Design a digital Butterworth filter satisfying the constraints

$$0.707 \le \left|H(e^{j\omega})\right| \le 1 \qquad for\ 0 \le \omega \le \frac{\pi}{2}$$

$$\left|H(e^{jw})\right| \le 0.2 \quad for\ \frac{3\pi}{4} \le \omega \le \pi$$

With *T=1 sec using Impulse invariant method. [Nov/Dec-13]*

Solution:

Given data:

Pass band attenuation $\alpha_P = 0.707$; *Pass band frequency* $\omega_P = \frac{\pi}{2}$;

Stop band attenuation $\alpha_S = 0.2$; *Stops band frequency* $\omega_S = \frac{3\pi}{4}$;

Step 1: Specifying the pass band and stop band attenuation in dB.

$$Pass\ band\ attenuation\ \alpha_P = -20\log\delta 1 = -20\log(0.707) = 3.0116dB$$

$$Stop\ band\ attenuation\ \alpha_S = -20\log\delta 2 = -20\log(0.2) = 13.9794dB$$

Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency

$$\omega_P = \Omega_P T = \frac{\pi}{2} Rad / Sec$$

$$\omega_S = \Omega_S T = \frac{3\pi}{4} Rad / Sec$$

Step3. To find order of the filter

$$N \ge \left[\frac{\log_{10}\sqrt{\left(\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}\right)}}{\log_{10}\left(\frac{\Omega_s}{\Omega_P}\right)} \right]$$

$$N \geq \frac{\log\sqrt{\left(\frac{10^{0.1*3.01}-1}{10^{0.1*13.97}-1}\right)}}{\log\left(\frac{\frac{3\pi}{4}}{\frac{\pi}{2}}\right)}$$

$$\geq \frac{log\sqrt{\left(\frac{0.9998}{23.945}\right)}}{log(1.5)}$$

$$\geq \frac{\log(0.20433)}{\log(1.5)}$$

$$\geq \frac{-0.6896}{0.17609}$$

$$\geq 3.924$$

Rounding the next higher value N=4

Step 4: The normalized transfer function

$$H_a(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)}$$

Step 5: Cut off frequency

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha p}-1)^{1/2N}}$$

$$\Omega_c = \frac{\frac{\pi}{2}}{(10^{0.1*3.01}-1)^{\frac{1}{2*4}}} = \frac{\frac{\pi}{2}}{(0.9998)^{\frac{1}{8}}} = 1.57 Rad/Sec$$

Step 6: To find Transfer function of *H(s)*:

$$H(s) = H_a(s)\Big|_{S->\frac{s}{1.57}}$$

$$H(s) = \frac{1}{(s^2+0.76537s+1)(s^2+1.8477s+1)}\Bigg|_{S\to\frac{s}{1.57}}$$

$$= \frac{1}{\left(\left(\frac{s}{1.57}\right)^2 + 0.76537\left(\frac{s}{1.57}\right)+1\right)\left(\left(\frac{s}{1.57}\right)^2 + 1.8477\left(\frac{s}{1.57}\right)+1\right)}$$

$$H(s) = \frac{(1.57)^4}{(s^2 + 1.202s + 2.465)(s^2 + 2.902s + 2.465)}$$

Step 7: Using partial fraction expansion, expand H(s) into

$$H(s) = \frac{A}{(s+1.45+j0.6)} + \frac{A^*}{(s+1.45-j0.6)} + \frac{B}{(s+0.6+1.45j)} + \frac{B^*}{(s+0.6-1.45j)}$$

To find A and A:*

$$A = H(s)\big|_{S=-1.45-j0.6}$$

$$= (s+1.45-j0.6)\frac{(1.57)^4}{(s+1.45+j0.6)(s+1.45-j0.6)(s^2+2.902s+2.465)}\Bigg|_{S=-1.45-j0.6}$$

$$= \frac{(1.57)^4}{(s+1.45-j0.6)(s^2+1.202s+2.465)}\Bigg|_{S=-1.45-j0.6}$$

$$= \frac{(1.57)^4}{(-1.45-j0.6+1.45-j0.6)((-1.45-j0.6)^2+1.202(-1.45-j0.6)+2.465)}$$

$$= \frac{(1.57)^4}{-j(1.2)[1.7425+1.74j-1.7429-j0.7212+2.465]}$$

$$= \frac{(1.57)^4}{-j(1.2)[2.465+j1.0188]} = \frac{5.063}{1.0188-j2.465}$$

$$= \frac{5.063(1.0188-j2.465)}{7.114}$$

$$A = 0.7253 + j1.754;\ A^* = 0.7253 - j1.754$$

To find B and B:*

$$A = H(s)\big|_{S=-0.6-j1.45}$$

$$= (s+0.6+j1.45)\frac{(1.57)^4}{(s+1.45+j0.6)(s+1.45-j0.6)(s+0.6+1.45j)(s+0.6-1.45j)}\Bigg|_{S=-0.6-j1.45}$$

$$= \frac{\quad}{(-0.6-j1.45+1.45+j0.6)(-0.6-j1.45+1.45-j0.6)(-0.6-j1.45+0.6-1.45j)}\Bigg|_{S=-0.6-j1.45}$$

$$= \frac{(1.57)^4}{(0.85-j0.85)(0.85-j0.85)(-2.9j)}$$

$$= \frac{(1.57)^4}{-j[-1.0187-j2.468]}$$

$$= \frac{2.095}{-2.468+j1.0187}$$

$$B = -0.7253-j0.3; \quad B^* = -0.7253+j0.3$$

$$H(s) = \frac{0.7253+j1.754}{s-(-1.45-j0.6)} + \frac{0.7253-j1.754}{s-(-1.45+j0.6)} + \frac{-0.7253-0.3j}{s-(-0.6-j1.45)} + \frac{-0.7253+0.3j}{s-(-0.6+j1.45)}$$

we know for $T = 1\ sec$

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{p_k} z^{-1}}$$

$$\therefore \quad = \frac{0.7253+j1.754}{1-e^{-1.45}e^{-j0.6}z^{-1}} + \frac{0.7253-j1.754}{1-e^{-1.45}e^{j0.6}z^{-1}} + \frac{-0.7253-0.3j}{1-e^{-0.6}e^{-j1.45}} + \frac{-0.7253+0.3j}{1-e^{-0.6}e^{j1.45}}$$

$$H(z) = \frac{1.454+0.1839z^{-1}}{1-0.387z^{-1}+0.055z^{-2}} + \frac{-1.454+0.2307z^{-1}}{1-0.1322z^{-1}+0.301z^{-2}}$$

Design a chebyshev filter for the following specification using bilinear transformation.

$$0.8 \le \left|H\left(e^{j\omega}\right)\right| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$\left|H\left(e^{j\omega}\right)\right| \le 0.2 \qquad 0.6\pi \le \omega \le \pi.$$

Solution:

Given data:

Pass band attenuation $\alpha_P = 0.8$; *Pass band frequency* $\omega_P = 0.2\pi$;

Stop band attenuation $\alpha_S = 0.2$; *Stops band frequency* $\omega_S = 0.6\pi$;

Step 1: Specifying the pass band and stop band attenuation in dB.

Pass band attenuation $\alpha_P = -20\log\delta 1 = -20\log(0.8) = 1.938dB$

Stop band attenuation $\alpha_S = -20\log\delta 2 = -20\log(0.2) = 13.9794dB$

Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency

$$\Omega_P = \frac{2}{T}\tan\left(\frac{\omega_P}{2}\right) = 2\tan\left(\frac{0.2\pi}{2}\right) = 0.649dB$$

$$\Omega_S = \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) = 2\tan\left(\frac{0.6\pi}{2}\right) = 2.75dB$$

Step3. To find order of the filter

$$N \ge \frac{Cosh^{-1}\sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\alpha_P}-1}}}{Cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\geq \frac{Cosh^{-1}\sqrt{\frac{10^{0.1*13.97}-1}{10^{0.1*1.938}-1}}}{Cosh^{-1}\left(\frac{2.75}{0.649}\right)}$$

$$\geq \frac{Cosh^{-1}\sqrt{\frac{23.945}{0.562}}}{Cosh^{-1}\left(\frac{2.75}{0.649}\right)}$$

$$\geq \frac{Cosh^{-1}(6.5273)}{Cosh^{-1}(4.2372)}$$

$$\geq \frac{2.5632}{2.1228}$$

$$\geq 1.207$$

Rounding the next higher integer value N=2

Step4. The poles of chebyshev filter can be determined by

$$S_k = a\cos\varphi_k + jb\sin\varphi_k, k = 0,1,\cdots,N$$

Where,

$$\phi_k = \left[\frac{(2k+N-1)\pi}{2N}\right]$$ And calculate a, b, ε, μ

$$\varepsilon = \sqrt{10^{0.1\alpha p}-1},$$

$$= \sqrt{10^{0.1*1.938}-1}$$

$$\varepsilon = 0.75$$

$$\mu = \varepsilon^{-1} + \left[\sqrt{1+\varepsilon^{-2}}\right]$$

$$= (0.75)^{-1} + \left[\sqrt{1+(0.75)^{-2}}\right]$$

$$\mu = 3$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$= 0.649\left[\frac{(3)^{\frac{1}{2}} - (3)^{-\frac{1}{2}}}{2}\right]$$

$$a = 0.375$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$= 0.649\left[\frac{(3)^{\frac{1}{2}} - (3)^{-\frac{1}{2}}}{2}\right]$$

$$b = 0.75$$

$$\phi_k = \left[\frac{(2k+N-1)\pi}{2N}\right] \; ; \quad k = 1,2$$

$$\phi_1 = \left[\frac{(2(1)+2-1)\pi}{2*2}\right] = \frac{3\pi}{4} = 135^0$$

$$\phi_2 = \left[\frac{(2(2)+2-1)\pi}{2*2}\right] = \frac{5\pi}{4} = 225^0$$

$$S_K = a\cos\varphi_k + jb\sin\varphi_k , \quad k = 1,2$$

$$for\, k = 1,$$

$$S_1 = 0.375\cos\varphi_1 + j(0.75)\sin\varphi_1$$

$$= 0.375cos135^0 + j(0.75)\sin 135^0$$

$$S_1 = -0.265 + j0.53$$

$$for\, k = 2,$$

$$S_1 = 0.375\cos\varphi_2 + j(0.75)\sin\varphi_2$$

$$= 0.375cos225^0 + j(0.75)\sin 225^0$$

$$S_1 = -0.265 - j0.53$$

Step.5 Find the denominator polynomial of the transfer function using above poles.

$$H(s) = \{S + 0.265 - j0.53\}\{S + 0.265 - j0.53\}$$

$$= \{(S + 0.265)^2 - (j0.53)^2\}$$

$$= (S + 0.265)^2 + (0.53)^2$$

$$= S^2 + 0.5306s + 0.3516$$

Step 6 : The numerator of the transfer function depends on the value of N.

- If N is Even substitute s=0 in the denominator polynomial and divide the result by $\sqrt{1+\varepsilon^2}$ Find the value. This value is equal to numerator

$$= \frac{0.3516}{\sqrt{1+\varepsilon^2}} = \frac{0.3516}{\sqrt{1+(0.75)^2}}$$

$$H(s) = 0.28$$

Step 7: The Transfer function is

$$H(s) = \frac{NM}{DM}$$

$$H(s) = \frac{0.28}{s^2 + 0.5306s + 0.3516}$$

Step 8: Apply bilinear transformation with to obtain the digital filter

$$H(z) = H(s)\big|\, s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{0.28}{s^2 + 0.5306s + 0.3516}\Bigg|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{0.28}{s^2 + 0.5306s + 0.3516}\Bigg|_{s = 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{0.28}{\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 0.5306\left(2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right) + 0.3516}$$

$$H(z) = \frac{0.28(1+z^{-1})^2}{1 - 1.348z^{-1} + 0.608z^{-2}}$$

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Design a chebyshev filter for the following specification using impulse invariance method.

$$0.8 \le \left|H\left(e^{j\omega}\right)\right| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$\left|H\left(e^{j\omega}\right)\right| \le 0.2 \qquad 0.6\pi \le \omega \le \pi. \text{ [May/June – 2016]}$$

Solution:

Given data:

Pass band attenuation $\alpha_P = 0.8$; *Pass band frequency* $\omega_P = 0.2\pi$;

Stop band attenuation $\alpha_S = 0.2$; *Stops band frequency* $\omega_S = 0.6\pi$;

Step 1: Specifying the pass band and stop band attenuation in dB.

Pass band attenuation $\alpha_P = -20\log\delta 1 = -20\log(0.8) = 1.938dB$

Stop band attenuation $\alpha_S = -20\log\delta 2 = -20\log(0.2) = 13.9794dB$

Step2. Choose *T* and determine the analog frequencies (i.e) Prewarp band edge frequency

$$\Omega_P = \frac{\omega_P}{T} = 0.2\pi \ Rad/Sec$$

$$\Omega_S = \frac{\omega_S}{T} = 0.6\pi \ Rad/Sec$$

Step3. To find order of the filter

$$N \ge \frac{Cosh^{-1}\sqrt{\frac{10^{0.1\alpha_S} - 1}{10^{0.1\alpha_P} - 1}}}{Cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\geq \frac{Cosh^{-1}\sqrt{\frac{10^{0.1*13.97}-1}{10^{0.1*1.938}-1}}}{Cosh^{-1}\left(\frac{0.6\pi}{0.2\pi}\right)}$$

$$\geq \frac{Cosh^{-1}\sqrt{\frac{23.945}{0.562}}}{Cosh^{-1}(3)}$$

$$\geq \frac{Cosh^{-1}(6.5273)}{Cosh^{-1}(3)}$$

$$\geq \frac{2.5632}{1.7627}$$

$$\geq 1.454$$

Rounding the next higher integer value N=2

Step4. The poles of chebyshev filter can be determined by

$$S_k = a\cos\varphi_k + jb\sin\varphi_k, k = 0,1,\cdots,N$$

Where,

$$\phi_k = \left[\frac{(2k+N-1)\pi}{2N}\right]$$ And calculate a, b, ε, μ

$$\varepsilon = \sqrt{10^{0.1\alpha p}-1},$$

$$= \sqrt{10^{0.1*1.938}-1}$$

$$\varepsilon = 0.75$$

$$\mu = \varepsilon^{-1} + \left\lfloor\sqrt{1+\varepsilon^{-2}}\right\rfloor$$

$$= (0.75)^{-1} + \left[\sqrt{1+(0.75)^{-2}}\right]$$

$$\mu = 3$$

$$a = \Omega_p\left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$= 0.2\pi\left[\frac{(3)^{\frac{1}{2}} - (3)^{-\frac{1}{2}}}{2}\right]$$

$$a = 0.362$$

$$b = \Omega_p\left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

$$= 0.2\pi\left[\frac{(3)^{\frac{1}{2}} - (3)^{-\frac{1}{2}}}{2}\right]$$

$$b = 0.7255$$

$$\phi_k = \left[\frac{(2k+N-1)\pi}{2N}\right] \;;\quad k = 1,2$$

$$\phi_1 = \left[\frac{(2(1)+2-1)\pi}{2*2}\right] = \frac{3\pi}{4} = 135^0$$

$$\phi_2 = \left[\frac{(2(2)+2-1)\pi}{2*2}\right] = \frac{5\pi}{4} = 225^0$$

$$S_K = a\cos\varphi_k + jb\sin\varphi_k\,,\quad k = 1,2$$

$for\, k = 1,$

$$S_1 = 0.362\cos\varphi_1 + j(0.7255)\sin\varphi_1$$

$$= 0.362cos135^0 + j(0.7255)\sin 135^0$$

$$S_1 = -0.256 + j0.513$$

$for\, k = 2,$

$$S_1 = 0.362\cos\varphi_2 + j(0.7255)\sin\varphi_2$$

$$= 0.362cos225^0 + j(0.7255)\sin 225^0$$

$$S_1 = -0.256 - j0.513$$

Step.5 Find the denominator polynomial of the transfer function using above poles.

$$H(s) = \{S + 0.256 - j0.513\}\{S + 0.256 - j0.513\}$$

$$= \{(S + 0.256)^2 - (j0.513)^2\}$$

$$= (S + 0.256)^2 + (0.513)^2$$

$$= S^2 + 0.513s + 0.33$$

Step 6 : The numerator of the transfer function depends on the value of N.

- If N is Even substitute s=0 in the denominator polynomial and divide the result by $\sqrt{1+\varepsilon^2}$ Find the value. This value is equal to numerator

$$= \frac{0.33}{\sqrt{1+\varepsilon^2}} = \frac{0.33}{\sqrt{1+(0.75)^2}}$$

$$H(s) = 0.264$$

Step 7: The Transfer function is

$$H(s) = \frac{NM}{DM}$$

$$H(s) = \frac{0.264}{s^2 + 0.513s + 0.33}$$

Step 8: Using partial fraction expansion, expand H(s) into

$$H(s) = \sum_{k=1}^{2}\frac{A_k}{s-p_k} = \frac{A_1}{s-p_1} + \frac{A_2}{s-p_2}$$

$$\frac{0.264}{s^2 + 0.513s + 0.33} = \frac{A_1}{s-(-0.256+j0.514)} + \frac{A_2}{s-(-0.256-j0.514)}$$

$$= \frac{0.257j}{s-(-0.256+j0.514)} - \frac{0.257j}{s-(-0.256-j0.514)}$$

Step 9: Now transform analog poles $\{P_k\}$ into digital poles $\{e^{p_kT}\}$ to obtain the digital filter

$$H(z) = \sum_{k=1}^{N} \frac{A_k}{1 - e^{p_k T} z^{-1}}$$

$$= \sum_{k=1}^{2} \frac{A_k}{1 - e^{p_k T} z^{-1}}$$

$$= \frac{0.257 j}{s - e^{-0.256T} e^{j0.513T} z^{-1}} - \frac{0.257 j}{s - e^{-0.256T} e^{-j0.513T} z^{-1}}$$

$$H(z) = \frac{0.1954z^{-1}}{1 - 1.3483z^{-1} + 0.5987z^{-2}}$$

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