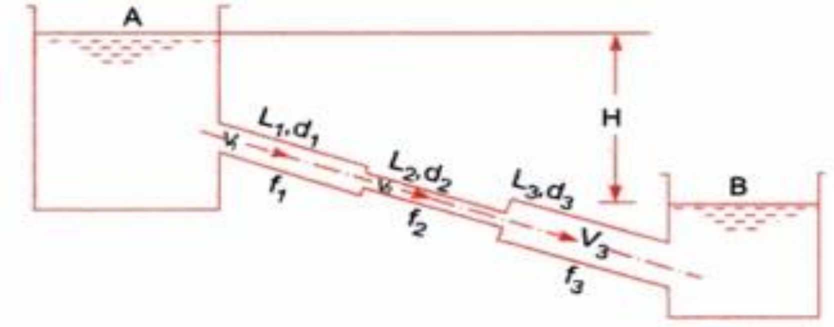
4.6 PIPES IN SERIES AND IN PARALLEL

PIPES IN SERIES:

When pipes of different lengths and different diameters are connected end to end to form a pipe line, such arrangement or connection of pipes will be considered as pipes in series or compound pipes. Following figure, displayed here, indicates the arrangement of connection of three pipes in series.



Let us consider the following terms from above figure

L_1, L_2 and L_3: Length of pipes 1, 2 and 3 respectively
d_1, d_2 and d_3: Diameter of pipes 1, 2 and 3 respectively
V_1, V_2 and V_3: Velocity of flow through pipes 1, 2 and 3 respectively
f_1, f_2 and f_3: Co-efficient of friction for pipes 1, 2 and 3 respectively
H = Difference of water level in two tanks

We must note it here that difference in liquid surface level will be equal to the sum of total head loss in the pipes.

If we neglect the minor head losses, we will have following equation for total head loss as mentioned here.

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

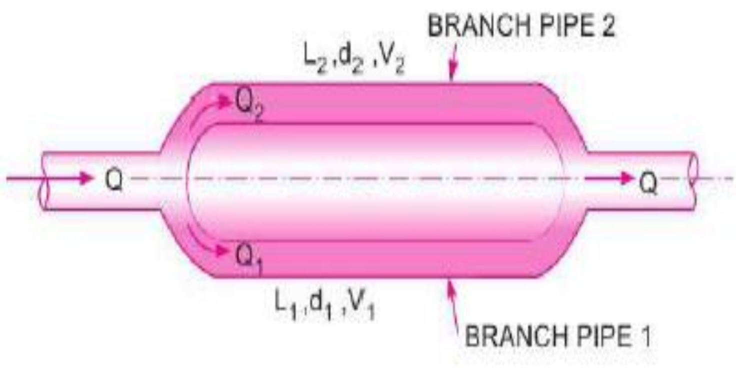
Let us consider that co-efficient of friction i.e. f is same for all three pipes and therefore we can write the equation for head loss as mentioned here.

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

PIPES IN PARALLEL:

When a main pipeline divides in to two or more parallel pipes, which may again join together downstream and continue as main line, the pipes are said to be in parallel. The pipes are connected in parallel in order to increase the discharge passing through the main.



It is analogous to parallel electric current in which the drop in potential and flow of electric current can be compared to head loss and rate of discharge in a fluid flow respectively.
The rate of discharge in the main line is equal to the sum of the discharges in each of the parallel pipes.

Thus Q = Q1 + Q2

The flow of liquid in pipes (1) and (2) takes place under the difference of head between the sections A and B and hence the loss of head between the sections A and B will be the same whether the liquid flows through pipe (1) or pipe (2). Thus if D_1, D_2 and L_1, L_2 are the diameters and lengths of the pipes (1) and (2) respectively, then the velocities of flow V_1 and V_2 in the two pipes must be such as to give

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$f_1 = f_2, \text{ then } \frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}$$

EQUIVALENT PIPE

In practice adopting pipes in series may not be feasible due to the fact that they may be of unistandard size (ie. May not be comemercially available) and they experience other minor losses. Hence, the entire system will be replaced by a single pipe of uniform diameter D, but of the same length $L=L_1+ L_2+ L_3$ such that the head loss due to friction for both the pipes, viz equivalent pipe & the compound pipe are the same.

For a compound pipe or pipes in series

$$h_f = hf_1 + hf_2 + hf_3$$

$$h_f = \frac{8 f L_1 Q^2}{g\pi^2 D_1^5} + \frac{8 f L_2 Q^2}{g\pi^2 D_2^5} + \frac{8 f L_3 Q^2}{g\pi^2 D_3^5} --- (1)$$

for an equivalent pipe $h_f = \frac{8fLQ^2}{g\pi^2 D^5} ---(2)$

Equating (1) & (2) and simplifying $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$

$$\text{or} \quad D = \left\{ \frac{L}{\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}} \right\}^{\frac{1}{5}}$$

PROBLEM 1:The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 400 m, 200 m and 300 m and of diameters 400 mm, 300 mm and 200 mm respectively, is 16m. Estimate the rate of flow of water if co-efficient of friction for these pipes is same and equal to 0.005, considering: (i) minor losses also (ii) neglecting minor losses.

Solution. Given :

Difference of water levels, $H = 16$ m

Length and dia. of pipe 1, $L_1 = 400$ m and $d_1 = 400$ mm $= 0.4$ m

Length and dia. of pipe 2, $L_2 = 200$ m and $d_2 = 200$ mm $= 0.2$ m

Length and dia. of pipe 3, $L_3 = 300$ m and $d_3 = 300$ mm $= 0.3$ m

Also $f_1 = f_2 = f_3 = 0.005$

(*i*) **Discharge through the compound pipe first neglecting minor losses.**

Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2} \times V_1 = \frac{d_1^2}{d_2^2} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

$$V_3 = \frac{A_1V_1}{A_3} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_3^2} \times V_1 = \frac{d_1^2}{d_3^2} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 1.77V_1$$

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300}{0.3 \times 2 \times 9.81} \times (1.77\ V_1)^2$$

$$= \frac{V_1^2}{2 \times 9.81}\left(\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2 \times 9.81}(20 + 320 + 63.14) = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$$\therefore \quad V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

$\therefore$ Discharge, $Q = A_1 \times V_1 = \frac{\pi}{4}(0.4)^2 \times 0.882 = \mathbf{0.1108\ m^3/s.}$

(*ii*) **Discharge through the compound pipe considering minor losses also.**
Minor losses are :

(*a*) At inlet, $h_i = \frac{0.5\, V_1^2}{2g}$

(*b*) Between 1st pipe and 2nd pipe, due to contraction,

$$h_c = \frac{0.5\, V_2^2}{2g} = \frac{0.5\left(4V_1^2\right)}{2g} \qquad (\because V_2 = 4V_1)$$

$$= \frac{0.5 \times 16 \times V_1^2}{2g} = 8 \times \frac{V_1^2}{2g}$$

(*c*) Between 2nd pipe and 3rd pipe, due to sudden enlargement,

$$h_e = \frac{\left(V_2 - V_3\right)^2}{2g} = \frac{\left(4V_1 - 1.77V_1\right)^2}{2g} \qquad (\because V_3 = 1.77\, V_1)$$

$$= (2.23)^2 \times \frac{V_1^2}{2g} = 4.973\, \frac{V_1^2}{2g}$$

(*d*) At the outlet of 3rd pipe, $h_o = \frac{V_3^2}{2g} = \frac{(1.77V_1)^2}{2g} = 1.77^2 \times \frac{V_1^2}{2g} = 3.1329\, \frac{V_1^2}{2g}$

The major losses are $= \frac{4 f_1 \times L_1 \times V_1^2}{d_1 \times 2g} + \frac{4 f_2 \times L_2 \times V_2^2}{d_2 \times 2g} + \frac{4 f_3 \times L_3 \times V_3^2}{d_3 \times 2g}$

$$= \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300 \times (1.77V_1)^2}{0.3 \times 2 \times 9.81}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

$\therefore$ Sum of minor losses and major losses

$$= \left[\frac{0.5\, V_1^2}{2g} + 8 \times \frac{V_1^2}{2g} + 4.973\, \frac{V_1^2}{2g} + 3.1329\, \frac{V_1^2}{2g}\right] + 403.14\, \frac{V_1^2}{2g}$$

$$= 419.746 \frac{V_1^2}{2g}$$

But total loss must be equal to H (or 16 m)

$$\therefore \quad 419.746 \times \frac{V_1^2}{2g} = 16 \quad \therefore \quad V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.746}} = 0.864 \text{ m/s}$$

$$\therefore \text{ Discharge,} \quad Q = A_1 V_1 = \frac{\pi}{4} (0.4)^2 \times 0.864 = \mathbf{0.1085 \ m^3/s.}$$

PROBLEM 2:Three pipes of length 800m,500m and 400m of diameter 500mm,400mm and 300mm respectively are connected are connected in series these pipes are to be replaced by a single pipe of length 1700m.find the diameter of single pipe.

Solution. Given :

Length of pipe 1, $L_1 = 800$ m and dia., $d_1 = 500$ mm $= 0.5$ m

Length of pipe 2, $L_2 = 500$ m and dia., $d_2 = 400$ mm $= 0.4$ m

Length of pipe 3, $L_3 = 400$ m and dia., $d_3 = 300$ mm $= 0.3$ m

Length of single pipe, $L = 1700$ m

Let the diameter of equivalent single pipe $= d$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{.5^5} + \frac{500}{.4^5} + \frac{400}{0.3^5}$$

$$= 25600 + 48828.125 + 164609 = 239037$$

$$\therefore \quad d^5 = \frac{1700}{239037} = .007118$$

$$\therefore \quad d = (.007188)^{0.2} = 0.3718 = \mathbf{371.8 \ mm.}$$