

UNIT IV

ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

CHAPTER 5

Connecting Rod

The connecting rod is the intermediate member between the piston and the crankshaft. Its primary function is to transmit the push and pull from the piston pin to the crankpin and thus convert the reciprocating motion of the piston into the rotary motion of the crank. The usual form of the connecting rod in internal combustion engines is shown in Fig. 5.1. It consists of a long shank, a small end and a big end. The cross-section of the shank may be rectangular, circular, tubular, I-section or H-section. Generally circular section is used for low speed engines while I-section is preferred for high speed engines.

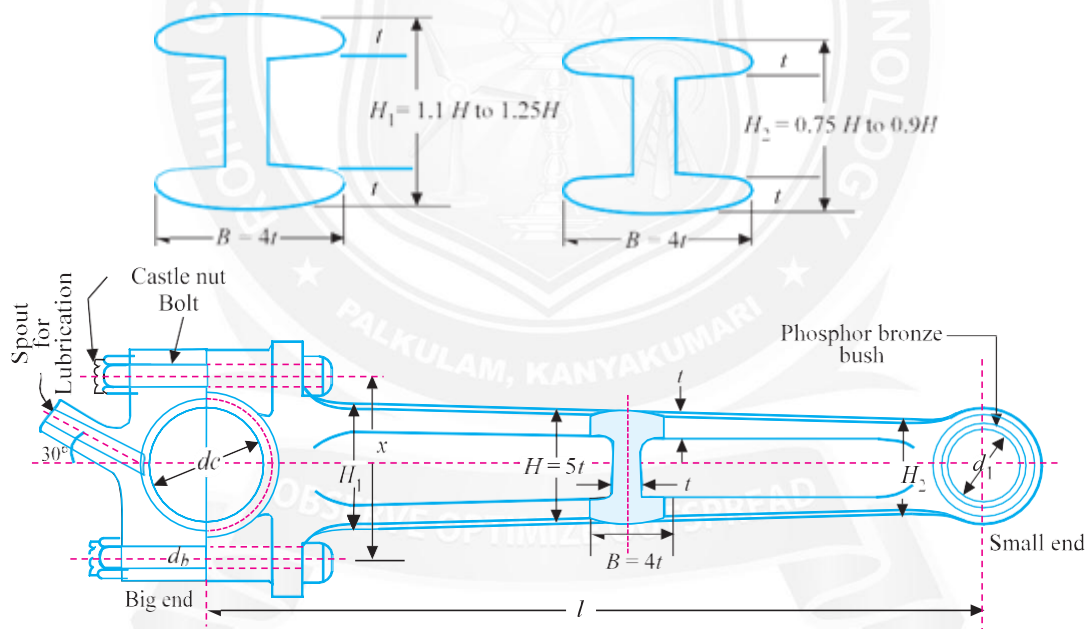


Fig 5.1 Connecting rod.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1144]

The length of the connecting rod (l) depends upon the ratio of l/r , where r is the radius of crank. It may be noted that the smaller length will decrease the ratio l/r . This increases the angularity of the connecting rod which increases the side thrust of the piston against the cylinder liner which in turn increases the wear of the liner. The larger length of the connecting rod will increase the ratio l/r . This decreases the angularity of the

connecting rod and thus decreases the side thrust and the resulting wear of the cylinder. But the larger length of the connecting rod increases the overall height of the engine. Hence, a compromise is made and the ratio l/r is generally kept as 4 to 5. The small end of the connecting rod is usually made in the form of an eye and is provided with a bush of phosphor bronze. It is connected to the piston by means of a piston pin. The big end of the connecting rod is usually made split (in two halves) so that it can be mounted easily on the crankpin bearing shells. The split cap is fastened to the big end with two cap bolts. The bearing shells of the big end are made of steel, brass or bronze with a thin lining (about 0.75 mm) of white metal or babbitt metal. The wear of the big end bearing is allowed for by inserting thin metallic strips (known as shims) about 0.04 mm thick between the cap and the fixed half of the connecting rod. As the wear takes place, one or more strips are removed and the bearing is trued up.

The connecting rods are usually manufactured by drop forging process and it should have adequate strength, stiffness and minimum weight. The material mostly used for connecting rods varies from mild carbon steels (having 0.35 to 0.45 percent carbon) to alloy steels (chrome-nickel or chromemolybdenum steels). The carbon steel having 0.35 percent carbon has an ultimate tensile strength of about 650 MPa when properly heat treated and a carbon steel with 0.45 percent carbon has an ultimate tensile strength of 750 MPa. These steels are used for connecting rods of industrial engines. The alloy steels have an ultimate tensile strength of about 1050 MPa and are used for connecting rods of aero engines and automobile engines. The bearings at the two ends of the connecting rod are either splash lubricated or pressure lubricated. The big end bearing is usually splash lubricated while the small end bearing is pressure lubricated. In the splash lubrication system, the cap at the big end is provided with a dipper or spout and set at an angle in such a way that when the connecting rod moves downward, the spout will dip into the lubricating oil contained in the sump. The oil is forced up the spout and then to the big end bearing. Now when the connecting rod moves upward, a splash of oil is produced by the spout. This splashed up lubricant find its way into the small end bearing through the widely chamfered holes provided on the upper surface of the small end.

In the pressure lubricating system, the lubricating oil is fed under pressure to the big end bearing through the holes drilled in crankshaft, crank webs and crank pin. From

the big end bearing, the oil is fed to small end bearing through a fine hole drilled in the shank of the connecting rod. In some cases, the small end bearing is lubricated by the oil scrapped from the walls of the cylinder liner by the oil scraper rings.

Forces Acting on the Connecting Rod

The various forces acting on the connecting rod are as follows:

1. Force on the piston due to gas pressure and inertia of the reciprocating parts,
2. Force due to inertia of the connecting rod or inertia bending forces,
3. Force due to friction of the piston rings and of the piston, and
4. Force due to friction of the piston pin bearing and the crankpin bearing.

1. Force on the piston due to gas pressure and inertia of reciprocating parts

Consider a connecting rod PC as shown in Fig. 5.2.

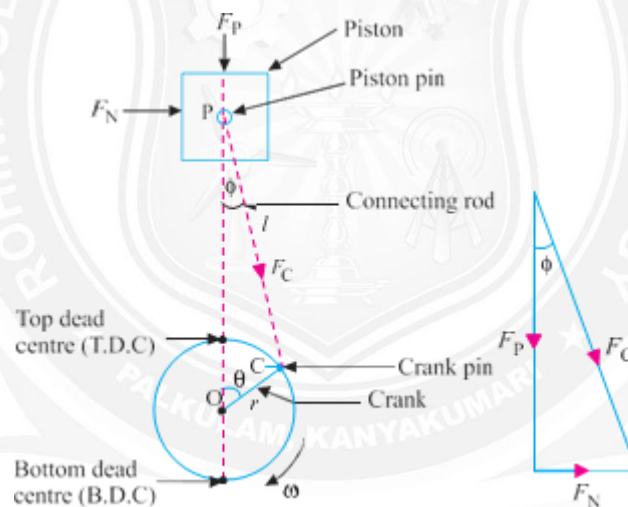


Fig 5.2 Forces on the connecting rod.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1145]

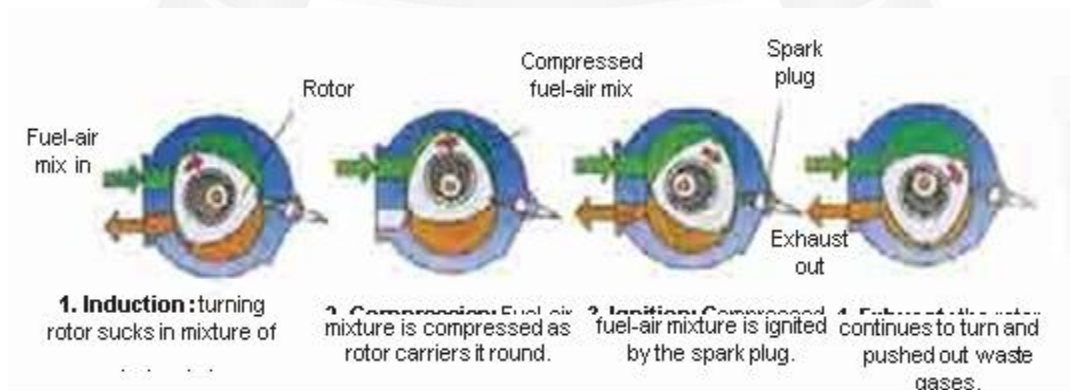


Fig 5.3

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1146]

Let p = Maximum pressure of gas,
 D = Diameter of piston,
 A = Cross-section area of piston = $\frac{\pi D^2}{4}$
 m_R = Mass of reciprocating parts,
= Mass of piston, gudgeon pin etc. + 1/3 rd mass of connecting rod,
 ω = Angular speed of crank,
 ϕ = Angle of inclination of the connecting rod with the line of stroke,
 θ = Angle of inclination of the crank from top dead centre,
 r = Radius of crank,
 l = Length of connecting rod, and
 n = Ratio of length of connecting rod to radius of crank = l / r .

We know that the force on the piston due to pressure of gas,

$$F_L = \text{Pressure} \times \text{Area} = p \cdot A = p \times \pi D^2 / 4$$

and inertia force of reciprocating parts,

$$F_I = \text{Mass} \times \text{Acceleration} = m_R \cdot \omega^2 \cdot R \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

It may be noted that the inertia force of reciprocating parts opposes the force on the piston when it moves during its downward stroke (i. e. when the piston moves from the top dead centre to bottom dead centre). On the other hand, the inertia force of the reciprocating parts helps the force on the piston when it moves from the bottom dead centre to top dead centre.

∴ Net force acting on the piston or piston pin (or gudgeon pin or wrist pin),

$$F_P = \text{Force due to gas pressure} \pm \text{Inertia force}$$

$$F_P = F_L \pm F_I$$

The –ve sign is used when piston moves from TDC to BDC and +ve sign is used when piston moves from BDC to TDC. When weight of the reciprocating parts ($W_R = m_R \cdot g$) is to be taken into consideration, then

$$F_P = F_L \pm F_I \pm W_R$$

The force F_P gives rise to a force F_C in the connecting rod and a thrust F_N on the sides of the cylinder walls. From Fig. 5.2, we see that force in the connecting rod at any instant,

$$F_C = \frac{F_P}{\cos \phi}$$

$$F_C = \frac{F_P}{\sqrt{1 - \frac{\sin^2 \theta}{n^2}}}$$

The force in the connecting rod will be maximum when the crank and the connecting rod are perpendicular to each other (i.e. when $\theta = 90^\circ$). But at this position, the gas pressure would be decreased considerably. Thus, for all practical purposes, the force in the connecting rod (F_C) is taken equal to the maximum force on the piston due to pressure of gas (F_L), neglecting piston inertia effects.

2. Force due to inertia of the connecting rod or inertia bending forces

Consider a connecting rod PC and a crank OC rotating with uniform angular velocity ω rad / s. In order to find the acceleration of various points on the connecting rod, draw the Klien's acceleration diagram CQNO as shown in Fig. 5.4 (a). CO represents the acceleration of C towards O and NO represents the acceleration of P towards O. The acceleration of other points such as D, E, F and G etc., on the connecting rod PC may be found by drawing horizontal lines from these points to intersect CN at d, e, f, and g respectively. Now dO, eO, fO and gO represents the acceleration of D, E, F and G all towards O. The inertia force acting on each point will be as follows:

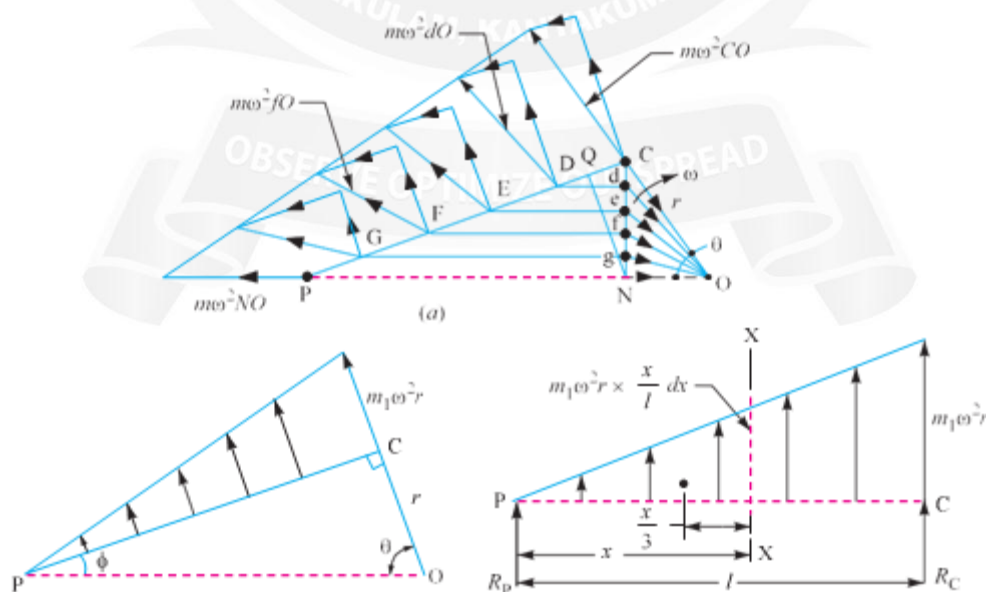


Fig 5.4 Inertia bending forces.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1147]

$$\text{Inertia force at C} = m \times \omega^2 \times CO$$

$$\text{Inertia force at D} = m \times \omega^2 \times dO$$

$$\text{Inertia force at E} = m \times \omega^2 \times eO, \text{ and so on.}$$

The inertia forces will be opposite to the direction of acceleration or centrifugal forces. The inertia forces can be resolved into two components, one parallel to the connecting rod and the other perpendicular to rod. The parallel (or longitudinal) components adds up algebraically to the force acting on the connecting rod (FC) and produces thrust on the pins. The perpendicular (or transverse) components produce bending action (also called whipping action) and the stress induced in the connecting rod is called whipping stress. It may be noted that the perpendicular components will be maximum, when the crank and connecting rod are at right angles to each other. The variation of the inertia force on the connecting rod is linear and is like a simply supported beam of variable loading as shown in Fig. 5.4 (b) and (c). Assuming that the connecting rod is of uniform cross-section and has mass m_1 kg per unit length, therefore,

$$\text{Inertia force per unit length at the crankpin} = m_1 \times \omega^2 r$$

$$\text{and inertia force per unit length at the piston pin} = 0$$

Inertia force due to small element of length dx at a distance x from the piston pin P,

$$dF_1 = m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

\therefore Resultant inertia force,

$$F_I = \int_0^l m_1 \times \omega^2 r \times \frac{x}{l} \times dx$$

$$F_I = \frac{m_1 \times \omega^2 r}{l} \left[\frac{x^2}{2} \right]_0^l$$

$$F_I = \frac{m_1 l \times \omega^2 r}{2}$$

$$F_I = \frac{m_1 \times \omega^2 r}{2} \quad \text{..(Substituting } m_1 \cdot l = m)$$

This resultant inertia force acts at a distance of $2l / 3$ from the piston pin P.

Since it has been assumed that $1/3^{\text{rd}}$ mass of the connecting rod is concentrated at piston pin P (i.e. small end of connecting rod) and $2/3^{\text{rd}}$ at the crankpin (i.e. big end of connecting rod), therefore, the reaction at these two ends will be in the same proportion.
i.e.

$$R_P = \frac{1}{3} F_I \text{ and } R_C = \frac{2}{3} F_I$$

Now the bending moment acting on the rod at section X – X at a distance x from P,

$$M_X = R_P \times x - m_1 \times \omega^2 r \times \frac{x}{l} \times \frac{1}{2} x \times \frac{x}{3}$$

$$M_X = \frac{1}{3} F_I \times x - \frac{m_1 l}{2} \times \omega^2 r \times \frac{x^3}{3l^2}$$

... (Multiplying and dividing the latter expression by l)

$$M_X = \frac{F_I \times x}{3} - F_I \frac{x^3}{3l^2}$$

$$M_X = \frac{F_I}{3} \left(x - \frac{x^3}{l^2} \right)$$

For maximum bending moment, differentiate M_X with respect to x and equate to zero, i.e.

$$\frac{dM_X}{dx} = 0$$

$$\frac{F_I}{3} \left(1 - \frac{3x^2}{l^2} \right) = 0$$

$$1 - \frac{3x^2}{l^2} = 0$$

$$3x^2 = l^2$$

$$x = \frac{l}{\sqrt{3}}$$

Maximum bending moment,

$$M_{\max} = \frac{F_I}{3} \left[\frac{l}{\sqrt{3}} - \frac{\left(\frac{l}{\sqrt{3}} \right)^3}{l^2} \right]$$

$$M_{\max} = \frac{F_I}{3} \left[\frac{l}{\sqrt{3}} - \frac{l}{3\sqrt{3}} \right]$$

$$M_{\max} = \frac{F_I \times l}{3\sqrt{3}} \times \frac{2}{3}$$

$$M_{\max} = \frac{2 F_I \times l}{9\sqrt{3}}$$

$$M_{\max} = 2 \times \frac{m}{2} \times \omega^2 r \frac{l}{9\sqrt{3}}$$

$$M_{\max} = m \times \omega^2 r \frac{l}{9\sqrt{3}}$$

and the maximum bending stress, due to inertia of the connecting rod,

$$\sigma_{\max} = \frac{M_{\max}}{Z}$$

where

Z = Section modulus.

From above we see that the maximum bending moment varies as the square of speed, therefore, the bending stress due to high speed will be dangerous. It may be noted that the maximum axial force and the maximum bending stress do not occur simultaneously. In an I.C. engine, the maximum gas load occurs close to top dead centre whereas the maximum bending stress occurs when the crank angle $\theta = 65^\circ$ to 70° from top dead centre. The pressure of gas falls suddenly as the piston moves from dead centre. Thus the general practice is to design a connecting rod by assuming the force in the connecting rod (F_C) equal to the maximum force due to pressure (F_L), neglecting piston inertia effects and then checked for bending stress due to inertia force (i.e. whipping stress).

3. Force due to friction of piston rings and of the piston

The frictional force (F) of the piston rings may be determined by using the following expression

$$F = \pi D \cdot t_R \cdot n_R \cdot p_R \cdot \mu$$

where

D = Cylinder bore,

t_R = Axial width of rings,

n_R = Number of rings,

p_R = Pressure of rings (0.025 to 0.04 N/mm²), and

μ = Coefficient of friction (about 0.1).

Since the frictional force of the piston rings is usually very small, therefore, it may be neglected. The friction of the piston is produced by the normal component of the piston pressure which varies from 3 to 10 percent of the piston pressure. If the coefficient of friction is about 0.05 to 0.06, then the frictional force due to piston will be about 0.5 to 0.6 of the piston pressure, which is very low.

Thus, the frictional force due to piston is also neglected.

4. Force due to friction of the piston pin bearing and crankpin bearing

The force due to friction of the piston pin bearing and crankpin bearing, is to bend the connecting rod and to increase the compressive stress on the connecting rod due to the direct load. Thus, the maximum compressive stress in the connecting rod will be

$$\sigma_{c(\max)} = \text{Direct compressive stress} + \text{Maximum bending or whipping stress due to inertia bending stress}$$

Design of Connecting Rod

In designing a connecting rod, the following dimensions are required to be determined:

1. Dimensions of cross-section of the connecting rod,
2. Dimensions of the crankpin at the big end and the piston pin at the small end,
3. Size of bolts for securing the big end cap, and
4. Thickness of the big end cap.

1. Dimensions of cross-section of the connecting rod

A connecting rod is a machine member which is subjected to alternating direct compressive and tensile forces. Since the compressive forces are much higher than the tensile forces, therefore, the cross-section of the connecting rod is designed as a strut and the Rankine's formula is used. A connecting rod, as shown in Fig. 5.5, subjected to an axial load W may buckle with X-axis as neutral axis (i.e. in the plane of motion of the connecting rod) or Y-axis as neutral axis (i.e. in the plane perpendicular to the plane of motion). The connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis.

A connecting rod should be equally strong in buckling about both the axes.

Let A = Cross-sectional area of the connecting rod,
 l = Length of the connecting rod,
 σ_c = Compressive yield stress,
 W_B = Buckling load,
 I_{xx} and I_{yy} = Moment of inertia of the section about X-axis and Y-axis respectively, and
 k_{xx} and k_{yy} = Radius of gyration of the section about X-axis and Y-axis respectively.

According to Rankine's formula,

$$W_B \text{ about X-axis} = \frac{\sigma_c A}{1 + a \left(\frac{L}{k_{xx}} \right)^2} = \frac{\sigma_c A}{1 + a \left(\frac{l}{k_{xx}} \right)^2} \quad \dots (\text{For both ends hinged, } L = l)$$

$$W_B \text{ about Y-axis} = \frac{\sigma_c A}{1 + a \left(\frac{L}{k_{yy}} \right)^2} = \frac{\sigma_c A}{1 + a \left(\frac{l}{2k_{yy}} \right)^2} \quad \dots (\text{For both ends hinged, } L = l/2)$$

where L = Equivalent length of the connecting rod, and
 a = Constant

= 1 / 7500, for mild steel

= 1 / 9000, for wrought iron

= 1 / 1600, for cast iron

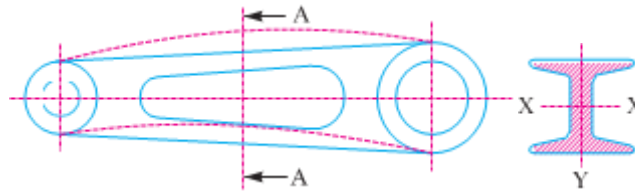


Fig 5.5 Buckling of connecting rod.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1151]

In order to have a connecting rod equally strong in buckling about both the axes, the buckling loads must be equal, i.e.

$$\frac{\sigma_c \cdot A}{1 + a \left(\frac{1}{k_{xx}} \right)^2} = \frac{\sigma_c \cdot A}{1 + a \left(\frac{1}{2k_{yy}} \right)^2}$$

$$\left(\frac{1}{k_{xx}} \right)^2 = \left(\frac{1}{2k_{yy}} \right)^2$$

$$(k_{xx}^2) = (4k_{yy}^2)$$

$$I_{xx}^2 = I_{yy}^2 \quad \dots (I = A \cdot k^2)$$

This shows that the connecting rod is four times strong in buckling about Y-axis than about X-axis. If $I_{xx} > 4 I_{yy}$, then buckling will occur about Y- axis and if $I_{xx} < 4 I_{yy}$, buckling will occur about X-axis. In actual practice, I_{xx} is kept slightly less than $4 I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis. The design will always be satisfactory for buckling about Y-axis. The most suitable section for the connecting rod is I-section with the proportions as shown in Fig. 5.6 (a). Let thickness of the flange and web of the section = t

Width of the section, $B = 4 t$

and depth or height of the section,

$$H = 5t$$

From Fig. 5.6 (a), we find that area of the section,

$$A = 2 (4 t \times t) + 3 t \times t$$

$$A = 11 t^2$$

Moment of inertia of the section about X-axis,

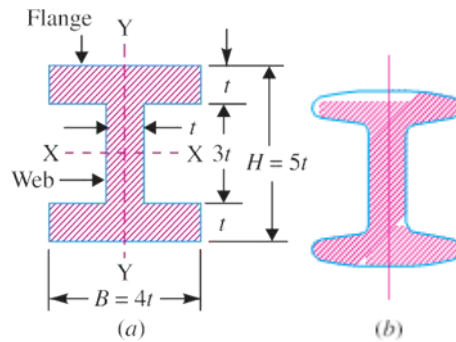


Fig 5.6 I-section of connecting rod.

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1152]

$$I_{xx} = \frac{1}{12} [4t(5t)^3 - 3t(3t)^3]$$

$$I_{xx} = \frac{419}{12} t^4$$

and moment of inertia of the section about Y-axis,

$$I_{yy} = \frac{1}{12} \left[2 \times \frac{1}{12} t(4t)^3 + \frac{1}{12} (3t)t^3 \right]$$

$$I_{yy} = \frac{131}{12} t^4$$

$$\frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since the value of $\frac{I_{xx}}{I_{yy}}$ lies between 3 and 3.5, therefore, I-section chosen is quite satisfactory. After deciding the proportions for I-section of the connecting rod, its dimensions are determined by considering the buckling of the rod about X-axis (assuming both ends hinged) and applying the Rankine's formula. We know that buckling load,

$$W_B = \frac{\sigma_c A}{1 + a \left(\frac{L}{k_{xx}} \right)^2}$$

The buckling load (W_B) may be calculated by using the following relation, i.e.

$$W_B = \text{Max. gas force} \times \text{Factor of safety}$$

The factor of safety may be taken as 5 to 6.

Notes:

(a) The I-section of the connecting rod is used due to its lightness and to keep the inertia forces as low as possible specially in case of high speed engines. It can also withstand high gas pressure.

(b) Sometimes a connecting rod may have rectangular section. For slow speed engines, circular section may be used.

(c) Since connecting rod is manufactured by forging, therefore the sharp corner of I-section is rounded off as shown in Fig. 5.6 (b) for easy removal of the section from dies. The dimensions $B = 4t$ and $H = 5t$, as obtained above by applying the Rankine's formula, are at the middle of the connecting rod. The width of the section (B) is kept constant throughout the length of the connecting rod, but the depth or height varies. The depth near the small end (or piston end) is taken as $H_1 = 0.75H$ to $0.9H$ and the depth near the big end (or crank end) is taken $H_2 = 1.1H$ to $1.25H$.

2. Dimensions of the crankpin at the big end and the piston pin at the small end

Since the dimensions of the crankpin at the big end and the piston pin (also known as gudgeon pin or wrist pin) at the small end are limited, therefore, fairly high bearing pressures have to be allowed at the bearings of these two pins. The crankpin at the big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1 mm or less) of bearing metal (such as tin, lead, babbitt, copper, lead) on the inner surface of the shell. The allowable bearing pressure on the crankpin depends upon many factors such as material of the bearing, viscosity of the lubricating oil, method of lubrication and the space limitations.

The value of bearing pressure may be taken as 7 N/mm^2 to 12.5 N/mm^2 depending upon the material and method of lubrication used. The piston pin bearing is usually a phosphor bronze bush of about 3 mm thickness and the allowable bearing pressure may be taken as 10.5 N/mm^2 to 15 N/mm^2 . Since the maximum load to be carried by the crankpin and piston pin bearings is the maximum force in the connecting rod (F_C), therefore the dimensions for these two pins are determined for the maximum force in the connecting rod (F_C) which is taken equal to the maximum force on the piston due to gas pressure (F_L) neglecting the inertia forces.

We know that maximum gas force,

$$F_L = \frac{\pi D^2}{4} \times p$$

where

D = Cylinder bore or piston diameter in mm, and

p = Maximum gas pressure in N/mm^2

Now the dimensions of the crankpin and piston pin are determined as discussed below:

Let d_c = Diameter of the crank pin in mm,
 l_c = Length of the crank pin in mm,
 p_{bc} = Allowable bearing pressure in N/mm², and
 d_p , l_p and p_{bp} = Corresponding values for the piston pin,

We know that load on the crank pin

$$\begin{aligned} &= \text{Projected area} \times \text{Bearing pressure} \\ &= d_c \cdot l_c \cdot p_{bc} \end{aligned}$$

Similarly, load on the piston pin

$$= d_p \cdot l_p \cdot p_{bp}$$

Equating equations, we have

$$F_L = d_c \cdot l_c \cdot p_{bc}$$

Taking $l_c = 1.25 d_c$ to $1.5 d_c$, the value of d_c and l_c are determined from the above expression.

Again, equating equations and we have

$$F_L = d_p \cdot l_p \cdot p_{bp}$$

Taking $l_p = 1.5 d_p$ to $2 d_p$, the value of d_p and l_p are determined from the above expression.

3. Size of bolts for securing the big end cap

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke.

We know that inertia force of the reciprocating parts,

$$F_I = m_R \cdot \omega^2 \cdot r \left(\cos \theta + \frac{\cos 2\theta}{l/r} \right)$$

We also know that at the top dead centre, the angle of inclination of the crank with the line of stroke, $\theta = 0$

$$F_I = m_R \cdot \omega^2 \cdot r \left(1 + \frac{r}{l} \right)$$

where m_R = Mass of the reciprocating parts in kg,
 ω = Angular speed of the engine in rad / s,
 r = Radius of the crank in metres, and
 l = Length of the connecting rod in metres.

The bolts may be made of high carbon steel or nickel alloy steel. Since the bolts are under repeated stresses but not alternating stresses, therefore, a factor of safety may be taken as 6.

Let d_{cb} = Core diameter of the bolt in mm,
 σ_t = Allowable tensile stress for the material of the bolts in MPa, and
 n_b = Number of bolts. Generally, two bolts are used.

\therefore Force on the bolts

$$= \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b$$

Equating the inertia force to the force on the bolts, we have

$$F_I = \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b$$

From this expression, d_{cb} is obtained. The nominal or major diameter (d_b) of the bolt is given by

$$d_b = \frac{d_{cb}}{0.84}$$

4. Thickness of the big end cap

The thickness of the big end cap (t_c) may be determined by treating the cap as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (i.e. F_I when $\theta = 0$). This load is assumed to act in between the uniformly distributed load and the centrally concentrated load. Therefore, the maximum bending moment acting on the cap will be taken as

where x = Distance between the bolt centres.

x = Dia. of crankpin or big end bearing (d_c) + 2 \times Thickness of bearing liner (3 mm) + Clearance (3 mm)

Let b_c = Width of the cap in mm. It is equal to the length of the crankpin or big end bearing (l_c), and

σ_b = Allowable bending stress for the material of the cap in MPa.

We know that section modulus for the cap,

$$Z_C = \frac{b_c(t_1)^2}{6}$$

$$\therefore \text{Bending stress, } \sigma_b = \frac{M_c}{Z_c} = \frac{F_I \times x}{6} \times \frac{6}{b(t_1)^2} = \frac{F_I \times x}{b(t_1)^2}$$

From this expression, the value of t_c is obtained.

Problem 5.1

Design a connecting rod for an I.C. engine running at 1800 r.p.m. and developing a maximum pressure of 3.15 N/mm². The diameter of the piston is 100 mm; mass of the reciprocating parts per cylinder 2.25 kg; length of connecting rod 380 mm; stroke of piston 190 mm and compression ratio 6: 1. Take a factor of safety of 6 for the design. Take length to diameter ratio for big end bearing as 1.3 and small end bearing as 2 and the corresponding bearing pressures as 10 N/mm² and 15 N/mm². The density of material of the rod may be taken as 8000 kg/m³ and the allowable stress in the bolts as 60 N/mm² and in cap as 80 N/mm². The rod is to be of I-section for which you can choose your own proportions. Draw a neat dimensioned sketch showing provision for lubrication. Use Rankine formula for which the numerator constant may be taken as 320 N/mm² and the denominator constant 1 / 7500.

Given Data:

$$N = 1800 \text{ r.p.m.}$$

$$p = 3.15 \text{ N/mm}^2$$

$$D = 100 \text{ mm}$$

$$m_R = 2.25 \text{ kg}$$

$$l = 380 \text{ mm} = 0.38 \text{ m}$$

$$\text{Stroke} = 190 \text{ mm}$$

$$\text{Compression ratio} = 6: 1$$

$$F. S. = 6.$$

The connecting rod is designed as discussed below:

1. Dimension of I- section of the connecting rod

Let us consider an I-section of the connecting rod, as shown in Fig. 5.7 (a), with the following proportions:

Flange and web thickness of the section = t

Width of the section, $B = 4t$

and depth or height of the section, $H = 5t$

First of all, let us find whether the section chosen is satisfactory or not.

We have already discussed that the connecting rod is considered like both ends hinged for buckling about X-axis and both ends fixed for buckling about Y-axis. The

connecting rod should be equally strong in buckling about both the axes. We know that in order to have a connecting rod equally strong about both the axes, $I_{xx} = 4 I_{yy}$

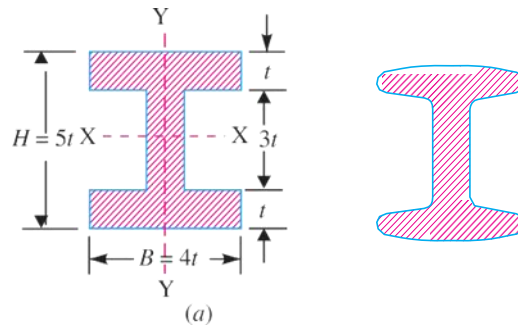


Fig 5.7

[Source: "A Textbook of Machine Design by R.S. Khurmi J.K. Gupta, Page: 1156]

where I_{xx} = Moment of inertia of the section about X-axis, and

I_{yy} = Moment of inertia of the section about Y-axis.

In actual practice, I_{xx} is kept slightly less than $4 I_{yy}$. It is usually taken between 3 and 3.5 and the connecting rod is designed for buckling about X-axis.

Now, for the section as shown in Fig. 5.7 (a), area of the section,

$$A = 2 (4 t \times t) + 3t \times t$$

$$A = 11 t^2$$

$$I_{xx} = \frac{1}{12} [4t(5t)^3 - 3t(3t)^3]$$

$$I_{xx} = \frac{419}{12} t^4$$

$$I_{yy} = \frac{1}{12} \left[2 \times \frac{1}{12} t(4t)^3 + \frac{1}{12} (3t)t^3 \right]$$

$$I_{yy} = \frac{131}{12} t^4$$

$$\frac{I_{xx}}{I_{yy}} = \frac{419}{12} \times \frac{12}{131} = 3.2$$

Since $\frac{I_{xx}}{I_{yy}} = 3.2$ therefore the section chosen is quite satisfactory. Now let us find the dimensions of this I-section. Since the connecting rod is designed by taking the force on the connecting rod (F_C) equal to the maximum force on the piston (F_L) due to gas pressure, therefore,

$$F_C = F_L = \frac{\pi D^2}{4} \times p$$

$$F_C = F_L = \frac{\pi \times 100^2}{4} \times 3.15$$

$$F_C = F_L = 24740 \text{ N}$$

We know that the connecting rod is designed for buckling about X-axis (i.e. in the plane of motion of the connecting rod) assuming both ends hinged. Since a factor of safety is given as 6, therefore, the buckling load,

$$W_B = F_C \times F. S. = 24\,740 \times 6$$

$$W_B = 148440 \text{ N}$$

We know that radius of gyration of the section about X-axis,

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{419t^4}{12} \times \frac{1}{11t^2}}$$

$$K_{xx} = 1.78t$$

$$\text{Length of crank, } r = \frac{\text{Stroke of piston}}{2} = \frac{190}{2} = 95 \text{ mm.}$$

$$\text{Length of the connecting rod, } l = 380 \text{ mm ... (Given)}$$

$$\therefore \text{Equivalent length of the connecting rod for both ends hinged, } L = l = 380 \text{ mm}$$

Now according to Rankine's formula, we know that buckling load (W_B),

$$148440 = \frac{\sigma_c \cdot A}{1 + a \left(\frac{L}{K_{xx}} \right)^2}$$

$$148440 = \frac{320 \times 11t^2}{1 + \frac{1}{7500} \left(\frac{380}{1.78t} \right)^2}$$

... (It is given that $\sigma_c = 320 \text{ MPa}$ or N/mm^2 and $a = 1 / 7500$)

$$\frac{148440}{320} = \frac{11t^2}{1 + \frac{6.1}{t^2}}$$

$$464 = \frac{11t^4}{t^2 + 6.1}$$

$$464(t^2 + 6.1) = 11t^4$$

$$t^4 - 42.2 t^2 - 257.3 = 0$$

$$t^2 = \frac{42.2 \pm \sqrt{(42.2)^2 + 4 \times 257.3}}{2}$$

$$t^2 = \frac{42.2 \pm 53}{2}$$

$$t^2 = 47.6$$

... (Taking +ve sign)

$$\text{or } t = 6.9 \text{ say } 7 \text{ mm.}$$

Thus, the dimensions of I-section of the connecting rod are:

Thickness of flange and web of the section = $t = 7$ mm.

Width of the section, $B = 4 t = 4 \times 7 = 28$ mm.

and depth or height of the section, $H = 5 t = 5 \times 7 = 35$ mm.

These dimensions are at the middle of the connecting rod. The width (B) is kept constant throughout the length of the rod, but the depth (H) varies. The depth near the big end or crank end is kept as $1.1H$ to $1.25H$ and the depth near the small end or piston end is kept as $0.75H$ to $0.9H$. Let us take

Depth near the big end, $H_1 = 1.2H = 1.2 \times 35$

$$H_1 = 42 \text{ mm}$$

and depth near the small end,

$$H_2 = 0.85H = 0.85 \times 35$$

$$H_2 = 29.75 \text{ say } 30 \text{ mm}$$

\therefore Dimensions of the section near the big end = $42 \text{ mm} \times 28 \text{ mm}$.

and dimensions of the section near the small end = $30 \text{ mm} \times 28 \text{ mm}$.

Since the connecting rod is manufactured by forging, therefore the sharp corners of I-section are rounded off, as shown in Fig. 5.7 (b), for easy removal of the section from the dies.

2. Dimensions of the crankpin or the big end bearing and piston pin or small end bearing

Let d_c = Diameter of the crankpin or big end bearing,

l_c = length of the crankpin or big end bearing = $1.3 d_c$...(Given)

p_{bc} = Bearing pressure = 10 N/mm^2 ...(Given)

We know that load on the crankpin or big end bearing

$$= \text{Projected area} \times \text{Bearing pressure}$$

$$= d_c \cdot l_c \cdot p_{bc} = d_c \times 1.3 d_c \times 10 = 13 (d_c)^2$$

Since the crankpin or the big end bearing is designed for the maximum gas force (F_L), therefore, equating the load on the crankpin or big end bearing to the maximum gas force, i.e.

$$13 (d_c)^2 = F_L = 24740 \text{ N}$$

$$\therefore (d_c)^2 = 24740 / 13 = 1903$$

$$d_c = 43.6 \text{ say } 44 \text{ mm. and}$$

$$l_c = 1.3 d_c = 1.3 \times 44$$

$$l_c = 57.2 \text{ say } 58 \text{ mm.}$$

The big end has removable precision bearing shells of brass or bronze or steel with a thin lining (1mm or less) of bearing metal such as babbitt.

Again,

let d_p = Diameter of the piston pin or small end bearing,

l_p = Length of the piston pin or small end bearing = $2d_p$...(Given)

p_{bp} = Bearing pressure = 15 N/mm^2 ...(Given)

We know that the load on the piston pin or small end bearing

$$= \text{Project area} \times \text{Bearing pressure}$$

$$= d_p \cdot l_p \cdot p_{bp}$$

$$= d_p \times 2 d_p \times 15$$

$$= 30 (d_p)^2$$

Since the piston pin or the small end bearing is designed for the maximum gas force (F_L), therefore, equating the load on the piston pin or the small end bearing to the maximum gas force, i.e.

$$30 (d_p)^2 = 24740 \text{ N}$$

$$\therefore (d_p)^2 = 24740 / 30 = 825$$

$$d_p = 28.7 \text{ say } 29 \text{ mm.}$$

and

$$l_p = 2 d_p = 2 \times 29$$

$$l_p = 58 \text{ mm Ans.}$$

The small end bearing is usually a phosphor bronze bush of about 3 mm thickness.

3. Size of bolts for securing the big end cap

Let d_{cb} = Core diameter of the bolts,

σ_t = Allowable tensile stress for the material of the bolts = 60 N/mm^2

...(Given)

and n_b = Number of bolts. Generally, two bolts are used.

We know that force on the bolts

$$F_I = \frac{\pi}{4} (d_{cb})^2 \sigma_t \times n_b$$

$$F_I = \frac{\pi}{4} (d_{cb})^2 60 \times 2$$

$$F_I = 94.26 (d_{cb})^2$$

The bolts and the big end cap are subjected to tensile force which corresponds to the inertia force of the reciprocating parts at the top dead centre on the exhaust stroke.

We know that inertia force of the reciprocating parts,

$$F_I = m_R \cdot \omega^2 \cdot r \left(\cos \theta + \frac{\cos 2\theta}{l/r} \right)$$

We also know that at the top dead centre, the angle of inclination of the crank with the line of stroke, $\theta = 0$

$$F_I = m_R \cdot \omega^2 \cdot r \left(1 + \frac{r}{l} \right)$$

$$F_I = 2.25 \left(\frac{2\pi \times 1800}{60} \right)^2 \times 0.095 \left(1 + \frac{0.095}{0.38} \right)$$

$$F_I = 9490 \text{ N}$$

Equating the inertia force to the force on the bolts, we have

$$9490 = 94.26 (d_{cb})^2$$

$$(d_{cb})^2 = 9490 / 94.26 = 100.7$$

$$\therefore d_{cb} = 10.03 \text{ mm}$$

and nominal diameter of the bolt,

$$d_b = \frac{d_{cb}}{0.84} = \frac{10.03}{0.84}$$

$$d_b = 11.94 \text{ say } 12 \text{ mm.}$$

4. Thickness of the big end cap

Let t_c = Thickness of the big end cap,

b_c = Width of the big end cap.

It is taken equal to the length of the crankpin or big end bearing (l_c) = 58 mm (calculated above)

$$\sigma_b = \text{Allowable bending stress for the material of the cap} = 80 \text{ N/mm}^2$$

...(Given)

The big end cap is designed as a beam freely supported at the cap bolt centres and loaded by the inertia force at the top dead centre on the exhaust stroke (i.e. F_I when $\theta = 0$). Since the load is assumed to act in between the uniformly distributed load and the centrally concentrated load, therefore, maximum bending moment is taken as

$$M_C = \frac{F_I \times x}{6}$$

where

x = Distance between the bolt centres

x = Dia. of crank pin or big end bearing + $2 \times$ Thickness of bearing liner + Nominal dia. of bolt + Clearance

$$x = (d_c + 2 \times 3 + d_b + 3) \text{ mm}$$

$$x = 44 + 6 + 12 + 3$$

$$x = 65 \text{ mm}$$

\therefore Maximum bending moment acting on the cap,

$$M_C = \frac{F_I \times x}{6}$$

$$M_C = \frac{9490 \times 65}{6}$$

$$M_C = 102810 \text{ mm.}$$

Section modulus for the cap

$$Z_C = \frac{b_c(t_c)^2}{6}$$

$$Z_C = \frac{58(t_c)^2}{6}$$

$$Z_C = 9.7(t_c)^2$$

We know that bending stress (σ_b),

$$80 = \frac{M_C}{Z_C}$$

$$80 = \frac{102810}{9.7(t_c)^2}$$

$$80 = \frac{10600}{(t_c)^2}$$

$$(t_c)^2 = 10600 / 80 = 132.5$$

$$t_c = 11.5 \text{ mm.}$$

Let us now check the design for the induced bending stress due to inertia bending forces on the connecting rod (i.e. whipping stress).

We know that mass of the connecting rod per metre length,

$$m_l = \text{Volume} \times \text{density} = \text{Area} \times \text{length} \times \text{density}$$

$$m_l = A \times 1 \times \rho = 11t^2 \times 1 \times \rho \quad \dots (A = 11t^2)$$

$$m_l = 11(0.007)^2 (0.38) 8000$$

$$m_1 = 1.64 \text{ kg}$$

$$\dots [\rho = 8000 \text{ kg/m}^3 \text{ (given)}]$$

∴ Maximum bending moment,

$$M_{\max} = m \times \omega^2 r \frac{1}{9\sqrt{3}}$$

$$M_{\max} = m_1 \times \omega^2 r \frac{l^2}{9\sqrt{3}} \quad \dots (m = m_1 \cdot l)$$

$$M_{\max} = 1.64 \times \left(\frac{2\pi \times 1800}{60} \right)^2 (0.095) \frac{(0.38)^2}{9\sqrt{3}}$$

$$M_{\max} = 51.3 \text{ N-m.}$$

$$M_{\max} = 51300 \text{ N-mm.}$$

and section modulus,

$$Z_{xx} = \frac{I_{xx}}{5t/2} = \frac{419t^4}{12} \times \frac{2}{5t}$$

$$Z_{xx} = 13.97t^3$$

$$Z_{xx} = 13.97(7)^3$$

$$Z_{xx} = 4792 \text{ mm}^3.$$

∴ Maximum bending stress (induced) due to inertia bending forces or whipping stress,

$$\sigma_{b(\max)} = \frac{M_{\max}}{Z_{xx}} = \frac{51300}{4792}$$

$$\sigma_{b(\max)} = 10.7 \text{ N/mm}^2$$

Since the maximum bending stress induced is less than the allowable bending stress of 80 N/mm^2 , therefore the design is safe.