# Irrotational and Solenoidal vector fields

## Solenoidal vector

A vector $\vec{F}$ is said to be solenoidal if $div\,\vec{F} = 0$ (i.e) $\nabla.\vec{F} = 0$

## Irrotational vector

A vector is said to be irrotational if Curl $\vec{F} = 0$ $(i.e)$ $\nabla \times \vec{F} = 0$

**Example: Prove that the vector $\vec{F} = z\,\vec{i} + x\,\vec{j} + y\,\vec{k}$ is solenoidal.**

**Solution:**

Given $\vec{F} = z\,\vec{i} + x\,\vec{j} + y\,\vec{k}$

To prove $\nabla \cdot \vec{F} = 0$

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(y)$$

$$= 0$$

$\therefore$ $\vec{F}$ is solenoidal.

**Example: If $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + \lambda z)\vec{k}$ is solenoidal, then find the value of $\lambda$.**

**Solution:**

Given $\vec{F}$ is solenoidal.

$$(ie)\nabla \cdot \vec{F} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(x + 3y) + \frac{\partial}{\partial y}(y - 2z) + \frac{\partial}{\partial z}(x + \lambda z) = 0$$

$$\Rightarrow 1 + 1 + \lambda = 0$$

$$\therefore \lambda = -2$$

**Example: Find a such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.**

**Solution:**

Given $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.

$$(ie)\nabla \cdot \vec{F} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(3x - 2y + z) + \frac{\partial}{\partial y}(4x + ay - z) + \frac{\partial}{\partial z}(x - y + 2z) = 0$$

$$\Rightarrow 3 + a + 2 = 0$$

$$\therefore a = -5$$

**Example: Show that the vector $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational.**

**Solution:**

Given $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$

To prove $curl\,\vec{F} = 0$

$(i.e) To\ prove\ \nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy + z^3 & 3x^2 - 3z^2 & 3xz^2 - y \end{vmatrix}$$

$$= \vec{i}(-1 + 1) - \vec{j}(3z^2 - 3z^2) + \vec{k}(6x - 6x) = \vec{0}$$

$\therefore\ \vec{F}$ is irrotational.

**Example: Find the constants $a, b, c$ so that the vectors is irrotational $\vec{F} = (x + 2y + az)\vec{i} + (bx + 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$.**

**Solution:**

**Given $\vec{F} = (x + 2y + az)\vec{i} + (bx + 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.**

$(ie) \nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x + 2y + az & bx + 3y - z & 4x + cy + 2z \end{vmatrix} = \vec{0}$$

$$\Rightarrow \vec{i}(c + 1) - \vec{j}(4 - a) + \vec{k}(b - 2) = \vec{0}$$

$$\Rightarrow c + 1 = 0\ ; \quad 4 - a = 0\ ; \quad b - 2 = 0$$

$$\Rightarrow c = -1\ ; \quad 4 = a\ ; \quad b = 2$$