UNIT – III

APPLICATIONS OF PARTIAL DIFFERENTIALEQUATIONS

Solution of the heat equation

The heat equation is

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \text{ ----------------(1).}$$

Let u = X(x) . T(t) be the solution of (1), where „X‟ is a function of „x‟ alone and „T‟ is afunction of „t‟ alone.

Substituting these in (1), we get

$$X T' = \alpha^2 X'' T.$$

$$\text{i.e,} \quad \frac{X''}{X} = \frac{T'}{\alpha^2 T} \text{ ---------------------------------- (2).}$$

Now the left side of (2) is a function of „x‟ alone and the right side is a function of „t‟alone. Since „x‟ and „t‟ are independent variables, (2) can be true only if each side is equal to a constant.

$$\text{Therefore,} \quad \frac{X''}{X} = \frac{T'}{\alpha^2 T} = k \text{ (say).}$$

Hence, we get $X'' - kX = 0$ and $T' - \alpha^2 kT = 0$---------------------------(3).

Solving equations (3), we get

(i) when „k‟ is positive and $k = \lambda^2$, say

$$X = c_1 e^{\lambda x} + c_2 e^{-\lambda x}$$

$$T = c_3 e^{\alpha^2 \lambda^2 t}$$

(ii) when „k‟ is negative and $k = -\lambda^2$, say

$$X = c_4 \cos\lambda x + c_5 \sin\lambda x$$

$$T = c_6 e^{-\alpha^2 \lambda^2 t}$$

(iii) when „k‟ is zero.

$$X = c_7 x + c_8 T$$
$$= c_9$$

Thus the various possible solutions of the heat equation (1) are

$$u = (c_1 e^{\lambda x} + c_2 e^{-\lambda x}) c_3 e^{\alpha^2 \lambda^2 t} \text{ ---------------------------- (4)}$$

$$u = (c_4 \cos\lambda x + c_5 \sin\lambda x) c_6 e^{-\alpha^2 \lambda^2 t} \text{ ---------------------- (5)}$$

$$u = (c_7 x + c_8) c_9 \text{ --- (6)}$$

Of these three solutions, we have to choose that solution which suits the physical nature of the problem and the given boundary conditions. As we are dealing with problems on heat flow, u(x,t) must be a transient solution such that „u‟ is to decrease withthe increase of time „t‟.

Therefore, the solution given by (5),

$$u = (c_4 \cos\lambda x + c_5 \sin\lambda x) c_6 e^{-\alpha^2 \lambda^2 t}$$

is the only suitable solution of the heat equation.

Illustrative Examples

Example 7

A rod „ℓ" cm with insulated lateral surface is initially at temperature f(x) at an inner point of distance x cm from one end. If both the ends are kept at zero temperature,find the temperature at any point of the rod at any subsequent time.

Let the equation for the conduction of heat be

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \qquad \text{------------- (1)}$$

The boundary conditions are

(i) $u(0,t) = 0, \quad \forall\ t \geq 0$

(ii) $u(\ell,t) = 0, \quad \forall\ t \geq 0$

(iii) $u(x,0) = f(x), \ 0 < x < \ell$

The solution of equation (1) is given by

$$u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2\lambda^2 t} \qquad \text{---------- (2)}$$

Applying condition (i) in (2), we have

$$0 = A.e^{-\alpha^2\lambda^2 t} \text{ which gives } A = 0$$

$\therefore$ u (x,t) = B sinλx $e^{-\alpha^2\lambda^2 t}$ ------------------------------------ (3)

Applying condition (ii) in the above equation, we get $0 = B\sin\lambda\ell\, e^{-\alpha^2\lambda^2 t}$

i.e, $\lambda\ell = n\pi$ or $\lambda = \dfrac{n\pi}{\ell}$ (n is an integer)

$$\therefore u(x,t) = B \sin\frac{n\pi x}{\ell}\, e^{-\frac{n^2\pi^2\alpha^2}{\ell^2}t}$$

Thus the most general solution is

$$u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{\ell}\, e^{-\frac{n^2\pi^2\alpha^2}{\ell^2}t} \qquad (4)$$

By condition (iii),

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{\ell} = f(x).$$

The LHS series is the half range Fourier sine series of the RHS function.

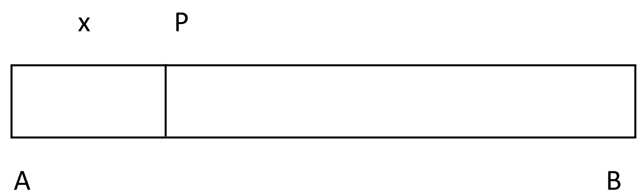
$$\therefore B_n = \frac{2}{\ell}\int_0^{\ell} f(x)\sin\frac{n\pi x}{\ell}\,dx$$

Substituting in (4), we get the temperature function

$$u(x,t) = \sum_{n=1}^{\infty} \left[\frac{2}{\ell}\int_0^{\ell} f(x)\sin\frac{n\pi x}{\ell}\,dx\right] \sin\frac{n\pi x}{\ell}\, e^{-\frac{n^2\pi^2\alpha^2}{\ell^2}t}$$

Example 8

The equation for the conduction of heat along a bar of length ℓ is $\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$, neglecting radiation. Find an expression for u, if the ends of the bar are maintained at zero temperature and if, initially, the temperature is T at the centre of the bar and falls uniformly to zero at its ends.



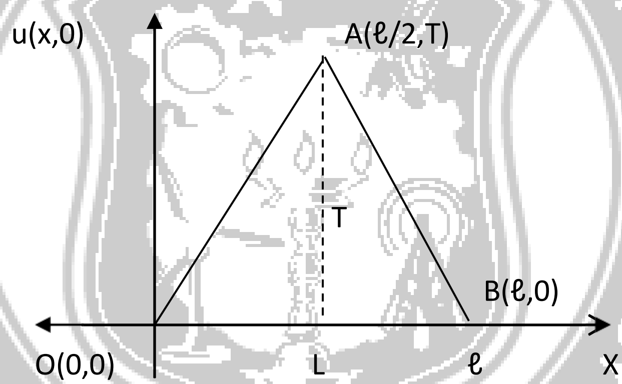
Let u be the temperature at P, at a distance x from the end A at time t.

The temperature function u (x,t) is given by the equation
$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad \text{------------------------(1)}$$

The boundary conditions are

(i) $u(0,t) = 0, \ \forall \ t \geq 0.$

(ii) $u(\ell,t) = 0, \ \forall \ t \geq 0.$



$$u(x,0) = \frac{2Tx}{\ell}, \quad \text{for } 0 \leq x \leq \frac{\ell}{2}$$
$$= \frac{2T}{\ell}(\ell - x), \quad \text{for } \frac{\ell}{2} \leq x \leq \ell$$

The solution of (1) is of the form

$$u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2\lambda^2 t} \quad \text{------------------------ (2)}$$

Applying conditions (i) and (ii) in (2), we get

$$A = 0 \;\&\; \lambda = \frac{n}{\ell}$$

$$\therefore\; u(x,t) = B \sin \frac{n\pi x}{\ell}\, e^{\frac{-n^2\pi^2\alpha^2}{\ell^2} t}$$

Thus the most general solution is

$$\therefore\; u(x,t) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{\ell}\, e^{\frac{-n^2\pi^2\alpha^2}{\ell^2} t} \quad \text{------------ (3)}$$

Using condition (iii) in (3), we have

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{\ell} \quad \text{------------ (4)}$$

We now expand u (x,0) given by (iii) in a half – range sine series in (0,ℓ)

$$\text{Here } B_n = \frac{2}{\ell} \int_0^{\ell} u(x,0) \sin \frac{n\pi x}{\ell}\, dx$$

$$\text{ie, } B_n = \frac{2}{\ell} \left\{ \int_0^{\ell/2} \frac{2Tx}{\ell} \sin \frac{n\pi x}{\ell}\, dx + \int_{\ell/2}^{\ell} \frac{2T}{\ell} (\ell - x) \sin \frac{n\pi x}{\ell}\, dx \right\}$$

$$= \frac{4T}{\ell^2} \left\{ \int_0^{\ell/2} x\, d\left(\frac{-\cos \frac{n\pi x}{\ell}}{n\pi/\ell} \right) + \int_{\ell/2}^{\ell} (\ell - x)\, d\left(\frac{-\cos \frac{n\pi x}{\ell}}{n\pi/\ell} \right) \right\}$$

$$= \frac{4T}{\ell^2} \left\{ \left[(x) \left(\frac{-\cos \frac{n\pi x}{\ell}}{n\pi/} \right) - (1) \left(\frac{-\sin \frac{n\pi x}{\ell}}{n^2\pi^2/\ell} \right) \right]_0^{\ell/2} + \right.$$

$$(\ell - x)\frac{-\cos\dfrac{}{\ell}}{n\pi/\ell} - (-1)\left(\frac{-\sin\dfrac{}{\ell}}{n^2\pi^2/\ell^2}\right)\Bigg]_{\ell/2}^{\ell}$$

$$= \frac{4T}{\ell^2}\left\{\frac{-\ell^2}{2n\pi}\cos\frac{n\pi}{2} + \frac{\ell^2}{n^2\pi^2}\sin\frac{n\pi}{2} + \frac{\ell^2}{2n\pi}\cos\frac{n\pi}{2} + \frac{\ell^2}{n^2\pi^2}\sin\frac{n\pi}{2}\right.$$

$$= \frac{4T}{\ell^2}\;\frac{2\ell^2}{n^2\pi^2}\sin\frac{n\pi}{2}$$

$$\therefore B_n = \frac{8T}{n^2\pi^2}\sin\frac{n\pi}{2}$$

Hence the solution is

$$u(x,t) = \sum_{n=1}^{\infty} \frac{8T}{n^2\pi^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{\ell}\, e^{\frac{-n^2\pi^2\alpha^2}{\ell^2}t}$$

or

$$u(x,t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{8T}{n^2\pi^2}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{\ell}\, e^{\frac{-n^2\pi^2\alpha^2}{\ell^2}t}$$

or

$$u(x,t) = \frac{8T}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\frac{(2n-1)\pi x}{\ell}\, e^{\frac{-\alpha^2(2n-1)^2\pi^2}{\ell^2}t}$$

Steady - state conditions and zero boundary conditions

Example 9

A rod of length „ℓ" has its ends A and B kept at 0°C and 100°C until steady state conditions prevails. If the temperature at B is reduced suddenly to 0°C and kept so while that of A is maintained, find the temperature u(x,t) at a distance x from A and at time „t".

The heat-equation is given by

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad \text{------------- (1)}$$

Prior to the temperature change at the end B, when t = 0, the heat flow was independent of time (steady state condition).

When the temperature u depends only on x, equation(1) reduces to

$$\frac{\partial^2 u}{\partial x^2} = 0$$

Its general solution is $u = ax + b$ ---------------------------------- (2)

Since u = 0 for x = 0 & u = 100 for $x = \ell$, therefore (2) gives $b = 0$ & $a = \frac{100}{\ell}$

$$\therefore u(x,0) = \frac{100}{\ell} x, \text{ for } 0 < x < \ell$$

Hence the boundary conditions are

(i) $u(0,t) = 0, \quad \forall\ t \geq 0$

(ii) $u(\ell,t) = 0, \forall\ t \geq 0100x$

(iii) $u(x,0) = \frac{\quad}{\ell}$, for $0 \leq x \leq \ell$

The solution of (1) is of the form

$$u(x,t) = (A \cos\lambda x + B \sin \lambda x)\ e^{-\alpha^2\lambda^2 t} \quad \text{------------- (3)}$$

Using, conditions (i) and (ii) in (3), we get

$$A = 0 \ \&\ \lambda = \frac{n\pi}{\ell}$$

$$\therefore u(x,t) = B \sin \frac{n\pi x}{\ell}\ e^{-\frac{n^2\pi^2\alpha^2}{\ell^2} t}$$

Thus the most general solution is

$-n^2\pi^2\alpha^2$

$$\therefore\ u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{\ell}\ e^{\frac{\quad t}{\ell^2}} \qquad (4)$$

Applying (iii) in (4), we get

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{\ell}$$

$$\text{ie, } \frac{100x}{\ell} = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{\ell}$$

$$\Longrightarrow B_n = \frac{2}{\ell}\int_0^{\ell} \frac{100x}{\ell} \sin\frac{n\pi x}{\ell}\, dx$$

$$= \frac{200}{\ell^2}\int_0^{\ell} x\, d\left(\frac{-\cos\frac{n\pi x}{\ell}}{\frac{n\pi}{\ell}}\right)$$

$$= \frac{200}{\ell^2}\left\{ (x)\frac{-\cos\frac{n\pi x}{\ell}}{\frac{n\pi}{\ell}} - (1)\frac{-\sin\frac{n\pi x}{\ell}}{\frac{n^2\pi^2}{\ell^2}} \right\}_0^{\ell}$$

$$= \frac{200}{\ell^2}\ \frac{-\ell^2}{n\pi}\cos n\pi$$

$$B_n = \frac{200(-1)^{n+1}}{n\pi}$$

Hence the solution is

$$u(x,t) = \sum_{n=1}^{\infty} \frac{200(-1)^{n+1}}{n\pi} \sin \frac{n\pi x}{\ell} \, e^{\frac{-n^2\pi^2\alpha^2 t}{\ell^2}}$$

Example 10

A rod, 30 c.m long, has its ends A and B kept at 20°C and 80°C respectively, untilsteady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function u (x,t) taking x = 0 at A.

The one dimensional heat flow equation is given by

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \qquad \text{-----------} (1)$$

In steady-state, $\frac{\partial u}{\partial t} = 0$.

Now, equation (1) reduces to $\frac{\partial^2 u}{\partial x^2} = 0$ ------------------------ (2)

Solving (2), we get $u = ax + b$ ------------------------------------ (3)

The initial conditions, in steady – state, are

u = 20, when x = 0 u
= 80, when x = 30

Therefore, (3) gives b = 20, a = 2.

$\therefore u(x) = 2x + 20$ ------------------------------------- (4)

Hence the boundary conditions are (i)
$u(0,t) = 0, \quad \forall \; t \geq 0$
(ii) $u(30,t) = 0, \quad \forall \; t \geq 0$
(iii) $u(x,0) = 2x + 20$, for $0 < x < 30$

The solution of equation (1) is given by

$$u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2\lambda^2 t} \qquad \text{----------------------------} (5)$$

Applying conditions (i) and (ii), we get

$A = 0, \lambda = \frac{n\pi}{30}$, where „n" is an integer

$$\therefore\ u(x,t) = B \sin\frac{n\pi x}{30}\, e^{-\frac{\alpha^2 n^2\pi^2}{900}t} \qquad \text{......(6)}$$

The most general solution is

$$\therefore\ u(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{30}\, e^{-\frac{\alpha^2 n^2\pi^2}{900}t} \qquad \text{------------- (7)}$$

Applying (iii) in (7), we get

$$u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{30} = 2x + 20,\ 0 < x < 30.$$

$$\therefore\ B_n = \frac{2}{30}\int_0^{30} (2x+20) \sin\frac{n\pi x}{30}\, dx$$

$$= \frac{1}{15}\int_0^{30} (2x+20)\, d\left(\frac{-\cos\frac{n\pi x}{30}}{\frac{n\pi}{30}}\right)$$

$$= \frac{1}{15}\left[(2x+20)\left(\frac{-\cos\frac{n\pi x}{30}}{\frac{n\pi}{30}}\right) - (2)\left(\frac{-\sin\frac{n\pi x}{30}}{\frac{n^2\pi^2}{900}}\right)\right]_0^{30}$$

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$$= \frac{1}{15}\left\{\frac{-2400\cos n\pi}{n\pi} + \frac{600}{n\pi}\right\}$$

$$B_n = \frac{40}{n\pi}\{1 - 4(-1)^n\}$$

Hence, the required solution is

$$u(x,t) = \sum_{n=1}^{\infty} \frac{40}{n\pi}\{1 - 4(-1)^n\} \sin \frac{n\pi x}{30} e^{-\frac{\alpha^2 n^2 \pi^2}{900}t}$$

Steady–state conditions and non–zero boundary conditions

Example 11

The ends A and B of a rod 30cm. long have their temperatures kept at 20°C and80°C, until steady–state conditions prevail. The temperature of the end B is suddenly reduced to 60°C and kept so while the end A is raised to 40°C. Find the temperature distribution in the rod after time t.

Let the equation for the heat- flow be

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \quad \text{------------ (1)}$$

In steady–state, equation (1) reduces to $\frac{\partial^2 u}{\partial x^2} = 0.$

Solving, we get $u = ax + b$ -- (2)

The initial conditions, in steady–state, areu =

20, when x = 0
u = 80, when x = 30

From (2), b = 20 & a = 2.

Thus the temperature function in steady–state is

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$$u(x) = 2x + 20 \text{ -- } (3)$$

Hence the boundary conditions in the transient–state are(i)

$u(0,t) = 40, \quad \forall\ t > 0$

(ii) $u(30,t) = 60, \quad \forall\ t > 0$

(iii) $u(x,0) = 2x + 20$, for $0 < x < 30$

we break up the required funciton $u(x,t)$ into two parts and write

$$u(x,t) = u_s(x) + u_t(x,t) \text{ -- } (4)$$

where $u_s(x)$ is a solution of (1), involving x only and satisfying the boundary condition (i) and (ii). $u_t(x,t)$ is then a function defined by (4) satisfying (1).

Thus $u_s(x)$ is a steady state solution of (1) and $u_t(x,t)$ may therefore be regardedas a transient solution which decreases with increase of t.

To find $u_s(x)$

we have to solve the equation $\dfrac{\partial^2 u}{\partial x^2} = 0$

Solving, we get $u_s(x) = ax + b$ -- (5)

Here $u_s(0) = 40$, $u_s(30) = 60$.

Using the above conditions, we get $b = 40$, $a = 2/3$.

$$\therefore u_s(x) = \frac{2}{3} x + 40 \text{ -- } (6)$$

To find $u_t(x,t)$

$u_t(x,t) = u(x,t) - u_s(x)$

Now putting $x = 0$ and $x = 30$ in (4), we have u_t

$(0,t) = u(0,t) - u_s(0) = 40 - 40 = 0$

and $u_t(30,t) = u(30,t) - u_s(30) = 60 - 60 = 0$

Also $u_t(x,0) = u(x,0) - u_s(x)$

$$= 2x + 20 - \frac{2}{3} 40$$

$$= \frac{4}{} x - 203$$

Hence the boundary conditions relative to the transient solution $u_t(x,t)$ are

$u_t(0,t) = 0$ ---------------------------------- (iv)

$u_t(30,t) = 0$ -------------------------------- (v)

and $u_t(x,0) = (4/3)x - 20$ -------------------- (vi)

We have

$$u_t(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{-\alpha^2\lambda^2 t} \qquad \text{.......(7)}$$

Using condition (iv) and (v) in (7), we get

$$= 0 \;\&\; \lambda = \frac{n\pi}{30}$$

Hence equation (7) becomes

$$u_t(x,t) = B \sin\frac{n\pi x}{30}\, e^{-\frac{\alpha^2 n^2\pi^2}{900} t}$$

The most general solution of (1) is

$$u_t(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{30}\, e^{-\frac{\alpha^2 n^2\pi^2}{900} t} \qquad \text{(8)}$$

Using condition (vi) ,

$$u_t(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{30} = (4/3)x - 20,\; 0 < x < 30.$$

$$\therefore B_n = \frac{2}{30}\int_0^{30} \{(4/3)x - 20\} \sin\frac{n\pi x}{30}\, dx$$

$$= \frac{1}{15}\int_0^{30}\left(\frac{4}{3}x-20\right) d\left(\frac{-\cos\frac{n\pi x}{30}}{\frac{n\pi}{30}}\right)$$

$$= \frac{1}{15}\left\{\left(\frac{4}{3}x-20\right)\left(\frac{-\cos\frac{n\pi x}{30}}{\frac{n\pi}{30}}\right) - \frac{4}{3}\left(\frac{-\sin\frac{n\pi x}{30}}{\frac{n^2\pi^2}{900}}\right)\right\}$$

$$= \frac{1}{15}\left\{\frac{-600\cos n\pi}{n\pi} - \frac{600}{n\pi}\right\}$$

$$= \frac{-40}{n\pi}\{1+\cos n\pi\}$$

$$B_n = \frac{-40\{1+(-1)^n\}}{n\pi}$$

$$\text{or } B_n = \begin{cases} 0, & \text{when } n \text{ is odd} \\ \dfrac{-80}{n\pi}, & \text{when } n \text{ is even} \end{cases}$$

$$\therefore u_t(x,t) = \sum_{n=2,4,6,\ldots}^{\infty} \frac{-80}{n\pi} \sin\frac{n\pi x}{30}\, e^{-\frac{\alpha^2 n^2\pi^2}{900}t}$$

$$\therefore u(x,t) = u_s(x) + u_t(x,t)$$

$$\text{ie, } u(x,t) = \frac{2}{3}x + 40 - 80 \sum_{n=2,4,6,..}^{\infty} \frac{1}{n} \sin\frac{n\pi x}{30}\, e^{-\frac{\alpha^2 n^2\pi^2}{900}t}$$

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Exercises

(1) Solve $\partial u/\partial t = \alpha^2 (\partial^2 u / \partial x^2)$ subject to the boundary conditions u(0,t) = 0, u(l,t) = 0, u(x,0) = x, 0< x< l.

(2) Find the solution to the equation $\partial u/\partial t = \alpha^2 (\partial^2 u / \partial x^2)$ that satisfies the conditionsi. u(0,t) = 0,

ii. u(l,t) = 0, $\forall$ t > 0,

iii. u(x,0) = x for 0< x< l/ 2.

= l – x for l/ 2< x< l.

(3) Solve the equation $\partial u/\partial t = \alpha^2 (\partial^2 u / \partial x^2)$ subject to the boundary conditionsi. u(0,t) = 0,

ii. u(l,t) = 0, $\forall$ t > 0,

iii. u(x,0) = kx(l – x), k > 0, 0 ≤ x ≤ l.

(4) A rod of length „l" has its ends A and B kept at 0^o C and 120^o C respectively until steady state conditions prevail. If the temperature at Bis reduced to 0^o C and kept so whilethat of A is maintained, find the temperature distribution in the rod.

(5) A rod of length „l" has its ends A and B kept at 0^o C and 120^o C respectively until steady state conditions prevail. If the temperature at Bis reduced to 0^o C and kept so while10^o C and at the same instant that at A is suddenly raised to 50^o C. Find the temperature distribution in the rod after time „t".

(6) A rod of length „l" has its ends A and B kept at 0^o C and 100^o C respectively until steady state conditions prevail. If the temperature of A is suddenly raised to 50^o C andthat of B to 150^o C, find the temperature distribution at the point of the rod and at any time.

(7) A rod of length 10 cm. has the ends A and B kept at temperatures 30^o C and 100^o C, respectively until the steady state conditions prevail. After some time, the temperature at A is lowered to 20^o C and that of B to 40^o C, and then these temperatures are maintained.Find the subsequent temperature distribution.

(8) The two ends A and B of a rod of length 20 cm. have the temperature at 30^o C and80^o C respectively until th steady state conditions prevail. Then the temperatures at theends A and B are changed to 40^o C and 60^o C respectively. Find u(x,t).

(9) A bar 100 cm. long, with insulated sides has its ends kept at 0^o C and 100^o C until steady state condition prevail. The two ends are then suddenly insulated and kept so. Findthe temperature distribution

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(10) Solve the equation $\partial u/ \partial t = \alpha^2 (\partial^2 u / \partial x^2)$ subject to the conditions (i) „u‟ is notinfinite
as $t \to \infty$ (ii) $u = 0$ for $x = 0$ and $x = \pi$, $\forall$ t (iii) $u = \pi x - x^2$ for $t = 0$ in $(0, \pi)$.