1.7 ELECTRIC FIELD INTENSITY

ELECTRIC FILED OR ELECTRIC FIELD INTENSITY:

The electric field or electric field intensity is defined as the electric force per unit charge .It is given by

$$E = \frac{F}{q}$$

According to coulomb's law

$$F = \frac{Qq}{4\pi\varepsilon r^2}$$

Electric Filed

$$E = \frac{F}{q}$$

Substitute F value in above equation

$$E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}$$

$$E = \frac{Q\cancel{q}}{4\pi\varepsilon r^2 \cancel{q}}$$

$$E = \frac{Q}{4\pi\varepsilon r^2}\ V/m$$

The another unit of electric field is $Volts/meter$

ELECTRIC FIELD INTENSITY DUE TO LINE CHARGE:

Considered uniformly charged line of length L whose linear charge density is ρ_l Coulomb/meter. Consider a small element dl at a distance l from one end of the charged line as shown in figure 1.7.1 .Let P be any point at a distance r from the element dl.

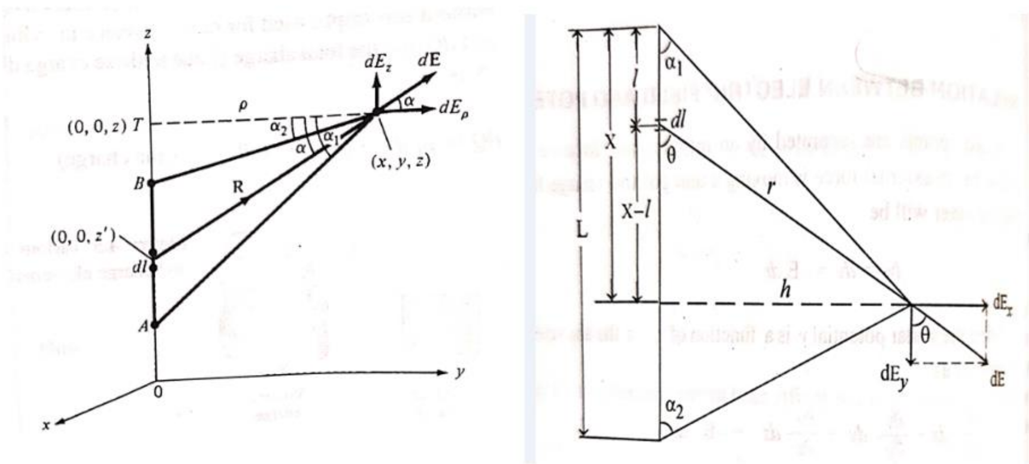


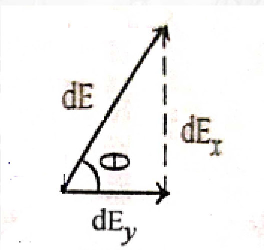
Figure 1.7.1 Evaluation of the E field due to a line charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-114]

The electric field at a point $\boldsymbol{P}$ due to the charge element $\boldsymbol{\rho dl}$ is given

$$dE = \frac{\rho_l dl}{4\pi\varepsilon r^2}$$

The $\boldsymbol{x}$ and $\boldsymbol{y}$ components of electric field $\boldsymbol{dE}$ are given by



From the above diagram find $\sin\theta$ and $\cos\theta$

$$\sin\theta = \frac{dE_x}{dE}$$

$$dE_x = dE\sin\theta$$

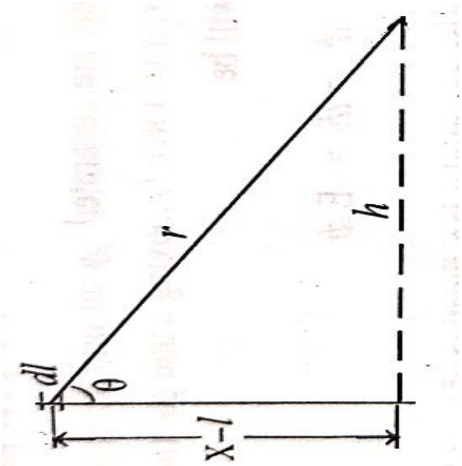
$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_y = dE\cos\theta$$

Substitute $\boldsymbol{dE}$ expression in $\boldsymbol{dE_x}$

$$dE_x = \frac{\rho_l dl\sin\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_l dl\cos\theta}{4\pi\varepsilon r^2}$$



From the above diagram find $\mathbf{tan}\,\boldsymbol{\theta}$

$$tan\,\theta = \frac{h}{x-l}$$

$$x-l = \frac{h}{\tan\theta}$$

$$x-l = h\cot\theta$$

Differentiate above equation on both sides

$$0 - dl = h(-\operatorname{cosec}^2\theta)$$

$$-dl = -h(\operatorname{cosec}^2\theta)$$

$$dl = h(\operatorname{cosec}^2\theta).d\theta$$

From the above diagram find $\mathbf{sin}\,\boldsymbol{\theta}$

$$\sin\theta = \frac{h}{r}$$

$$r = \frac{h}{\sin\theta}$$

$$r = h\operatorname{cosec}\theta$$

Substitute $\boldsymbol{dl}$ and $\boldsymbol{r}$ value in $\boldsymbol{dE_x}$

$$dE_x = \frac{\rho_l dl\sin\theta}{4\pi\varepsilon r^2}$$

$$dE_x = \frac{\rho_l h(\operatorname{cosec}^2\theta)\,d\theta\sin\theta}{4\pi\varepsilon\,(h\operatorname{cosec}\theta)^2}$$

$$dE_x = \frac{\rho_l h(\mathrm{cosec}^2\,\theta)\,d\theta \sin\theta}{4\pi\varepsilon h^2 \,\mathrm{cosec}^2\,\theta}$$

$$dE_x = \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

Integrate the above equation dE_x considered the limit as α_1 *to* $\pi - \alpha_2$

The electric field E_x due to the entire length of line charge is given by

$$\int dE_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

$$E_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}\int_{\alpha_1}^{\pi-\alpha_2} \sin\theta d\theta$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[-\cos\theta]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[-\cos(\pi-\alpha_2) - (\cos\ \alpha_1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos\alpha_2) + (\cos\ \alpha_1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos\alpha_1) + (\cos\ \alpha_2)]$$

Substitute dl and r value in dE_x

$$dE_y = \frac{\rho_l h(\mathrm{cosec}^2\,\theta) d\theta \cos\theta}{4\pi\varepsilon\,(h\,\mathrm{cosec}\,\theta)^2}$$

$$dE_y = \frac{\rho_l h(\mathrm{cosec}^2\,\theta)\,d\theta \cos\theta}{4\pi\varepsilon h^2 \,\mathrm{cosec}^2\,\theta}$$

$$dE_y = \frac{\rho_l h(\mathrm{cosec}^2\,\theta)\,d\theta\cos\theta}{4\pi\varepsilon h^2 \,\mathrm{cosec}^2\,\theta}$$

$$dE_y = \frac{\rho_l \, d\theta \cos\theta}{4\pi\varepsilon h}$$

$$dE_y = \frac{\rho_l \cos\theta d\theta}{4\pi\varepsilon h}$$

Similarly for $\boldsymbol{y}$ component of $\boldsymbol{E}$

Integrate the above equation $\boldsymbol{dE_y}$ considered the limit as $\boldsymbol{\alpha_1}$ *to* $\boldsymbol{\pi - \alpha_2}$

The electric field $\boldsymbol{E_y}$ due to the entire length of line charge is given by

$$\int dE_y = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \cos\theta \, d\theta}{4\pi\varepsilon h}$$

$$E_y = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \cos\theta \, d\theta}{4\pi\varepsilon h}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \cos\theta \, d\theta$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [\sin\theta]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [\sin(\pi - \alpha_2) - (\sin \ \alpha_1)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [(\sin\alpha_2) - (\sin \ \alpha_1)]$$

Case (i): If the point $\boldsymbol{P}$ is at bisector of a line, then $\boldsymbol{\alpha_1 = \alpha_2 = \alpha}$

$\boldsymbol{E_y = 0}$ $\quad$ $\boldsymbol{E}$ becomes $\boldsymbol{E_x}$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha_1) + (\cos \ \alpha_2)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha) + (\cos\alpha)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} (2\cos\alpha)$$

$$E_x = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha_2) - (\sin\ \alpha_1)]$$

Substitute $\alpha_1 = \alpha_2 = \alpha$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha) - (\sin\alpha)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[0]$$

$$E_y = 0$$

$\boldsymbol{E}$ becomes $\boldsymbol{E_x}$ $\quad E = E_x$

$$E = E_x = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

$$E = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

Case (ii): If the line is infinitely long then $\alpha_1 = \alpha_2 = \alpha = 0$

$E_y = 0$ $\quad E$ becomes E_x

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos\alpha_1) + (\cos\ \alpha_2)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos 0) + (\cos 0)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(1) + (1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[2]$$

$$E_x = \frac{\rho_l}{2\pi\varepsilon h}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha_2) - (\sin\ \alpha_1)]$$

Substitute $\boldsymbol{\alpha_1 = \alpha_2 = \alpha = 0}$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin 0) - (\sin 0)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(0) - (0)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[0]$$

$$E_y = 0$$

$\boldsymbol{E}$ becomes $\boldsymbol{E_x}$ $\quad E = E_x$

$$E = E_x = \frac{\rho_l}{2\pi\varepsilon h}$$

$$E = \frac{\rho_l}{2\pi\varepsilon h}$$

ELECTRIC FIELD INTENSITY DUE TO CIRCULAR DISC:

Consider a circular disc of radius $\boldsymbol{R}$ is charged uniformly with a charge density of $\boldsymbol{\rho_s\ coulomb/m^2}$ .Let $\boldsymbol{P}$ be any point on the axis of the disc at a distance from the centre. Consider an annular ring of radius $\boldsymbol{r}$ and of radial thickness $\boldsymbol{dr}$ as shown in figure 1.7.2.The area of the annular ring is $\boldsymbol{ds = 2\pi r dr}$**.**

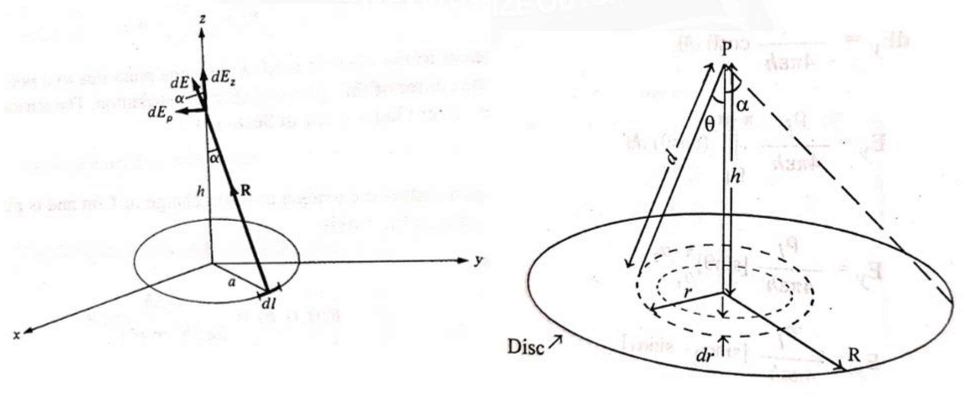


Figure 1.7.2 Evaluation of the $\boldsymbol{E}$ field due to a charged ring

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-120]

The field intensity at point $\boldsymbol{P}$ due to the charged annular ring is given by

$$dE = \frac{\rho_S ds}{4\pi\varepsilon d^2}$$

Since the horizontal component of electric field intensity is zero, The horizontal components and vertical components are dE_x and dE_y

The horizontal components of angular ring is zero

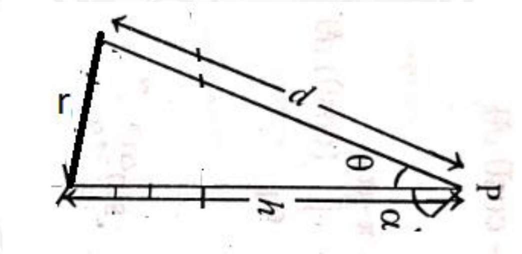
$$dE_x = 0$$

$$E_x = 0$$

The horizontal components of angular ring E_y have to find for circular ring.

the vertical component is given by

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$



From the above diagram find $\tan\theta$ *and* $\sin\theta$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$\sin\theta = \frac{r}{d}$$

$$d = \frac{r}{\sin\theta}$$

Assume $$ds = 2\pi r dr$$

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$

Substitute ds in dE_y

$$dE_y = \frac{\rho_S 2\pi r dr \cos\theta}{4\pi\varepsilon d^2}$$

$$r = h \tan\theta$$

Differentiate above equation

$$dr = h \sec^2\theta d\theta$$

Substitute dr and d in dE_y

$$dE_y = \frac{\rho_S (2\pi r) h \sec^2\theta d\theta \cos\theta}{4\pi\varepsilon d^2}$$

$$dE_y = \frac{\rho_S (2\pi r) h \sec^2\theta d\theta \cos\theta}{4\pi\varepsilon \left(\frac{r}{\sin\theta}\right)^2}$$

$$dE_y = \frac{\rho_S (2\pi r)(h \sec^2\theta\ ) d\theta \cos\theta\ \sin^2\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S (2\pi r)(h \sec^2\theta\ ) \sin^2\theta \cos\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S (2\pi r)(h) \sin^2\theta \cos\theta\, d\theta}{4\pi\varepsilon r^2\ \cos^2\theta}$$

$$dE_y = \frac{\rho_S (2\pi r)(h) \sin^2\theta d\theta}{4\pi\varepsilon r^2\ \cos\theta}$$

$$dE_y = \frac{\rho_S (2\pi r)(h) \tan\theta \sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S (2\pi r)(h) \tan\theta \sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S (h) \tan\theta \sin\theta\, d\theta}{2\varepsilon r}$$

Substitute r in dE_y

$$dE_y = \frac{\rho_S (h) \tan\theta \sin\theta\, d\theta}{2\varepsilon r}$$

$$dE_y = \frac{\rho_S (h) \tan\theta \sin\theta\, d\theta}{2\varepsilon\, h \tan\theta}$$

$$dE_y = \frac{\rho_S \sin\theta\, d\theta}{2\varepsilon}$$

Integrate the above equation $\boldsymbol{dE_y}$ considered the limit as $\mathbf{0}\ to\ \boldsymbol{\alpha}$

$$\int dE_y = \int_0^{\alpha} \frac{\rho_S \sin\theta \, d\theta}{2\varepsilon}$$

$$\int dE_y = \frac{\rho_S}{2\varepsilon} \int_0^{\alpha} \sin\theta \, d\theta$$

$$E_y = \frac{\rho_S}{2\varepsilon} [-\cos\theta]_0^{\alpha}$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) - (-\cos 0)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) + (1)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(1) + (-\cos\alpha)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The total electric field $E = E_x + E_y$

$$E = E_x + E_y$$

$$E_x = 0$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = 0 + \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

ELECTRIC FIELD INTENSITY DUE TO INFINITE SHEET OF CHARGE:

Consider an infinite plane sheet which is uniformly charged with a charge density of

$\rho_s\ Coulom/m^2$ as shown in figure 1.7.3.

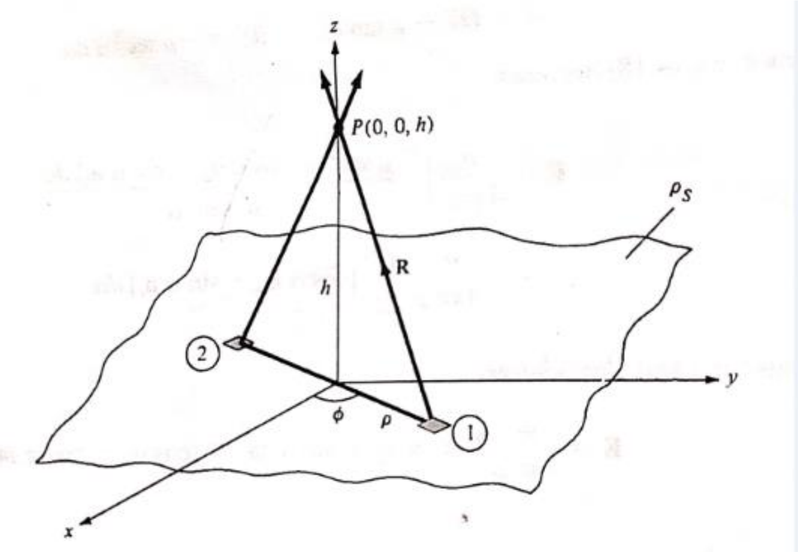


Figure 1.7.2 Evaluation of the E field due to an infinite sheet of charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-116]

The field intensity at any point P due to infinite plane sheet of charge can be evaluated

by applying expression of charged circular disc.

The field intensity at point P due to the charged annular ring is given by

$$dE = \frac{\rho_S ds}{4\pi\varepsilon d^2}$$

Since the horizontal component of electric field intensity is zero, The horizontal components and vertical components are dE_x and dE_y

The horizontal components of angular ring is zero

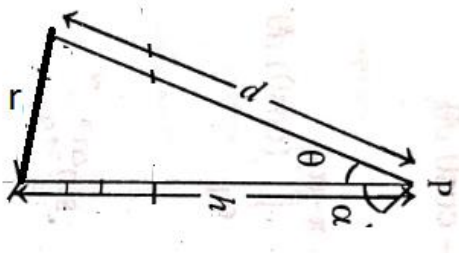
$$dE_x = 0$$

$$E_x = 0$$

The horizontal components of angular ring E_y have to find for circular ring.

the vertical component is given by

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$



From the above diagram find $\tan\theta$ *and* $\sin\theta$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$\sin\theta = \frac{r}{d}$$

$$d = \frac{r}{\sin\theta}$$

Assume $$ds = 2\pi r dr$$

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$

Substitute ds in dE_y

$$dE_y = \frac{\rho_S 2\pi r dr \cos\theta}{4\pi\varepsilon d^2}$$

$$r = h\tan\theta$$

Differentiate above equation

$$dr = h\sec^2\theta d\theta$$

Substitute dr and d in dE_y

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon d^2}$$

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon\left(\frac{r}{\sin\theta}\right)^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)d\theta\cos\theta\ \sin^2\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)\sin^2\theta\cos\theta\,d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta\cos\theta\,d\theta}{4\pi\varepsilon r^2\ \cos^2\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta d\theta}{4\pi\varepsilon r^2\ \cos\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\,d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\,d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\,d\theta}{2\varepsilon r}$$

Substitute $\boldsymbol{r}$ in $\boldsymbol{dE_y}$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\,d\theta}{2\varepsilon r}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\,d\theta}{2\varepsilon\, h\tan\theta}$$

$$dE_y = \frac{\rho_S\sin\theta\,d\theta}{2\varepsilon}$$

Integrate the above equation $\boldsymbol{dE_y}$ consider the limit as $\mathbf{0}\ to\ \boldsymbol{\alpha}$

$$\int dE_y = \int_0^{\alpha}\frac{\rho_S\, sin\,\theta\,d\theta}{2\varepsilon}$$

$$\int dE_y = \frac{\rho_S}{2\varepsilon}\int_0^{\alpha} sin\,\theta\,d\theta$$

$$E_y = \frac{\rho_S}{2\varepsilon}\,[-\cos\theta]_0^{\alpha}$$

$$E_y = \frac{\rho_S}{2\varepsilon}\,[(-\cos\alpha)-(-\cos 0)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) + (1)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(1) + (-\cos\alpha)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The total electric field $E = E_x + E_y$

$$E = E_x + E_y$$

$$E_x = 0$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = 0 + \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The electric field due to infinite uniformly charge sheet $\alpha = 90°$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos 90°]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - 0]$$

$$E = \frac{\rho_S}{2\varepsilon} [1]$$

$$E = \frac{\rho_S}{2\varepsilon}$$