frac{895}{0.1732} = 33596$$

$\therefore$ $N_1 = 183.3$ r.p.m.

Now from Fig. 18.7 (*b*), we find that maximum radius of rotation

$$r_2 = BG = BP \sin 40° = 0.2 \times 0.643 = 0.1268 \text{ m}$$

Height of the governor,

$$h_2 = PG = BP \cos 40° = 0.2 \times 0.766 = 0.1532 \text{ m}$$

and $DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1268)^2} = 0.2154$ m

$\therefore$ $\tan \beta_2 = BG/DG = 0.1268 / 0.2154 = 0.59$

and $\tan \alpha_2 = \tan 40° = 0.839$

$\therefore$ $q_2 = \dfrac{\tan \beta_2}{\tan \alpha_2} = \dfrac{0.59}{0.839} = 0.703$

We know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by,

$$(N_2)^2 = \frac{m.g + \left(\frac{m.g + F}{2}\right)(1 + q_2)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 + 24}{2}\right)(1 + 0.703)}{2 \times 9.81} \times \frac{895}{0.1532} = 49\,236$$

$\therefore \quad N_2 = 222$ r.p.m.

We know that range of speed

$= N_2 - N_1 = 222 - 183.3 = 38.7$ r.p.m. **Ans.**

7.In a spring loaded governor of the Hartnell type, the mass of each ball is 1kg, length of vertical arm of the bell crank lever is 100 mm and that of the horizontal arm is 50 m m. The distance of fulcrum of each bell crank lever is 80 mm from the axis of rotation of the governor. The extreme radii of rotation of the balls are 75 mm and 112.5 m m. The maximum equilibrium speed is 5 per cent greater than the minimum equilibrium speed which is 360 r.p.m. Find, neglecting obliq-uity of arms, initial compression of the spring and equilibrium speed corresponding to the radius of rotation of 100 m m.

Solution. Given : $m = 1$ kg ; $x = 100$ mm $= 0.1$ m ; $y = 50$ mm $= 0.05$ m ; $r = 80$ mm $= 0.08$ m ; $r_1 = 75$ mm $= 0.075$ m ; $r_2 = 112.5$ mm $= 0.1125$ m ; $N_1 = 360$ r.p.m. or $\omega_1 = 2\pi \times 360/60 = 37.7$ rad/s

Since the maximum equilibrium speed is 5% greater than the minimum equilibrium speed (ω_1), therefore maximum equilibrium speed,

$$\omega_2 = 1.05 \times 37.7 = 39.6 \text{ rad/s}$$

We know that centrifugal force at the minimum equilibrium speed,

$$F_{C1} = m(\omega_1)^2 r_1 = 1 (37.7)^2\, 0.075 = 106.6 \text{ N}$$

and centrifugal force at the maximum equilibrium speed,

$$F_{C2} = m(\omega_2)^2 r_2 = 1 (39.6)^2\, 0.1125 = 176.4 \text{ N}$$

Initial compression of the spring

Let S_1 = Spring force corresponding to ω_1, and

S_2 = Spring force corresponding to ω_2.

Since the obliquity of arms is neglected, therefore for minimum equilibrium position

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 106.6 \times \frac{0.1}{0.05} = 426.4 \text{ N}$$

$\therefore \quad S_1 = 426.4$ N ...($\because M = 0$)

and for maximum equilibrium position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 176.4 \times \frac{0.1}{0.05} = 705.6 \text{ N}$$

$\therefore \quad S_2 = 705.6$ N ...($\because M = 0$)

We know that lift of the sleeve,

$$h = (r_2 - r_1)\frac{y}{x} = (0.1125 - 0.075)\frac{0.05}{0.1} = 0.018\,75 \text{ m}$$

and stiffness of the spring $s = \dfrac{S_2 - S_1}{h} = \dfrac{705.6 - 426.4}{0.018\,75} = 14\,890 \text{ N/m} = 14.89 \text{ N/mm}$

∴ Initial compression of the spring

$$= \frac{S_1}{s} = \frac{426.4}{14.89} = 28.6 \text{ mm} \quad \textbf{Ans.}$$

Equilibrium speed corresponding to radius of rotation r = 100 mm = 0.1 m

Let N = Equilibrium speed in r.p.m.

Since the obliquity of the arms is neglected, therefore the centrifugal force at any instant,

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right)$$

$$= 106.6 + (176.4 - 106.6)\left(\frac{0.1 - 0.075}{0.1125 - 0.075}\right) = 153 \text{ N}$$

We know that centrifugal force (F_C),

$$153 = m.\omega^2.r = 1\left(\frac{2\pi N}{60}\right)^2 0.1 = 0.0011\,N^2$$

∴ $N^2 = 153 / 0.0011 = 139\,090$ or $N = 373$ r.p.m. **Ans.**

5.20 REVIEW QUESTIONS

(1) What are the effects of friction and of adding a central weight to the sleeve of a Watt governor ?

(2) What is stability of a governor ? Sketch the controlling force *versus* radius diagrams for a stable, unstable and isochronous governor. Derive the conditions for stability.

(3) Prove that the sensitiveness of a Proell governor is greater than that of a Porter governor

(4) When the sleeve of a Porter governor moves upwards, the governor speed

(5) When the relation between the controlling force (F_C) and radius of rotation (r) for a spring controlled governor is $F_C = a.r + b$, then the governor will be

5.21 TUTORIAL PROBLEMS

1. In a governor of the Hartnell type, the mass of each ball is 1.5 kg and the lengths of the vertical and horizontal arms of the bell crank lever are 100 mm and 50 mm respectively. The fulcrum of the bell crank lever is at a distance of 90 mm from the axis of rotation. The maximum and minimum radii of rotation of balls are 120 mm and 80 mm and the corresponding equilibrium speeds are 325 and 300 r.p.m. Find the stiffness of the spring and the equilibrium speed when the radius of rotation is 100 mm.

[Ans. 18 kN/m, 315 r.p.m.]

2. A governor of the Hartnell type has equal balls of mass 3 kg, set initially at a radius of 200 mm. The arms of the bell crank lever are 110 mm vertically and 150 mm horizontally. Find : 1. the initial compressive force on the spring, if the speed for an initial ball radius of 200 mm is 240 r.p.m. ; and 2. the stiffness of the spring required to

permit a sleeve movement of 4 mm on a fluctuation of 7.5 per cent in the engine speed. **[Ans. 556 N ; 23.75 N/mm]**

3. A four wheel trolley car of total mass 2000 kg running on rails of 1 m gauge, rounds a curve of 25 m

radius at 40 km / h. The track is banked at 10°. The wheels have an external diameter of 0.6 m and each pair of an axle has a mass of 200 kg. The radius of gyration for each pair is 250 mm. The height of C.G. of the car above the wheel base is 0.95 m. Allowing for centrifugal force and gyroscopic couple action, determine the pressure on each rail. **[Ans. 4328 N ; 16 704 N]**

4. Each paddle wheel of a steamer have a mass of 1600 kg and a radius of gyration of 1.2 m. The steamer turns to port in a circle of 160 m radius at 24 km / h, the speed of the paddles being 90 r.p.m. Find the

magnitude and effect of the gyroscopic couple acting on the steamer. **[Ans. 905.6 N-m]**

5. The rotor of the turbine of a yacht makes 1200 r.p.m. clockwise when viewed from stern. The rotor has

a mass of 750 kg and its radius of gyration is 250 mm. Find the maximum gyroscopic couple transmit-ted to the hull (body of the yacht) when yacht pitches with maximum angular velocity of 1 rad /s. What is the effect of this couple ? **[Ans. 5892 N-m]**

UNIT-I FORCE ANALYSIS

Part-A(2 Marks)

1. **Write D'Alembert's principle. What is the use of it? (AU - APRIL/MAY-2011)**
   D'Alembert's principle states that the inertia forces and torques, and the external forces and torques acting on a body together result in statical equilibrium. **Use (or) Application (or) significance:**
   By applying D'Alembert's principle to a dynamic analysis problem, we can reduce it into an equivalent problem of static equilibrium.

2. **Distinguish between static force and inertia force. (AU - MAY /JUNE -2013)**
   - While analyzing the mechanism, if mass of the body and inertia force are not considered, then it is called static force.
   - The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position.

   Inertia force = - Acceleration force = - m. a

3. **What are the conditions for a body to be in equilibrium under the action of (a) two forces, (b) two forces and torque (AU - MAY /JUNE -2012)**
   a) Condition for two forces:
   - The forces are of the same magnitude.
   - The forces act along the same line.
   - The forces are in opposite direction.

   b) Condition for two forces & Torque:
   - The forces are equal in magnitude, parallel in direction and opposite in sense.
   - The forces from a couple, which is equal and opposite to the applied torque.

4. **What is engine shaking force? (AU - MAY /JUNE -2013)**
   The force produces in an engine due to the mass of piston, and mass of the connecting rod is called engine shaking force.

5. **Differentiate between static & dynamic equilibrium. (or) What are the conditions for a body to be in static and dynamic equilibrium? (AU-NOV/DEC-2007)**
   Necessary and sufficient conditions for static and dynamic equilibrium are:
   1.Vector sum of all the forces acting on a body is zero.
   2. The vector sum of all the moments of all the forces acting about any arbitrary point or axis is zero.
   First conditions are sufficient conditions for static equilibrium together with second condition is necessary for dynamic equilibrium.

6. **What is free body diagram? (AU- APRIL/MAY -2005)**
   A free body diagram is a sketch of the isolated or free body which shows all the pertinent weight force, the externally applied loads, and the reaction from its supports &connections acting upon it by the removed elements.

7. **Define piston effort and crank effort. (AU-NOV/DEC-2012)**
   - **Piston effort** is defined as the net or effective force applied on the piston, along the line of stroke. It is also known as effective driving force (or) net load on the gudgeon pin.
   - **Crank effort** is the net effort (force) applied at the crank pin perpendicular to the crank, which gives the required turning moment on the crankshaft.
8. **Define crank pin effort. (AU-NOV/DEC-2006)**
   The component of force acting along connecting rod perpendicular to the crank is known as crank-pin effort.

9. **What are the requirements of an equivalent dynamical system? (or) Write the conditions for any disturbed mass have the same dynamical properties.**
   (AU-NOV/DEC-2013) The mass of the rigid body must be equal to the sum of masses of two concentrated masses.

$$\text{i.e. } m1 + m2 = m$$

 The centre of gravity of the two masses must coincide with the centre of gravity of the rigid body.

$$\text{i.e. } m1l1 = m2l2$$

 The sum of mass moment of inertia of two masses about their centre of gravity is equal to the mass moment of inertia of the rigid body.

$$\text{i.e. } l1 \, . \, l2 = (kG)^2$$

10. **What is the function of a flywheel? how does it differ from that of a governor? (AU-NOV/DEC-2012)** The function of flywheel is to reduce the fluctuations of speed during a cycle above and below the mean value for constant load from prime mover. The function of governor is to control the mean speed over a period for output load variations.
   - Flywheel works continuously from cycle to cycle. Governor works intermittently, i.e. only when there is change in the load.
   - Flywheel has no influence on mean speed of the prime mover. Governor has no influence over cyclic speed fluctuations.
11. **Differentiate between the usage of flywheel in engines and punching presses with turning moment diagrams. (AU- APRIL/MAY -2011)**
   - In the engines, the output of the flywheel was constant and input torque was varying during each cycle.
   - In case of punching press, the input of the flywheel is constant and output torque is varying cyclically.

12. Define coefficient of fluctuation of energy. (AU-NOV/DEC-2010)

It is the ratio of maximum fluctuation of energy to the work done per cycle.
C_E=Maximum fluctuation of energy/Work done per cycle.

13. Why flywheels are needed in forging and pressing operations? (AU- APRIL/MAY -2005)

In both forging and pressing operations, flywheels are required to control the variations in speed during each cycle of an engine.

14. Define unbalance and spring surge? (AU- APRIL/MAY -2003)

Unbalance: A disc cam produces unbalance because its mass is not symmetrical with the axis of rotation.
Spring surge: Spring surge means vibration of the retaining spring.

15. Define windup. What is the remedy for camshaft windup?

- Twisting effect produced in the camshaft during the raise of heavy load follower is called as windup
- Camshaft windup can be prevented to a large extend by mounting the flywheel as close as possible to the cam.

Part – B

1. (i) Derive the equation of forces on the reciprocating parts of an engine, neglecting the weight of the connecting rod. (12) (AU-NOV/DEC-2013)

The various forces acting on the reciprocating parts of a horizontal engine are shown in Fig. The expressions for these forces, neglecting the weight of the connecting rod, may be derived as discussed below :

1. *Piston effort*. It is the net force acting on the piston or crosshead pin, along the line of stroke. It is denoted by F_P in Fig.

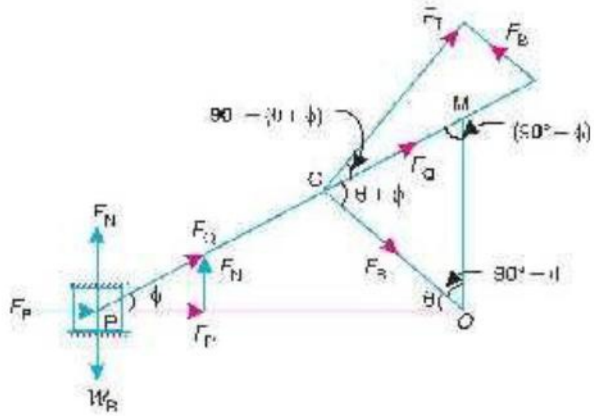


Fig. Forces on the reciprocating parts of an engine.

Let m_R = Mass of the reciprocating parts, *e.g.* piston, crosshead pin or gudgeon pin etc., in kg, and

W_R = Weight of the reciprocating parts in Newton = $m_R g$

We know that acceleration of the reciprocating parts,

$$a_R = a_P = \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

Accelerating force or inertia force of the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

Therefore,

Piston effort, F_P = Net load on the piston ∓ Inertia force

$= F_L \mp F_I$ ...(Neglecting frictional resistance)

$= F_L \mp F_I - R_F$ ...(Considering frictional resistance)

where R_F = Frictional resistance.

The –ve sign is used when the piston is accelerated, and +ve sign is used when the piston is retarded.

In a double acting reciprocating steam engine, net load on the piston,

$$F_L = p_1 A_1 - p_2 A_2 = p_1 A_1 - p_2 (A_1 - a)$$

where p_1, A_1 = Pressure and cross-sectional area on the back end side of the piston,

p_2, A_2 = Pressure and cross-sectional area on the crank end side of the piston,

a = Cross-sectional area of the piston rod.

2.*Force acting along the connecting rod.* It is denoted by F_Q in Fig. 15.8. From the geom-etry of the figure, we find that

$$F_Q = \frac{F_P}{\cos\varphi}$$

We know that $\cos\varphi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$

$$\therefore \quad F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

3. *Thrust on the sides of the cylinder walls or normal reaction on the guide bars.* It is denoted by F_N in Fig. 15.8. From the figure, we find that

$$F_N = F_Q \sin\varphi = \frac{F_P}{\cos\varphi} \times \sin\varphi = F_P \tan\varphi \quad \text{... i.e., } F_Q = \frac{F_P}{\cos\varphi}$$

4. ***Crank-pin effort and thrust on crank shaft bearings.***

The force acting on the connecting rod F_Q may be resolved into two components, one perpendicular to the crank and the other along the crank. The component of F_Q perpendicular to the crank is known as crank-pin effort and it is denoted by F_T in Fig. 15.8. The component of F_Q along the crank produces a thrust on the crank shaft bearings and it is denoted by F_B in Fig. 15.8. Resolving F_Q perpendicular to the crank,

Crank effort,

$$T = F_T \times r = \frac{F_P \sin(\theta + \phi)}{\cos\phi} \times r$$

$$= \frac{F_P (\sin\theta \cos\phi + \cos\theta \sin\phi)}{\cos\phi} \times r$$

$$= F_P \left(\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi} \right) \times r$$

$$= F_P (\sin\theta + \cos\theta \tan\phi) \times r \quad \text{...(i)}$$

We know that $l \sin\phi = r \sin\theta$

$$\sin\phi = \frac{r}{l} \sin\theta = \frac{\sin\theta}{n} \quad \text{...}\left(\because n = \frac{l}{r}\right)$$

and

$$\cos\phi = \sqrt{1 - \sin^2\phi} = \sqrt{1 - \frac{\sin^2\theta}{n^2}} = \frac{1}{n}\sqrt{n^2 - \sin^2\theta}$$

$\therefore$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{n} \times \frac{n}{\sqrt{n^2 - \sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Substituting the value of $\tan\phi$ in equation (i), we have crank effort,

$$T = F_P \left(\sin\theta + \frac{\cos\theta \sin\theta}{\sqrt{n^2 - \sin^2\theta}} \right) \times r$$

$$= F_P \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right) \quad \text{...(ii)}$$

$$\text{...}(\because 2\cos\theta \sin\theta = \sin 2\theta)$$

$$F_T = F_Q \sin(\theta + \varphi) = \frac{F_P}{\cos\varphi} \sin(\theta + \varphi)$$

and resolving F_Q along the crank,

$$F_B = F_Q \cos(\theta + \varphi) = \frac{F_P}{\cos\varphi} \cos(\theta + \varphi)$$

5. **Crank effort** or **turning moment or torque on the crank shaft.** The product of the crankpin effort (F_T) and the crank pin radius (r) is known as **crank effort** or **turning moment** or **torque on the crank shaft**. Mathematically,

1 (ii) What is turning moment diagram and draw it's for four stroke IC engine? (AU-NOV/DEC-2013)

The turning moment diagram is the graphical representation of the turning moment (T) for various position of the crank (θ).
A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

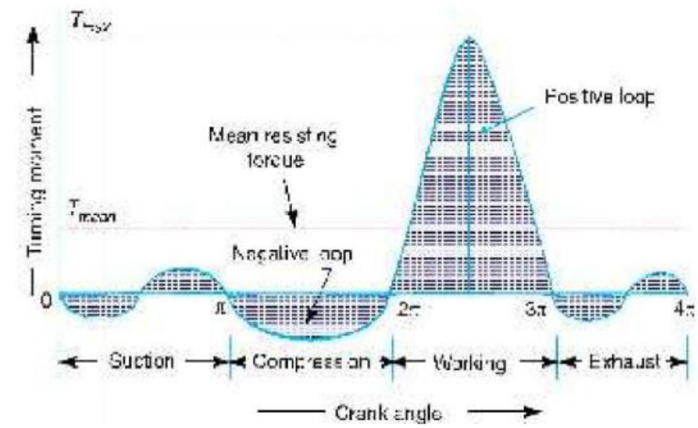


Fig. Turning moment diagram for a four stroke cycle internal combustion engine.

2. The crank and connecting rod of a petrol engine, running at 1800 r.p.m.are 50 mm and

ing parts is 1 kg. At a point during the power stroke, the pressure N/mm², when it has moved 10 mm from the inner dead centre. Determ the gudgeon pin, 2. Thrust in the connecting rod, 3. Reaction betw cylinder, and 4. The engine speed at which the above values b
NOV/DEC-2011)

200 mm respectively. The diameter of the piston is 80 mm and the mass of the reciprocaton

Solution. Given : N = 1800 r.p.m. or $\omega = 2\pi \times 1800/60 = 188.52$ rad/s ; r = 50 mm = 0.05 m; l = 200 mm ; D = 80 mm ; m_R = 1 kg ; p = 0.7 N/mm^2 ; x = 10 mm

1. *Net load on the gudgeon pin*

We know that load on the piston,

$$F_L = \frac{\pi}{4} D^2 \times p = \frac{\pi}{4} \times (80)^2 \times 0.7 = 3520 \text{ N}$$

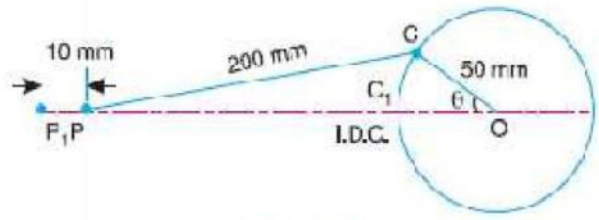


Fig. 15.10

When the piston has moved 10 mm from the inner dead centre, *i.e.* when $P_1P = 10$ mm, the crank rotates from OC_1 to OC through an angle θ as shown in Fig. 15.10.

By measurement, we find that *θ = 33°.

We know that ratio of lengths of connecting rod and crank,

$$n = l/r = 200/50 = 4$$

and inertia force on the reciprocating parts,

$$F_I = m_R . a_R = m_R . \omega^2 . r \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

$$= 1 \times (188.52)^2 \times 0.05 \left(\cos 33° + \frac{\cos 66°}{4} \right) = 1671 \text{ N}$$

We know that net load on the gudgeon pin,

$$F_P = F_L - F_I = 3520 - 1671 = 1849 \text{ N} \textbf{ Ans.}$$

2. Thrust in the connecting rod

Let ϕ = Angle of inclination of the connecting rod to the line of stroke.

We know that, $$\sin \phi = \frac{\sin \theta}{n} = \frac{\sin 33°}{4} = \frac{0.5446}{4} = 0.1361$$

$\therefore$ $\phi = 7.82°$

We know that thrust in the connecting rod,

$$F_Q = \frac{F_P}{\cos \phi} = \frac{1849}{\cos 7.82°} = 1866.3\text{N} \textbf{ Ans.}$$

3. Reaction between the piston and cylinder

We know that reaction between the piston and cylinder,

$$F_N = F_P \tan \phi = 1849 \tan 7.82° = 254 \text{ N} \textbf{ Ans.}$$

4. Engine speed at which the above values will become zero

A little consideration will show that the above values will become zero, if the inertia force on the reciprocating parts (F_I) is equal to the load on the piston (F_L). Let ω_1 be the speed in rad/s, at which $F_I - F_L$.

$$\therefore \quad m_R (\omega_1)^2 r \left(\cos \theta + \frac{\cos 2\theta}{n} \right) - \frac{\pi}{4} D^2 \times p$$

$$1 (\omega_1)^2 \times 0.05 \left(\cos 33° + \frac{\cos 66°}{4} \right) = \frac{\pi}{4} \times (80)^2 \times 0.7 \quad \text{or} \quad 0.0 47 (\omega_1)^2 = 3520$$

$$\therefore \quad (\omega_1)^2 = 3520 / 0.047 = 74\ 894 \text{ or } \omega_1 = 273.6 \text{ rad/s}$$

$\therefore$ Corresponding speed in r.p.m.,

$$N_1 = 273.6 \times 60 / 2\pi = 2612 \text{ r.p.m.} \textbf{ Ans.}$$

3. The crank and connecting rod lengths of an engine are 125 mm and 500 mm respectively. The mass of the connecting rod is 60 kg and its centre of gravity is 275 mm from the crosshead pin centre, the radius of gyration about centre of gravity being 150 mm.

If the engine speed is 600 r.p.m. for a crank position of 45° from the inner dead centre, determine, using Klien's or any other construction 1. the acceleration of the piston; 2. the magni-tude, position and direction of inertia force due to the mass of the connecting rod.

(AU-APR/MAY-2011)

Solution. Given : $r = OC = 125$ mm ; $l = PC = 500$ mm; $m_C = 60$ kg ; $PG = 275$ mm ; $m_C = 60$ kg ; $PG = 275$ mm ; $k_G = 150$ mm ; $N = 600$ r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad/s ; $\theta = 45°$

1. ***Acceleration of the piston***

Let a_P = Acceleration of the piston.

First of all, draw the configuration diagram *OCP*, as shown in Fig. 15.24, to some suitable scale, such that

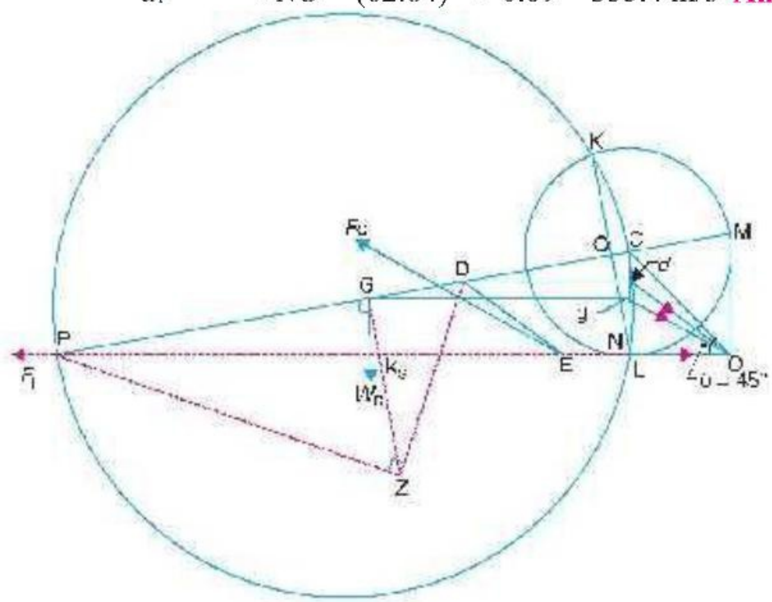
$$OC = r = 125 \text{ mm} ; PC = l = 500 \text{ mm} ; \text{and } \theta = 45°.$$

Now, draw the Klien's acceleration diagram *OCQN*, as shown in Fig, in the same manner as already discussed. By measurement,

$$NO = 90 \text{ mm} = 0.09 \text{ m}$$

Acceleration of the piston,

$$a_P = \omega^2 \times NO = (62.84)^2 \times 0.09 = 355.4 \text{ m/s} \quad \textbf{Ans.}$$



2. ***The magnitude, position and direction of inertia force due to the mass of the connecting rod***

The magnitude, postition and direction of the inertia force may be obtained as follows:

(i) Replace the connecting rod by dynamical equivalent system of two masses, assuming that one of the masses is placed at *P* and the other mass at *D*.

(ii) Locate the points *G* and *D* on *NC* which is the acceleration image of the connecting rod. Let these points are *g* and *d* on *NC*. Join *gO* and *dO*. By measurement, $gO = 103$ mm $= 0.103$ m

Acceleration of *G*, $a_G = \omega^2 \times gO$, acting in the direction from *g* to *O*.

(iii) From point *D*, draw *DE* parallel to *dO*. Now *E* is the point through which the inertia force of the connecting rod passes. The magnitude of the inertia force of the connecting rod is given by $F_C = m_C \times \omega^2 \times gO = 60 \times (62.84)^2 \times 0.103 = 24\,400$ N $= 24.4$ kN **Ans.**

(iv) From point E, draw a line parallel to gO, which shows the position of the inertia force of the connecting rod and acts in the opposite direction of gO.

4 (i) A connecting rod is suspended from a point 25 mm above the centre of small end, and 650 mm above its centre of gravity, its mass being 37.5 kg. When permitted to oscil-late, the time period is found to be 1.87 seconds. Find the dynamical equivalent system constituted of two masses, one of which is located at the small end centre.

(AU-APR/MAY-2010)

Solution. Given : $h = 650$ mm $= 0.65$ m ; $l_1 = 650 - 25 = 625$ mm $= 0.625$ m ; $m = 37.5$ kg ; $t_p = 1.87$ s

First of all, let us find the radius of gyration (k_G) of the connecting rod (considering it is a compound pendulum), about an axis passing through its centre of gravity, G.

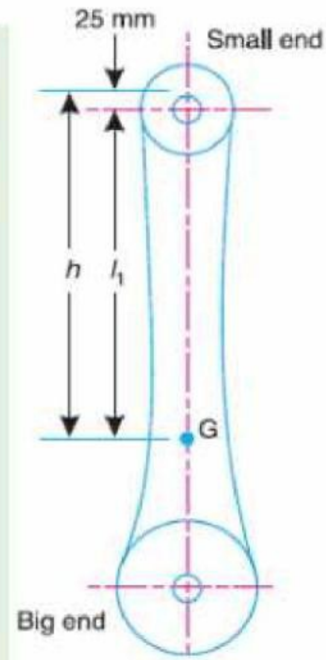


Fig. 15.18

We know that for a compound pendulum, time period of oscillation (t_p).

$$1.87 = 2\pi\sqrt{\frac{(k_G)^2 + h^2}{g.h}} \quad \text{or} \quad \frac{1.87}{2\pi} = \sqrt{\frac{(k_G)^2 + (0.65)^2}{9.81 \times 0.65}}$$

Squaring both sides, we have

$$0.0885 = \frac{(k_G)^2 + 0.4225}{6.38}$$

$$(k_G)^2 = 0.0885 \times 6.38 - 0.4225 = 0.1425 \text{ m}^2$$

$$\therefore \quad k_G = 0.377 \text{ m}$$

It is given that one of the masses is located at the small end centre. Let the other mass is located at a distance l_2 from the centre of gravity G, as shown in Fig. 15.19. We know that, for a dynamically equivalent system,

$$l_1.l_2 = (k_G)^2$$

$$\therefore \quad l_2 = \frac{(k_G)^2}{l_1} = \frac{0.1425}{0.625} = 0.228 \text{ m}$$

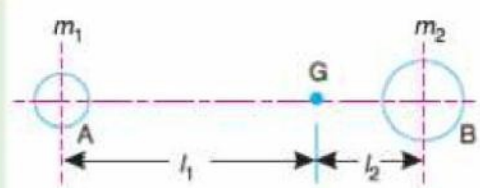


Fig. 15.19

Let m_1 = Mass placed at the small end centre A, and

m_2 = Mass placed at a distance l_2 from G, *i.e.* at B.

We know that, for a dynamically equivalent system,

$$m_1 = \frac{l_2.m}{l_1 + l_2} = \frac{0.228 \times 37.5}{0.625 + 0.228} = 10 \text{ kg} \quad \textbf{Ans.}$$

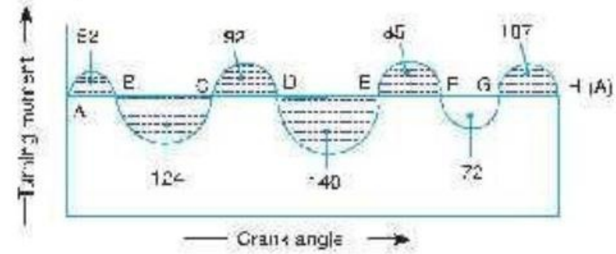
and

$$m_2 = \frac{l_1.m}{l_1 + l_2} = \frac{0.625 \times 37.5}{0.625 + 0.228} = 27.5 \text{ kg} \quad \textbf{Ans.}$$

4(ii) The turning moment diagram for a multicylinder engine has been drawn to a scale 1 m m = 600 N-m vertically and 1 m m = 3° horizontally. The intercepted areas between the output torque curve and the mean resistance line, taken in order from one end, are as follows :

+ 52, – 124, + 92, – 140, + 85, – 72 and + 107 m m^2, when the engine is running at a speed of 600 r.p.m. If the total fluctuation of speed is not to exceed ± 1.5% of the mean, find the necessary mass of the flywheel of radius 0.5 m. (AU-MAY/JUNE-2012)

Solution. Given : N = 600 r.p.m. or $\omega = 2\pi \times 600 / 60 = 62.84$ rad / s ; R = 0.5 m



Since the total fluctuation of speed is not to exceed 1.5% of the mean speed,

therefore $\omega_1 - \omega_2$ = 3%, ω = 0.03 ω

and coefficient of fluctuation of speed,

$$C_s = \frac{\omega_1 - \omega_2}{\omega} = 0.03$$

The turning moment diagram is shown in Fig. 16.7.

Since the turning moment scale is 1 mm = 600 N-m and crank angle scale is 1 mm = 3° = 3° × π/180 = π / 60 rad, therefore

1 mm^2 on turning moment diagram

$$= 600 \times \pi/60 = 31.42 \text{ N-m}$$

Let the total energy at $A = E$, then referring to Fig. 16.7,

Energy at $B = E + 52$ ...(Maximum energy)

Energy at $C = E + 52 - 124 = E - 72$

Energy at $D = E - 72 + 92 = E + 20$

Energy at $E = E + 20 - 140 = E - 120$ ...(Minimum energy)

Energy at $F = E - 120 + 85 = E - 35$

Energy at $G = E - 35 - 72 = E - 107$

Energy at $H = E - 107 + 107 = E$ = Energy at A

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + 52) - (E - 120) = 172 = 172 \times 31.42 = 5404 \text{ N-m}$$

Let m = Mass of the flywheel in kg.

We know that maximum fluctuation of energy (ΔE),

$$5404 = m.R^2.\omega^2.C_S = m \times (0.5)^2 \times (62.84)^2 \times 0.03 = 29.6\, m$$

$$\therefore \quad m = 5404 / 29.6 = 183 \text{ kg} \quad \textbf{Ans.}$$

5. A shaft fitted with a flywheel rotates at 250 r.p.m. and drives a machine. The torque of machine varies in a cyclic manner over a period of 3 revolutions. The torque rises from 750 N-m to 3000 N-m uniformly during 1/2 revolution and remains constant for the following revolution. It then falls uniformly to 750 N-m during the next 1/2 revolution and remains constant for one revolution, the cycle being repeated thereafter.

Determine the power required to drive the machine and percentage fluctuation in speed, if the driving torque applied to the shaft is constant and the mass of the flywheel is 500 kg with radius of gyration of 600 m m. (AU-MAY/JUNE-2013)

Solution. Given : N = 250 r.p.m. or $\omega = 2\pi \times 250/60 = 26.2$ rad/s ; m = 500 kg ; k = 600 mm = 0.6 m

The turning moment diagram for the complete cycle is shown in Fig.

We know that the torque required for one complete cycle

$$= \text{Area of figure } OABCDEF$$

$$= \text{Area } OAEF + \text{Area } ABG + \text{Area } BCHG + \text{Area } CDH$$

$$= OF \times OA + \frac{1}{2} \times AG \times BG + GH \times CH + \frac{1}{2} \times HD \times CH$$

$$= 6\pi \times 750 + \frac{1}{2} \times \pi(3000 - 750) + 2\pi(3000 - 750) + \frac{1}{2} \times \pi(3000 - 750)$$

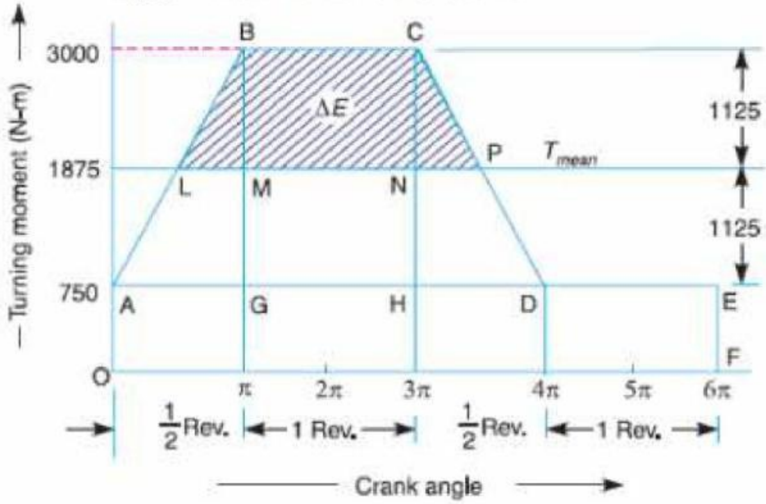
$$= 11\,250\,\pi \text{ N-m} \qquad ...(i)$$

If T_{mean} is the mean torque in N-m, then torque required for one complete cycle

$$= T_{mean} \times 6\,\pi \text{ N-m} \qquad ...(ii)$$

From equations *(i)* and *(ii)*,

$$T_{mean} = 11\,250\,\pi / 6\,\pi = 1875 \text{ N-m}$$



Power required to drive the machine

We know that power required to drive the machine,

$$P = T_{mean} \times \omega = 1875 \times 26.2 = 49\,125 \text{ W} = 49.125 \text{ kW} \textbf{ Ans.}$$

Coefficient of fluctuation of speed

Let C_S = Coefficient of fluctuation of speed.

First of all, let us find the values of LM and NP. From similar triangles ABG and BLM,

$$\frac{LM}{AG} = \frac{BM}{BG} \quad \text{or} \quad \frac{LM}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad LM = 0.5\,\pi$$

Now, from similar triangles CHD and CNP,

$$\frac{NP}{HD} = \frac{CN}{CH} \quad \text{or} \quad \frac{NP}{\pi} = \frac{3000 - 1875}{3000 - 750} = 0.5 \quad \text{or} \quad NP = 0.5\,\pi$$

$$BM = CN = 3000 - 1875 = 1125 \text{ N-m}$$

Since the area above the mean torque line represents the maximum fluctuation of energy, therefore, maximum fluctuation of energy,

$$\Delta E = \text{Area } LBCP = \text{Area } LBM + \text{Area } MBCN + \text{Area } PNC$$

$$= \frac{1}{2} \times LM \times BM + MN \times BM + \frac{1}{2} \times NP \times CN$$

$$= \frac{1}{2} \times 0.5\pi \times 1125 + 2\pi \times 1125 + \frac{1}{2} \times 0.5\pi \times 1125$$

$$= 8837 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$$8837 = m.k^2.\omega^2.C_S = 500 \times (0.6)^2 \times (26.2)^2 \times C_S = 123\,559\, C_S$$

$$C_S = \frac{8837}{123559} = 0.071 \textbf{ Ans.}$$

6. A single cylinder, single acting, four stroke gas engine develops 20 kW at 300 r.p.m. The work done by the gases during the expansion stroke is three times the work done on the gases during the compression stroke, the work done during the suction and exhaust strokes being negligible. If the total fluctuation of speed is not to exceed ± 2 per cent of the mean speed and the turning moment diagram during compression and expansion is assumed to be triangular in shape, find the moment of inertia of the flywheel.

(AU-NOV/DEC-2013)

Solution. Given : $P = 20 \text{ kW} = 20 \times 10^3 \text{ W}$; $N = 300$ r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

Since the total fluctuation of speed ($\omega_1 - \omega_2$) is not to exceed ± 2 per cent of the mean speed (ω), therefore

$$\omega_1 - \omega_2 = 4\%\ \omega$$

and coefficient of fluctuation of speed,

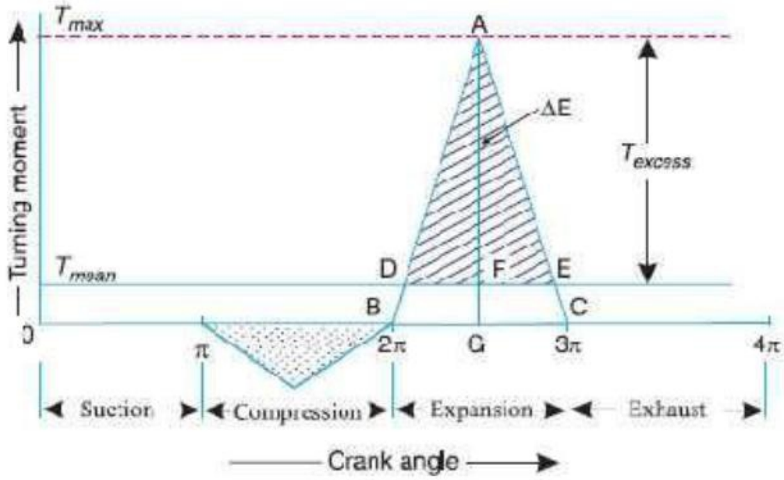
$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 4\% = 0.04$$

The turning moment-crank angle diagram for a four stroke engine is shown in Fig. It is assumed to be triangular during compression and expansion strokes, neglecting the suction and exhaust strokes.

We know that for a four stroke engine, number of working strokes per cycle,

$$n = N/2 = 300 / 2 = 150$$

$\therefore$ Work done/cycle $= P \times 60/n = 20 \times 10^3 \times 60/150 = 8000$ N-m ...(*i*)



Since the work done during suction and exhaust strokes is negligible, therefore net work done per cycle (during compression and expansion strokes)

$$= W_E \quad W_C = W_E \quad \frac{W_E}{3} = \frac{2}{3} W_E \qquad \dots (\because W_E = 3W_C) \dots (ii)$$

Equating equations (*i*) and (*ii*), work done during expansion stroke,

$$W_E = 8000 \times 3/2 = 12\,000 \text{ N-m}$$

We know that work done during expansion stroke (W_E),

$$12\,000 = \text{Area of triangle } ABC = \frac{1}{2} \times BC \times AG = \frac{1}{2} \times \pi \times AG$$

$$\therefore \quad AG = T_{mus} = 12\,000 \times 2/\pi = 7638 \text{ N-m}$$

and mean turning moment,

$$*T_{mean} = FG = \frac{\text{Work done/cycle}}{\text{Crank angle/cycle}} = \frac{8000}{4\pi} = 637 \text{ N-m}$$

$\therefore$ Excess turning moment,

$$T_{excess} = AF = AG - FG = 7638 - 637 = 7001 \text{ N-m}$$

Now, from similar triangles ADE and ABC,

$$\frac{DE}{BC} = \frac{AF}{AG} \quad \text{or} \quad DE = \frac{AF}{AG} \times BC = \frac{7001}{7638} \times \pi = 2.88 \text{ rad}$$

Since the area above the mean turning moment line represents the maximum fluctuation of energy, therefore maximum fluctuation of energy,

$$\Delta E = \text{Area of } \Delta ADE = \frac{1}{2} \times DE \times AF = \frac{1}{2} \times 2.88 \times 7001 = 10081 \text{ N-m}$$

Let I = Moment of inertia of the flywheel in kg-m^2.

We know that maximum fluctuation of energy (ΔE),

$$10\,081 = I.\omega^2.C_S = I \times (31.42)^2 \times 0.04 = 39.5\, I$$

$$\therefore \quad I = 10081/39.5 = 255.2 \text{ kg-m}^2 \quad \textbf{Ans.}$$

7. The turning moment curve for an engine is represented by the equation, T = (20 000 + 9500 sin 2θ – 5700 cos 2θ) N-m, where θ is the angle moved by the crank from inner dead centre. If the resisting torque is constant, find: 1. Power developed by the engine ; 2. Moment of inertia of flywheel in kg-m^2, if the total fluctuation of speed is not exceed 1% of mean speed which is 180 r.p.m; and 3. Angular acceleration of the flywheel when the crank has turned through 45° from inner dead centre. (AU-NOV/DEC-2012)

Solution. Given : $T = (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta)$ N-m ; $N = 180$ r.p.m. or $\omega = 2\pi \times 180/60 = 18.85$ rad/s

Since the total fluctuation of speed ($\omega_1 - \omega_2$) is 1% of mean speed (ω), therefore coefficient of fucruation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 1\% = 0.01$$

1. *Power developed by the engine*

We know that work done per revolution

$$= \int_0^{2\pi} T\, d\theta = \int_0^{2\pi} (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta)\, d\theta$$

$$= \left[20\,000\theta - \frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2} \right]_0^{2\pi}$$

$$= 20\,000 \times 2\pi = 40\,000\,\pi \text{ N-m}$$

and mean resisting torque of the engine,

$$T_{mean} = \frac{\text{Work done per revolution}}{2\pi} = \frac{40\,000}{2\pi} = 20000 \text{ N-m}$$

We know that power developed by the engine

$$= T_{mean} \cdot \omega = 20\,000 \times 18.85 = 377\,000 \text{ W} = 377 \text{ kW} \quad \textbf{Ans.}$$

2. *Moment of inertia of the flywheel*

Let I = Moment of inertia of the flywheel in kg-m^2.

The turning moment diagram for one stroke (*i.e.* half revolution of the crankshaft) is shown in Fig. 16.13. Since at points *B* and *D*, the torque exerted on the crankshaft is equal to the mean resisting torque on the flywheel, therefore,

$$T = T_{mean}$$

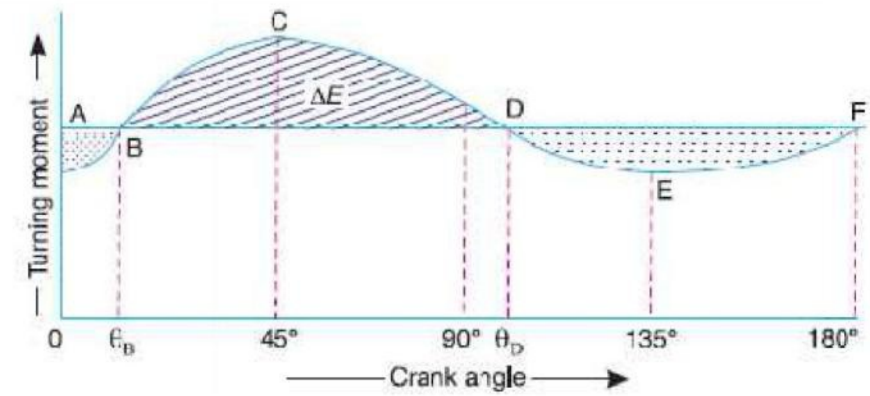
$$20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta = 20\,000$$

or
$$9500 \sin 2\theta = 5700 \cos 2\theta$$

$$\tan 2\theta = \sin 2\theta / \cos 2\theta = 5700/9500 = 0.6$$

$\therefore$ $2\theta = 31°$ or $\theta = 15.5°$

$\therefore$ $\theta_B = 15.5°$ and $\theta_D = 90° + 15.5° = 105.5°$



Maximum fluctuation of energy,

$$\Delta E = \int_{\theta_D}^{\theta_D} \left(T - T_{mean}\right) d\theta$$

$$= \int_{15.5°}^{105.5°} (20\,000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20\,000)\, d\theta$$

$$= \left[-\frac{9500 \cos 2\theta}{2} - \frac{5700 \sin 2\theta}{2}\right]_{15.5°}^{105.5°} = 11\,078 \text{ N-m}$$

We know that maximum fluctuation of energy (ΔE),

$11\,078 = I.\omega^2.C_S = I \times (18.85)^2 \times 0.01 = 3.55\, I$

$I = 11078/3.55 = 3121$ kg-m^2 **Ans.**

3. Angular acceleration of the flywheel Let α = Angular acceleration of the flywheel, and θ = Angle turned by the crank from inner dead centre = 45°

The angular acceleration in the flywheel is produced by the excess torque over the mean torque.

We know that excess torque at any instant,

$T_{excess} = T - T_{mean}$

$= 20000 + 9500 \sin 2\theta - 5700 \cos 2\theta - 20000$

$= 9500 \sin 2\theta - 5700 \cos 2\theta$

$\therefore$ Excess torque at 45° = 9500 sin 90° – 5700 cos 90° = 9500 N-m . . . **(i)** We also know that excess torque

$= I.\alpha = 3121 \times \alpha$ . . . **(ii)**

From equations **(i)** and **(ii)**, α = 9500/3121 = 3.044 rad /s^2 **Ans.**

8(i) A punching press is driven by a constant torque electric motor. The press is provided with a flywheel that rotates at maximum speed of 225 r.p.m. The radius of gyration of the flywheel is 0.5 m. The press punches 720 holes per hour; each punching operation takes 2 second and requires 15 kN-m of energy. Find the power of the motor and the minimum mass of the flywheel if speed of the same is not to fall below 200 r. p. m.

(AU-MAY/JUNE-2012)

Solution. Given $N_1 = 225$ r.p.m ; $k = 0.5$ m ; Hole punched = 720 per hr; $E_1 = 15$ kN-m $= 15 \times 10^3$ N-m ; $N_2 = 200$ r.p.m.

Power of the motor

We know that the total energy required per second

= Energy required / hole × No. of holes / s

$= 15 \times 10^3 \times 720/3600 = 3000$ N-m/s

∴ Power of the motor = 3000 W = 3 kW **Ans.** (∵ 1 N-m/s = 1 W)

Minimum mass of the flywheel

Let m = Minimum mass of the flywheel.

Since each punching operation takes 2 seconds, therefore energy supplied by the motor in 2 seconds,

$$E_2 = 3000 \times 2 = 6000 \text{ N-m}$$

∴ Energy to be supplied by the flywheel during punching or maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = 15 \times 10^3 - 6000 = 9000 \text{ N-m}$$

Mean speed of the flywheel,

$$N = \frac{N_1 + N_2}{2} = \frac{225 + 200}{2} = 212.5 \text{ r.p.m}$$

We know that maximum fluctuation of energy (ΔE),

$$9000 = \frac{\pi^2}{900} \times m.k^2.N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times m \times (0.5)^2 \times 212.5 \times (225 - 200) = 14.565\, m$$

∴ $m = 9000/14.565 = 618$ kg **Ans.**

8 (ii) A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at the rate of 30 holes per minute. It requires 6 N-m of energy per mm² of sheared area. If the punching takes 1/10 of a second and the r.p.m. of the flywheel varies from 160 to 140, determine the mass of the flywheel having radius of gyration of 1 metre.

(AU-APR/MAY-2011)

Solution. Given: $d = 40$ mm; $t = 15$ mm; No. of holes = 30 per min.; Energy required = 6 Nm/mm²; Time = 1/10 s = 0.1 s; $N_1 = 160$ r.p.m.; $N_2 = 140$ r.p.m.; $k = 1$m We know that sheared area per hole

$$= \pi\, d.t = \pi \times 40 \times 15 = 1885 \text{ mm}^2$$

∴ Energy required to punch a hole,

$E_1 = 6 \times 1885 = 11\,310$ N-m and energy required for punching work per second

= Energy required per hole × No. of holes per second

$= 11\,310 \times 30/60 = 5655$ N-m/s

Since the punching takes 1/10 of a second, therefore, energy supplied by the motor in 1/10 second,

$$E_2 = 5655 \times 1/10 = 565.5 \text{ N-m}$$

∴ Energy to be supplied by the flywheel during punching a hole or maximum fluctuation of energy of the flywheel,

$$\Delta E = E_1 - E_2 = 11\,310 - 565.5 = 10\,744.5 \text{ N-m}$$

UNIT-II BALANCING Part-A(2 Marks)

1. **Write the importance of balancing?**

   If the moving part of a machine are not balanced completely then the inertia forces are set up which may cause excessive noise, vibration, wear and tear of the system. So balancing of machine is necessary.

2. **Why rotating masses are to be dynamically balanced? (or) Why balancing of dynamic forces is necessary? (AU-NOV/DEC-2008)**

   If the rotating masses are not dynamically balanced, the unbalanced dynamic forces will cause worse effects such as wear and tear on bearings and excessive vibrations on machines. It is very common in cam shafts, steam turbine rotors, engine crank shafts, and centrifugal pumps, etc.

3. **Differentiate: static and dynamic balancing. (AU-NOV/DEC-2012)**

S.No	Static Balancing	Dynamic Balancing
1	The net dynamic force acting on the shaft is equal to zero	The net dynamic force and the net couple due to the dynamic force is equal to zero.
2	It deals only with balancing of dynamic forces.	It deals with balancing of dynamic force and balancing of couple due to dynamic forces.

4. **Why is only a part of the unbalanced force due to reciprocating masses balanced by revolving mass? (Or) Why complete balancing is not possible in reciprocating engine? (AU-NOV/DEC-2009)**

   Balancing of reciprocating masses is done by introducing the balancing mass opposite to the crank. The vertical component of the dynamic force of this balancing mass gives rise to "Hammer blow". In order to reduce the Hammer blow, a part of the reciprocating mass is balanced. Hence complete balancing is not possible in reciprocating engines.

5. **Can a single cylinder engine be fully balanced? Why? (AU-NOV/DEC-2011)**

   No. A single cylinder engine can't be fully balanced. Because the unbalanced forces due to reciprocating masses remains constant in magnitude (because of variation in Θ).

6. **What are the effects of hammer blow and swaying couple?**

   - The effect of hammer blow is to cause the variation in pressure between the wheel and the rail, such that vehicle vibrates vigorously.
   - The effect of swaying couple is to make the leading wheels sway from side to side.

7. **List the effects of partial balancing of locomotives? (AU-NOV/DEC-2010)**

   - Variation in tractive force along the line of stroke,
   - Swaying couple, and
   - Hammer blow

8. **Define tractive force. (AU-APR/MAY-2010)**

   The resultant unbalanced force due to the two cylinders along the line of stroke is known as tractive force.

9. What is swaying couple?

The unbalanced force acting at a distance between the line of stroke of two cylinders, constitute a couple in the horizontal direction. This couple is known as swaying couple.

10. Define hammer blow with respect to locomotives? (AU-NOV/DEC-2013)

The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as hammer blow.

11. State the conditions for static and dynamic balancing? (or) Write the condition for complete balancing? (AU-NOV/DEC-2013)

- The net dynamic force acting on the shaft is zero. This is called as conditions for static balancing.
- The net couple due to dynamic force acting on the shaft is zero.

12. What are the condition to be satisfied for complete balance of in- line engine?

- The algebraic sum of the primary and secondary forces must be zero, and
- The algebraic sum of the couples due to primary and secondary forces must be zero.

13. Whether grinding wheels are balanced or not? If so why?

Yes, the grinding wheels are properly balanced by inserting some low density materials. If not the required surface finish won't be attained and the vibration will cause much noise.

14. Differentiate between the unbalanced force caused due to rotating and reciprocating masses?

- Complete balancing of revolving mass can be possible. But fraction of reciprocating mass only balanced.
- The unbalanced force due to reciprocating mass varies in magnitude but constant in direction. But in the case of revolving masses, the unbalanced force is constant in magnitude but varies in direction.

15. Unbalanced effects of shafts in high speed machines are to be closely looked into – Why? (AU-MAY/JUNE-2013)

The dynamic forces of centrifugal forces or a result of unbalanced masses are a function the angular velocity of rotation.

16. Write a note on the effect of firing order on balancing of reciprocating mass in multicylinder engines. (AU-APR/MAY-2011)

In multi-cylinder engines, there are several possibilities of the order in which firing takes place. To overcome the problem of vibrations, fuel distribution, exhaust distribution, etc., the designers are selecting different firing orders.

17. Two masses in different planes are necessary to correct the dynamic unbalance – Justify. (AU-MAY/JUNE-2012)

- If the balancing mass and disturbing mass lie in different planes, disturbing mass cannot be balanced by a single mass as there will be a couple left unbalanced.
- In such cases, at least two balancing masses are required for complete balancing and the three masses are arranged in such a way that the resultant force and couple on the shaft are zero.

PART-B

1. A shaft has three eccentrics, each 75 mm diameter and 25 mm thick, machined in one piece with the shaft. The central planes of the eccentric are 60 mm apart. The distance of the centres from the axis of rotation are 12 mm, 18 mm and 12 mm and their angular positions are 120° apart. The density of metal is 7000 kg/m^3. Find the amount of out-of-balance force and couple at 600 r.p.m. If the shaft is balanced by adding two masses at a radius 75 mm and at distances of 100 mm from the central plane of the middle eccentric, find the amount of the masses and their angular positions. (AU-MAY/JUNE-2013)

Solution. Given : $D = 75$ mm $= 0.075$ m ; $t = 25$ mm $= 0.025$ m ; $r_A = 12$ mm $= 0.012$ m ; $r_B = 18$ mm $= 0.018$ m ; $r_C = 12$ mm $= 0.012$ mm ; $\rho = 7000$ kg/m^3 ; $N = 600$ r.p.m. or $\omega = 2\pi \times 600/60 = 62.84$ rad/s ; $r_L = r_M = 75$ mm $= 0.075$ m

We know that mass of each eccentric,

$$m_A = m_B = m_C = \text{Volume} \times \text{Density} = \frac{\pi}{4} \times D^2 \times t \times \rho$$

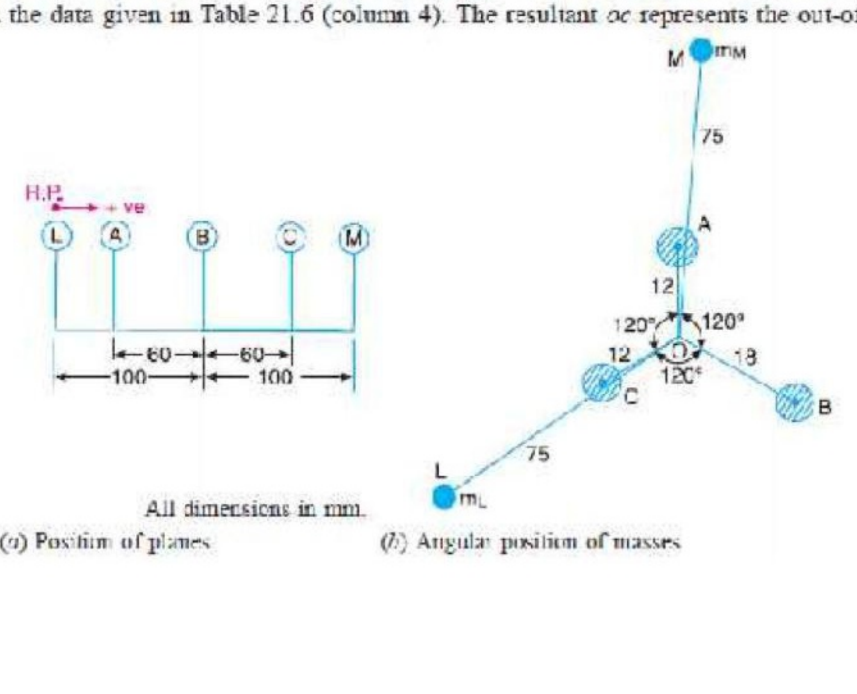
$$= \frac{\pi}{4}(0.075)^2(0.025)7000 = 0.77 \text{ kg}$$

Let *L* and *M* be the planes at distances of 100 mm from the central plane of middle eccentric. The position of the planes and the angular position of the three eccentrics is shown in Fig. 21.12 (*a*) and (*b*) respectively. Assuming *L* as the reference plane and mass of the eccentric *A* in the vertical direction, the data may be tabulated as below :

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane L.(l)m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m^2* (6)
L (*R.P.*)	m_L	0.075	$75 \times 10^{-3}\, m_L$	0	0
A	0.77	0.012	9.24×10^{-3}	0.04	0.3696×10^{-3}
B	0.77	0.018	13.86×10^{-3}	0.1	1.386×10^{-3}
C	0.77	0.012	9.24×10^{-3}	0.16	1.4784×10^{-3}
M	m_M	0.075	$75 \times 10^{-3}\, m_M$	0.20	$15 \times 10^{-3}\, m_M$

Out-of-balance force

The out-of-balance force is obtained by drawing the force polygon, as shown in Fig. 21.12 (*c*), from the data given in Table 21.6 (column 4). The resultant *oc* represents the out-of-balance force.



(*a*) Position of planes (*b*) Angular position of masses

Since the centrifugal force is proportional to the product of mass and radius (*i.e.* $m.r$), therefore by measurement.

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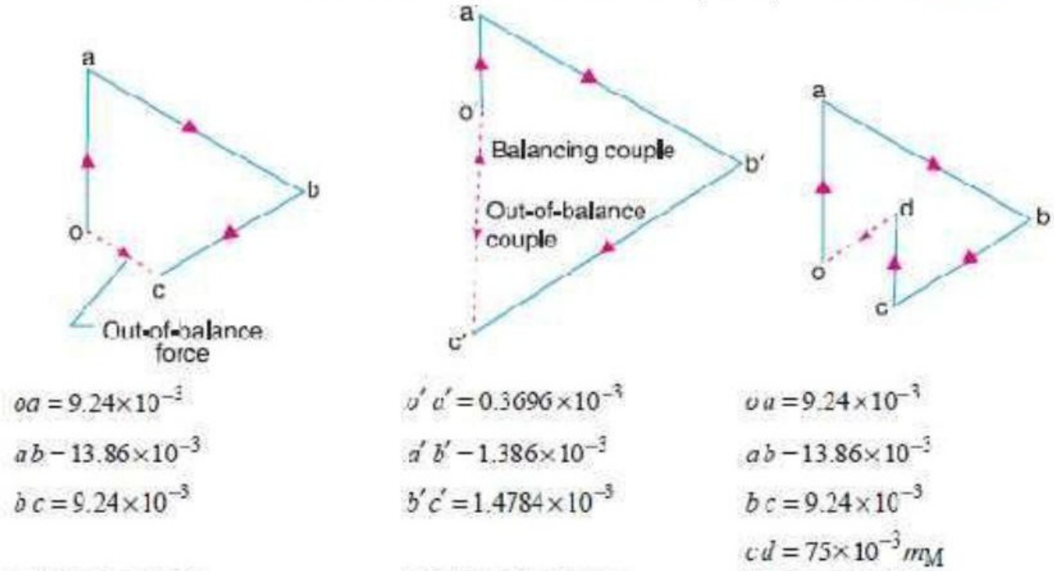
Out-of-balance force = vector $oc = 4.75 \times 10^{-3}$ kg-m

$$= 4.75 \times 10^{-3} \times \omega^2 = 4.75 \times 10^{-3} (62.84)^2 = 18.76 \text{ N}$$ **Ans.**



(*c*) Force polygon. (*d*) Couple polygon. (*e*) Force polygon.

Fig. 21.12

Out-of-balance couple

The out-of-balance couple is obtained by drawing the couple polygon from the data given in Table 21.6 (column 6), as shown in Fig. 21.12 (d). The resultant o ' c' represents the out-of-balance couple. Since the couple is proportional to the product of force and distance (m.r.l), there-fore by measurement,

Out-of-balance couple = vector $o'c' = 1.1 \times 10^{-3}$ kg-m^2

$$= 1.1 \times 10^{-3} \times \omega^2 = 1.1 \times 10^{-3} (62.84)^2 = 4.34 \text{ N-m}$$ **Ans.**

Amount of balancing masses and their angular positions

The vector $c'o'$ (in the direction from c' to o'), as shown in Fig. 21.12 (*d*) represents the balancing couple and is proportional to $15 \times 10^{3}\, m_M$, *i.e.*

$15 \times 10^{-3}\, m_M$ = vector $c'o' = 1.1 \times 10^{-3}$ kg-m^2 or

$m_M = 0.073$ kg **Ans.**

Draw OM in Fig. 21.12 (*b*) parallel to vector $c'o'$. By measurement, we find that the angular position of balancing mass (m_M) is 5° from mass A in the clockwise direction. **Ans.**

In order to find the balancing mass (m_L), a force polygon as shown in Fig. 21.12 (*e*) is drawn. The closing side of the polygon *i.e.* vector do (in the direction from d to o) represents the balancing force and is proportional to $75 \times 10^{3}\, m_L$. By measurement, we find that,

$75 \times 10^{-3}\, m_L$ = vector $do = 5.2 \times 10^{-3}$ kg-m or $m_L = 0.0693$ kg **Ans.**

Draw OL in Fig. 21.12 (*b*), parallel to vector do.

By measurement, we find that the angular position of mass (m_L) is 124° from mass A in the clockwise direction. **Ans.**

2.(i) A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass

of B, C and D are 10 kg, 5 kg, and 4 kg respectively.Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

(AU-NOV/DEC-2012) ***(8)***

Solution. Given : $r_A = 100$ mm = 0.1 m ; $r_B = 125$ mm = 0.125 m ; $r_C = 200$ mm = 0.2 m ; $r_D =$ 150 mm = 0.15 m ; $m_B = 10$ kg ; $m_C = 5$ kg ; $m_D = 4$ kg

1. The position of planes is shown in Fig. 21.10 (*a*). Assuming the plane of mass *A* as the reference plane (*R.P.*), the data may be tabulated as below :

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. Force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A (R.P.)	m_A	0.1	0.1 m_A	0	0
B	10	0.125	1.25	0.5	0.75
C	5	0.2	1	1.2	1.2
D	4	0.15	0.6	1.8	1.08

First of all, the angular setting of masses *C* and *D* is obtained by drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass *B* in the horizontal direction *OB* as shown in Fig. 21.10 (*b*). Now the couple polygon as shown in Fig.(*c*) is drawn as discussed below :

1. Draw vector *o*′ *b*′ in the horizontal direction (*i.e.* parallel to *OB*) and equal to 0.75 kgm^2, to some suitable scale.

2. From points *o*′ and *b*′, draw vectors *o*′ *c*′ and *b*′ *c*′ equal to 1.2 kg-m^2 and 1.08 kg-m^2 respectively. These vectors intersect at *c*′.

3. Now in Fig. 21.10 (*b*), draw *OC* parallel to vector *o*′ *c*′ and *OD* parallel to vector *b*′ *c*′.

By measurement, we find that the angular setting of mass *C* from mass *B* in the anticlockwise direction, *i.e.* $\angle BOC = 240°$ **Ans.**

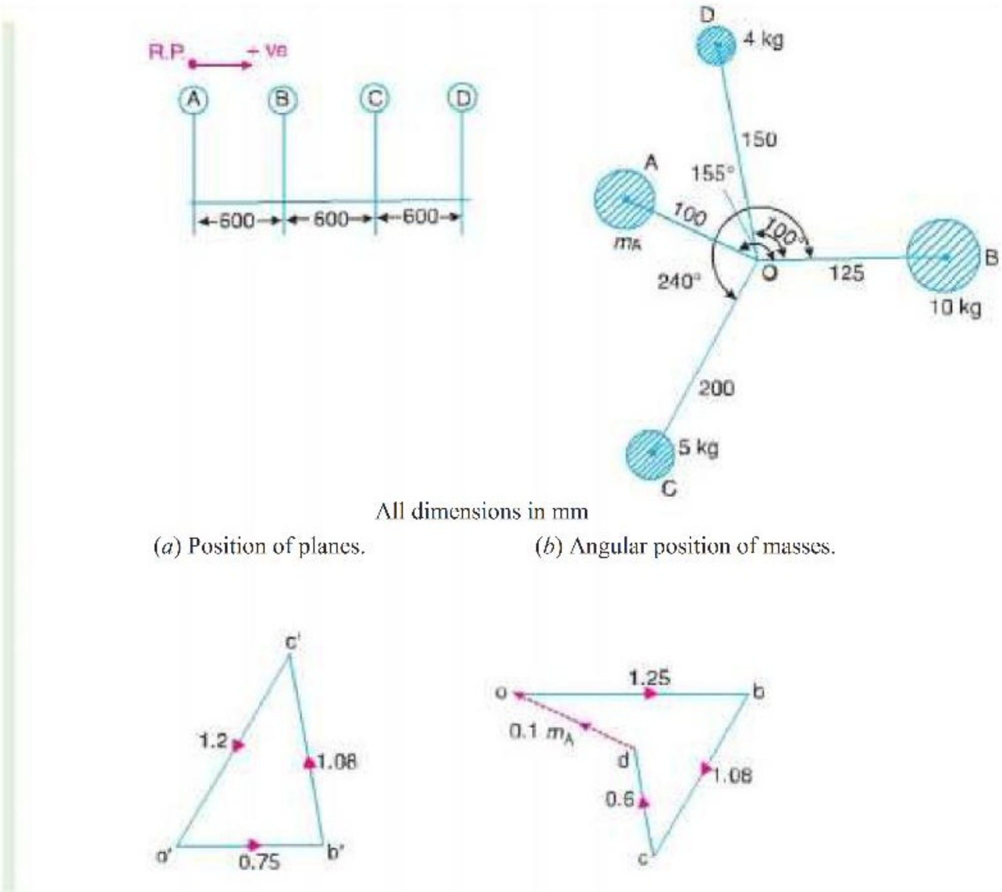
and angular setting of mass *D* from mass *B* in the anticlockwise direction, *i.e.* $\angle BOD = 100°$ **Ans.**

In order to find the required mass *A* (m_A) and its angular setting, draw the force polygon to some suitable scale, as shown in Fig. 21.10 (*d*), from the data given in Table 21.4 (column 4). Since the closing side of the force polygon (vector *do*) is proportional to 0.1 m_A, therefore by measurement,

$0.1\ m = 0.7$ kg-m^2 or $mA = 7$ kg **Ans.**

Now draw *OA* in Fig. 21.10 (*b*), parallel to vector *do*. By measurement, we find that the angular setting of mass *A* from mass *B* in the anticlockwise direction, *i.e.*

$\angle BOA = 155°$ **Ans.**



All dimensions in mm

(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 21.10

2(ii) Derives the expressions for the following: (i) Variation in tractive force and (ii) Swaying couple. (8) (AU-NOV/DEC-2009)

Variation in tractive force

The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as ***tractive force***. Let the crank for the first cylinder be inclined at an angle θ with the line of stroke, as shown in Fig. 22.4.

Since the crank for the second cylinder is at right angle to the first crank, therefore the angle of inclination for the second crank will be (90° + θ).

Let m = Mass of the reciprocating parts per cylinder, and
c = Fraction of the reciprocating parts to be balanced.

$= (1-c)\,m\omega^2.r\cos\theta$

Similarly, unbalanced force along the line of stroke for cylinder 2,

$= (1-c)\,m.\omega^2 \cdot r\cos(90° + \theta)$

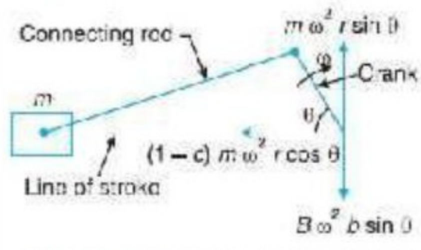


Fig. 22.4 Variation of tractive force.

∴ As per definition, the tractive force,

F_1 = Resultant unbalanced force along the line of stroke

$= (1-c)\,m\omega^2.r\cos\theta + (1-c)\,m\omega^2.r\cos(90° + \theta)$

$= (1-c)\,m\omega^2.r(\cos\theta - \sin\theta)$

We know that unbalanced force along the line of stroke for cylinder 1

The tractive force is maximum or minimum when $(\cos\theta - \sin\theta)$ is maximum or minimum. For $(\cos\theta - \sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0 \quad \text{or} \quad -\sin\theta - \cos\theta = 0 \quad \text{or} \quad -\sin\theta = \cos\theta$$

$$\therefore \quad \tan\theta = -1 \quad \text{or} \quad \theta = 135° \quad \text{or} \quad 315°$$

Thus, the tractive force is maximum or minimum when θ = 135° or 315°.

∴ Maximum and minimum value of the tractive force or the variation in tractive force

$$= \pm(1-c)\,m.\omega^2.r(\cos 135° - \sin 135°) = \pm\sqrt{2}\,(1-c)\,m.\omega^2.r$$

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line *YY* between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.*** *a* = Distance between the centre lines of the two cylinders.

Let a = Distance between the centre lines of the two cylinders.

∴ Swaying couple

$= (1-c)\,m\omega^2.r\cos\theta \times \frac{a}{2}$

$\quad - (1-c)\,m\omega^2.r\cos(90° + \theta)\,\frac{a}{2}$

$= (1-c)\,m.\omega^2.r \times \frac{a}{2}(\cos\theta + \sin\theta)$

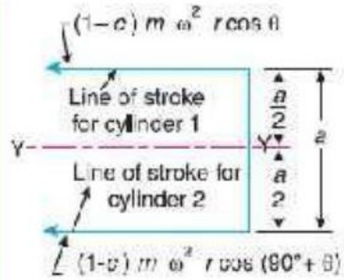


Fig. 22.5. Swaying couple.

The swaying couple is maximum or minimum when $(\cos\theta + \sin\theta)$ is maximum or minimum. For $(\cos\theta + \sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0 \quad \text{or} \quad \sin\theta + \cos\theta = 0 \quad \text{or} \quad -\sin\theta = -\cos\theta$$

$$\therefore \quad \tan\theta = 1 \quad \text{or} \quad \theta = 45° \quad \text{or} \quad 225°$$

Thus, the swaying couple is maximum or minimum when θ = 45° or 225°.

∴ Maximum and minimum value of the swaying couple

$$= \pm(1-c)\,m.\omega^2.r \times \frac{a}{2}(\cos 45° + \sin 45°) = \pm\frac{a}{\sqrt{2}}(1-c)\,m\omega^2.r$$

3. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A

to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions. (AU-MAY/JUNE-2012)

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm $= 0.08$m ;

$r_B = 70$ mm $= 0.07$ m ; $r_C = 60$ mm $= 0.06$ m ; $r_D = 80$ mm $= 0.08$ m ; $r_X = r_Y = 100$ mm $= 0.1$ m

Let m_X = Balancing mass placed in plane X, and m_Y = Balancing mass placed in plane Y.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 21.8 (*a*) and (*b*) respectively.

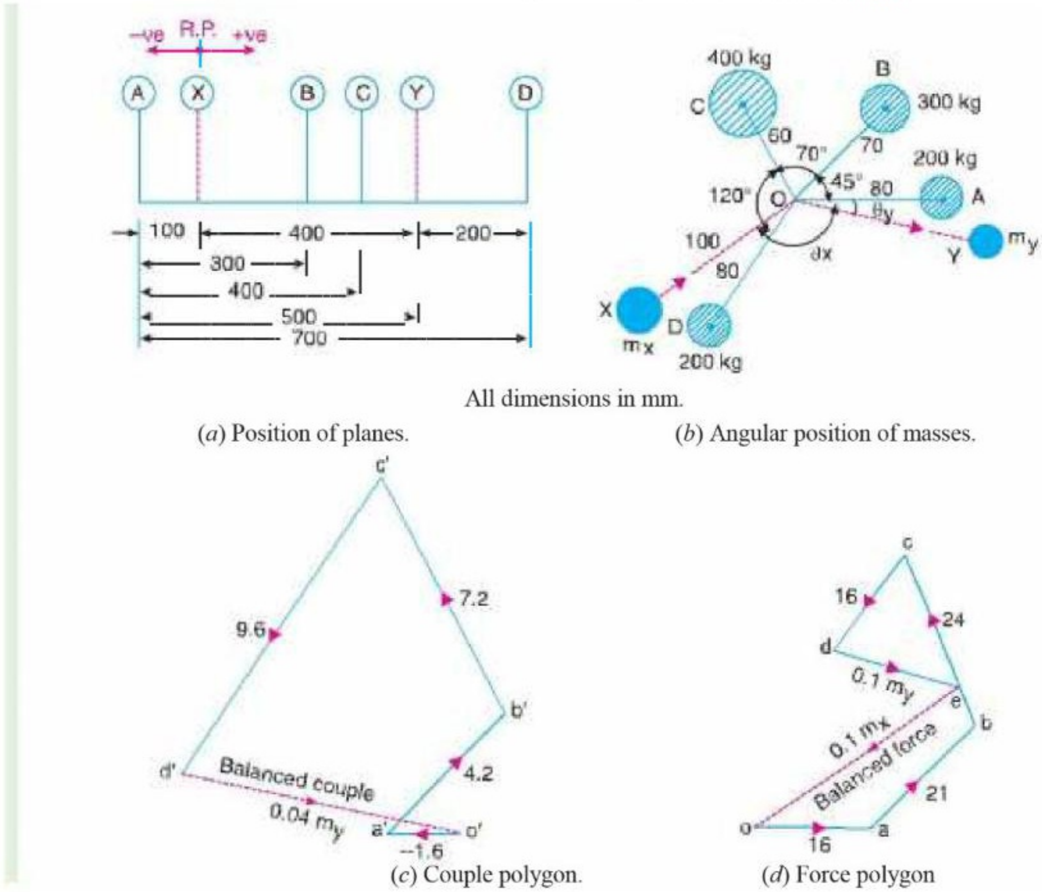
Assume the plane X as the reference plane (*R.P.*). The distances of the planes to the right of plane X are taken as + ve while the distances of the planes to the left of plane X are taken as – ve. The data may be tabulated as shown in Table 21.2.

Plane (1)	*Mass (m)* kg (2)	*Radius (r)* m (3)	*Cent.force* $\div \omega^2$ (m.r) kg-m (4)	*Distance from Plane x(l)* m (5)	*Couple* $\div \omega^2$ (m.r.l) kg-m^2 (6)
A	200	0.08	16	– 0.1	– 1.6
X(*R.P.*)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

The balancing masses m_X and m_Y and their angular positions may be determined graphi-cally as discussed below :

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (*c*) to some suitable scale. The vector $d'o'$ represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by measurement,

$0.04\, m_Y$ = vector $d'o'$ = 7.3 kg-m^2 or m_Y = 182.5 kg **Ans.**



All dimensions in mm.

(a) Position of planes. (b) Angular position of masses.

(c) Couple polygon. (d) Force polygon

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 21.8 (b), parallel to vector $d'o'$.

By measurement, the angular position of m_Y is θ_Y □ 12□ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. 21.8 (d). The vector eo represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

$0.1\, m_X$ □ vector eo □ 35.5 kg-m or m_X = **355 kg Ans.**

The angular position of the mass m_X is obtained by drawing Om_X in Fig. 21.8 (b), parallel to vector eo. By measurement, the angular position of m_X is θ_X □ 145□ in the clockwise direction from mass m_A (*i.e.* 200 kg). **Ans.**

4. Four masses A, B, C and D as shown below are to be completely balanced.

	A	B	C	D
Mass (kg)	**—**	**30**	**50**	**40**
Radius (mm)	**180**	**240**	**120**	**150**

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. **The magnitude and the angular position of mass A ; and**
2. **The position of planes A and D. (AU-NOV/DEC-2011)**

Solution. Given : $r_A = 180$ mm = 0.18 m ; $m_B = 30$ kg ; $r_B = 240$ mm = 0.24 m ; $m_C = 50$ kg ; $r_C = 120$ mm = 0.12 m ; $m_D = 40$ kg ; $r_D = 150$ mm = 0.15 m ; $\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$

1. ***The magnitude and the angular position of mass A***

Let m_A = Magnitude of Mass *A*,

x = Distance between the planes *B* and *D,* and y = Distance between the planes *A* and *B*.

The position of the planes and the angular position of the masses is shown in Fig. 21.9 (*a*) and (*b*) respectively.

Assuming the plane *B* as the reference plane (*R.P.*) and the mass *B* (m_B) along the horizon-tal line as shown in Fig. 21.9 (*b*), the data may be tabulated as below :

Plane (1)	*Mass* (*m*) *kg* (2)	*Radius* (*r*) *m* (3)	*Cent.force* ÷ ω^2 (*m.r*) *kg-m* (4)	*Distance from plane B* (*l*) *m* (5)	*Couple* ÷ ω^2 (*m.r.l*) *kg-m*^{2} (6)
A	m_A	0.18	0.08 m_A	$-y$	$-0.18\, m_A y$
B (*R.P.*)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$

The magnitude and angular position of mass *A* may be determined by drawing the force polygon from the data given in Table 21.3 (Column 4), as shown in Fig. 21.9 (*c*), to some suitable scale. Since the masses are to be completely balanced, therefore the force polygon must be a closed figure. The closing side (*i.e.* vector *do*) is proportional to 0.18 m_A. By measurement,

$0.18\, m_A$ = Vector do = 3.6 kg-m or m_A = 20 kg **Ans.**

In order to find the angular position of mass *A*, draw *OA* in Fig. 21.9 (*b*) parallel to vector *do*. By measurement, we find that the angular position of mass *A* from mass *B* in the anticlockwise

21130

direction is $\angle AOB$ = **236° Ans.**

2. *Position of planes A and D*

The position of planes *A* and *D* may be obtained by drawing the couple polygon, as shown in Fig. 21.9 (*d*), from the data given in Table 21.3 (column 6). The couple polygon is drawn as discussed below :

1. Draw vector $o'c'$ parallel to *OC* and equal to 1.8 kg-m^2, to some suitable scale.
2. From points c' and o' , draw lines parallel to *OD* and *OA* respectively, such that they intersect at point d' . By measurement, we find that

$6x$ = vector $c'd'$ = 2.3 kg-m^2 or x = 0.383 m

We see from the couple polygon that the direction of vector $c'd'$ is opposite to the direction of mass D. Therefore the plane of mass D is 0.383 m or 383 mm towards left of plane B and not towards right of plane B as already assumed. **Ans.**

Again by measurement from couple polygon,

$$-0.18\, m_A . y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

$$-0.18 \times 20\, y = 3.6 \quad \text{or } y = -1 \text{ m}$$

The negative sign indicates that the plane A is not towards left of B as assumed but it is 1 m or 1000 mm towards right of pne B. **Ans.**

5. An inside cylinder locomotive has its cylinder centre lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel centre lines are 1.5 m apart. The cranks are at right angles.

The whole of the rotating and 2/3 of the recipro-cating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m. (AU-NOV/DEC-2008) Solution. Given : $a = 0.7$ m; $l_B = l_C = 0.6$ m or

$r_B = r_C = 0.3$ m; $m_1 = 150$ kg; $m_2 = 180$ kg; $c = 2/3$; $r_A = r_D = 0.6$ m; $N = 300$ r.p.m. or ω

□ 2□□ 300 / 60 = 31.42 rad/s

We know that the equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2 = 150 + \tfrac{2}{3} \times 180 = 270 \text{ kg}$$

Magnitude and direction of the balancing masses Let m_A and m_D = Magnitude of the balancing masses θ_A and θ_D = Angular position of the balancing masses m_A and m_D from the first crank B.

The magnitude and direction of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 22.7 (*a*). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder B in the horizontal direc-tion, draw OC and OB at right angles to each other as shown in Fig. 22.7 (*b*).

Tabulate the data as given in the following table. Assume the plane of wheel A as the reference plane.

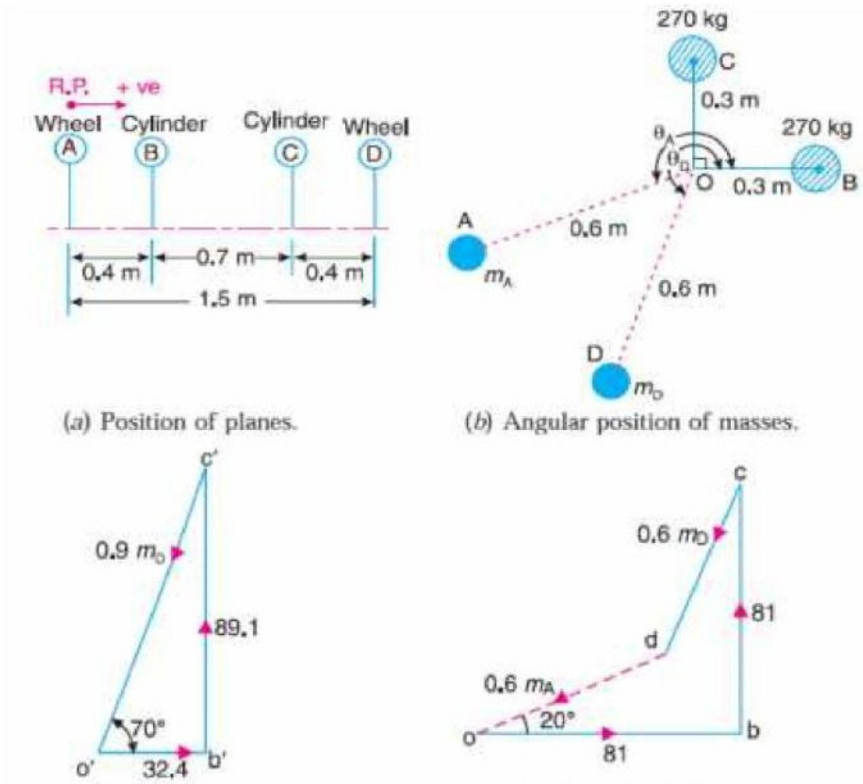
Plane (1)	*mass. (m) kg* (2)	*Radius (r) m* (3)	*Cent. force ÷ ω^2 (m.r) kg-m* (4)	*Distance from plane A (l) m* (5)	*Couple ÷ ω^2 (m.r.l) kg-m²* (6)
A (R.P.)	m_A	0.6	0.6 m_A	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6 m_D$	1.5	0.9 m_D

3. Now, draw the couple polygon from the data given in Table 22.1 (column 6), to some suitable scale, as shown in Fig 22.7 (*c*). The closing side $c'o'$ represents the balancing couple and it is proportional to 0.9 m_D. Therefore, by measurement,

$0.9\ m_D$ = vector $c'o'$ = 94.5 kg-m^2 or m_D = 105 kg **Ans.**

$$D = \frac{m_1}{m} \times 105 = \frac{150}{270} \times 105 = 58.3 \text{ kg}$$

and balancing mass for reciprocating masses.



(*a*) Position of planes. (*b*) Angular position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.7

4. To determine the angular position of the balancing mass *D*, draw *OD* in Fig. 22.7 (*b*) parallel to vector $c'o'$. By measurement,

$$\theta_D = 250° \textbf{ Ans.}$$

5. In order to find the balancing mass *A*, draw the force polygon from the data given in Table 22.1 (column 4), to some suitable scale, as shown in Fig. 22.7 (*d*), The vector *do* represents the balancing force and it is proportional to $0.6\ m_A$. Therefore by measurement, $0.6 m_A$ = vector *do* = 63 kg-m or m_A = 105 kg **Ans.**

6. To determine the angular position of the balancing mass *A*, draw *OA* in Fig. 22.7 (*b*) parallel to vector *do*. By measurement,

$$\theta_A = 200° \textbf{ Ans.}$$

Fluctuation in rail pressure

We know that each balancing mass105 kg

∴ Balancing mass for rotating masses,

$$B = \frac{c.m_2}{m} \times 105 = \frac{2}{3} \times \frac{180}{270} \times 105 = 46.6 \text{ kg}$$

This balancing mass of 46.6 kg for reciprocating masses gives rise to the centrifugal force.

∴ Fluctuation in rail pressure or hammer blow

$$= B.\omega^2.b = 46.6\,(31.42)^2\,0.6 = 27\,602 \text{ N. Ans.} \qquad \ldots (\because b = r_A = r_D)$$

Variation of tractive effort

We know that maximum variation of tractive effort

,

$$= \pm\sqrt{2}(1-c)\,m_2.\omega^2.r = \pm\sqrt{2}\left(1-\frac{2}{3}\right)180(31.42)^2\,0.3 \text{N}$$

$$= \pm 25\,127 \text{ N Ans.} \qquad \ldots (\because r = r_B = r_C)$$

Swaying couple

We know that maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2.\omega^2.r = \frac{0.7\left(1-\frac{2}{3}\right)}{\sqrt{2}} \times 180(31.42)^2\,0.3 \text{ N-m}$$

$$= 8797 \text{ N-m Ans.}$$

6. The following data apply to an outside cylinder uncoupled locomotive : Mass of rotating parts per cylinder = 360 kg ; Mass of reciprocating parts per cylinder = 300 kg ; Angle between cranks = 90° ; Crank radius = 0.3 m ; Cylinder centres = 1.75 m ; Radius of balance masses = 0.75 m ; Wheel centres = 1.45 m. If whole of the rotating and two-thirds of reciprocating parts are to be balanced in planes of the driving wheels, find :

1.Magnitude and angular positions of balance masses,

2.Speed in kilometres per hour at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and

3.Swaying couple at speed arrived at in (2) above. (AU-NOV/DEC-2013)

Solution : Given : $m_1 = 360$ kg ; $m_2 = 300$ kg ; ∠ $AOD = 90°$; $r_A = r_D = 0.3$ m ; $a = 1.75$ m ; $r_B = r_C = 0.75$ m ; $c = 2 / 3$.

We know that the equivalent mass of the rotating parts to be balanced per cylinder,

$$m = m_A = m_D = m_1 + c.m_2 = 360 + \frac{2}{3} \times 300 = 560 \text{ kg}$$

1. ***Magnitude and angular position of balance masses***

Let m_B and m_C = Magnitude of the balance masses, and θ_B and θ_C = angular position of the balance masses m_B and m_C from the crank A.

The magnitude and direction of the balance masses may be determined, graphically, as discussed below :

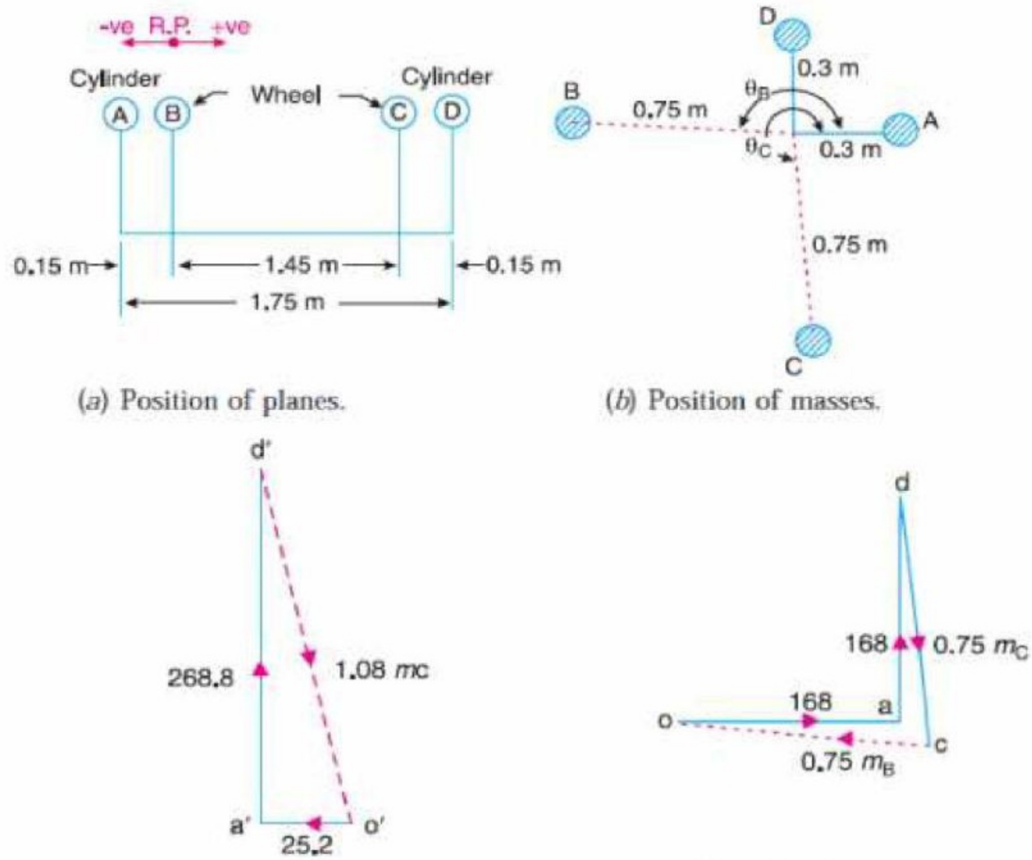
1. First of all, draw the positions of the planes of the wheels and the cylinders as shown in Fig. 22.11 (*a*). Since the cranks of the two cylinders are at right angles, therefore assum-ing the position of the cylinder A in the horizontal direction, draw OA and OD at right angles to each other as shown in Fig. 22.11 (*b*).

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane B (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
A	560	0.3	168	– 0.15	– 25.2
B (R.P.)	m_B	0.75	0.75 m_B	0	0
C	m_C	0.75	0.75 m_C	1.45	1.08 m_C
D	560	0.3	168	1.6	268.8

2. Assuming the plane of wheel *B* as the reference plane, the data may be tabulated as be-low:

3. Now draw the couple polygon with the data given in Table 22.4 column (6), to some suitable scale as shown in Fig. 22.11(*c*). The closing side *d* '*o*' represents thebalancing couple and it is proportional to 1.08 m_C. Therefore, by measurement,

$$1.08\ m_C = 269.6 \text{ kg-m}^2 \quad \text{or} \quad m_C = 249 \text{ kg} \quad \textbf{Ans.}$$



(*a*) Position of planes. (*b*) Position of masses.

(*c*) Couple polygon. (*d*) Force polygon.

Fig. 22.11

4. To determine the angular position of the balancing mass *C*, draw *OC* parallel to vector *d'o* ' as shown in Fig. 22.11 (*b*). By measurement, $\theta_C = 275°$ **Ans.**

5. In order to find the balancing mass *B*, draw the force polygon with the data given in Tablecolumn

(4), to some suitable scale, as shown in Fig. 22.11 (*d*). The vector *co* represents the balancing force and it is proportional to 0.75 m_B. Therefore, by measurement, 0.75 m_B = 186.75 kg-m or m_B = 249 kg **Ans.**

4. To determine the angular position of the balancing mass *B*, draw *OB* parallel to vector *oc* as shown Fig. 22.11 (*b*). By measurement, $\theta_B = 174.5°$ **Ans.**

2. ***Speed at which the wheel will lift off the rails***

Given : $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$; $D = 1.8$ m

Let ω = Angular speed at which the wheels will lift off the rails in rad/s, and *v* = Corresponding linear speed in km/h.

We know that each balancing mass, $m_B = m_C = 249$ kg

∴ Balancing mass for reciprocating parts,

$$B = \frac{c.m_1}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 249 = 89 \text{ kg}$$

We know that $$\omega = \sqrt{\frac{P}{Bb}} = \sqrt{\frac{30 \times 10^3}{89 \times 0.75}} = 21.2 \text{ rad/s} \qquad \ldots(\because b = r_B = r_C)$$

and $$v = \omega \times D/2 = 21.2 \times 1.8/2 = 19.08 \text{ m/s}$$
$$= 19.08 \times 3600/1000 = 68.7 \text{ km/h}$$ **Ans.**

3. ***Swaying couple at speed ω = 21.1 rad/s***

We know that the swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m_2 . \omega^2 . r = \frac{1.75\left[1-\frac{2}{3}\right]}{\sqrt{2}} \times 300 (21.2)^2 0.3 \text{ N-m}$$
$$= 16\,687 \text{ N-m} = 16.687 \text{ kN-m}$$ **Ans.**

7. The three cranks of a three cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 meter and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank. If 40% of the reciprocating parts are to be balanced, find :

1. the magnitude and the position of the balancing masses required at a radius of 0.6 m ;

And

2. the hammer blow per wheel when the axle makes 6 r.p.s. (AU-MAY/JUNE-2013)

Solution. Given : ∠ *AOB* = ∠ *BOC* = ∠ *COA* = 120° ; $l_A = l_B = l_C = 0.6$ m or $r_A = r_B = r_C = 0.3$ m ; $m_I = 300$ kg ; $m_O = 260$ kg ; $c = 40\% = 0.4$; $b_1 = b_2 = 0.6$ m ; $N = 6$ r.p.s.

$= 6 \times 2\pi = 37.7$ rad/s

Since 40% of the reciprocating masses are to be balanced, therefore mass of the reciprocat-ing parts to be balanced for each outside cylinder,

$m_A = m_C = c \times m_O = 0.4 \times 260 = 104$ kg and mass of the reciprocating parts to be balanced for inside cylinder,

$m_B = c \times m_I = 0.4 \times 300 = 120$ kg

1. ***Magnitude and position of the balancing masses***

Let B_1 and B_2 = Magnitude of the balancing masses in kg, θ_1 and θ_2 = Angular position of the balancing masses B_1 and B_2 from crank A.

The magnitude and position of the balancing masses may be determined graphically as discussed below :

1. First of all, draw the position of planes and cranks as shown in Fig. 22.8 (*a*) and (*b*) respectively. The position of crank *A* is assumed in the horizontal direction.

2. Tabulate the data as given in the following table. Assume the plane of balancing mass B_1 (*i.e.* plane 1) as the reference plane.

Table 22.2

Plane (1)	Mass (m)kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane1 (l)m (5)	Couple ÷ ω^2 (m.r.l.) kg-m^2 (6)
A	104	0.3	31.2	0.2	6.24
1 (R.P.)	B_1	0.6	0.6 B_1	0	0
B	120	0.3	36	0.8	28.8
2	B_2	0.6	0.6 B_2	1.6	0.96 B_2
C	104	0.3	31.2	1.8	56.16

3. Now draw the couple polygon with the data given in Table 22.2 (column 6), to some suitable scale, as shown in Fig. 22.8 (*c*). The closing side *c* '*o*' represents the balancing couple and it is proportional to 0.96 B_2. Therefore, by measurement,

$$0.96\, B_2 = \text{vector } c'o' = 55.2 \text{ kg-m}^2 \text{ or } B_2 = 57.5 \text{ kg}$$ **Ans.**

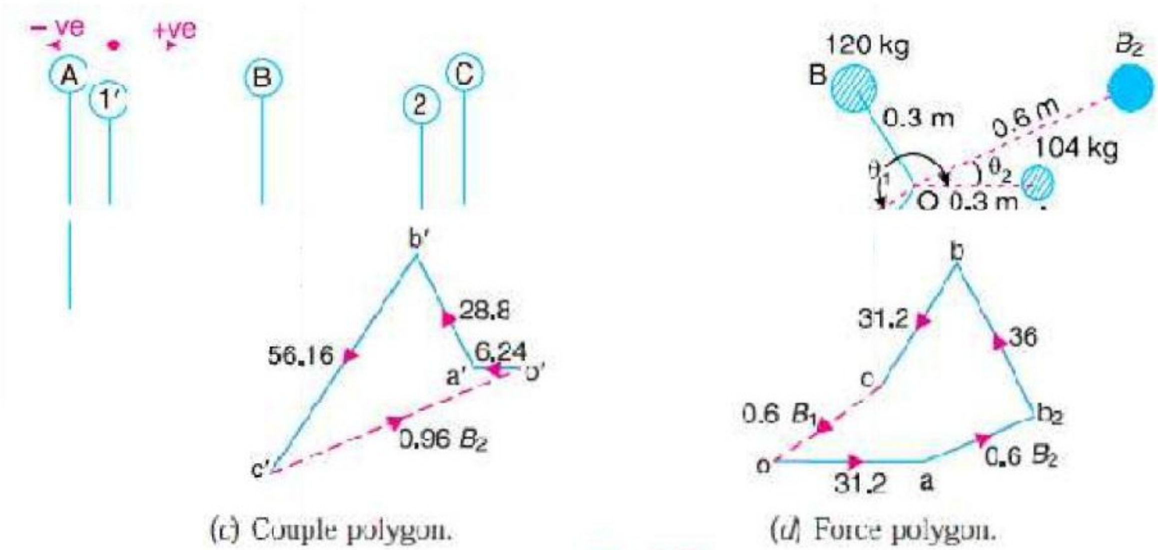
4. To determine the angular position of the balancing mass B_2, draw OB_2 parallel to vector *c* '*o*' as shown in Fig. 22.8 (*b*). By measurement, $\theta_2 = 24°$ **Ans.**

5. In order to find the balance mass B_1, draw the force polygon with the data given in Table 22.2(column 4), to some suitable scale, as shown in Fig. 22.8 (*d*). The closing side *co* represents the balancing force and it is proportional to 0.6 B_1. Therefore, by measurement,

$$0.6\, B_1 = \text{vector } co = 34.5 \text{ kg-m or } B_1 = 57.5 \text{ kg}$$ **Ans.**

6. To determine the angular position of the balancing mass B_1, draw OB_1 parallel to vector *co*, as shown in Fig. 22.8 (*b*). By measurement,

$\theta_1 = 215°$ **Ans.**



(c) Couple polygon. (d) Force polygon.

Fig. 22.8

2. ***Hammer blow per wheel***

We know that hammer blow per wheel

$= B_1.\omega^2.b_1 = 57.5\ (37.7)^2\ 20.6 = 49\ 035$ N **Ans.**

8. The following data refer to two cylinder locomotive with cranks at 90° :

Reciprocating mass per cylinder = 300 kg ; Crank radius = 0.3 m ; Driving wheel diameter = 1.8 m ; Distance between cylinder centre lines = 0.65 m ; Distance between the driving wheel central planes = 1.55 m.

Determine : 1. the fraction of the reciprocating masses to be balanced, if the hammer blow is not to exceed 46 kN at 96.5 km. p.h. ; 2. the variation in tractive effort ; and 3. the maximum swaying couple. (AU-MAY/JUNE-2009)

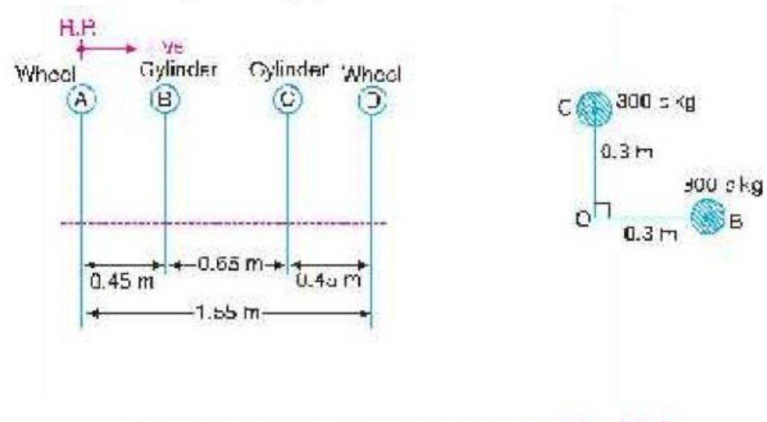
Solution. Given : $m = 300$ kg ; $r = 0.3$ m ; $D = 1.8$ m or $R = 0.9$ m ; $a = 0.65$ m ; Hammer blow = 46 kN = 46×10^3 N ; $v = 96.5$ km/h = 26.8 m/s

1. ***Fraction of the reciprocating masses to be balanced***

Let c = Fraction of the reciprocating masses to be balanced, and

B = Magnitude of balancing mass placed at each of the driving wheels at radius b.

We know that the mass of the reciprocating parts to be balanced □ $c.m$ □ $300c$ kg



(*a*) Position of planes. (*b*) Position of cranks. Fig. 22.9

The position of planes of the wheels and cylinders is shown in Fig. 22.9 (*a*), and the position of

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A (l) m (5)	Couple ÷ ω^2 (m.r.l) kg-m² (6)
A (R.P.)	B	b	B.b	0	0
B	300 c	0.3	90 c	0.45	40.5 c
C	300 c	0.3	90 c	1.1	99 c
D	B	b	B.b	1.55	1.55 B.b

cranks is shown in Fig 22.9 (*b*). Assuming the plane of wheel *A* as the reference plane, the data may be tabulated as below :

Now the couple polygon, to some suitable scale, may be drawn with the data given in Table 22.3 (column 6), as shown in Fig. 22.10. The closing side of the polygon (vector *c'o'*) represents the balancing couple and is proportional to 1.55 *B.b*.

From the couple polygon,

$$1.55\;B.b = \sqrt{(40.5c)^2 + (99\,c)^2} = 107c$$

$\therefore\; B.b = 107\,c / 1.55 = 69\,c$

We know that angular speed,

$$\omega = v/R = 26.8/0.9 = 29.8 \text{ rad/s}$$

∴ Hammer blow,

$$46 \times 10^3 = B.\,\omega^2.b$$

$$= 69\,c\,(29.8)^2 = 61\,275\,c$$

$\therefore \quad c = 46 \times 10^3/61\,275 = 0.751$ **Ans.**

2. *Variation in tractive effort*

We know that variation in tractive effort,

$$= \pm\sqrt{2}(1-c)\,m\omega^2.r = \pm\sqrt{2}(1-0.751)\,300\,(29.8)^2\,0.3$$

$$= 28\,140 \text{ N} = 28.14 \text{ kN}$$ **Ans.**

3.Maximum swaying couple

We know the maximum swaying couple

$$= \frac{a(1-c)}{\sqrt{2}} \times m.\omega^2.r = \frac{0.65\,(1-0.751)}{\sqrt{2}} \times 300\,(29.8)^2\,0.3 = 9148 \text{ N-m}$$

$$= 9.148 \text{ kN-m}$$ **Ans.**

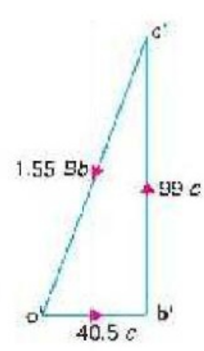


Fig. 22.10

UNIT-III SINGLE DEGREE FREE VIBRATION Part-A(2 Marks)

1. **How will you classify vibration? (Or) what are the different type f vibratory motions? (AU-MAY/JUNE-2009)**
   - Free vibrations
   - Longitudinal vibration,
     Transverse vibration, and
     Torsional vibration.
   - Forced vibrations, and
     Damped vibration.

2. **What are the causes and effect of vibration?**

   The causes of vibration are unbalanced forces, elastic nature of the system, self excitation, wind and earthquakes.

   The existence of vibration elements in any mechanical system produces unwanted noise, high stress, poor reliability and premature failure of one or more of the parts.

3. **What do you mean by a degree of freedom or movability?** (AU-NOV/DEC-2007)

   The number of independent coordinates required to completely define the motion of a system is known as degree of freedom of the system.

4. **What is the limit beyond which damping is detrimental and why?** (MAY/JUNE-2013)

   When damping factor $\zeta > 1$, the aperiodic motion is resulted. That is, aperiodic motion means the system cannot vibrate due to over damping. Once the system is disturbed, it will take infinite time to come back to equilibrium position.

5. **What is meant by critical damping? (AU-MAY/JUNE-2012)**

   The system is said to be critically damped when the damping factor $\zeta = 1$. If the system is critically damped, the mass moves back very quickly to its equilibrium position within no time.

6. **Define critical or whirling or whipping speed of a shaft? Give one application of critical damping. (AU-NOV/DEC-2012)**

   The speed at which resonance occurs is called critical speed of the shaft. In other words, the speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite is known as critical speed.

   The property of critical damping is used in designing elecrical instruments, hydraulic door closers and large guns.

7. **What are the factors that affect the critical speed of a shaft?** (AU-NOV/DEC-2007) The critical speed essentially depends on:
   - The eccentricity of the C.G of the rotating masses from the axis of rotation of the shaft,
   - Diameter of the disc,
   - Span of the shaft, and
   - Type of supports connections at its ends.

8. What is the effect of inertia on the shaft in longitudinal and transverse vibrations?

In longitudinal vibrations, he inertia effect of the shaft is equal to the that of a mass one third of the mass of the shaft concentrated at its free end.

9. Define logarithmic decrement. (AU-APR/MAY-2010)

Logarithmic decrement is defined as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position.

10. Define damping factor and damping co-efficient. (AU-NOV/DEC-2013)

- The damping factor or damping ratio is defined as the ratio of actual damping coefficient (c) to the critical damping co-efficient(c_c).

The damping force per unit velocity is known as damping co-efficient.

11. Define node in torsional vibration. (or) what is nodal section in two rotor system.
(AU-NOV/DEC-2013)

Node is the point or the section of the shaft at which amplitude of the torsional vibration is zero. At nodes, the shaft remains unaffected by the vibration.

12. What is difference between damping, viscous damping and Coloumb damping? (AU-NOV/DEC-2012)

- **Damping:** The resistance against the vibration is called damping.
- **Viscous Damping** is the damping provided by fluid resistance.
- **Coloumb damping** is the dampin results from two dry or unlubricated surfaces rubbing together.

13. Define torsional equivalent shaft?

A shaft having diameter for different lengths can be theoretically replaced by an equivalent shaft of uniform diameter such that they have the same total angle of twist when equal opposing torques are applied at their ends. Such a theoretically replaced shaft is known as torsion ally equivalent shaft.

14. Determine the natural frequency of mass of 10kgsuspended at the bottom of two springs of stiffness: 5 N/mm and 8 N/mm in series. (AU-MAY/JUN-2013)

Natural Frequency ,$f_{n=1/2\pi}\sqrt{s/m}$

$f_n = 1/2\pi\sqrt{13000/10}$

=.74 Hz

15. State natural frequency of torsional vibration of a simple system?

Natural frequency of torsional vibration,

Where C = Rigidity modulus of shaft, I = Mass M.I. of rotor, J = polar M.I of shaft, and l = Length of node from rotor.

PART-B

1. A machine of mass 75 kg is mounted on springs and is fitted with a dashpot to damp out

amplitude of vibration diminishes from 38.4 mm to 6.4 mm in two Assuming that the damping force varies as the velocity, determine : 1. dash-pot at unit velocity ; 2. the ratio of the frequency of the dam frequency of the undamped vibration ; and 3. the periodic time of t (AU-MAY/JUNE-2013)

vibrations. There are three springs each of stiffness 10 N/mm and it is found that the complete oscillations.

Solution. Given : $m = 75$ kg ; $s = 10$ N/mm $= 10 \times 10^3$ N/m ; $x_1 = 38.4$ mm $= 0.0384$ m ; $x_3 = 6.4$ mm $= 0.0064$ m

Since the stiffness of each spring is 10×10^3 N/m and there are 3 springs, therefore total stiffness,

$$s = 3 \times 10 \times 10^3 = 30 \times 10^3 \text{ N/m}$$

We know that natural circular frequency of motion,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{30 \times 10^3}{75}} = 20 \text{ rad/s}$$

1. *Resistance of the dashpot at unit velocity*

Let c = Resistance of the dashpot in newtons at unit velocity *i.e.* in N/m/s,

x_2 = Amplitude after one complete oscillation in metres, and

x_3 = Amplitude after two complete oscillations in metres.

We know that $\frac{x_1}{x_2} = \frac{x_2}{x_3}$

$$\therefore \quad \left(\frac{x_1}{x_2}\right)^2 = \frac{x_1}{x_3} \qquad \ldots \left[\because \frac{x_1}{x_3} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} = \frac{x_1}{x_2} \times \frac{x}{x_2} = \left(\frac{x_1}{x_2}\right)^2\right]$$

or

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_3}\right)^{1/2} = \left(\frac{0.0384}{0.0064}\right)^{1/2} = 2.45$$

We also know that

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

2. 2. Ratio of the frequency of the damped vibration to the frequency of undamped vibration

$$\log_e 2.45 = a \times \frac{2\pi}{\sqrt{(20)^2 - a^2}}$$

$$0.8951 = \frac{a \times 2\pi}{\sqrt{400 - a^2}} \quad \text{or} \quad 0.8 = \frac{a^2 \times 39.5}{400 - a^2} \qquad \ldots \text{(Squaring both sides)}$$

$$\therefore \quad a^2 = 7.94 \quad \text{or} \quad a = 2.8$$

We know that $a = c / 2m$

$$\therefore \quad c = a \times 2m = 2.8 \times 2 \times 75 = 420 \text{ N/m/s} \quad \textbf{Ans.}$$

Let f_{n1} = Frequency of damped vibration = $\frac{\omega_d}{2\pi}$

f_{n2} = Frequency of undamped vibration = $\frac{\omega_n}{2\pi}$

$$\therefore \quad \frac{f_{n1}}{f_{n2}} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \frac{\sqrt{(\omega_n)^2 - a^2}}{\omega_n} = \frac{\sqrt{(20)^2 - (2.8)^2}}{20} = 0.99 \text{ Ans.}$$

3. Periodic time of damped vibration

We know that periodic time of damped vibration

$$= \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{2\pi}{\sqrt{(20)^2 - (2.8)^2}} = 0.32 \text{ s Ans.}$$

2.The mass of a single deg ree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds wh en disturbed from its equilibrium positio n. The amplitude of vibration reduces to 0.25 of i ts initial value after five oscillations. Dete rmine : 1. stiffness of the spring, 2. logarithmic d ecrement, and 3. damping factor, i.e. the ratio of the system damping to critical damping. (AU-NOV/DEC-2012)

Solution. Given : m = 7.5 kg

Since 24 oscillations are made in 14 seconds, therefore frequency of free vibrations,

$$f_n = 24/14 = 1.7$$

and

$$\omega_n = 2\pi \times f_n = 2\pi \times 1.7 = 10.7 \text{ rad/s}$$

1. Stiffness of the spring

Let s = Stiffness of the spring in N/m.

We know that $(\omega_n)^2 = s/m$ or $s = (\omega_n)^2 m = (10.7)^2 7.5 = 860$ N/m Ans.

2. Logarithmic decrement

Let x_1 = Initial amplitude,

x_6 = Final amplitude after five oscillations = $0.25\, x_1$ ... (Given)

$$\therefore \quad \frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} = \left(\frac{x_1}{x_2}\right)^5 \quad \ldots \left[\because \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}\right]$$

or

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5} = \left(\frac{x_1}{0.25\, x_1}\right)^{1/5} = (4)^{1/5} = 1.32$$

We know that logarithmic decrement,

$$\delta = \log_e \left(\frac{x_1}{x_2}\right) = \log_e 1.32 = 0.28 \text{ Ans.}$$

3. Damping factor

Let c = Damping coefficient for the actual system, and

c_c = Damping coefficient for the critical damped system.

We know that logarithmic decrement (δ),

$$0.28 = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{a \times 2\pi}{\sqrt{(10.7)^2 - a^2}}$$

$$0.0784 = \frac{a^2 \times 39.5}{114.5 - a^2}$$ ... (Squaring both sides)

$$8.977 - 0.0784\, a^2 = 39.5\, a^2 \quad \text{or} \quad a^2 = 0.227 \quad \text{or} \quad a = 0.476$$

We know that $a = c / 2m$ or $c = a \times 2m = 0.476 \times 2 \times 7.5 = 7.2$ N/m/s **Ans.**

and $c_c = 2m\omega_n = 2 \times 7.5 \times 10.7 = 160.5$ N/m/s **Ans.**

$\therefore$ Damping factor $= c/c_c = 7.2 / 160.5 = 0.045$ **Ans.**

3(i) The measurements on a mechanical vibrating system show that it has a mass of 8 kg that the springs can be combined to give an equivalent spring of stiffn vibrating system have a dashpot attached which exerts a force of 40 N whe velocity of 1 m/s, find : 1. critical damping coefficient, 2. damping factor, decrement, and 4. ratio of two consecutive amplitudes. (AU-MAY/JUN-2012)

Solution. Given : $m = 8$ kg ; $s = 5.4$ N/mm $= 5400$ N/m

Since the force exerted by dashpot is 40 N, and the mass has a velocity of 1 m/s , therefore Damping coefficient (actual),

1. Critical damping coefficient

We know that critical damping coefficient,

$$c_c = 2m\omega_n = 2m \times \sqrt{\frac{s}{m}} = 2 \times 8\sqrt{\frac{5400}{8}} = 416 \text{ N/m/s } \textbf{Ans.}$$

2. Damping factor

We know that damping factor

$$= \frac{c}{c_c} = \frac{40}{416} = 0.096 \textbf{ Ans.}$$

3. Logarithmic decrement

We know that logarithmic decrement,

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}} = \frac{2\pi \times 40}{\sqrt{(416)^2 - (40)^2}} = 0.6 \textbf{ Ans.}$$

4. Ratio of two consecutive amplitudes

Let x_n and x_{n-1} = Magnitude of two consecutive amplitudes,

We know that logarithmic decrement,

$$\delta = \log_e\left[\frac{x_n}{x_{n+1}}\right] \quad \text{or} \quad \frac{x_n}{x_{n+1}} = e^{\delta} = (2.7)^{1.6} = 1.82 \textbf{ Ans.}$$

3 (ii) An instrument vibrates with a frequency of 1 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 0.9 Hz. Find 1. the damping factor, and 2. logarithmic decrement.

Solution. Given : $f_n = 1$ Hz ; $f_d = 0.9$ Hz

1. ***Damping factor***

Let m = Mass of the instrument in kg, c = Damping coefficient or damping force per unit velocity in N/m/s, and

c_c = Critical damping coefficient in N/m/s.

We know that natural circular frequency of undamped vibrations,

$$\omega_n = 2\pi \times f_n = 2\pi \times 1 = 6.284 \text{ rad/s}$$

and circular frequency of damped vibrations,

$$\omega_d = 2\pi \times f_d = 2\pi \times 0.9 = 5.66 \text{ rad/s}$$

We also know that circular frequency of damped vibrations (ω_d),

$$5.66 = \sqrt{(\omega_n)^2 - a^2} = \sqrt{(6.284)^2 - a^2}$$

Squaring both sides,

$$(5.66)^2 = (6.284)^2 - a^2 \text{ or } 32 = 39.5 - a^2$$

$\therefore$ $a^2 = 7.5$ or $a = 2.74$

We know that, $a = c/2m$ or $c = a \times 2m = 2.74 \times 2m = 5.48\, m$ N/m/s

and $c_c = 2m\omega_n = 2m \times 6.284 = 12.568\, m$ N/m/s

$\therefore$ Damping factor,

$$c/c_c = 5.48m/12.568\,m = 0.436 \text{ Ans.}$$

2. ***Logarithmic decrement***

We know that logarithmic decrement,

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}} = \frac{2\pi \times 5.48\,m}{\sqrt{(12.568\,m)^2 - (5.48\,m)^2}} = \frac{34.4}{11.3} = 3.04 \text{ Ans.}$$

4(i) A coil of spring stiffness 4 N/mm supports vertically a mass of 20 kg at the free end.

beginning of the fourth cycle is 0.8 times the amplitude of the previous the damping force per unit velocity. Also find the ratio of the frequ undamped vibrations. (AU-APR/MAY-2010)

The motion is resisted by the oil dashpot. It is found that the amplitude at the vibration. Determine ency of damped and

Solution. Given : $s = 4$ N/mm $= 4000$ N/m ; $m = 20$ kg

Damping force per unit velocity

Let c = Damping force in newtons per unit velocity *i.e.* in N/m/s x_n = Amplitude at the beginning of the third cycle,

x_{n+1} = Amplitude at the beginning of the fourth cycle = 0.8 x_n

We know that natural circular frequency of motion,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{4000}{20}} = 14.14 \text{ rad/s}$$

and

$$\log_e\left(\frac{x_n}{x_{n+1}}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

or

$$\log_e\left(\frac{x_n}{0.8\,x_n}\right) = a \times \frac{2\pi}{\sqrt{(14.14)^2 - a^2}}$$

$$\log_e 1.25 = a \times \frac{2\pi}{\sqrt{200 - a^2}} \quad \text{or} \quad 0.223 = a \times \frac{2\pi}{\sqrt{200 - a^2}}$$

Squaring both sides

$$0.05 = \frac{a^2 \times 4\pi^2}{200 - a^2} = \frac{39.5\,a^2}{200 - a^2}$$

$$0.05 \times 200 - 0.05\,a^2 = 39.5a^2 \quad \text{or} \quad 39.55\,a^2 = 10$$

$\therefore$ $a^2 = 10 / 39.55 = 0.25$ or $a = 0.5$

We know that $a = c / 2m$

$\therefore$ $c = a \times 2m = 0.5 \times 2 \times 20 = 20$ N/m/s **Ans.**

Ratio of the frequencies

Let f_{n1} = Frequency of damped vibrations = $\frac{\omega_d}{2\pi}$

f_{n2} = Frequency of undamped vibrations = $\frac{\omega_n}{2\pi}$

$\therefore$

$$\frac{f_{n1}}{f_{n2}} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \frac{\sqrt{(\omega_n)^2 - a^2}}{\omega_n} = \frac{\sqrt{(14.14)^2 - (0.5)^2}}{14.14}$$

$$\ldots \left(\because\ \omega_d = \sqrt{(\omega_n)^2 - a^2}\right)$$

= 0.999 **Ans.**

4(ii) Derive an expression for the natural frequency of single degrees of freedom system. (AU-MAY/JUNE-2009)

We know that the kinetic energy is due to the motion of the body and the potential energy is with respect to a certain datum position which is equal to the amount of work required to move the body from the datum position. In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero.

In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times. In other words, We know that kinetic energy, $\therefore \frac{d}{dt}(K.E.+P.E.)=0$

$$K.E.=\frac{1}{2}\times m\left(\frac{dx}{dt}\right)^2$$

and potential energy, $P.E.=\left(\frac{0+s.x}{2}\right)x=\frac{1}{2}\times s.x^2$

... ($\because$ P.E. = Mean force × Displacement)

$$\therefore \frac{d}{dt}\left[\frac{1}{2}\times m\left(\frac{dx}{dt}\right)^2+\frac{1}{2}\times s.x^2\right]=0$$

$$\frac{1}{2}\times m\times 2\times\frac{dx}{dt}\times\frac{d^2x}{dt^2}+\frac{1}{2}\times s\times 2x\times\frac{dx}{dt}=0$$

or $m\times\frac{d^2x}{dt^2}+s.x=0$ or $\frac{d^2x}{dt^2}+\frac{s}{m}\times x=0$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2}+\omega^2.x=0$$

Comparing equations,

$$\omega=\sqrt{\frac{s}{m}}$$

$\therefore$ Time period, $t_p=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{s}}$

and natural frequency, $f_n=\frac{1}{t_p}=\frac{1}{2\pi}\sqrt{\frac{s}{m}}=\frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ ... ($\because m.g=s.\delta$)

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n=\frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}}=\frac{0.4985}{\sqrt{\delta}}\text{ Hz}$$

5. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E = 200 GN/m². (AUMAY/JUN-2013)

Solution. Given : d = 5 mm = 0.005 m ; l = 200 mm = 0.2 m ; m = 50 kg ; e = 0.25 mm = 0.25 × 10^{-3} m ; E = 200 GN/m^2 = 200 × 10^9 N/m^2

Critical speed of rotation

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.005)^4 = 30.7 \times 10^{-12}\,\text{m}^4$$

Since the shaft is supported in long bearings, it is assumed to be fixed at both ends. We know that the static deflection at the centre of the shaft due to a mass of 50 kg,

$$\delta = \frac{Wl^3}{192\,EI} = \frac{50 \times 9.81(0.2)^3}{192 \times 200 \times 10^9 \times 30.7 \times 10^{-12}} = 3.33 \times 10^{-3}\,\text{m}$$

$$\ldots (\because W = mg)$$

We know that critical speed of rotation (or natural frequency of transverse vibrations),

$$N_c = \frac{0.4985}{\sqrt{3.33 \times 10^{-3}}} = 8.64 \text{ r.p.s. } \textbf{Ans.}$$

Maximum bending stress

Let σ = Maximum bending stress in N/m², and

N = Speed of the shaft = 75% of critical speed = 0.75 N_c . . . (Given)

When the shaft starts rotating, the additional dynamic load (W_1) to which the shaft is subjected, may be obtained by using the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y_1} \quad \text{or} \quad M = \frac{\sigma . I}{y_1}$$

We know that for a shaft fixed at both ends and carrying a point load (W_1) at the centre, the maximum bending moment

$$M = \frac{W_1 . l}{8}$$

$$\therefore \quad \frac{W_1 . l}{8} = \frac{\sigma \cdot I}{d/2} \qquad \ldots (\because y_1 = d/2)$$

and

$$W_1 = \frac{\sigma . I}{d/2} \times \frac{8}{l} = \frac{\sigma \times 30.7 \times 10^{-12}}{0.005/2} \times \frac{8}{0.2} = 0.49 \times 10^{-6}\,\sigma\,\text{N}$$

$\therefore$ Additional deflection due to load W_1,

$$y = \frac{W_1}{W} \times \delta = \frac{0.49 \times 10^{-6}\sigma}{50 \times 9.81} \times 3.33 \times 10^{-3} = 3.327 \times 10^{-12}\sigma$$

We know that

$$y = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1} = \frac{\pm e}{\left(\frac{N_c}{N}\right)^2 - 1} \qquad \ldots (\text{Substituting } \omega_c = N_c \text{ and } \omega = N)$$

$$3.327 \times 10^{-12}\sigma = \frac{\pm 0.25 \times 10^{-3}}{\left(\frac{N_c}{0.75\,N_c}\right)^2 - 1} = \pm 0.32 \times 10^{-3}$$

$$\sigma = 0.32 \times 10^{-3} / 3.327 \times 10^{-12} = 0.0962 \times 10^9\,\text{N/m}^2 \quad \ldots (\text{Taking + ve sign})$$

$$= 96.2 \times 10^6\,\text{N/m}^2 = 96.2\,\text{MN/m}^2 \textbf{ Ans.}$$

6.(i) A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support. The Young's modulus for shaft material is 200 GN/m^2. Find the frequency of transverse vibration. (AU-NOV/DEC-2011)

Solution. Given :^{2} = 200 × d = 50 mm 10^9 N/= 0.05 m ;m^2l = 3 m, W_1 = 1000 N ; W_2 = 1500 N ; W_3 = 750 N;

E = 200 GN/m

The shaft carrying the loads is shown in Fig. 23.13

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6} \text{ m}^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3\,EIl}$$

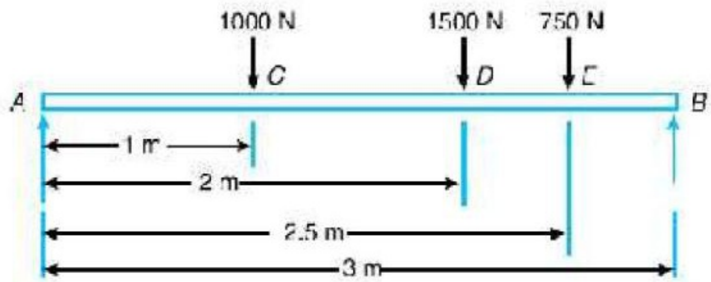


Fig. 23.13

∴ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3} \text{ m}$$

. . . (Here a = 1 m, and b = 2 m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3} \text{ m}$$

We know that frequency of transverse vibration,

anc

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz Ans.}$$

6.(ii) Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m³, and Young's modulus is 200 GN/m². Assume the shaft to be freely supported. (AU-MAY/JUNE-2009)

Solution. Given : d = 20 mm = 0.02 m ; l = 0.6 m ; m_1 = 1 kg ; ρ = 40 Mg/m³ = 40 × 10⁶ g/m³ = 40 × 10³ kg/m³ ; E = 200 GN/m² = 200 × 10⁹ N/m²

The shaft is shown in Fig. 23.15.

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.02)^4 \text{ m}^4$$

$$= 7.855 \times 10^{-9} \text{ m}^4$$

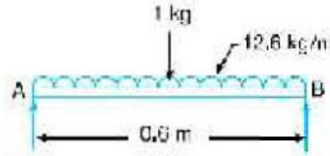


Fig. 23.15

Since the density of shaft material is 40 × 10³ kg/m³, therefore mass of the shaft per metre length,

$$m_S = \text{Area} \times \text{length} \times \text{density} = \frac{\pi}{4}(0.02)^2 \times 1 \times 40 \times 10^3 = 12.6 \text{ kg/m}$$

We know that static deflection due to 1 kg of mass at the centre,

$$\delta = \frac{Wl^3}{48\,EI} = \frac{1 \times 9.81\,(0.6)^3}{48 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 28 \times 10^{-6} \text{ m}$$

and static deflection due to mass of the shaft,

$$\delta_S = \frac{5\,wl^4}{384\,EI} = \frac{5 \times 12.6 \times 9.81\,(0.6)^4}{384 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 0.133 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta + \dfrac{\delta_S}{1.27}}} = \frac{0.4985}{\sqrt{28 \times 10^{-6} + \dfrac{0.133 \times 10^{-3}}{1.27}}}$$

$$= \frac{0.4985}{11.52 \times 10^{-3}} = 43.3 \text{ Hz}$$

Let N_c = Whirling speed of a shaft.

We know that whirling speed of a shaft in r.p.s. is equal to the frequency of transverse vibration in Hz , therefore

$$N_c = 43.3 \text{ r.p.s.} = 43.3 \times 60 = 2598 \text{ r.p.m.} \textbf{ Ans.}$$

UNIT-IV FORCED VIBRATION Part-A(2 Marks)

1. **What is meant by forced vibrations ? Give Examples of forced vibrations.** **(NOV/DEC-2011)**

When the body vibrates under the influence of external force, then the body is said to be under forced vibrations.

Examples:

- Ringing of electrical bell
- The vibrations of air compressors, internal combustion engines, machine tools and various other machinery.

2. **What are the types of external excitation? (NOV/DEC-2011)**

- Periodic forces
- Impulsive forces and Random forces.

3. **What is the vibration isolation? (AU-MAY/JUNE-2009)**

The term vibration isolation refers to the prevention or minimization of vibrations and their transmission due to the unbalanced machines.

4. **Specify the importance of vibration isolation? (AU-NOV/DEC-2009)**

When an unbalanced machine is installed on the foundation, it produces vibration in the foundation. So, in order to prevent these vibrations or to minimize the transmission of forces to the foundation, vibration isolation is important.

5. **When does resonance takes place in a system?(AU-NOV/DEC-2010)**

When the frequency of external force is equal to the natural frequency of a vibrating body, the resonance occurs. At resonance, the amplitude of vibration becomes exclusively large.

6. **Define transmissibility? (AU-NOV/DEC-2013)**

When a machine is supported by a spring, the spring transmits the force applied on the machine to the fixed support or foundation. This is called as transmissibility.

7. **What is meant by harmonic forcing? (AU-NOV/DEC-2013)**

The term harmonic refers to a spring-mass system with viscous damping, excited by a sinusoidal harmonic force.

$$F = F_0 \sin \omega t$$

8. **Define transmissibility ratio or isolation factor? (AU-NOV/DEC-2012)**

The ratio of force transmitted (FT) to the force applied (F) is known as transmissibility ratio

9. **What is the role of transmission ratio?**

Transmission ratio, also known as transmissibility, is a measure of the effectiveness of the vibration isolation material.

10. **What is meant by dynamic magnifier or magnification factor? What are the factors on which it depend? (AU-NOV/DEC-2010)**

It is the ratio of maximum displacement of the forced vibration (Xmax) to the deflection due to the static force F (x0)

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

It depends on: (i) the ratio of circular frequencies

(ii) The damping factor (□)

11. **Write down the expression for amplitude of forced vibration. ? (AU-NOV/DEC-2012)**

Amplitude of forced vibration, ________

12. **List out the materials used for vibration isolation. Laso which material is most suitable for compressive loads? (AU-MAY/JUN-2012)**

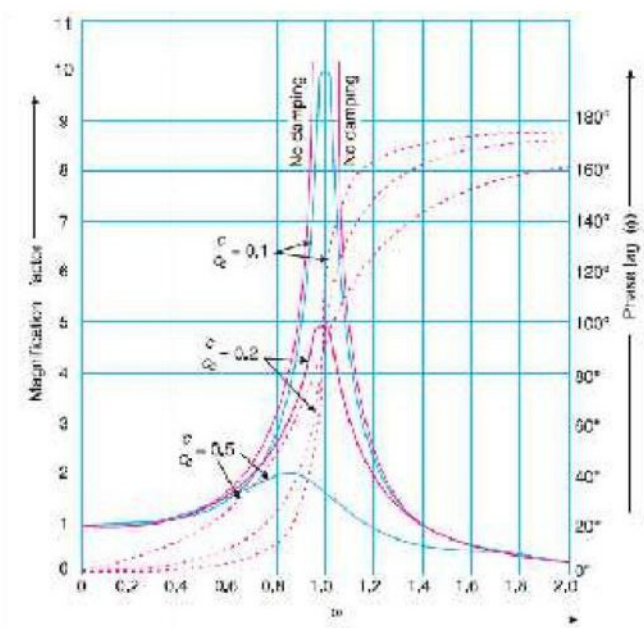
- Rubber
- Felt
- Cork
- Metallic Springs

Cork is suitable for compressive loads because it is not perfectly elastic. At high loads it becomes more flexible.

13. **Show that for effective isolation of vibration, frequency ratio r >√2. (AU-MAY/JUN-2013)**

When r >√2, then transmissibility is less than one for all values of damping factor. This means that the transmitted force is always less than the excited force.

14. **Sketch the graph for (ω/ω_n) Vs Transmissibility for different values of damping factor.**



15. What are the methods of isolating the vibration?

- High speed engines/machines mounted on foundation and supports cause vibrations of excessive amplitude because of the unbalanced forces. It can be minimized by providing "spring-damper" , etc.
- The materials used for vibration isolation are rubber, felt cork, etc. These are placed between the foundation and vibrating body.

PART-B

1. Derive the relation for the displacement of mass from the equilibrium position of the damped vibration system with harmonic forcing. ? (AU-NOV/DEC-2013)

Consider a system consisting of spring, mass and damper as shown in Fig. 23.19. Let the system is acted upon by an external periodic (*i.e.* simple harmonic) disturbing force,

$$F_x = F \cos \omega .t$$

where F = Static force, and

ω = Angular velocity of the periodic disturbing force.

When the system is constrained to move in vertical guides, it has only one degree of freedom. Let at sometime t, the mass is displaced downwards through a distance x *from its mean position.*

The equation of motion may be written as,

$$m \times \frac{d^2x}{dt^2} = -c \times \frac{dx}{dt} - s.x + F\cos\omega t$$

or

$$m \times \frac{d^2x}{dt^2} + c \times \frac{dx}{dt} + s.x = F\cos\omega t$$

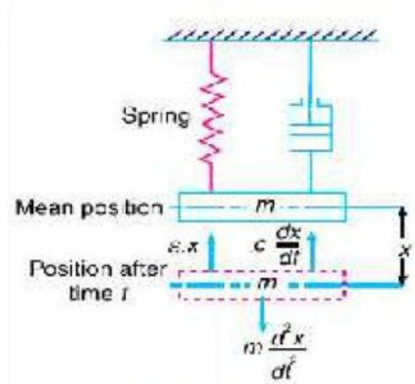


Fig. 23.19. Frequency of under damped forced vibrations.

This equation of motion may be solved either by differential equation method or by graphical method as discussed below :

1. ***Differential equation method***

The equation **(*i*)** is a differential equation of the second degree whose right hand side is some function in t. The solution of such type of differential equation consists of two parts ; one part is the complementary function and the second is particular integral. Therefore the solution may be written as

$$x = x_1 + x_2$$

where x_1 = Complementary function, and x_2 = Particular integral.

The complementary function is same as discussed in the previous article, *i.e.* $x_1 = Ce^{-at} \cos$

$(\omega_d t - \theta)$ . . . (*ii*) where C and θ are constants. Let us

now find the value of particular integral as discussed below : Let the particular integral of equation (*i*) is given by

$$x_2 = B_1 \sin \omega.t + B_2 \cos \omega.t \qquad \text{. . . (where } B_1 \text{ and } B_2 \text{ are constants)}$$

$\therefore$
$$\frac{dx}{dt} = B_1.\omega \cos \omega.t - B_2.\omega \sin \omega.t$$

and
$$\frac{d^2x}{dt^2} = -B_1.\omega^2 \sin \omega.t - B_2.\omega^2 \cos \omega.t$$

Substituting these values in the given differential equation (*i*), we get

$$m(-B_1.\omega^2 \sin \omega.t - B_2.\omega^2 \cos \omega.t) + c(B_1.\omega \cos \omega.t - B_2.\omega \sin \omega.t) + s(B_1 \sin \omega.t + B_2 \cos \omega.t)$$
$$= F \cos \omega.t$$

or
$$(-m.B_1.\omega^2 - c.\omega.B_2 + s.B_1) \sin \omega.t + (-m.\omega^2.B_2 + c.\omega.B_1 + s.B_2) \cos \omega.t$$
$$= F \cos \omega.t$$

or
$$\left[(s - m.\omega^2) B_1 - c.\omega.B_2\right] \sin \omega.t + \left[c.\omega.B_1 + (s - m\omega^2) B_2\right] \cos \omega.t$$
$$= F \cos \omega.t + 0 \sin \omega.t$$

Comparing the coefficients of sin *ωt* and cos *ωt* on the **left** hand side and **right** hand side separately, we get

$$(s - m\omega^2) B_1 - c.\omega.B_2 = 0 \qquad \text{. . . (iii)}$$

and
$$c.\omega.B_1 + (s - m\omega^2) B_2 = F \qquad \text{. . . (iv)}$$

Now from equation (*iii*)

$$(s - m.\omega^2) B_1 = c.\omega.B_2$$

$\therefore$
$$B_2 = \frac{s - m.\omega^2}{c.\omega} \times B_1 \qquad \text{. . . (v)}$$

Substituting the value of B_2 in equation (*iv*)

$$c.\omega.B_1 + \frac{(s - m\omega^2)(s - m.\omega^2)}{c.\omega} \times B_1 = F$$

$$c^2.\omega^2.B_1 + (s - m.\omega^2)^2 B_1 = c.\omega.F$$

$$B_1\left[c^2.\omega^2 + (s - m.\omega^2)^2\right] = c.\omega.F$$

$\therefore$
$$B_1 = \frac{c.\omega.F}{c^2.\omega^2 - (s - m.\omega^2)^2}$$

and
$$B_2 = \frac{s - m.\omega^2}{c.\omega} \times \frac{c.\omega.F}{c^2.\omega^2 + (s - m.\omega^2)^2} \qquad \text{. . . [From equation (v)]}$$

$$= \frac{F(s - m.\omega^2)}{c^2.\omega^2 + (s - m.\omega^2)^2}$$

$\therefore$ The particular integral of the differential equation (*i*) is

$$x_2 = B_1 \sin \omega.t + B_2 \cos \omega.t$$

$$= \frac{c\omega F}{c^2.\omega^2 + (s - m\omega^2)^2} \times \sin \omega t - \frac{F(s - m\omega^2)}{c^2.\omega^2 + (s - m\omega^2)^2} \times \cos \omega t$$

$$= \frac{F}{c^2.\omega^2 + (s - m\omega^2)^2} \left[c\omega \sin \omega t + (s - m\omega^2) \cos \omega t \right] \quad \ldots (vi)$$

Let $\qquad c.\omega = X \sin \phi;$ and $s - m\omega^2 = X \cos \phi$

$\therefore \qquad X = \sqrt{c^2.\omega^2 + (s - m\omega^2)^2} \qquad \ldots$ (By squaring and adding)

and $\qquad \tan \phi = \dfrac{c.\omega}{s - m\omega^2} \qquad$ or $\qquad \phi = \tan^{-1}\left(\dfrac{c.\omega}{s - m.\omega^2} \right)$

Now the equation (*vi*) may be written as

$$x_2 = \frac{F}{c^2.\omega^2 + (s - m.\omega^2)^2} \left[X \sin \phi . \sin \omega t + X \cos \phi \cos \omega t \right]$$

$$= \frac{F.X}{c^2.\omega^2 + (s - m.\omega^2)^2} \times \cos (\omega t - \phi)$$

$$= \frac{F\sqrt{c^2.\omega^2 + (s - m\omega^2)^2}}{c^2.\omega^2 + (s - m\omega^2)^2} \times \cos (\omega t - \phi)$$

$$= \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}} \times \cos (\omega t - \phi)$$

$\therefore$ The complete solution of the differential equation (*i*) becomes

$$x = x_1 + x_2$$

$$= C.e^{-at} \cos (\omega_d.t - \theta) + \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}} \times \cos (\omega t - \phi)$$

In actual practice, the value of the complementary function x_1 at any time t is much smaller as compared to particular integral x_2. Therefore, the displacement x, at any time t, is given by the particular integral x_2 only.

$$\therefore \qquad x = \frac{F}{\sqrt{c^2.\omega^2 + (s - m\omega^2)^2}} \times \cos (\omega t - \phi) \quad \ldots (vii)$$

A little consideration will show that the frequency of forced vibration is equal to the angular velocity of the periodic force and the amplitude of the forced vibration is equal to the maximum displacement of vibration.

$\therefore$ Maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}} \quad \ldots (viii)$$

This equation shows that motion is simple harmonic whose circular frequency is ω and the amplitude is $\dfrac{F}{\sqrt{c^2.\omega^2 + (s - m.\omega^2)^2}}$.

2. A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to one-tenth of the initial value in four complete oscillations. If a periodic force of 150 cos 50 t N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations. What is its value of resonance ? ? (AU-NOV/DEC-2014)

Solution. Given : $m = 10$ kg ; $s = 10$ N/mm $= 10 \times 10^3$ N/m ; $x_5 = \frac{x_1}{10}$

Since the periodic force, $F_x = F\cos\omega t = 150\cos 50t$, therefore

Static force, $F = 150$ N

and angular velocity of the periodic disturbing force.

$$\omega = 50 \text{ rad/s}$$

We know that angular speed or natural circular frequency of free vibrations,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{10\times10^3}{10}} = 31.6 \text{ rad/s}$$

Amplitude of the forced vibrations

Since the amplitude decreases to 1/10th of the initial value in four complete oscillations, therefore, the ratio of initial amplitude (x_1) to the final amplitude after four complete oscillations (x_5) is given by

$$\frac{x_1}{x_5} = \frac{x_1}{x_2}\times\frac{x_2}{x_3}\times\frac{x_3}{x_4}\times\frac{x_4}{x_5} = \left(\frac{x_1}{x_2}\right)^4 \quad \ldots \left(\because \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5}\right)$$

$$\therefore \quad \frac{x_1}{x_2} = \left(\frac{x_1}{x_5}\right)^{1/4} = \left(\frac{x_1}{x_1/10}\right)^{1/4} = (10)^{1/4} = 1.78 \quad \ldots \left(x_5 = \frac{x_1}{10}\right)$$

We know that

$$\log_e\left(\frac{x_1}{x_2}\right) = a\times\frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

$$\log_e 1.78 = a\times\frac{2\pi}{\sqrt{(31.6)^2 - a^2}} \quad \text{or} \quad 0.576 = \frac{a\times 2\pi}{\sqrt{1000 - a^2}}$$

We know that amplitude of the forced vibrations,

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2\omega^2}{s^2} + \left[1 - \frac{\omega^2}{(\omega_n)^2}\right]^2}}$$

$$= \frac{0.015}{\sqrt{\frac{(57.74)^2(50)^2}{(10\times10^3)^2} + \left[1 - \left(\frac{50}{31.6}\right)^2\right]^2}} = \frac{0.015}{\sqrt{0.083 + 2.25}}$$

Squaring both sides and rearranging,

$$39.832\ a^2 = 332 \quad \text{or} \quad a^2 = 8.335 \quad \text{or} \quad a = 2.887$$

We know that $a = c/2m$ or $c = a \times 2m = 2.887 \times 2 \times 10 = 57.74$ N/m/s

and deflection of the system produced by the static force F,

$$x_o = F/s = 150/10 \times 10^3 = 0.015 \text{ m}$$

3. The mass of an electric motor is 120 kg and it runs at 1500 r.p.m. The armature mass is 35 kg and its C.G. lies 0.5 mm from the axis of rotation. The motor is mounted on five springs of negligible damping so that the force transmitted is one-eleventh of the impressed force. Assume that the mass of the motor is equally distributed among the five springs.

Determine : 1. stiffness of each spring; 2. dynamic force transmitted to the base at the operating speed; and 3. natural frequency of the system. ? (AU-MAY/JUN-2013)

Solution. Given $m_1 = 120$ kg ; $m_2 = 35$ kg; $r = 0.5$ mm $= 5 \times 10^{-4}$ m; $\varepsilon = 1/11$; $N = 1500$ r.p.m. or $\omega = 2\pi \times 1500 / 60 = 157.1$ rad/s ;

1. ***Stiffness of each spring***

Let s = Combined stiffness of the spring in N-m, and

ω_n = Natural circular frequency of vibration of the machine in rad/s.

$$= \frac{0.015}{1.53} = 9.8 \times 10^{-3} \text{ m} = 9.8 \text{ mm } \textbf{Ans.}$$

Amplitude of forced vibrations at resonance

We know that amplitude of forced vibrations at resonance.

$$x_{max} = x_0 \times \frac{s}{c.\omega_n} = 0.015 \times \frac{10 \times 10^3}{57.54 \times 31.6} = 0.0822 \text{ m} = 82.2 \text{ mm } \textbf{Ans.}$$

We know that $\omega_n = \sqrt{s / m_1}$

$$s = m_1 (\omega_n)^2 = 120 \times 2057 = 246\ 840 \text{ N/m}$$

Since these are five springs, therefore stiffness of each spring

$$= 246\ 840 / 5 = 49\ 368 \text{ N/m } \textbf{Ans.}$$

We know that transmissibility ratio (ε).

$$\frac{1}{11} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{(\omega_n)^2}{\omega^2 - (\omega_n)^2} = \frac{(\omega_n)^2}{(157.1)^2 - (\omega_n)^2}$$

or $$(157.1)^2 - (\omega_n)^2 = 11 (\omega_n)^2 \quad \text{or} \quad (\omega_n)^2 = 2057 \quad \text{or} \quad \omega_n = 45.35 \text{ rad/s}$$

2. ***Dynamic force transmitted to the base at the operating speed (i.e. 1500 r.p.m. or 157.1 rad/s)***

We know that maximum unbalanced force on the motor due to armature mass,

$$F = m_2\, \omega^2 \cdot r = 35\,(157.1)^2\, 5 \times 10^{-4} = 432 \text{ N}$$

∴ Dynamic force transmitted to the base.

$$F_T = \varepsilon . F = \frac{1}{11} \times 432 = 39.27 \text{ N } \textbf{Ans.}$$

3. ***Natural frequency of the system***

We have calculated above that the natural frequency of the system.

$$\omega_n = 45.35 \text{ rad/s } \textbf{Ans.}$$

4. What do you understand by transmissibility? Describe the method of finding the transmissibility ratio from unbalanced machine supported with foundation. **(AU-NOV/DEC-2012)**

A little consideration will show that when an unbalanced machine is installed on the foundation, it produces vibration in the foundation. In order to prevent these vibrations or to minimize the transmission of forces to the foundation, the machines are mounted on springs and dampers or on some vibration isolating material, as shown in Fig. 23.22. The arrangement is assumed to have one degree of freedom, *i.e.* it can move up and down only.

It may be noted that when a periodic (*i.e.* simple harmonic) disturbing force $F \cos \omega t$ is applied to a machine of mass m supported by a spring of stiffness s, then the force is transmitted by means of the spring and the damper or dashpot to the fixed support or foundation.

The ratio of the force transmitted (F_T) to the force applied (F) is known as the ***isolation factor*** or ***transmissibility ratio*** of the spring support.

We have discussed above that the force transmitted to the foundation consists of the following two forces :

1.Spring force or elastic force which is equal to $s.\, x_{max}$, and

2.Damping force which is equal to $c.\, \omega\, .x_{max}$.

Since these two forces are perpendicular to one another, as shown in Fig.23.23, therefore the force transmitted,

$$F_T = \sqrt{(s.x_{max})^2 + (c.\omega.x_{max})^2}$$

$$= x_{max}\sqrt{s^2 + c^2.\omega^2}$$

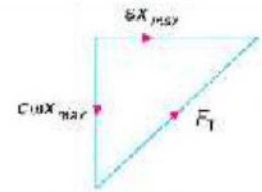


Fig. 23.23

∴ Transmissibility ratio,

$$\varepsilon = \frac{F_T}{F} = \frac{x_{max}\sqrt{s^2 + c^2.\omega^2}}{F}$$

We know that

$$x_{max} = x_o \times D = \frac{F}{s} \times D \qquad \ldots \left(\because\ x_o = \frac{F}{s}\right)$$

$$\therefore \quad \varepsilon = \frac{D}{s}\sqrt{s^2 + c^2.\omega^2} = D\sqrt{1 + \frac{c^2.\omega^2}{s^2}}$$

$$= D\sqrt{1 + \left(\frac{2c}{c_c} \times \frac{\omega}{\omega_n}\right)^2} \qquad \ldots \left(\because\ \frac{c.\omega}{s} = \frac{2c}{c_c} \times \frac{\omega}{\omega_n}\right)$$

magnification factor,

$$\therefore \quad \varepsilon = \frac{\sqrt{1 + \left(\frac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

When the damper is not provided, then $c = 0$, and

$$\varepsilon = \frac{1}{1 - (\omega/\omega_n)^2}$$

From above, we see that when $\omega/\omega_n > 1$, ε is negative. This means that there is a phase difference of 180° between the transmitted force and the disturbing force ($F \cos \omega t$). The value of ω/ω_n must be greater than $\sqrt{2}$ if ε is to be less than 1 and it is the numerical value of ε, independent of any phase difference between the forces that may exist which is important. It is therefore more convenient to use equation *(ii)* in the following form, *i.e.*

$$\varepsilon = \frac{1}{(\omega/\omega_n)^2 - 1} \qquad \ldots (iii)$$

Fig. 23.24 is the graph for different values of damping factor c/c_c to show the variation of transmissibility ratio (ε) against the ratio ω/ω_n

1. When $\omega/\omega_n = \sqrt{2}$, then all the curves pass through the point $\varepsilon = 1$ for all values of damping factor c/c_c.

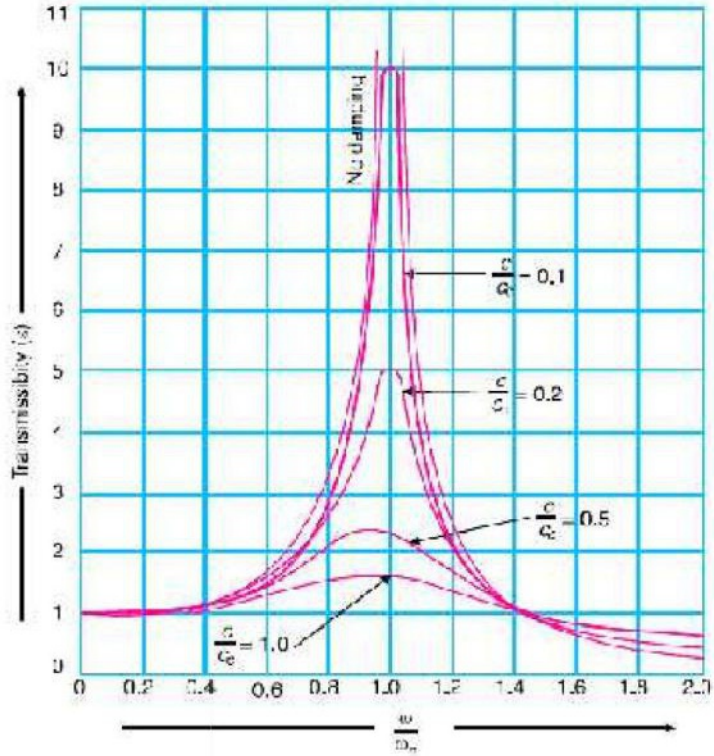


Fig. 23.24. Graph showing the variation of transmissibility ratio.

2. When $\omega/\omega_n < \sqrt{2}$, then $\varepsilon > 1$ for all values of damping factor c/c_c. This means that the force transmitted to the foundation through elastic support is greater than the force applied.

3. When $\omega/\omega_n > \sqrt{2}$, then $\varepsilon < 1$ for all values of damping factor c/c_c. This shows that the force transmitted through elastic support is less than the applied force. Thus vibration isolation is possible only in the range of $\omega/\omega_n > \sqrt{2}$.

5. A machine has a mass of 100 kg and unbalanced reciprocating parts of mass 2 kg which move through a vertical stroke of 80 mm with simple harmonic motion. The machine is mounted on four springs, symmetrically arranged with respect to centre of mass, in such a way that the machine has one degree of freedom and can undergo vertical displacements only.

Neglecting damping, calculate the combined stiffness of the spring in order that the force transmitted to the foundation is 1 / 25 th of the applied force, when the speed of rotation of ma-chine crank shaft is 1000 r.p.m.

When the machine is actually supported on the springs, it is found that the damping reduces the amplitude of successive free vibrations by 25%. Find : 1. the force transmitted

to foundation at 1000 r.p.m., 2. the force transmitted to the foundation at resonance, and 3. the amplitude of the forced vibration of the machine at resonance. (AUMAY/JUN-2012)

Solution. Given : $m_1 = 100$ kg ; $m_2 = 2$ kg ; $l = 80$ mm $= 0.08$ m ; $\varepsilon = 1/25$; $N = 1000$ r.p.m. or $\omega = 2\pi \times 1000/60 = 104.7$ rad/s

Combined stiffness of springs

Let s = Combined stiffness of springs in N/m, and

ω_n = Natural circular frequency of vibration of the machine in rad/s

We know that transmissibility ratio (ε),

$$\frac{1}{25} = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} = \frac{(\omega_n)^2}{\omega^2 - (\omega_n)^2} = \frac{(\omega_n)^2}{(104.7)^2 - (\omega_n)^2}$$

We know that $\omega_n = \sqrt{s/m}$

$\therefore$ $s = m_1 (\omega_n)^2 = 100 \times 421.6 = 42\ 160$ N/m **Ans.**

1. Force transmitted to the foundation at 1000 r.p.m.

Let F_T = Force transmitted, and

x_1 = Initial amplitude of vibration.

Since the damping reduces the amplitude of successive free vibrations by 25%, therefore final amplitude of vibration,

$$x_2 = 0.75\, x_1$$

We know that

$$\log_e\left(\frac{x_1}{x_2}\right) = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}} \quad \text{or} \quad \log_e\left(\frac{x_1}{0.75 x_1}\right) = \frac{a \times 2\pi}{\sqrt{421.6 - a^2}}$$

Squaring both sides,

$$(0.2877)^2 = \frac{a^2 \times 4\pi^2}{421.6 - a^2} \quad \text{or} \quad 0.083 = \frac{39.5 a^2}{421.6 - a^2}$$

$$\ldots\left[\because \log_e\left(\frac{1}{0.75}\right) = \log_e 1.333 = 0.2877\right]$$

$$35 - 0.083 a^2 = 39.5 a^2 \quad \text{or} \quad a^2 = 0.884 \quad \text{or} \quad a = 0.94$$

We know that damping coefficient or damping force per unit velocity,

$$c = a \times 2m_1 = 0.94 \times 2 \times 100 = 188 \text{ N/m/s}$$

and critical damping coefficient,

$$c_c = 2m.\omega_n = 2 \times 100 \times 20.5 = 4100 \text{ N/m/s}$$

∴ Actual value of transmissibility ratio,

$$\varepsilon = \frac{\sqrt{1+\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2}}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2+\left(1-\frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$= \frac{\sqrt{1+\left(\frac{2\times188\times104.7}{4100\times20.5}\right)^2}}{\sqrt{\left(\frac{2\times188\times104.7}{4100\times20.5}\right)^2+\left[1-\left(\frac{104.7}{20.5}\right)^2\right]^2}} = \frac{\sqrt{1+0.22}}{\sqrt{0.22+629}}$$

$$= \frac{1.104}{25.08} = 0.044$$

We know that the maximum unbalanced force on the machine due to reciprocating parts,

$$F = m_2.\omega^2.r = 2(104.7)^2\,(0.08/2) = 877 \text{ N} \qquad \ldots (\because\ r = l/2)$$

∴ Force transmitted to the foundation,

$$F_T = \varepsilon.F = 0.044 \times 877 = 38.6 \text{ N } \textbf{Ans.} \qquad \ldots (\because\ \varepsilon = F_T / F)$$

and maximum unbalanced force on the machine due to reciprocating parts at resonance speed ω_n,

$$F = m_2\,(\omega_n)^2 r = 2(20.5)^2 (0.08/2) = 33.6 \text{ N} \qquad \ldots (\because\ r = l/2)$$

∴ Force transmitted to the foundation at resonance,

$$F_T = \varepsilon.F = 10.92 \times 33.6 = 367 \text{ N } \textbf{Ans.}$$

3. Amplitude of the forced vibration of the machine at resonance

We know that amplitude of the forced vibration at resonance

$$= \frac{\text{Force transmitted at resonance}}{\text{Combined stiffness}} = \frac{367}{42\,160} = 8.7 \times 10^{-3} \text{ m}$$

$$= 8.7 \text{ mm } \textbf{Ans.}$$

6.(i) Derive the relation for magnification factor in case of forced vibration. ? (NOV/DEC-2013)

It is the ratio of *maximum displacement of the forced vibration (x_{max}) to the deflection due to the static force F(x_o)*. We have proved in the previous article that the maximum displacement or the amplitude of forced vibration,

$$x_{max} = \frac{x_o}{\sqrt{\frac{c^2.\omega^2}{s^2}+\left(1-\frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

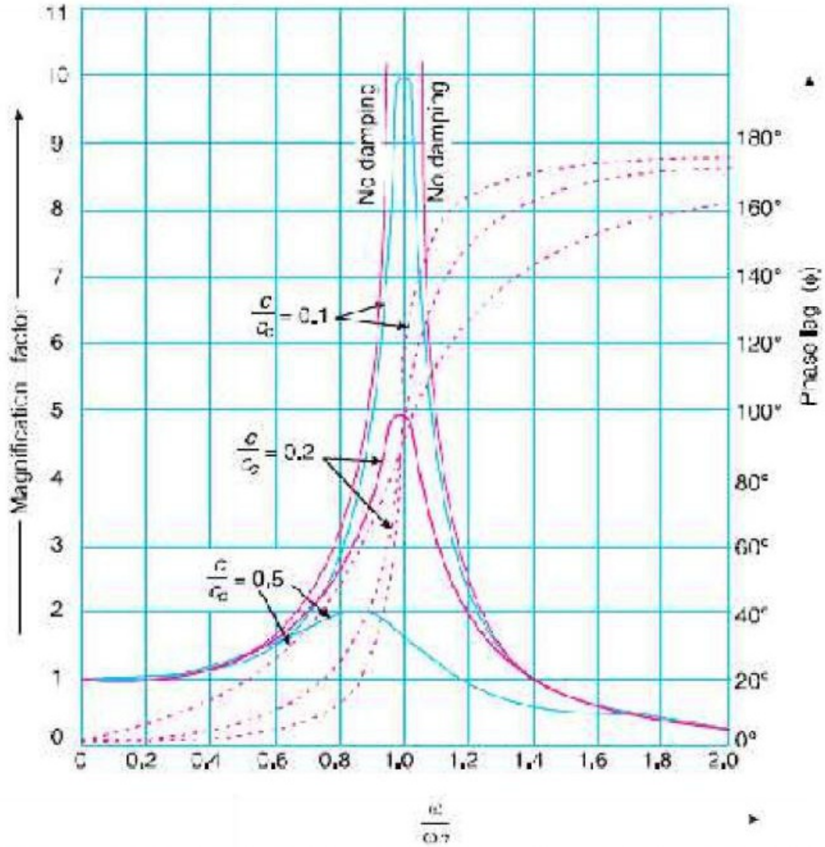


Fig. 23.21. Relationship between magnification factor and phase angle for different values of ω/ω_n.

$\therefore$ Magnification factor or dynamic magnifier,

$$D = \frac{x_{max}}{x_o} = \frac{1}{\sqrt{\frac{c^2.\omega^2}{s^2} + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}} \qquad \dots (i)$$

$$= \frac{1}{\sqrt{\left(\frac{2c.\omega}{c_c.\omega_n}\right)^2 + \left(1 - \frac{\omega^2}{(\omega_n)^2}\right)^2}}$$

$$\dots \left[\because \frac{c.\omega}{s} = \frac{2c.\omega}{2m \times \frac{s}{m}} = \frac{2c.\omega}{2m(\omega_n)^2} = \frac{2c.\omega}{c_c.\omega_n}\right]$$

The magnification factor or dynamic magnifier gives the factor by which the static deflection produced by a force F (*i.e.* x_o) must be multiplied in order to obtain the maximum amplitude of the forced vibration (*i.e.* x_{max}) by the harmonic force $F \cos \omega t$

$$\therefore \qquad x_{max} = x_o \times D$$

Fig. 23.21 shows the relationship between the magnification factor (D) and phase angle ϕ for different value of ω/ω_n and for values of damping factor c/c_c = 0.1, 0.2 and 0.5.

6.(ii) A single cylinder vertical petrol engine of total mass 300 kg is mounted upon a steel chassis frame and causes a vertical static deflection of 2 mm. The reciprocating parts of the engine has a mass of 20 kg and move through a vertical stroke of 150 mm with simple harmonic motion. A dashpot is provided whose damping resistance is directly proportional to the velocity and amounts to 1.5 kN per metre per second.

Considering that the steady state of vibration is reached ; determine : 1. the amplitude of forced vibrations, when the driving shaft of the engine rotates at 480 r.p.m., and 2. the speed of the driving shaft at which resonance will occur.

Solution : Given. m = 300 kg; δ = 2 mm = 2×10^{-3} m ; m_1 = 20 kg ; l = 150 mm = 0.15 m ; c = 1.5 kN/m/s = 1500 N/m/s ; N = 480 r.p.m. or ω □ 2□□ 480 / 60 = 50.3 rad/s

1. *Amplitude of the forced vibrations*

We know that stiffness of the frame,

$$s = m.g / \delta = 300 \times 9.81/2 \times 10^{-3} = 1.47 \times 10^{6} \text{ N/m}$$

Since the length of stroke (l) = 150 mm = 0.15 m, therefore radius of crank,

$$r = l / 2 = 0.15 / 2 = 0.075 \text{ m}$$

We know that the centrifugal force due to the reciprocating parts or the static force,

$$F = m_1 \omega^2 . r = 20\,(50.3)^2\, 0.075 = 3795 \text{ N}$$

∴ Amplitude of the forced vibration (maximum),

$$x_{max} = \frac{F}{\sqrt{c^2 \omega^2 + (s - m.\omega^2)^2}}$$

$$= \frac{3795}{\sqrt{(1500)^2 (50.3)^2 + [1.47 \times 10^6 - 300\,(50.3)^2]^2}}$$

$$= \frac{3795}{\sqrt{5.7 \times 10^9 + 500 \times 10^9}} = \frac{3795}{710 \times 10^3} = 5.3 \times 10^{-3} \text{ m}$$

2. *Speed of the driving shaft at which the resonance occurs*

Let N = Speed of the driving shaft at which the resonance occurs in r.p.m.

We know that the angular speed at which the resonance occurs,

$$\omega = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{1.47 \times 10^6}{300}} = 70 \text{ rad/s}$$

∴ $N = \omega \times 60 / 2\pi = 70 \times 60 / 2\pi$ = 668.4 r.p.m. **Ans.**

UNIT-V MECHANISMS FOR CONTROL Part-A(2 Marks)

1. **What is meant by sensitiveness of governors? (AU-NOV/DEC-2013)**

   The sensitiveness is defined as the ratio of the mean speed to the difference between the maximum and minimum speeds. A governor is said to be sensitive, when it really to a small change of speed.

2. **What is meant by hunting? (AU-NOV/DEC-2012)**

   The phenomenon of continuous fluctuations of the engine speed above and below the mean speed is termed as hunting. This occurs in oversensitive governors.

3. **What is meant by isochronous condition in governors? (AU-MAY/JUNE-2013)**

   A governor with zero range of speed is known as an isochronous governor. Actually the isochronism is the stake of Infinite sensitivity.

4. **What are centrifugal governor? How do they differ from inertia governor?**

   - The centrifugal governor controls the fuel supply by means of the centrifugal forces on the governor balls.
   - Inertia governor work s based on the inertia forces caused by an angular acceleration or retardation of the shaft.

5. **Explain the term stability of the governor? (AU-NOV/DEC-2010)**

   A governor is said to be stable if there is only one radius of rotation for all equilibrium speeds of the balls within the working range. If the equilibrium speed increases the radius of governor ball must also increase.

6. **Derive an expression for the height in the case of a Watt governor. (AU-APR/MAY-2010)**

   $FC \times h = w \times r = m.g.r$

   $m.\omega^2.r.h = m.g.r$ or $h = g / \omega^2$

   When g is expressed in m/s and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then $\omega = 2\pi N/60$

   $$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2}$$

7. **Define steering, pitching and rolling. (Or) list some of the terms related to motion of ships using gyroscopic principle. (AU-NOV/DEC-2013)**

   - Steering is the turning of a complete ship in a curve towards left or right, while it moves forward.
   - Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis.
   - Rolling is the movement of a ship in a linear fashion.

8. **Write the expression for gyroscopic couple?**

Gyroscopic couple, $C = I \cdot \omega \cdot \omega_P$

Where I = Moment of inertia of the disc,

ω = Angular velocity of the engine, and ω_P = Angular velocity of precession.

9. What will be the effect of gyroscopic couple on a disc fixed at a certain angle to a rotating shaft? (AU-MAY/JUNE-2013)

The gyroscopic couple is applied through the bearings which support the shaft. The bearings will resist equal and opposite couple.

10. What is the effect of gyroscopic couple on an automobile taking a turn? (AU-NOV/DEC-2012)

While an automobile will move in a straight line, there will not be any gyroscopic effect on it; but when it takes a turn (towards left or right), it will be subjected to gyroscopic couple. The tendency of this couple is to overturn the vehicle.

11. What is gyroscopic torque?

Whenever a rotating body changes its axis of rotation, a torque is applied on the rotating body. This torque is known as gyroscopic torque.

12. Which part of the automobile is subjected to the gyroscopic couple?

The rotating parts of automobile such as engine rotor, wheels and bearings are subjected to the gyroscopic couple.

13. What is meant by reactive gyroscopic couple? (AU-NOV/DEC-2011)

When the axis of spin itself moves with angular velocity ω_p, the disc is subjected to reactive couple whose magnitude is same but opposite in direction to that of active couple.

The reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as reactive gyroscopic couple.

14. What is meant by applied torque and reaction torque?(MAY/JUN-2012) The torque exerted by one body on another is called applied torque. When one body exerts torque on another body, then the opposite torque exerted by the second body on the first is called reaction torque.

15. What is the effect of gyroscopic couple on rolling of ship? Why?

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. In case of rolling of a ship, the axis of precession is always parallel to the axis of spin for all positions. Hence there is no effect of the gyroscopic couple acting on the body of the ship during rolling

PART-B

1.(i) Explain the function of the proell governor with the help of a neat sketch. Derive that relationship among the various forces acting on the link. (12) (AU-NOV/DEC-2013)

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig. 18.12 (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig. 18.12 (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The perpendicular *B M* is drawn on *ID*.

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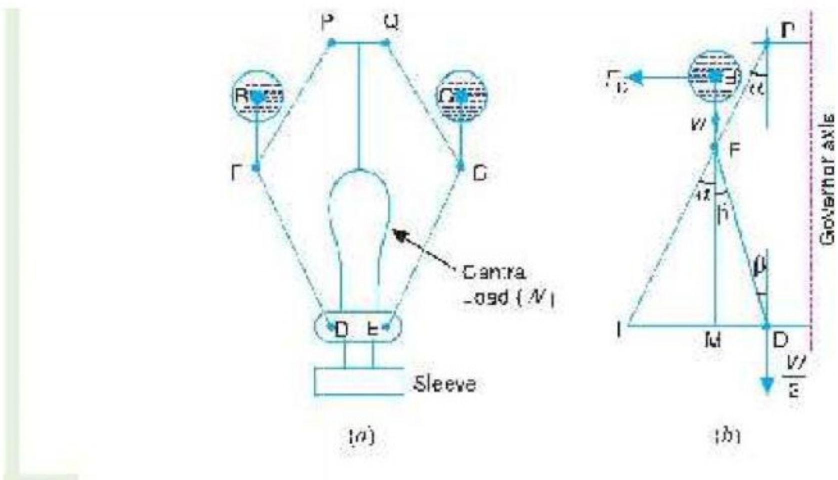


Fig. 18.12. Proell governor.

Taking moments about *I*,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \quad \ldots (i)$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \quad \ldots (\because ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1 + q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1 + q)}{m}\right]\frac{g}{h} \quad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s², we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1 + q)}{m}\right]\frac{895}{h} \quad \ldots (iii)$$

1 (ii). What are centrifugal governors? how do they differ from inertia governor? (4) (AU-NOV/DEC-2013)

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force.*** In inertia governors the positions of the balls are affected by the rate of change of speed. i.e., angular acceleration or retardation of the governor shaft. The amount of displacement of governor balls is controlled by suitable springs and the fuel supply to the engine is controlled by governor mechanism.

Though the sensitiveness of the inertia governors is more, there is a practical difficulty of balancing the inertia forces caused by the revolving parts of the governor to the controlling force. Hence these governors are not preferred when compared with the centifugal governors.

2. . A Proell governor has equal arms of length 300 m m. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 m m. The mass of each ball is 10

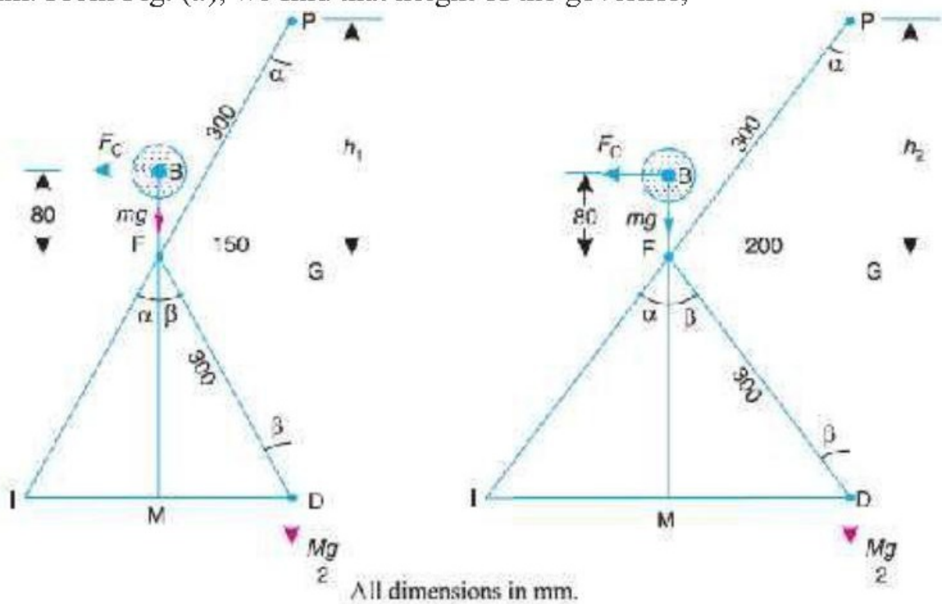
kg and the mass of the central load is 100 kg. Determine the range of speed of the governor. (AU-MAY/JUNE-2013)

Solution. Given : $PF = DF = 300$ mm ; $BF = 80$ mm ; $m = 10$ kg ; $M = 100$ kg ; $r_1 = 150$ mm; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.13.

Let N_1 = Minimum speed when radius of rotation, $r_1 = FG = 150$ mm ; and

N_2 = Maximum speed when radius of rotation , $r_2 = FG$ = 200 mm. From Fig. (*a*), we find that height of the governor,



(*a*) Minimum position. (*a*) Maximum position.

From Fig. (*a*), we find that

$\sin\alpha = \sin\beta = 150 / 300 = 0.5$ or $\alpha = \beta = 30°$

and $MD = FG = 150$ mm = 0.15 m

$FM = FD\cos\beta = 300\cos 30° = 260$ mm = 0.26 m

$IM = FM\tan\alpha = 0.26\tan 30° = 0.15$ m

$BM = BF + FM = 80 + 260 = 340$ mm = 0.34 m

$ID = IM + MD = 0.15 + 0.15 = 0.3$ m

We know that centrifugal force,

$$F_C = m(\omega_1)^2 r_1 = 10\left(\frac{2\pi N_1}{60}\right)^2 0.15 = 0.0165\,(N_1)^2$$

Now taking moments about point *I*,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID$$

or $$0.0165\,(N_1)^2\, 0.34 = 10 \times 9.81 \times 0.15 + \frac{100 \times 9.81}{2} \times 0.3$$

$$0.0056\,(N_1)^2 = 14.715 + 147.15 = 161.865$$

$$\therefore \quad (N_1)^2 = \frac{161.865}{0.0056} = 28\,904 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

$h = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224 \text{ m}$

$FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$

$BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$

We know that $(N_2)^2 = \frac{FM}{BM}\left(\frac{m + M}{m}\right)\frac{895}{h_2}$ ... ($\because \alpha = \beta$ or $q = 1$)

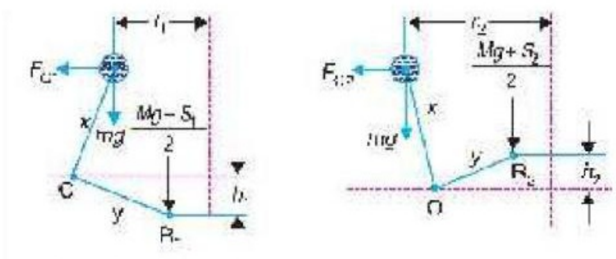
$= \frac{0.224}{0.304}\left(\frac{10 + 100}{10}\right)\frac{895}{0.224} = 32\,385$ or $N_2 = 180$ r.p.m.

We know that range of speed

$= N_2 - N_1 = 180 - 170 = 10$ r.p.m. **Ans.**

3. The radius of rotation of the balls of a Hartnell governor is 80 mm at the minimum speed of 300 r.p.m. Neglecting gravity effect, determine the speed after the sleeve has lifted by 60 m m. Also determine the initial compression of the spring, the governor effort and the power.The particulars of the governor are given below:

Length of ball arm = 150 mm ; length of sleeve arm = 100 mm ; mass of each ball = 4 kg ; and stiffness of the spring = 25 N/m m. (AU-NOV/DEC-2012) **Solution.** Given : $r_1 = 80$ mm = 0.08 m ; $N_1 = 300$ r.p.m. or $\omega_1 = 2\pi \times$ 300/60 = 31.42 rad/s ; $h = 60$ mm = 0.06 m ; $x = 150$ mm = 0.15 m ; $y = 100$ mm = 0.1 m ; $m = 4$ kg ; $s = 25$ N/mm



(a) Minimum position. (b) Maximum position.

The minimum and maximum position of the governor is shown in Fig. 18.36 (*a*) and (*b*) respectively. First of all, let us find the maximum radius of rotation (r_2). We know that lift of the sleeve,

$$h = (r_2 - r_1)\frac{y}{x}$$

or $$r_2 = r_1 + h \times \frac{x}{y} = 0.08 + 0.06 \times \frac{0.15}{0.1} = 0.17 \text{ m} \qquad \ldots (\because h = h_1 + h_2)$$

N

S_1 and S_2= Spring force at the minimum and maximum speed respectively, in newtons

.

We know centrifugal force at the minimum speed,

$$F_{C1} = m\,(\omega_1)^2 r_1 = 4\,(31.42)^2\,0.08 = 316 \text{ N}$$

Now taking moments about the fulcrum *O* of the bell crank lever when in minimum position as shown in Fig 18.36 (*a*). The gravity effect is neglected, *i.e.* the moment due to the weight of balls, sleeve and the bell crank lever arms is neglected.

$$\therefore \quad F_{C1} \times x = \frac{M.g + S_1}{2} \times y \quad \text{or} \quad S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 316 \times \frac{0.15}{0.1} = 948 \text{ N}$$

$$\ldots (\because M = 0)$$

We know that $S_2 - S_1 = h.s$ or $S_2 = S_1 + h.s = 948 + 60 \times 25 = 2448$ N

We know that centrifugal force at the maximum speed,

$$F_{C1} = m\,(\omega_2)^2\,r_2 = \left(\frac{2\pi N_2}{60}\right)^2 r_2 = m\left(\frac{2\pi N_2}{60}\right)^2 0.17 = 0.007\,46\,(N_2)^2$$

Initial compression of the spring

We know that initial compression of the spring $= \frac{S_1}{s} = \frac{948}{25} = 37.92$ mm **Ans.**

Governor effort

We know that the governor effort,

$$P = \frac{S_2 - S_1}{2} = \frac{2448 - 948}{2} = 750 \text{ N} \quad \textbf{Ans.}$$

Now taking moments about the fulcrum *O* when in maximum position, as shown in Fig. 18.36 (*b*).

$$F_{C2} \times x = \frac{M.g + S_2}{2} \times y$$

$$0.007\,46\,(N_2)^2\,0.15 = \frac{2448}{2} \times 0.1 \quad \text{or} \quad 0.001\,12\,(N_2)^2 = 122.4 \qquad \ldots (\because M = 0)$$

$$(N_2)^2 = \frac{122.4}{0.001\,12} = 109\,286 \quad \text{or} \quad N_2 = 331 \text{ r.p.m.} \quad \textbf{Ans.}$$

Governor power

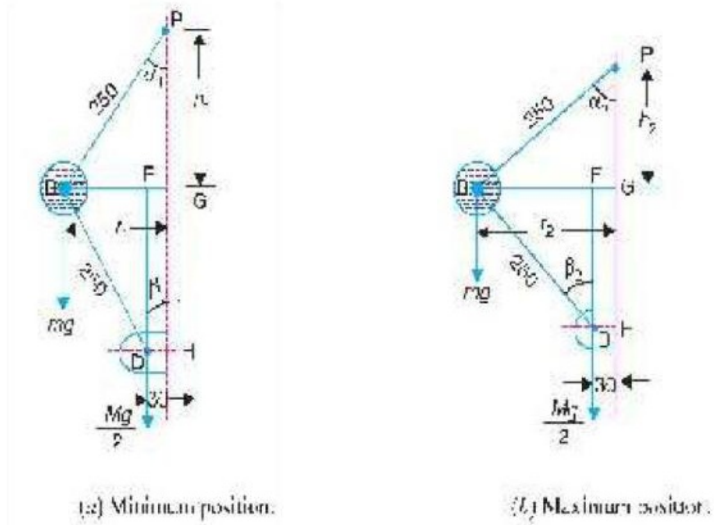
We know that the governor power

$= P \times h = 750 \times 0.06 = 45$ N-m **Ans.**

4. A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 m m from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 m m. Determine the range of speed of the governor. (AU-MAY/JUNE-2012)

Solution. Given : $BP = BD = 250$ mm ; $DH = 30$ mm ; $m = 5$ kg ; $M = 50$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.8 (*a*) and (*b*) respectively.



(a) Minimum position. (b) Maximum position.

Fig. 18.8

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm ; and N_2 = Maximum speed when $r_2 = BG = 200$ mm.

From Fig. 18.8 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$BF = BG - FG = 150 - 30 = 120 \text{ mm} \qquad \ldots (\because FG = DH)$$

and

$$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (120)^2} = 219 \text{ mm}$$

$\therefore$ $\tan \alpha_1 = BG/PG = 150 / 200 = 0.75$ and $\tan \beta_1 = BF/DF = 120/219 = 0.548$

$$\therefore \quad q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

We know that

$$(N_1)^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{5 + \frac{50}{2}(1 + 0.731)}{5} \times \frac{895}{0.2} = 43206$$

$$\therefore \quad N_1 = 208 \text{ r.p.m.}$$

From Fig. 18.8(*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$BF = BG \quad FG = 200 \quad 30 = 170 \text{ mm}$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (170)^2} = 183 \text{ mm}$$

∴ $$\tan\alpha_2 = BG/PG = 200/150 = 1.333$$

and $$\tan\beta_2 = BF/DF = 170/183 = 0.93$$

∴ $$q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.93}{1.333} = 0.7$$

We know that

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q_2)}{m} \times \frac{895}{h_2} = \frac{5 + \frac{50}{2}(1+0.7)}{5} \times \frac{895}{0.15} = 56\,683$$

∴ $N_2 = 238$ r.p.m.

We know that range of speed

$= N_2 \quad N_1 = 238 \quad 208 = 30$ r.p.m. **Ans.**

5. A spring loaded governor of the Hartnell type has arms of equal length. The masses rotate in a circle of 130 mm diameter when the sleeve is in the mid position and the ball arms are vertical. The equilibrium speed for this position is 450 r.p.m.,

mm and the maximum variation of speed taking in account the friction pe cent of the mid position speed. The mass of the sleeve is 4 kg friction may b considered equivalent to 30 N at the sleeve. The powe governor must b sufficient to over-come the friction by one per cent ch speed either way at mid position. Determine, neglecting obliquity ef arms ; 1. The value of eacl rotating mass : 2. The spring stiffness in N/mm and 3. The initial compressioi of spring. (AU-NOV/DEC-2011)

neglecting friction. The maximum sleeve movement is to be 25 to be 5 and the r of the ange of fect of

;

Solution. Given : $x = y$; $d = 130$ mm or $r = 65$ mm $= 0.065$ m ; $N = 450$ r.p.m. or $\omega = 2\pi \times 450/60 = 47.23$ rad/s ; $h = 25$ mm $= 0.025$ m ; $M = 4$ kg ; $F = 30$ N

1. Value of each rotating mass

Let m = Value of each rotating mass in kg, and

S = Spring force on the sleeve at mid position in newtons.

Since the change of speed at mid position to overcome friction is 1 per cent either way (*i.e.* ± 1%), therefore

Minimum speed at mid position,

$\omega = \omega - 0.01\omega = 0.99\omega = 0.99 \times 47.13 = 46.66$ rad/s and maximum speed at mid-position,

$$\omega_2 = \omega + 0.01\omega = 1.01\omega = 1.01 \times 47.13 = 47.6 \text{ rad/s}$$

∴ Centrifugal force at the minimum speed,

$$F_{C1} = m(\omega_1)^2 r = m(46.66)^2\, 0.065 = 141.5\, m \text{ N}$$

and centrifugal force at the maximum speed,

$$F_{C2} = m(\omega_2)^2 r = m(47.6)^2\, 0.0065 = 147.3\, m \text{ N}$$

We know that for minimum speed at mid-position,

$$S + (M.g + F) - 2\, F_{C1} \times \frac{x}{y}$$

or $$S + (4 \times 9.81 - 30) = 2 \times 141.5\, m \times 1 \quad \ldots (\because x = y)$$

∴ $$S \mid 9.24 = 283\, m \quad \ldots (i)$$

and for maximum speed at mid-position,

$$S + (M.g + F) = 2\, F_{C2} \times \frac{x}{y}$$

$$S + (4 \times 9.81 + 30) = 2 \times 147.3\, m \times 1 \quad \ldots (\because x = y)$$

∴ $$S + 69.24 = 294.6\, m \quad \ldots (ii)$$

From equations (*i*) and (*ii*),

$$m - 5.2 \text{ kg} \quad \textbf{Ans.}$$

2. *Spring stiffness in N/mm*

Let s = Spring stiffness in N/mm.

Since the maximum variation of speed, considering friction is ± 5% of the mid-position speed, therefore,

Minimum speed considering friction,

$$\omega_1' = \omega - 0.05\omega = 0.95\omega = 0.95 \times 47.13 = 44.8 \text{ rad/s}$$

and maximum speed considering friction,

$$\omega_2' = \omega + 0.05\omega = 1.05\omega = 1.05 \times 47.13 = 49.5 \text{ rad/s}$$

We know that minimum radius of rotation considering friction,

$$r_1 - r - h_1 \times \frac{x}{y} - 0.065 - \frac{0.025}{2} - 0.0525 \text{ m} \quad \ldots \left(\because x - y \text{ and } h_1 - \frac{h}{2}\right)$$

And maximum radius of rotation considering friction,

$$r_2 = r + h_2 \times \frac{x}{y} = 0.065 + \frac{0.025}{2} = 0.0775 \text{ m}$$

$$\ldots \left(\because x = y, \text{ and } h_2 = \frac{h}{2}\right)$$

∴ Centrifugal force at the minimum speed considering friction,

$$F_{C1}' = m (\omega'_1)^2 r_1 = 5.2 (44.8)^2\ 0.0525 = 548 \text{ N}$$

and centrifugal force at the maximum speed considering friction,

$$F_{C2}' = m (\omega_2)^2 r_2 = 5.2 (49.5)^2\ 0.0775 = 987 \text{ N}$$

Let S_1 = Spring force at minimum speed considering friction, and

S_2 = Spring force at maximum speed considering friction.

We know that for minimum speed considering friction,

$$S_1 + (M.g - F) = 2 F_{C1}' \times \frac{x}{y}$$

$$S_1 + (4 \times 9.81 - 30) = 2 \times 548 \times 1 \quad \ldots (\because x = y)$$

∴ $S_1 + 9.24 = 1096$ or $S_1 = 1096 - 9.24 = 1086.76$ N

and for maximum speed considering friction,

$$S_2 + (M.g + F) = 2 F_{C2}' \times \frac{x}{y}$$

3. *Initial compression of the spring*

$\ldots (\because x = y)$

We know that initial compression of the spring

$$= \frac{S_1}{s} = \frac{1086.76}{32.72} = 33.2 \text{ mm Ans.}$$

6. In an engine governor of the Porter type, the upper and lower arms are 200 m m and 250 mm respectively and pivoted on the axis of rotation. The mass of the central load is 15 kg, the mass of each ball is 2 kg and friction of the sleeve together with the resistance of the operating gear is equal to a load of 25 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, find, taking friction into account, range of speed of the governor.

(AU-APR/MAY-2011)

Solution . Given : BP = 200 mm = 0.2 m ; BD = 250 mm = 0.25 m ; M = 15 kg ; m = 2 kg ; F = 25 N ; α_1 = 30°; α_2 = 40°

First of all, let us find the minimum and maximum speed of the governor.

The minimum and maximum position of the governor is shown Fig. 18.7 (*a*) and (*b*) respectively.

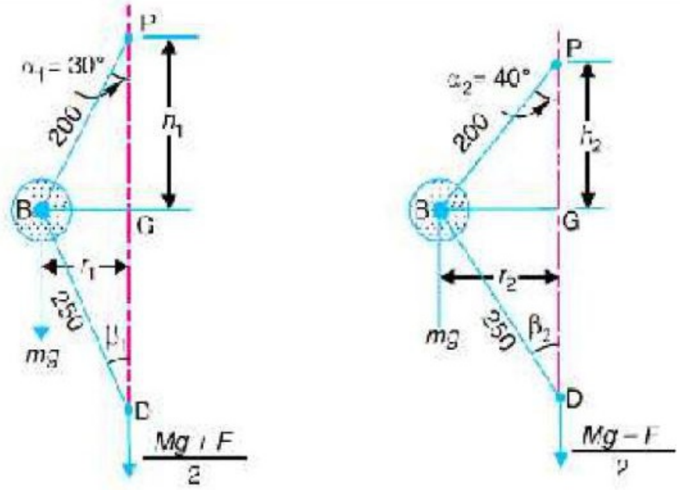
Let N_1 = Minimum speed, and N_2 = Maximum speed.

From Fig. 18.7 (*a*), we find that minimum radius of rotation, r_1 = $BG = BP \sin 30° = 0.2 \times 0.5 = 0.1$ m Height of the governor, h_1 = $PG = BP \cos 30° = 0.2 \times 0.866 = 0.1732$ m and $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1)^2} = 0.23\,\text{m}$$

∴ $$\tan \beta_1 = BG/DG = 0.1/0.23 = 0.4348$$

and $$\tan \alpha_1 = \tan 30° = 0.5774$$

∴ $$\eta = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.4348}{0.5774} = 0.753$$



All dimensions in mm.

(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.7

We know that when the sleeve moves downwards, the frictional force (*F*) acts upwards and the minimum speed is given by,

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - F}{2}\right)(1 + q_1)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 - 24}{2}\right)(1 + 0.753)}{2 \times 9.81} \times \frac{895}{0.1732} = 33596$$

$\therefore$ $N_1 = 183.3$ r.p.m.

Now from Fig. 18.7 (*b*), we find that maximum radius of rotation,

$$r_2 = BG = BP \sin 40° = 0.2 \times 0.643 = 0.1268 \text{ m}$$

Height of the governor,

$$h_2 = PG = BP \cos 40° = 0.2 \times 0.766 = 0.1532 \text{ m}$$

and $$DG = \sqrt{(BD)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.1268)^2} = 0.2154 \text{ m}$$

$\therefore$ $$\tan \beta_2 = BG/DG = 0.1268 / 0.2154 = 0.59$$

and $$\tan \alpha_2 = \tan 40° = 0.839$$

$\therefore$ $$q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.59}{0.839} = 0.703$$

We know that when the sleeve moves upwards, the frictional force (*F*) acts downwards and the maximum speed is given by,

$$(N_2)^2 = \frac{m.g + \left(\frac{m.g + F}{2}\right)(1 + q_2)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 + 24}{2}\right)(1 + 0.703)}{2 \times 9.81} \times \frac{895}{0.1532} = 49\ 236$$

$\therefore \quad N_2 = 222$ r.p.m.

We know that range of speed

$= N_2 - N_1 = 222 - 183.3 = 38.7$ r.p.m. **Ans.**

7.In a spring loaded governor of the Hartnell type, the mass of each ball is 1kg, length of vertical arm of the bell crank lever is 100 mm and that of the horizontal arm is 50 m m. The distance of fulcrum of each bell crank lever is 80 mm from the axis of rotation of the governor. The extreme radii of rotation of the balls are 75 mm and 112.5 m m. The maximum equilibrium speed is 5 per cent greater than the minimum equilibrium speed which is 360 r.p.m. Find, neglecting obliq-uity of arms, initial compression of the spring and equilibrium speed corresponding to the radius of rotation of 100 m m. (AU-APR/MAY-2010)

Solution. Given : $m = 1$ kg ; $x = 100$ mm $= 0.1$ m ; $y = 50$ mm $= 0.05$ m ; $r = 80$ mm $= 0.08$ m ; $r_1 = 75$ mm $= 0.075$ m ; $r_2 = 112.5$ mm $= 0.1125$ m ; $N_1 = 360$ r.p.m. or $\omega_1 = 2\pi \times 360/60 = 37.7$ rad/s

Since the maximum equilibrium speed is 5% greater than the minimum equilibrium speed (ω_1), therefore maximum equilibrium speed,

$$\omega_2 = 1.05 \times 37.7 = 39.6 \text{ rad/s}$$

We know that centrifugal force at the minimum equilibrium speed,

$$F_{C1} = m(\omega_1)^2 r_1 = 1(37.7)^2\, 0.075 = 106.6 \text{ N}$$

and centrifugal force at the maximum equilibrium speed,

$$F_{C2} = m(\omega_2)^2 r_2 = 1(39.6)^2\, 0.1125 = 176.4 \text{ N}$$

Initial compression of the spring

Let $\quad S_1$ = Spring force corresponding to ω_1, and

$\quad S_2$ = Spring force corresponding to ω_2.

Since the obliquity of arms is neglected, therefore for minimum equilibrium position

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 106.6 \times \frac{0.1}{0.05} = 426.4 \text{ N}$$

$\therefore \quad S_1 = 426.4$ N $\quad \ldots(\because M = 0)$

and for maximum equilibrium position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 176.4 \times \frac{0.1}{0.05} = 705.6 \text{ N}$$

$\therefore \quad S_2 = 705.6$ N $\quad \ldots(\because M = 0)$

We know that lift of the sleeve,

$$h = (r_2 - r_1)\,\frac{y}{x} = (0.1125 - 0.075)\,\frac{0.05}{0.1} = 0.018\ 75 \text{ m}$$

and stiffness of the spring $s = \dfrac{S_2 - S_1}{h} = \dfrac{705.6 - 426.4}{0.018\ 75} = 14\ 890 \text{ N/m} = 14.89 \text{ N/mm}$

∴ Initial compression of the spring

$$= \frac{S_1}{s} = \frac{426.4}{14.89} = 28.6 \text{ mm} \quad \textbf{Ans.}$$

Equilibrium speed corresponding to radius of rotation r = 100 mm = 0.1 m

Let N = Equilibrium speed in r.p.m.

Since the obliquity of the arms is neglected, therefore the centrifugal force at any instant,

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right)$$

$$= 106.6 + (176.4 - 106.6)\left(\frac{0.1 - 0.075}{0.1125 - 0.075}\right) = 153 \text{ N}$$

We know that centrifugal force (F_C),

$$153 = m.\omega^2.r = 1\left(\frac{2\pi N}{60}\right)^2 0.1 = 0.0011\,N^2$$

$$\therefore \quad N^2 = 153 / 0.0011 = 139\ 090 \quad \text{or} \quad N = 373 \text{ r.p.m.} \quad \textbf{Ans}$$