UNIT II

SHEAR FORCE AND BENDING MOMENT OF BEAM

2.1. INTRODUCTION

The algebraic sum of the vertical forces at any section of a beam to the right or left of the section is known as shear force. It is briefly written as S.F.

The algebraic sum of the moment of all the forces acting to the right or left of the section of the beam is known as bending moment. It is briefly written as B.M.

In this chapter the shear force and bending moment diagrams for different types of beam for different types of load acting on the beams, will be discussed.

2.2. SHEAR FORCE AND BENDING MOMENT DIAGRAMS

Shear force diagram is one which shows the variation of the shear force along the length of the beam.

Bending moment diagram is one which shows the variation of the bending moment along the length of the beam.

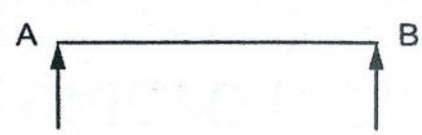
2.3. TYPES OF BEAMS

The following are the important types of beams. They are

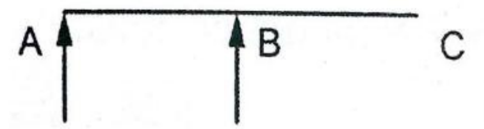
1. Cantilever beam
2. Simply supported beam
3. Overhanging beam
4. Fixed beam
5. Continuous beam

Cantilever beam: A beam which is fixed at one end and free at the other end, is known as cantilever beam.

Simply supported beam: A beam supported or resting freely on the supports at its both ends is known as simply supported beam.

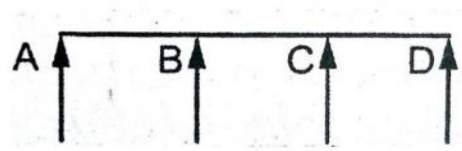


Overhanging beam: It is same as the SSB but the end portion of the beam is extend beyond the support is known as overhanging beam.



Fixed beam: A beam whose both ends are fixed or built in walls is known as fixed beam.

Continuous beam: A beam having more than two supports, such type of beam is known as continuous beam.

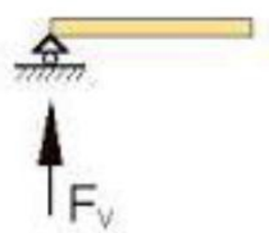


2.4. TYPES OF SUPPORTS

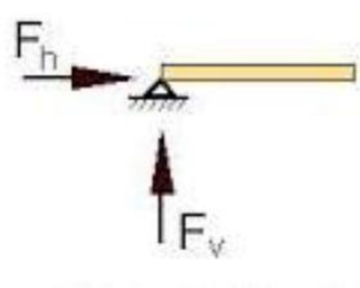
The following are the important type of supports. They are

1. Roller support
2. Hinged support
3. Fixed or built-in support

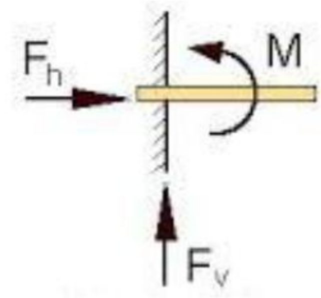
Roller Support: The roller is used as an external support since it allows rotation and horizontal translation. Therefore it will have a vertical support reaction. Here beam AB is supported on the rollers. The reaction will be normal to the surface on which rollers are placed.



Hinged support: Here the beam AB is hinged at point A. the reaction at the hinged end may be either vertical or inclined depending upon the type of loading. If load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at the hinged end will also be inclined. A hinge resists horizontal and vertical translation but allows rotation. Therefore a hinge consists of horizontal and vertical support reaction



Fixed or built-in support: In this type of support the beam should be fixed. The reaction will be inclined. Also the fixed support will provide a couple.

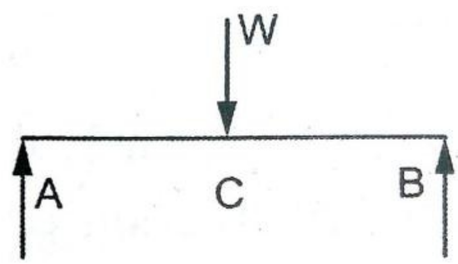


2.5. TYPES OF LOAD

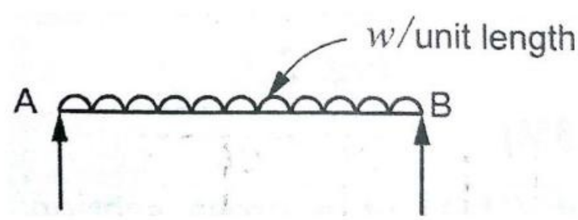
A beam is normally horizontal and the loads acting on the beams are generally vertical. the following are the important types of load acting on a beam

1. Concentrated or point load
2. Uniformly distributed load
3. Uniformly varying load

Concentrated or point load: A concentrated load is one which is considered to act at a point, although in practice it must really be distributed over a small area.

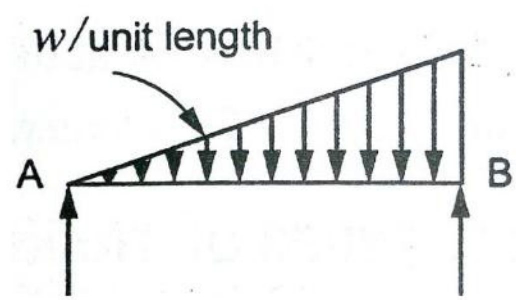


Uniformly distributed load: A uniformly distributed load is one which is spread over a beam in such a manner that rate of loading w is uniform along the length. It is expressed as w N/m run. It is denoted by UDL



For numerical problem solving, the total UDL is converted into a point load acting at the centre of load.

Uniformly varying load: A Uniformly varying load is one which is spread over abeam in such a manner that rate of loading varies from point to point along the beam from zero to rated at w N/m run. Such load is known as triangular load.



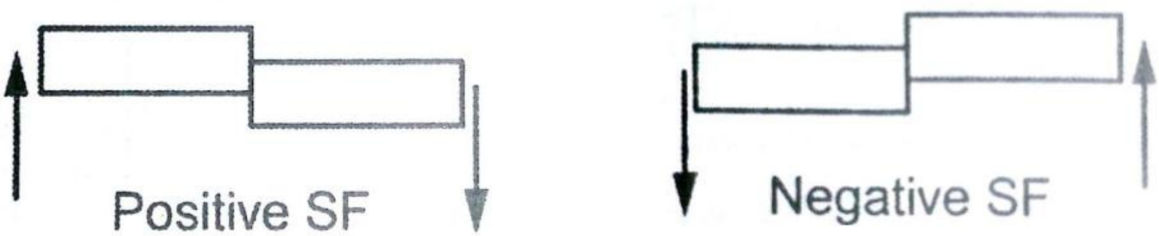
For numerical problem solving, the total UVL is converted into a point load as the area of the triangle and it acting at the C.G of the triangle. Take at a distance of 2/3 of total load acting from zero load.

2.6. CONCEPT AND SIGNIFICANCE OF SHEAR FORCE AND BENDING MOMENT SIGN CONVENTIONS FOR SHEAR FORCE AND BENDING MOMENT

(i) Shear force:

The shear force at a section will be considered positive when the resultant of the forces to the left to the section is upwards, or to the right of the section is downwards.

Similarly the shear force at the section will be considered negative if the resultant of the forces to the left of the section is downward, or to the right of the section is upwards. Here the resultant force to the left of the section is upwards and hence the shear force will be positive.



(ii) Bending moment:

The bending moment at a section is considered positive if the bending moment at that section is such that it tends to bend the beam to a curvature having concavity at the top as shown in Fig. The positive B.M. is often called sagging moment.

Similarly the bending moment at a section is considered negative if the bending moment at that section is such that it tends to bend the beam to a curvature haling convexity at the top. The negative B.M. as hogging Moment.

The bending moment will be considered positive when the moment of the forces and the reactions on the left portion is clockwise, and on the right portion anti-clockwise. In the given figure the bending moment at the section X-X is positive.

Similarly the bending moment will be considered negative when the moment of the forces and the reactions on the left portion is anti-clockwise, and on the right portion clockwise. In the given figure the bending moment at the section X-X is negative.

2.7. IMPORTANT POINTS FOR DRAWING SHEAR FORCE AND BENDING MOMENT DIAGRAMS

The shear force diagram is one which shows the variation of the shear force along the length of the beam. And a bending moment diagram is one which show the variation of the bending moment along the length of beam. In these diagrams, the shear force or bending moment are represented by ordinates whereas the length of the beam represents abscissa.

The following are the important points for drawing shear force and bending moment diagrams

1. Consider the left or the right portion of the section.

2. Add the forces (including reaction) normal to the beam on one of the portion. If right portion of the section is chosen, a force on the right portion acting downwards is positive while force acting upwards is negative. If the left portion of the section is chosen, a force on the left portion acting upwards is positive while force acting downwards is negative.

3. The positive values of shear force and bending moments are plotted above the base line, and negative values below the base line.

4. The shear force diagram will increase or decrease suddenly i.e., by a vertical straight line at a section where there is a vertical point load.

5. The shear force between any two vertical loads will be constant and hence the shear force diagram between two vertical loads will be horizontal.

6. The bending moment at the two supports of a simply supported beam and at the free end of a cantilever will be zero.

2.8. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A CANTILEVER CANTILEVER BEAM WITH SINGLE POINT LOAD AT FREE END

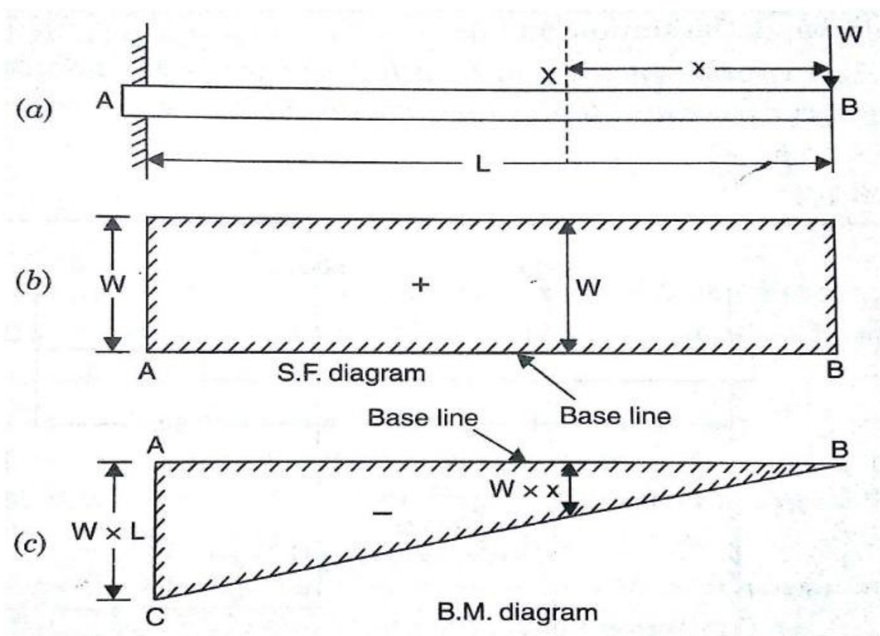
The following figure shows a cantilever AB of length L fixed at A and free at B and carrying a point load W at the free end B.

Let F_x = Shear force at X and M_x = Bending moment at X

Take a section X at a distance x from the free end. Consider the right portion of the section.

Shear Force Calculation

The shear force at the section B is equal to the resultant force acting on the right



portion at the given section B as W and it acting downward is considered as positive.

The shear force at this section X-X is equal to the resultant force acting on the right portion at the given section X-X as W and it acting downward is considered as positive.

The shear force will be constant at all section of the cantilever between A and B as there is no load between A and B. The shear force diagram is shown in fig.

SF at B = + W (+ve due to right side downward load)

SF at X-X = +W (Because no load between B and X-X)

SF at B = +W (due to same load as above)

Shear force diagram:

Bending Moment Calculation:

The Bending Moment at the section B is proportional to the distance of the section from the free end as (W x 0) and it acting clockwise about that section is considered as negative.

The Bending Moment at this section X-X is proportional to the distance of the section from the free end as *x* as (W x *x*) and it acting clockwise about that section is considered as negative.

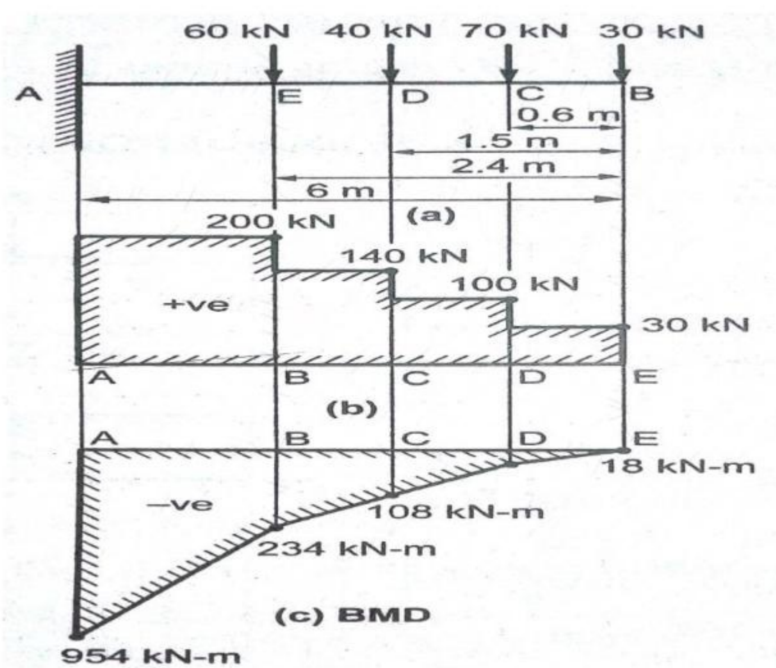
The Bending Moment at this section fixed end is proportional to the distance of the section from the free end as L as (W x L) and it acting clockwise about that section is considered as negative. The Bending Moment diagram is shown in fig.

Problem 2.1: A cantilever 6m long carries load of 30, 70, 40 and 60 kN at a distance of 0,0.6, 1.5 and 2m respectively from the free end. Draw the SF and BM diagram for the cantilever.

Given Data: shown in figure.

To find: SFD and BMD

Solution:



Shear Force Calculation: (Sum of vertical forces)

SF at B = + 30 kN

SF at C = +30 + 70 = +100 kN

SF at D = +30 + 70 + 40 = + 140 kN

SF at E = + 30 + 70 + 40 + 60 = + 200 kN

Shear Force Diagram:

Vertical downward point load are drawn as upward vertical line

No load are drawn as horizontal line.

Bending moment Calculation: [Sum of (Vertical force x Acting distance)]

BM at B = -(30 x 0) kN = 0 kNm

BM at C = -(30 x 0.6) – (70 x 0) = -18kNm

BM at D = -(30 x 1.5) – (70 x 0.9) – (40 x 0) = - 108kNm

BM at E = -(30 x 2.4) – (70 x 1.8) – (40 x 0.9) –(60 x 0) =- 234kNm

BM at A = -(30 x 6) – (70 x 5.4) – (40 x 4.5) –(60 x 3.6) = - 954 kNm

Bending moment Diagram:

Vertical downward point load are drawn as inclined line. All BM are in negative side.

Result: The SFD and BMD are drawn as shown in fig.

2.9. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A CANTILEVER BEAM WITH UNIFORMLY DISTRIBUTED LOAD

Consider a cantilever beam of length L fixed at A and carrying a uniformly distributed load of w per unit length over the entire length of the beam.

Take the section X at a distance of ' x' from the free end B. Here we have consider the right portion of the beam section.

Let F_x = Shear force at X and M_x = Bending moment at X

Shear Force Calculation:

The shear force at the section X will be equal to the resultant force acting on the right portion up to the section.

The resultant force on the right portion = load x distance of right portion = w x x

The resultant force acting on the right portion acting downward is considered positive.

∴ Shear force at X, $F_x = + w.x$

The above equation shows that the shear force follows a straight line law.

SF at B, when $x = 0$ hence = + (load x distance) = w x $0 = + 0$

SF at A, when $x = L$ hence = + (load x distance) = w x $L = + w.L$

Shear Force Diagram:

When an UDL acting on the beam is indicated in Shear force diagram as an ***inclined line***. The shear force diagram shown in fig.

Bending moment Calculation:

The UDL over a section of beam is converted into point load acting at the C.G of the section.

The bending moment will be negative as for the right portion of the section, the moment of the load at x is clockwise.

The bending moment at the section X is given by

M_x = -(total load on right portion) x (distance of C.G of right portion from X)

$= - (w.x) \times \left(\frac{x}{2}\right) = - w\frac{x^2}{2}$

From the above eqn. it is clear that B.M. at any section is proportional to the square of the distance from the free end. This follows a parabolic law.

BM at B, when $x = 0$ hence = $-\frac{w}{2} \times 0 = 0$

BM at A, when $x = L$ hence = $-\frac{w}{2} \times L^2 = -w.\frac{L^2}{2}$

Bending moment Diagram:

When an UDL acting on the beam is indicated in Bending Moment diagram as an ***parabolic curved line***. The Bending Moment diagram shown in fig.

Problem 2.2: A cantilever of length 2m carries a UDL of 3 kN/m. Draw the SF and BM diagram.

Given Data: shown in figure.

To find: SFD and BMD

Solution:

Shear Force Calculation: (Sum of vertical forces)

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2

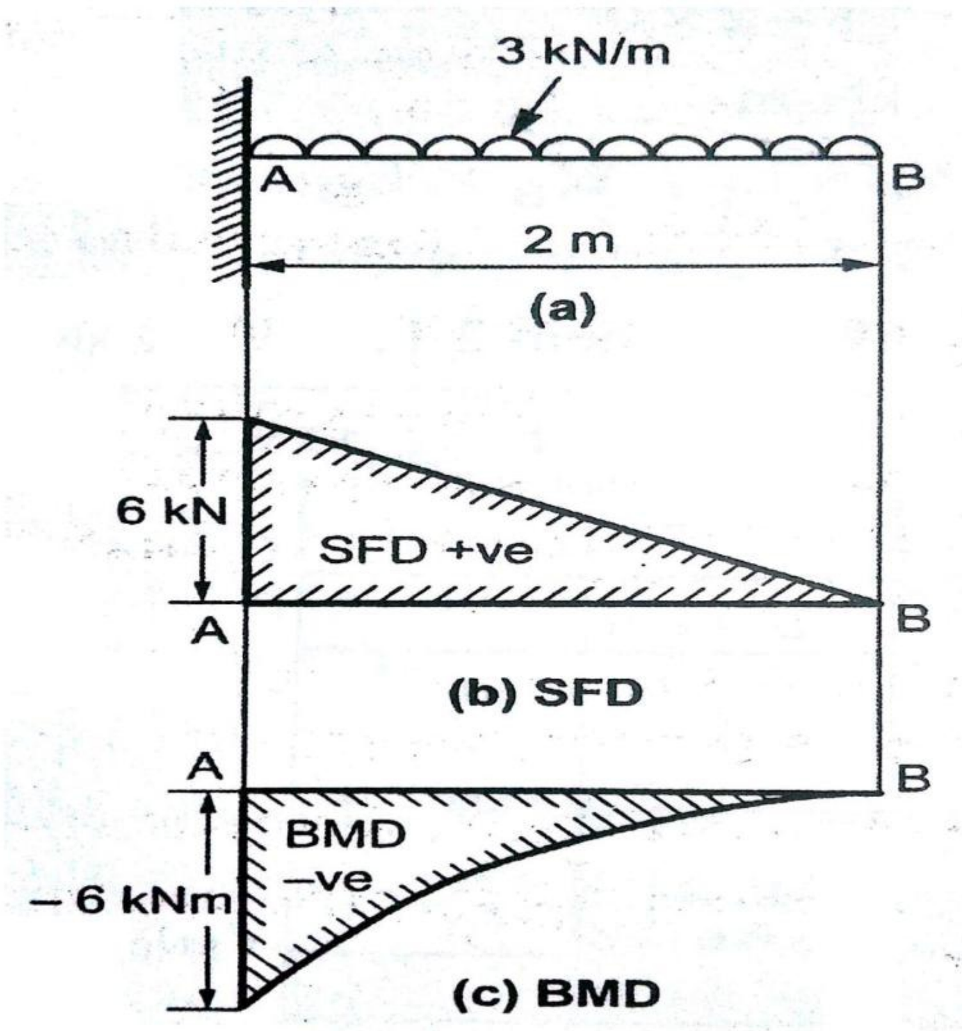
SF at B = + 0 kN

SF at A = + 0 +(3 x 2) = + 6kN

Shear Force Diagram:

Vertical downward UDL are drawn as inclined line based on sign

No load are drawn as horizontal line.



Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

For UDL, it will convert into point load and that PL act at its middle

BM at B = -(0 x 0) kN = 0 kNm

BM at C = -(0 x 2) – [(3 x 2) x $\frac{2}{2}$] = -6kNm

Bending moment Diagram:

Vertical downward UDL are drawn as parabolic curved line based on their sign.

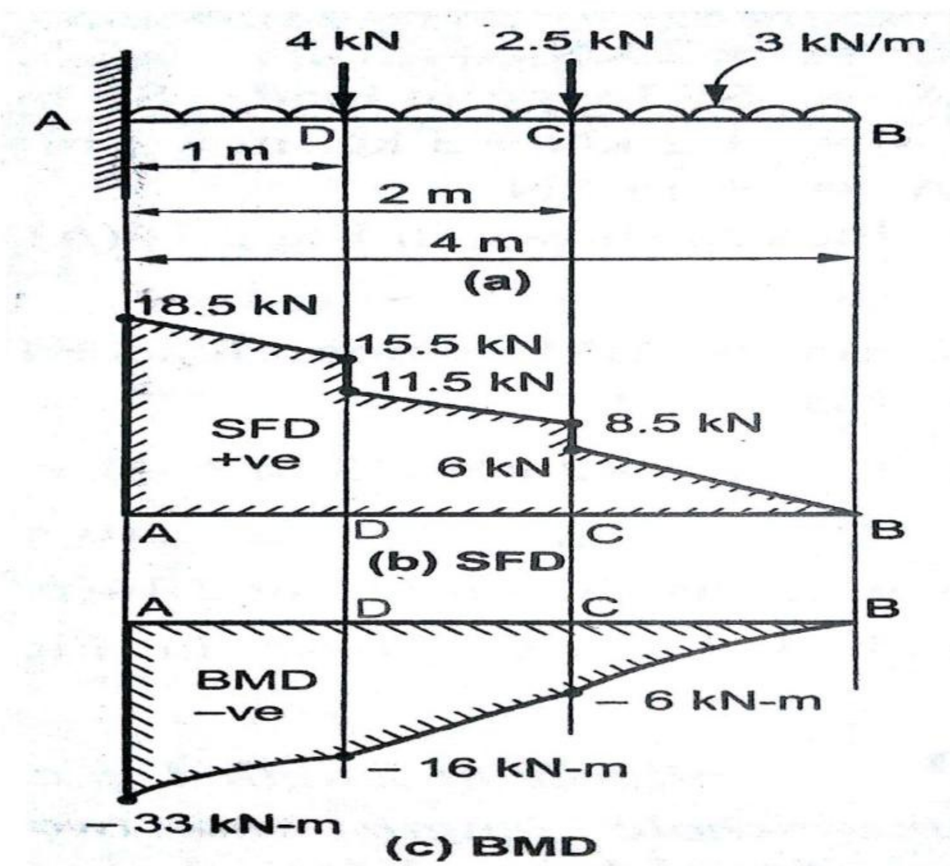
Result: The SFD and BMD are drawn as shown in fig.

Problem 2.3: A cantilever of length 4m carries a UDL of 3 kN/m run over the whole length and two point loads of 4kN and 2.5 kN are placed 1m and 2m respectively from the fixed end. Draw the SF and BM diagram.

Given Data: shown in figure.

To find: SFD and BMD

Solution:



Shear Force Calculation: (Sum of vertical forces)

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2

SF at B = + 0 kN

SF at C = +0 + 2.5 + (3 x 2) = + 8.5 kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force

SF at D = + 3 + 2 + (3 x 1.5) = + 9.5 kN (Without consider PL)

SF at D = + 3 + 2 + 2 + (3 x 1.5) = + 11.5 kN (With consider PL)

SF at A = +0 + 2.5 + 4 + (3 x 4) = + 18.5 kN

Shear Force Diagram:

Vertical downward point load are drawn as vertical line based on sign

Vertical downward UDL are drawn as inclined line based on sign

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

For UDL, it will convert into point load and that PL act at its middle

BM at B = -(0 x 0) = 0 kNm

BM at C = - (0 x 2)- (2.5 x 0)-[(3 x 2)x $\frac{2}{2}$] = -6kNm

BM at D = - (0 x 3)-(2.5 x 1)-(4 x 0)-[(3 x 3)x $\frac{3}{2}$] = -16kNm

BM at A = - (0 x 4)–(2.5 x 2)– (4 x 1)–[(3 x 4)x $\frac{4}{2}$] = -33kNm

Bending moment Diagram:

Vertical downward UDL are drawn as parabolic curved line based on their sign.

Result:

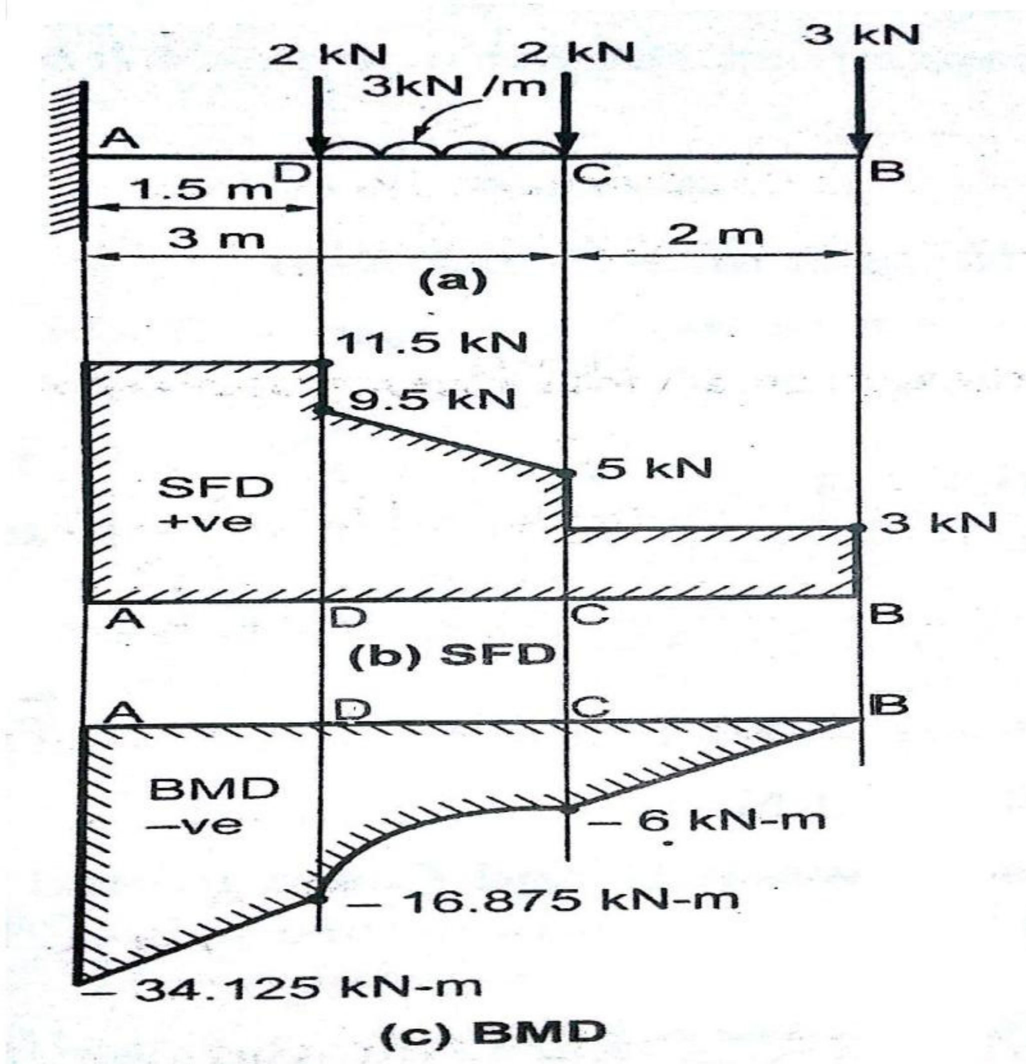
The SFD and BMD are drawn as shown in fig.

Problem 2.4: A cantilever of length 5m carries a UDL of 3 kN/m run over the length of 1.5m start from 1.5m from fixed end and two point load 2kN acting at 1.5m and 3m respectively from fixed end and another point load 3kN acting at the free end. Draw the SF and BM diagram.

Given Data: shown in figure.

To find: SFD and BMD

Solution:



Shear Force Calculation: (Sum of vertical forces)

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2

SF at B = + 3kN

SF at C = + 3 + 2 = + 5 kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force

SF at D = + 3 + 2 + (3 x 1.5) = + 9.5 kN (Without consider PL)

SF at D = + 3 + 2 + 2 + (3 x 1.5) = + 11.5 kN (With consider PL)

SF at A = + 3 + 2 + 2 + (3 x1.5) = + 11.5 kN

Shear Force Diagram:

Vertical downward point load are drawn as vertical line based on sign.

Vertical downward UDL are drawn as inclined line based on sign.

No load are drawn as horizontal line.

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

For UDL, it will convert into point load and that PL act at its middle

BM at B = - (3 x 0) = 0 kNm

BM at C = - (3 x 2) - (2. x 0) = - 6 kNm

BM at D = - (3 x 3.5) - (2 x 1.5) - (2 x 0) - [(3 x 1.5) x $\frac{1.5}{2}$] = - 16.875kNm

BM at A = - (3 x 5) – (2x 3) – (2 x 1.5) – [(3 x 1.5) x ($\frac{1.5}{2}$+1.5)] = - 34.125kNm

Bending moment Diagram:

Vertical downward PL are drawn as inclined line based on their sign. Vertical downward UDL are drawn as parabolic curved line based on their sign.

Result:

The SFD and BMD are drawn as shown in fig.

2.10. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A CANTILEVER BEAM WITH UNIFORMLY VARYING LOAD

Consider a cantilever beam of length L fixed at A and carrying a uniformly varying load from zero at the free and to *w* per unit length at the fixed end of beam.

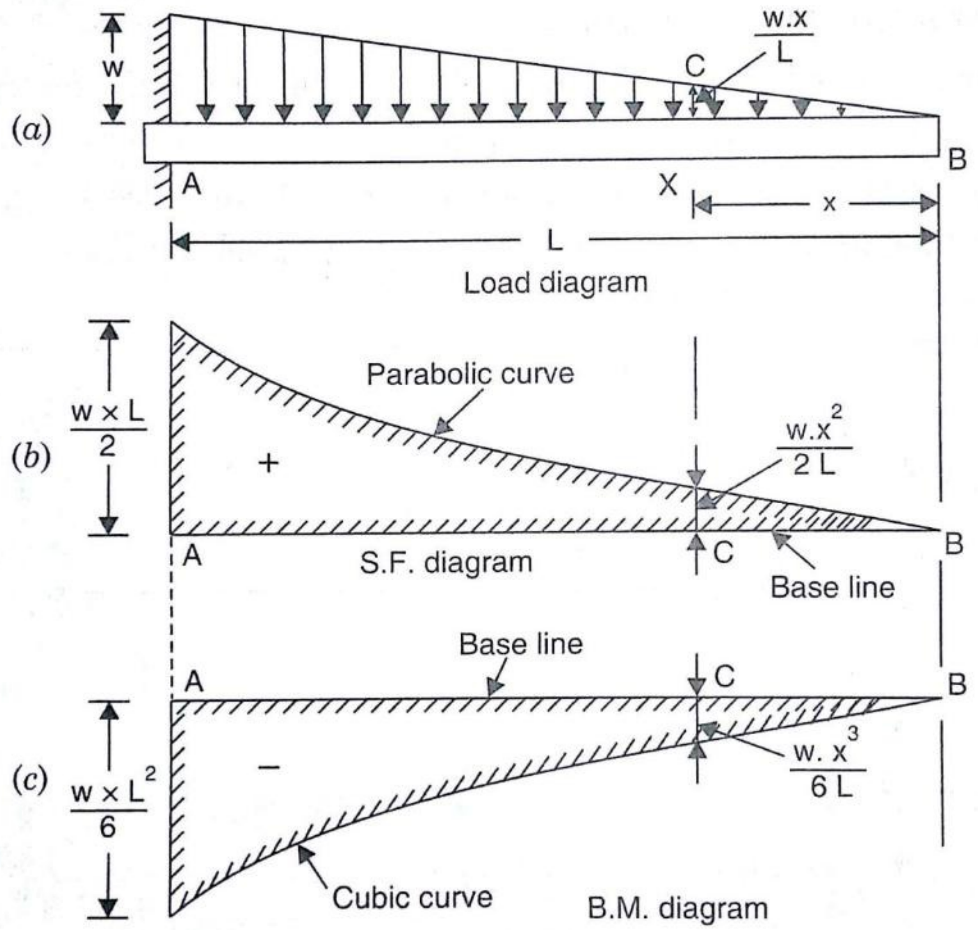
Take the section X at a distance of ' *x* ' from the free end B. Here we have consider the right portion of the beam section.

Let F_x = Shear force at X and M_x = Bending moment at X

Shear Force Calculation:

Let us first find the rate of loading at the section X. The rate of loading is zero at B and is 'w' per meter run at A. This means that rate of loading for a length L is w per unit length.

Hence rate of loading for a length of $x = \frac{w}{L}$ x x per unit length. Which is equal to the load acting at X = CX = $\frac{w}{L}$ x x



The resultant force acting on the right portion acting downward is considered positive.

The shear force at the section X at a distance x from free end is given by

F_x = +(Total load on the cantilever for a length x from the free end B)

= +(Area of triangle BCX)

= +($\frac{1}{2}$ x XB x XC)

$$=+\frac{1}{2}\text{ x } x \text{ x }\left(\frac{w}{L}\text{ x } x\right) = +\, w.\frac{x^2}{2L} \qquad \left(\therefore XB = x, XC = \frac{w}{L}\text{ x } x\right)$$

From the above eqn. shows that the SF varies according to the parabolic law.

SF at B, when $x = 0$ hence $= +\, w.\frac{0^2}{2L} = 0$

SF at A, when $x = L$ hence $= +\, w.\frac{L^2}{2L} \quad = +\frac{w.L}{2}$

Shear Force Diagram:

When an UVL acting on the beam is indicated in Shear force diagram as an ***Parabolic curved line***. The shear force diagram shown in fig.

Bending moment Calculation:

The UVL over a section of beam is converted into point load acting at the C.G of the section.

The bending moment will be negative as for the right portion of the section, the moment of the load at x is clockwise about the section.

The bending moment at the section X is given by

M_x = -(total load on right portion) x (Distance of the load from X)

= - (Area of triangle BCX) x (Distance of C.G of triangle from X)

$= -\,(w.\frac{x^2}{2L})\text{ x }\left(\frac{x}{3}\right) = -\,\frac{wx^3}{6L}$

From the above eqn. it is clear that B.M. at any section is proportional to the cube of the distance from the free end. This follows a cubic law.

BM at B, when $x = 0$ hence $= -\,\frac{w\text{ x }0}{6L} = 0$

BM at A, when $x = L$ hence $= -\,\frac{w\text{ x }L^3}{6L} = -\,w.\frac{L^2}{6}$

Bending moment Diagram:

When an UDL acting on the beam is indicated in Bending Moment diagram as an ***cubic curved line***. The Bending Moment diagram shown in fig.

Problem 2.5: A cantilever of length 4m carries a gradually increasing load, zero at the free end to 2kN/m at the fixed end. Draw the SF and BM diagrams for the cantilever.

Given Data: shown in figure.

To find: SFD and BMD

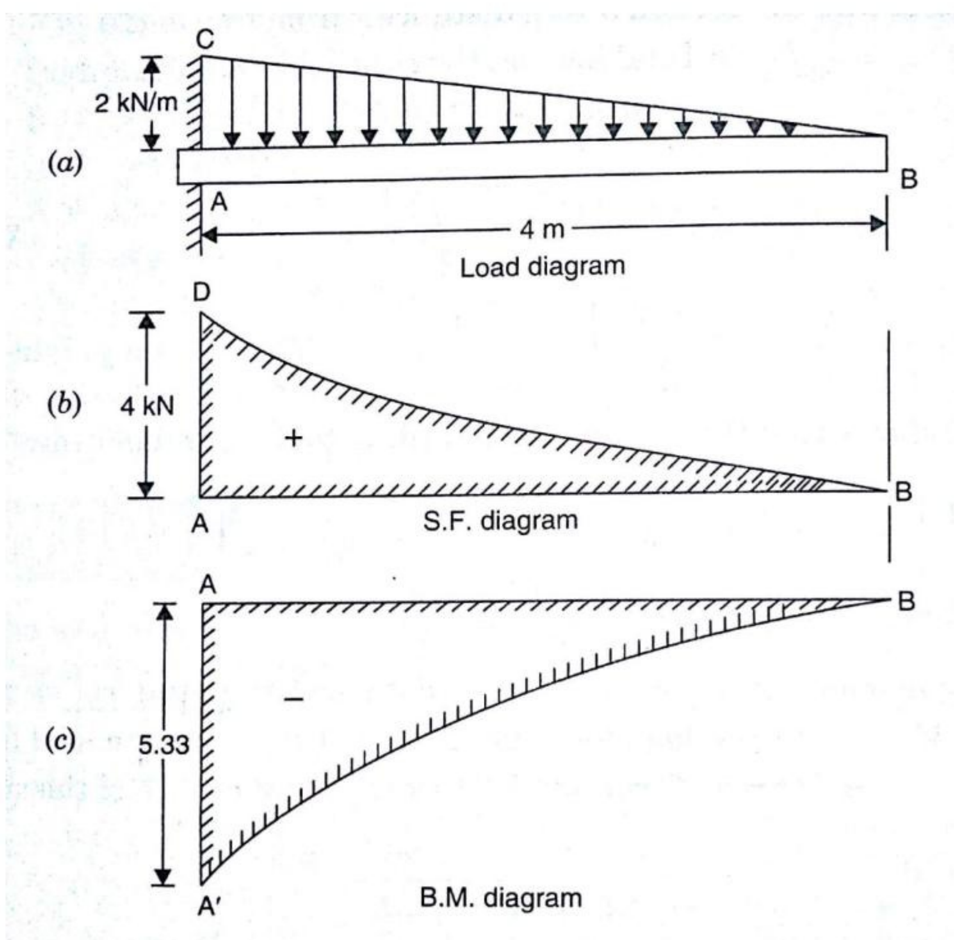
Solution:

Shear Force Calculation: (Sum of vertical forces)

For UVL, it will be converted into point load as (Point load = Area of triangle = $\frac{1}{2}$ x UVL x load acting distance) and the converted point load acting distance at its $\frac{l}{3}$ from the higher load end.

SF at B = + 0 kN

SF at A = + 0 + ($\frac{1}{2}$ x 2 x 4) = + 4kN



Shear Force Diagram:

Downward UVL are drawn as parabolic curved line based on sign.

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

For UVL, it will convert into point load and that PL act at its $\frac{l}{3}$ from the higher load end.

BM at B = - (0 x 0) = 0 kNm

BM at A = - (0 x 4) –[($\frac{1}{2}$ x 2 x 4) x $\frac{4}{3}$] = - 5.33kNm

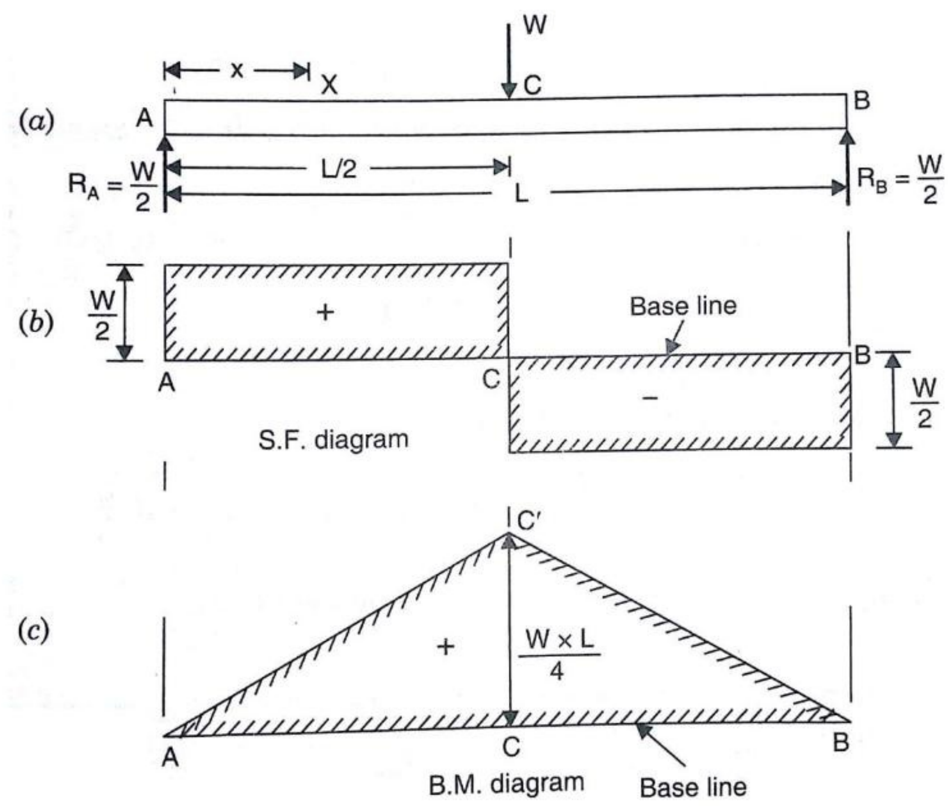
Bending moment Diagram:

Vertical downward UVL are drawn as cubic curved line based on their sign.

Result: The SFD and BMD are drawn as shown in fig.

2.11. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A SIMPLY SUPPORTED BEAM WITH CENTRAL POINT LOAD

Consider a beam AB of length L simply supported at the ends A and B and carrying a point load W at its middle point C.



First to find the reaction force at A and B as R_A and R_B by the two steps are followed.

Step1: Take moment about A = 0 ≫ R_B x L – W x $\frac{L}{2}$ = 0 ≫ $R_B = \frac{W}{2}$

Step2: Sum of upward force = Sum of downward force ≫ $R_A + R_B$ = W

then R_A = W - R_B ≫ R_A = W - $\frac{W}{2} = \frac{W}{2}$

Hence $R_A = R_B = \frac{W}{2}$

Take a section X at a distance x from the end B between B and C. Here we have consider the right portion of the beam section.

Let F_x = Shear force at X and M_x = Bending moment at X

Shear Force Calculation:

The shear force at X will be equal to the resultant force acting on the right of the portion of the section. But the resultant force on the right portion is $R_B = \frac{W}{2}$ acting upward. Hence the shear force at X is positive and its magnitude is $\frac{W}{2}$.

∴ Shear force at X, $F_x = +\frac{W}{2}$

Hence the shear force between B and C is constant and equal to $+\frac{W}{2}$

Now consider any section X between C and A at a distance x from end B. The resultant force on the right portion will be,

∴ Shear force at X between B and C, $F_x = +\frac{W}{2}$ - W = - $\frac{W}{2}$

At the section C the shear force is same as - $\frac{W}{2}$

Shear Force Diagram:

When the point load acting on the beam is indicated in Shear force diagram as an ***vertical line***. The shear force diagram shown in fig.

Bending moment Calculation:

The bending moment at any section between B and C at a distance x from the end B, is given by

The bending moment will be positive as for the right portion of the section, the moment of the load at x is anti-clockwise about the section.

The bending moment at the section X is given by

M_x = (total load on right portion) x (Distance of the load from X) = R_B x x

$= +\frac{W}{2} . x$

BM at B, when $x = 0$ hence $= +\frac{W}{2} . 0 = 0$

BM at C, when $x = \frac{L}{2}$ hence $= +\frac{W}{2} . \frac{L}{2} = +\frac{W.L}{4}$

From the above eqn. it is clear that B.M. at any section is proportional to the distance between B and C. This follows a ***straight line law***.

The bending moment at any section between C an A at a distance x from the end B, is given by

BM at X, at a distance $x = M_x = R_B$ x x - $W\left(x - \frac{L}{2}\right) = +\frac{W}{2} . x$ - $W\left(x - \frac{L}{2}\right)$

$$M_x = \frac{WL}{2} - \frac{Wx}{2}$$

BM at C, when $x = \frac{L}{2}$ hence $= +\frac{WL}{2} - \frac{W}{2} . \frac{L}{2} = +\frac{W.L}{4}$

BM at A, when $x = L$ hence $= +\frac{WL}{2} - \frac{WL}{2} = 0$

Hence bending moment at C is $+\frac{W.L}{4}$ and it decreases to zero at A.

Bending moment Diagram:

When point load acting on the beam is indicated in Bending Moment diagram as an ***inclined line***. The Bending Moment diagram shown in fig.

Problem 2.6: A simply supported beam of length 6m, carries point load of 3kN and 6kN of 2m and 4m from the left end. Draw the SF and BM diagrams for the beam.

Given Data: shown in figure.

To find: SFD and BMD

Solution:

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

R_B x 6 – (6 x 4) – (3 x 2) = 0 ≫ R_B x 6 = (6 x 4) + (3 x 2)/6 ≫ R_B = 5kN

Step2: Sum of upward force = Sum of downward force

$R_A + R_B = 6 + 3$ ≫ $R_A = 9 - R_B$ ≫ $R_A = 9 - 5 = 4$kN

Shear Force Calculation: (Sum of vertical forces)

SF at B = - R_B = - 5 kN

SF at C = -5 +6 = + 1kN

SF at D = - 5 +6 + 3 = + 4 kN

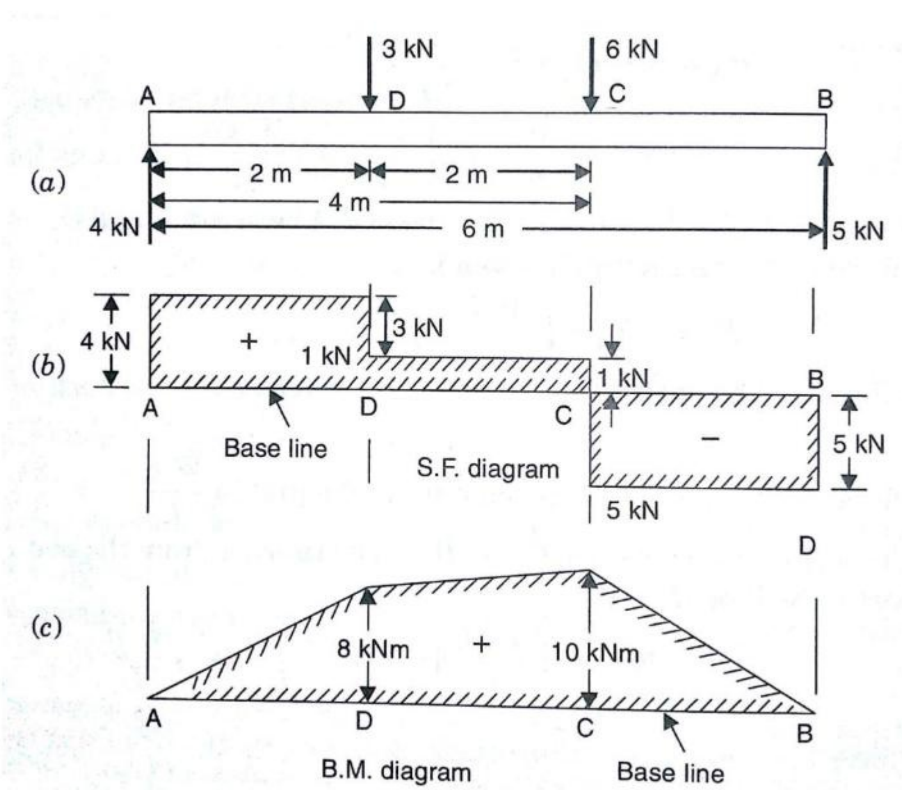
SF at A = - 5 +6 + 3 = + 4 kN

Shear Force Diagram:

Vertical downward point load are drawn as upward vertical line

Vertical upward reaction force are drawn as downward vertical line.

No load are drawn as horizontal line.



Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = + (R_B x 0) = 0 kNm

BM at C = +(5 x 2) – (6 x 0) = + 10 kN

BM at D = +(5 x 4) –(6 x 2) –(3 x 0) = + 8 kN

BM at A = +(5 x 6) –(6 x 4) –(3 x 2) = 0 kN

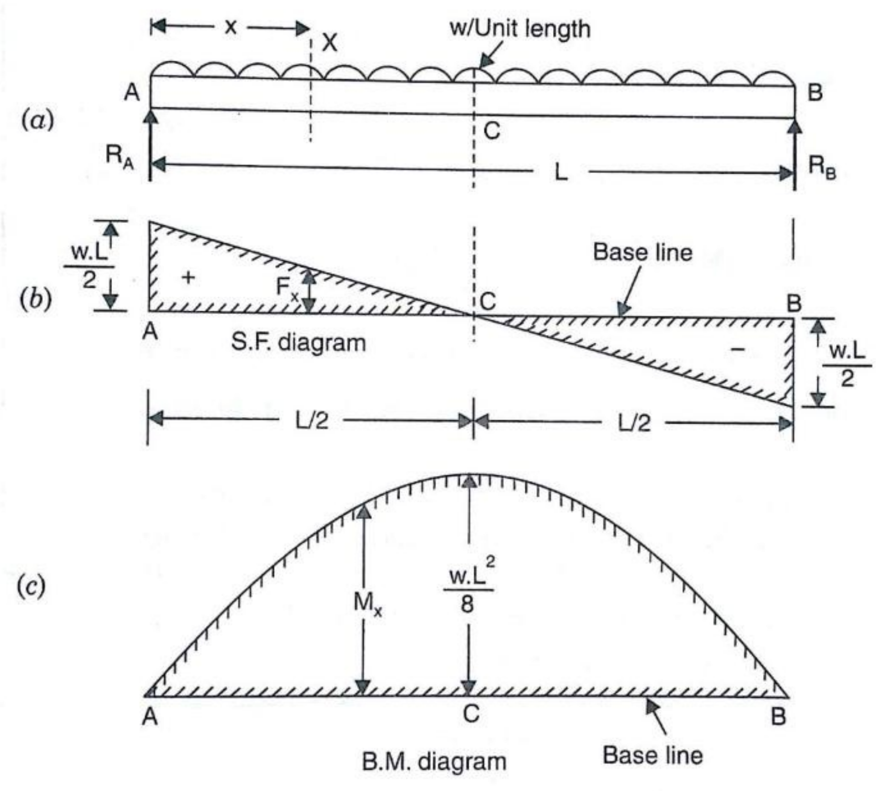
Bending moment Diagram:

Vertical downward point load are drawn as inclined line based on their sign.

Result: The SFD and BMD are drawn as shown in fig.

2.12. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A SIMPLY SUPPORTED BEAM WITH UNIFORMLY DISTRIBUTED LOAD

Consider a beam AB of length *L* Simply Supported at the end A and carrying a uniformly distributed load of *w* per unit length over the entire length of the beam.



First to find the reaction force at A and B as R_A and R_B by the two steps are followed.

Step1: Take moment about A = 0 ≫ R_B x L – $w.L$ x $\frac{L}{2}$ = 0 ≫ $R_B = \frac{WL}{2}$

Step2: Sum of upward force = Sum of downward force ≫ $R_A + R_B$ = W.L

then R_A = W.L - R_B ≫ R_A = W.L - $\frac{WL}{2} = \frac{WL}{2}$

Hence $R_A = R_B = \frac{WL}{2}$

Take a section X at a distance x from the end B, between B and C. Here we have consider the right portion of the beam section.

Let F_x = Shear force at X and M_x = Bending moment at X

Shear Force Calculation:

The shear force at X will be equal to the resultant force acting on the right of the portion of the section. But the resultant force on the right portion is $R_B = \frac{WL}{2}$ acting upward and downward UDL as W.x.

∴ Shear force at X, F_x = -R_B + $w.x$

$$F_x = -\frac{WL}{2} + w.x$$

The above equation shows that the shear force follows a straight line law.

SF at B, when $x = 0$ hence = $-\frac{WL}{2} + w.0 = -\frac{WL}{2}$

SF at C, when $x = \frac{L}{2}$ hence = $-\frac{WL}{2} + w.\frac{L}{2} = 0$

SF at A, when $x = L$ hence = $-\frac{WL}{2} + w.L = +\frac{WL}{2}$

Shear Force Diagram:

When an UDL acting on the beam is indicated in Shear force diagram as an ***inclined line***. The shear force diagram shown in fig.

Bending moment Calculation:

The bending moment at any section between B and C at a distance x from the end B, is given by

The bending moment will be positive as for the right portion of the section, the moment of the load at x is anti-clockwise about the section.

The bending moment at the section X is given by

M_x = (total load on right portion) x (Distance of the load from X)

$= + R_B \text{ x } x - w.x.\frac{x}{2} = \frac{WL}{2}.x - \frac{w.x^2}{2}$

From the above eqn. it is clear that B.M. at any section is proportional to the square of the distance from the free end. This follows a parabolic law.

BM at B, when $x = 0$ hence $= +\frac{WL}{2}.0 - \frac{w.0^2}{2} = 0$

BM at C, when $x = \frac{L}{2}$ hence $= +\frac{WL}{2}.\frac{L}{2} - \frac{w.\left(\frac{L}{2}\right)^2}{2} = +\frac{WL^2}{8}$

BM at A, when $x = L$ hence $= +\frac{WL}{2}.L - \frac{w.L^2}{2} = 0$

The maximum B.M. occurs at the centre of the beam, where S.F. becomes zero after changing its sign.

Bending moment Diagram:

When an UDL acting on the beam is indicated in Bending Moment diagram as an ***parabolic curved line***. The Bending Moment diagram shown in fig.

Problem 2.7: A beam of 8m span simply supported at its end carries loads of 2kN and 5kN at a distance of 3m and 6m from right support respectively. In addition the beam carries a UDL of 4kN/m for its entire length. Draw the shear force and bending moment diagram. Also find the maximum bending moment.

Given Data: shown in figure.

To find: SFD and BMD, Max. BM

Solution:

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2

$$R_B \text{ x8} - (2 \text{ x } 5) - (5 \text{ x } 2) - [(4 \text{ x } 8) \text{ x } \tfrac{8}{2}] = 0$$

$$R_B \text{ x8} = (2 \text{ x } 5) + (5 \text{ x } 2) + [(4 \text{ x } 8) \text{ x } \tfrac{8}{2}] \quad \gg R_B = 18.5\text{kN}$$

Step2: Sum of upward force = Sum of downward force

$$R_A + R_B = 2 + 5 + (4 \text{ x } 8) \quad \gg R_A = 39 - R_B \gg R_A = 39 - 18.5 = 20.5\text{kN}$$

Shear Force Calculation: (Sum of vertical forces)

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2

SF at B = - R_B = - 18.5 kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force

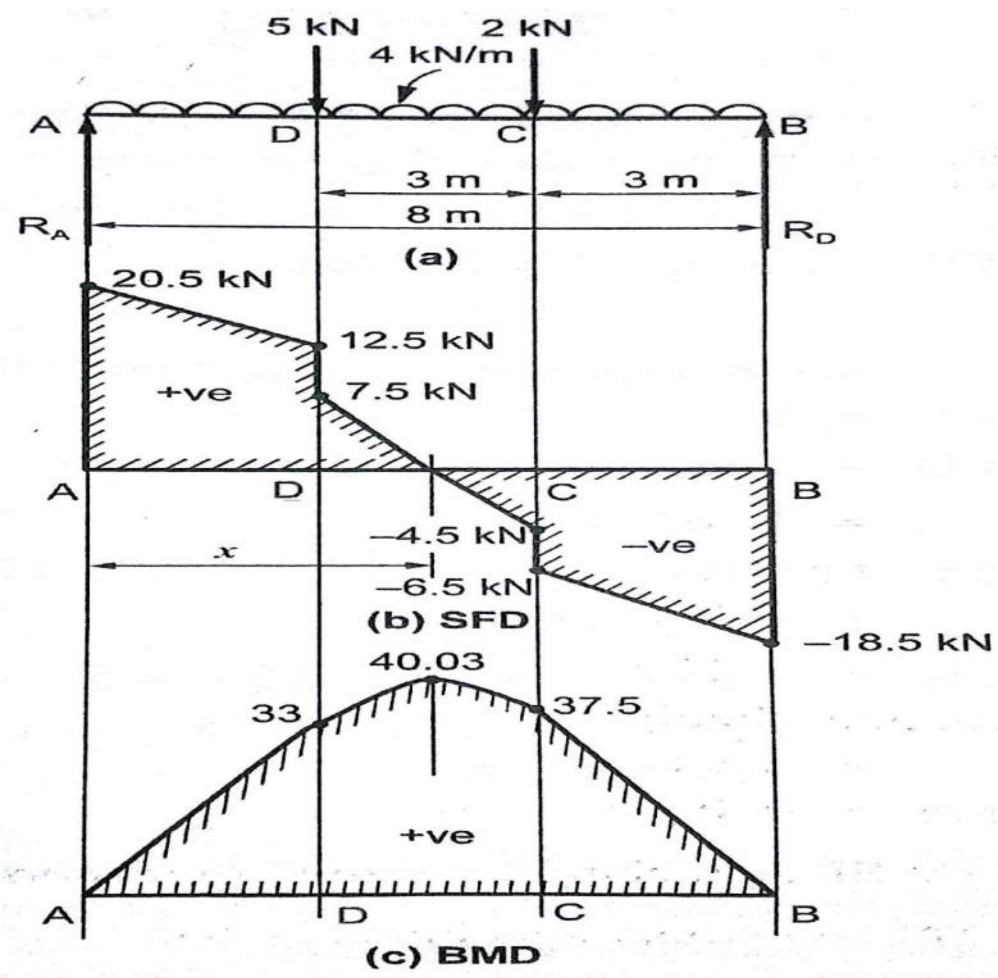
SF at C = - 18.5 +(4 x 3) = -6.5kN (Without consider PL)

SF at C = - 18.5 + 2 + (4 x 3) = - 4.5 kN (With consider PL)

SF at D = - 18.5 + 2 + (4 x 6) = + 4.5 kN (Without consider PL)

SF at D = - 18.5 + 2 + 5 + (4 x 6) = + 12.5 kN (With consider PL)

SF at A = - 18.5 + 2 + 5 + (4 x 8) = + 20.5 kN



Shear Force Diagram:

Vertical downward point load are drawn as vertical line based on sign

Vertical downward UDL are drawn as inclined line based on sign

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

For UDL, it will convert into point load and that PL act at its middle.

BM at B = +(18.5 x 0) = 0 kNm

BM at C = +(18.5 x 3) - (2 x 0) - [(4 x 3) x $\frac{3}{2}$] = + 37.5 kNm

BM at D = + (18.5 x 6) – (2 x 3) – (5 x 0) – [(4 x 6) x $\frac{6}{2}$] = + 33 kNm

BM at A = + (18.5 x 8) – (2 x 5) – (5 x 2) – [(4 x 8) x $\frac{8}{2}$] = 0 kNm

Bending moment Diagram:

Vertical downward PL are drawn as inclined line and UDL are drawn as parabolic curved line based on their sign.

Calculate Maximum Bending Moment:

Method 1

The maximum BM will occur where the shear force becomes zero. Let us take a point X at a distance of x from B where SF become zero is shown in fig.

SF at X = - R_B + 2 +(4 x x) = 0 ≫-18.5 + 2 + 4.x = 0 ≫x = 4.125m

Method 2

Distance x = distance BC + $\left(\frac{\text{Vertical Height of particular Triangle}}{\text{UDL}}\right)$

= 3 + $\left(\frac{4.5}{4}\right)$

= 3 + 1.125 = 4.125m

BM at X = +(18.5 x 4.125) – [2 x (4.125 - 3)] - [(4 x 4.125) x $\frac{4.125}{2}$] = 40.03 kNm

Result:

The SFD and BMD are drawn as shown in fig.

Maximum BM = **40.03 kNm** acting at a distance of **4.125m** from right support.

2.13. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A SIMPLY SUPPORTED BEAM WITH UNIFORMLY VARYING LOAD

Case1:UVL from zero at each end to w per unit length at the centre

A beam beam of length L simply supported at the ends A and B and carrying a uniformly varying load from zero at each end to w per unit length at the centre. The reactions at the supports will be equal and their magnitude will be half the total load on the entire length as the load is symmetrical on the beam.

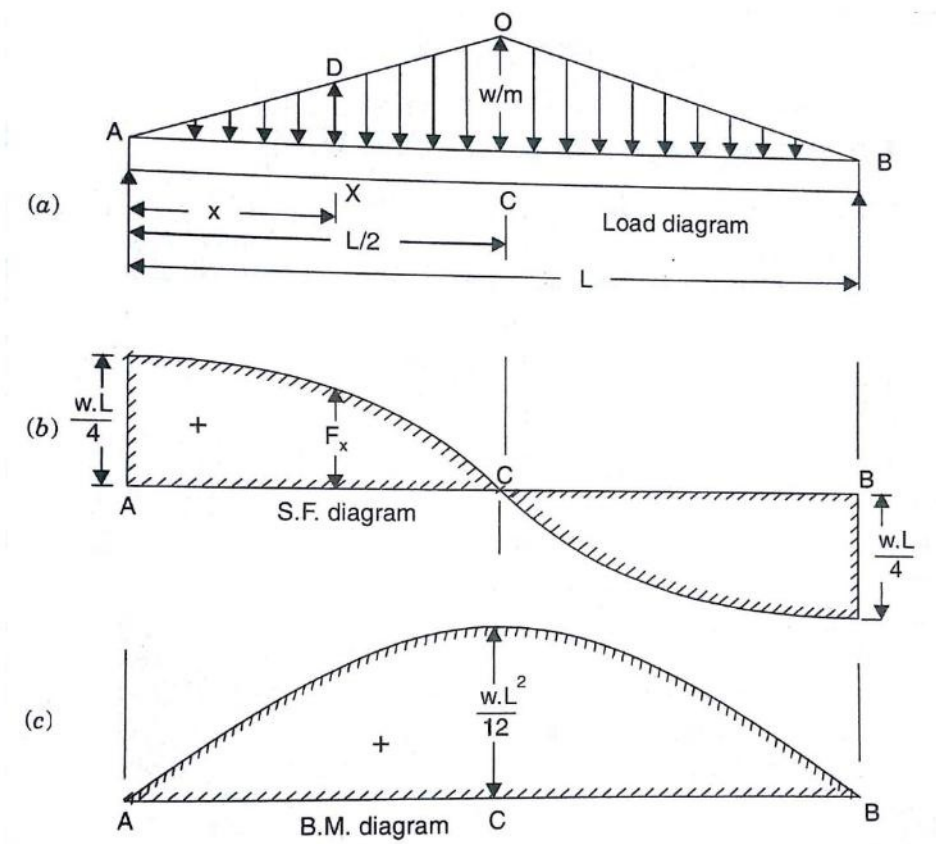
Total load on the beam = Area of triangle ABC

$$= \frac{1}{2} \text{ x AB x CO} = \frac{1}{2} \text{ x L x w} = \frac{w.L}{2}$$

$\therefore$ Reaction forces $R_A = R_B = \frac{1}{2} \text{ x } \frac{w.L}{2} = \frac{w.L}{4}$

Take a section X at a distance x from the end B, between B and C. Here we have considered the right portion of the beam section.

Let F_x = Shear force at X and M_x = Bending moment at X



Shear Force Calculation:

Let us first find the rate of loading at the section X. The rate of loading is zero at B and is '*w*' per meter run at C. This means that rate of loading for a length $\frac{L}{2}$ is *w* per unit length.

Hence rate of loading for a length of $x = \frac{w}{\frac{L}{2}}$ x x per unit length. ≫ DX = $\frac{2w}{L} . x$

$$\left[\frac{BX}{\frac{L}{2}} = \frac{DX}{w} \gg DX = \frac{x \,.\, w}{\frac{L}{2}} = \frac{2.w.x}{L}\right]$$

Now load on the length AX of the beam = Area of load diagram BXD

$$= \frac{1}{2} \text{ x } SX \text{ x } DX = \frac{1}{2} \text{ x } x \text{ x } \frac{2w}{L} . x = \frac{w}{L} . x^2$$

This load is acting at a distance of $\frac{x}{3}$ from X or $\frac{2x}{3}$from B.

The shear force at the section X at a distance *x* from free end is given by

F_x $= -R_B$ +load on the length BX

$= - \frac{w.L}{4} + \frac{w}{L} . x^2$

The above eqn. shows that the SF varies according to the parabolic law between B and C.

SF at B, when $x = 0$ hence = $= - \frac{w.L}{4} + \frac{w}{L} . 0^2 = - \frac{w.L}{4}$

SF at C, when $x = \frac{L}{2}$ hence = $= - \frac{w.L}{4} + \frac{w}{L} . \left(\frac{L}{2}\right)^2 = - \frac{w.L}{4} + \frac{w.L}{4} = 0$

SF at A, when $x = L$ hence = $+ R_A = + \frac{w.L}{4}$

Shear Force Diagram:

When an UVL acting on the beam is indicated in Shear force diagram as an ***parabolic curve line***. The shear force diagram shown in fig.

Bending moment Calculation:

The bending moment at both end become zero.

The bending moment will be positive as for the right portion of the section, the moment of the load at *x* is anti-clockwise about the section.

The bending moment at X at a distance of x from B is given by,

$M_x = + R_B \text{ x } x$ –load of length AX . $\frac{x}{3}$

$= \frac{w.L}{4} .x - \frac{w}{L} . x^2 . \frac{x}{3}$ $\qquad = \frac{w.L}{4} .x - \frac{w}{3L} . x^3$

From the above eqn. it is clear that B.M. at any section is proportional to the cube of the distance from the free end. This follows a cubic law.

BM at B, when $x = 0$ hence $= + \frac{w.L}{4} .0 - \frac{w}{3L} . 0^3 = 0$

BM at C, when $x = \frac{L}{2}$ hence $= + \frac{w.L}{4} .\frac{L}{2} - \frac{w}{3L} . \left(\frac{L}{2}\right)^3 = + \frac{WL^2}{8} - \frac{WL^2}{24} = + \frac{WL^2}{12}$

The maximum B.M. occurs at the centre of the beam, where S.F. becomes zero after changing its sign.

∴ Maximum B.M. is at C, $M_C = \frac{wL^2}{12}$

Bending moment Diagram:

When an UVL acting on the beam is indicated in Bending Moment diagram as a ***cubic curved line***. The Bending Moment diagram shown in fig.

Problem 2.8: The intensity of loading on a simply supported beam of 4m span increases gradually from 30 kN/m run at one end to 130 kN/m run at the other end. Draw the shear force and Bending Moment diagram. Also find the maximum BM.

Given Data:

shown in figure.

To find:

SFD and BMD, Max. BM

Solution:

The load will be consider as (i) UDL of 30kN/m run throughout the span. (ii) UVL which is zero at the right end B and increase to 100kN/m run at left end A.

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2.

For UVL, it will be converted into point load as (Point load = Area of triangle = $\frac{1}{2}$ x UVL x load acting distance) and the converted point load acting distance at its $\frac{l}{3}$ from the higher load end.

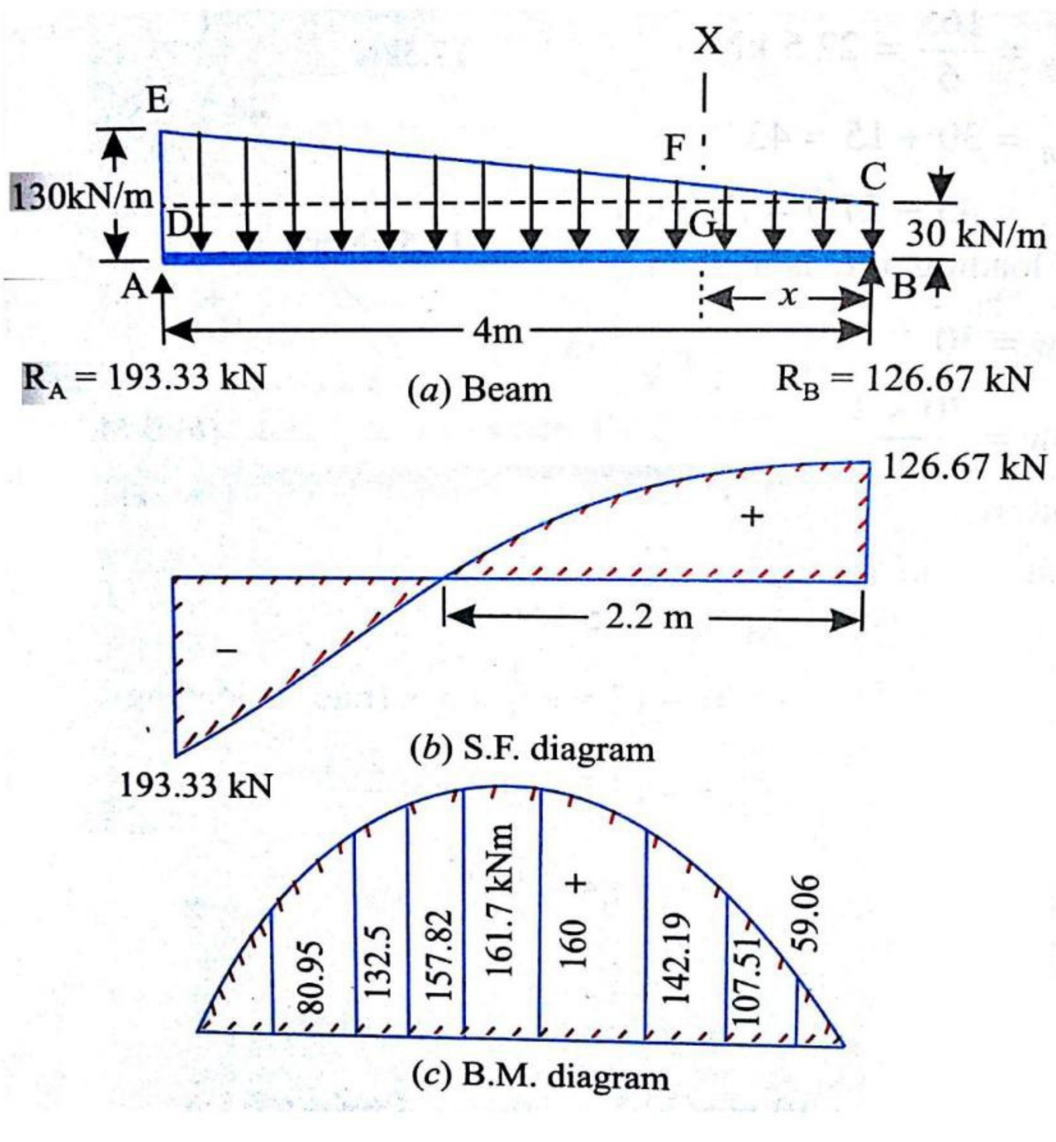
R_B x4 – [(30 x 4) x$\frac{4}{2}$]– [($\frac{1}{2}$ x 4 x 100) x $\frac{4}{3}$]= 0

R_B x4 = [(30 x 4) x$\frac{4}{2}$] +[($\frac{1}{2}$ x 4 x 100) x $\frac{4}{3}$] ≫ R_B = 126.67 kN

Step2: Sum of upward force = Sum of downward force

$R_A + R_B$ = (30 x 4) + ($\frac{1}{2}$ x 4 x 100) ≫ R_A = 320 - R_B

≫ R_A = 320 – 126.67 = 193.33 kN



(a) Beam

(b) S.F. diagram

(c) B.M. diagram

Shear Force Calculation: (Sum of vertical forces)

SF at B = - R_B = - 126.67kN

SF at A = - 126.67 +(30 x 4) + ($\frac{1}{2}$ x 4 x 100) = +193.33 kN

Shear Force Diagram:

Vertical upward reaction load are drawn as vertical line based on sign

Downward UVL are drawn as parabolic curved line based on sign

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at Both end B and Abecome Zero.

Bending moment Diagram:

Vertical downward PL are drawn as inclined line and UDL are drawn as parabolic curved line based on their sign.

Calculate Maximum Bending Moment:

The maximum BM will occur where the shear force becomes zero. Let us take a point X at a distance of x from B where SF become zero is shown in fig.

SF at X = - R_B + (30 x x) + ($\frac{1}{2}$ x xx FG) = 0 ≫

The rate of loading FG is calculate by the similar Δ^{le} CDE and CFG

$\frac{FG}{DE} = \frac{CG}{CD}$ ≫ FG = $\frac{CG}{CD}$ x DE ≫ FG = $\frac{x}{4}$ x 100 ≫ FG = 25 x

Substitute the FG value in shear force at X which gives

= - 126.67 + 30.x + ($\frac{1}{2}$ x xx 25 x) = 0

12.5 x^2 + 30 x – 126.67 = 0

Then $x = \frac{-30 \pm \sqrt{30^2 - 4 \text{ x } 12.5 \text{ x} -126.67}}{2\text{x } 12.5}$ = 2.2 m (neglect the –ve value)

Then BM at X = +(126.67 x 2.2) – [(30 x2.2) x $\frac{2.2}{2}$] - [($\frac{1}{2}$ x 2.2 x 100) x $\frac{2.2}{3}$]

= 161.7kNm

Result: The SFD and BMD are drawn as shown in fig.

Maximum BM = **161.7kNm** acting at a distance of **2.2m** from right support.

2.14. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR OVER-HANGING BEAM:

The beam having its portion is extended beyond the support, such beam is known as overhanging beam. In case of overhanging beam, the BM is positive between the two supports, whereas the BM is negative for the overhanging portion. Hence at some point, the BM is zero after changing its sign from positive to negative or vice versa. That point is known as **point of contraflexure or point of inflexion.**

Problem 2.9: Draw the S.F. and B.M. diagram for the loaded beam shown in fig

Given Data: shown in figure.

To find: SFD and BMD, Max. BM

Solution:

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2.

$$-(2 \times 7) + R_B \times 5 - [(2 \times 5) \times \frac{5}{2}] - (5.5 \times 2) = 0$$

$$R_B \times 5 = (2 \times 7) + [(2 \times 5) \times \frac{5}{2}] + (5.5 \times 2) \gg R_B = 10\text{kN}$$

Step2: Sum of upward force = Sum of downward force

$$R_A + R_B = 2 + (2 \times 5) + 5.5 \gg R_A = 17.5 - R_B$$

$$\gg R_A = 17.5 - 10 = 7.5 \text{ kN}$$

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 2 = +2kN

SF at C = +2 - 10 = - 8 kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force.

SF at D = +2 - 10 + (2 x 3) = - 2 kN (Without consider PL)

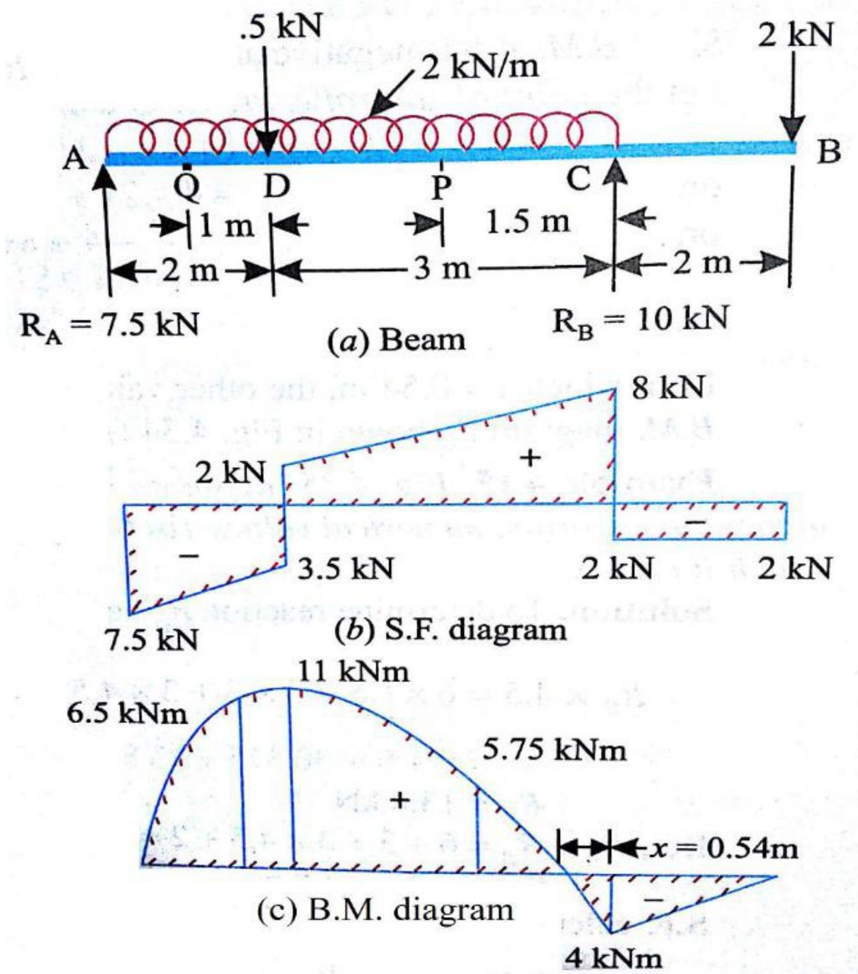
SF at D = +2 - 10 + 5.5 + (2 x 3) = +3.5 kN (With consider PL)

SF at A = +2 - 10 + 5.5 + (2 x 5) = + 7.5 kN

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign.

Vertical downward UDL are drawn as inclined line based on sign



Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = -(2 x 0) = - 0 kN

BM at C = -(2 x 2) + (10 x 0) = - 4kN

BM at D = -(2 x 5) + (10 x 3) - (5.5 x 0) – [(2 x 3) x $\frac{3}{2}$]= +11kN

BM at A = -(2 x 7) + (10 x 5) - (5.5 x 2) – [(2 x 5) x $\frac{5}{2}$]= 0kN

Bending moment Diagram:

Vertical downward and upward PL are drawn as inclined line and UDL are drawn as parabolic curved line based on their sign.

Calculate the position of point of contraflexure or point of inflexion:

The point of contraflexure or point of inflexion lie at a point where BM become zero. In the fig. the BM act at X at a distance x m from support C.

BM at X = - [2 x (x+ 2)] + (10 xx) -[(2 x x) x $\frac{x}{2}$] = 0 ≫

= - 2x - 4 +10x - x^2 ≫ x^2 – 8x + 4 = 0

Then $x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \text{ x } 1 \text{ x } 4}}{2\text{x } 1} = 0.54$or 7.45m

Now x = 0.54m and other value 7.45m being inadmissible.

Result:

The SFD and BMD are drawn as shown in fig.

Point of contraflexure or point of inflexion acts at a distance of **0.54m** from right support.

Problem 2.10:A beam AB 10 m long carries a UDL of 20 kN/m over its entire length together with concentrated load of 50 kN at the left end A and 80 kN at the right end B. The beam is to be supported at two props at the same level 6m apart, so that the reaction is the same at each. Determine the position of the supports and draw S.F. and B.M. diagram. Find the value of maximum B.M. Locate the point of contraflexure, if any.

Given Data: shown in figure.

To find: SFD and BMD, Max. BM and Locate the point of contraflexure

Solution:

Find the position of reactions:

Step1: Sum of upward force = Sum of downward force

$R_C + R_D$ = 80 + (20 x 10) + 50

Given $R_C = R_D$ ≫ $2R_C$ = 330 ≫ R_C = 330/2 ≫ R_C = 165 kN

≫ $R_C = R_D$ = 165 kN

Step2: Take moment about A is equal to zero. Let us take a distance x m form left end one reaction D will be acted.

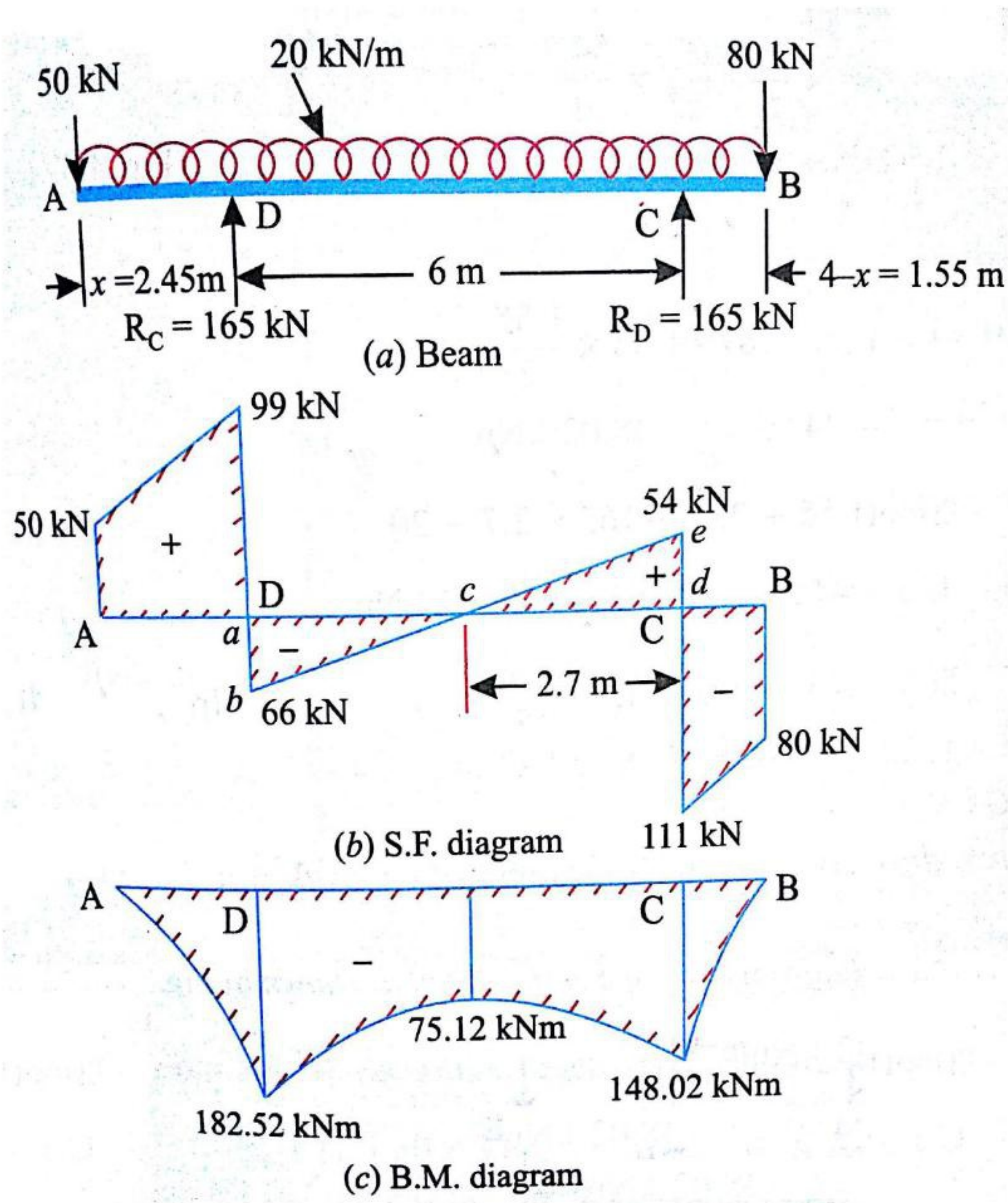
For UDL, it will be converted into point load as (Point load = UDL x load acting distance) and the converted point load acting at its middle means divided by 2.

$$-(80 \times 10) + R_c \times (6 + x) + R_D \times x - [(20 \times 10) \times \frac{10}{2}] = 0$$

$$2 \times 165x = 800 + 1000 - (6 \times 165)$$

$$x = 810/330 = 2.45\text{m}$$

Then distance of another reaction from right support = 4 – 2.45 = 1.55m



(a) Beam

(b) S.F. diagram

(c) B.M. diagram

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 80 = +80kN

When point load or reaction and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force.

SF at C = +80 + (20 x 1.55) = 111kN (Without consider PL)

SF at C = +80 - 165 + (20 x 1.55) = - 54 kN (With consider PL)

SF at D = +80 - 165 + (20 x 7.55) =+ 66kN (Without consider PL)

SF at D = +80 - 165 – 165 + (20 x 7.55) = - 99kN (With consider PL)

SF at A = +80 - 165 – 165 + (20 x 10) =- 50kN

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign.Vertical downward UDL are drawn as inclined line based on sign

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = -(80 x 0) = 0 kN

BM at C = -(80 x 1.55) + (165 x 0) – [(20 x 1.55) x $\frac{1.55}{2}$] = - 148.02kNm

BM at D = -(80 x 7.55)+(165 x 6)– (165 x 0) -[(20 x 7.55) x $\frac{7.55}{2}$] = - 182.52kNm

BM at A = -(80 x 10)+(165 x 8.45)– (165 x 2.45) - [(20 x 10) x $\frac{10}{2}$] = - 0 kNm

Bending moment Diagram:

UDL are drawn as parabolic curved line based on their sign.

Maximum B.M. Calculation:

Maximum B.M. available at the reaction D which is equal to **182.52kNm**

Calculate the position of point of contraflexure or point of inflexion:

The B.M remains negative throughout, hence there is no point of contraflexure or point of inflexion.

Result: The SFD and BMD are drawn as shown in fig.

There is no Point of contraflexure or point of inflexion.

Maximum B.M. = 182.52kNm

2.15. S.F AND B.M. DIAGRAM FOR BEAMS CARRYING INCLINED LOAD AND SUPPORTS

The shear force is defined as the algebraic sum of the vertical forces at any section of a beam to the right or left of the section. But when a beam carries inclined loads, then these inclined loads are resolved into their vertical and horizontal components. The vertical components only will cause shear force and bending moments.

The horizontal components of the inclined loads will introduce axial force or thrust in the beam. The variation of axial force for all sections of the beam can be shown by a diagram known as thrust diagram or axial force diagram.

In most of the cases, one end of the beam is hinged and the other end is supported on rollers. The roller support cannot provide any horizontal reaction. Hence only the hinged end will provide the horizontal reaction.

Problem 2.10:A beam AB 4 m long is hinged at A and supported on roller at B. The beam carries inclined loads of 100N, 200N and 300N inclined at 60°, 45° and 30° to the horizontal at a distance of 1m 2m and 3m respectively form left support. Draw the S.F., B.M. and thrust diagram for the beam.

Given Data: shown in figure.

To find: SFD and BMD and Thrust force diagram.

Solution:

First of all resolve the inclined force into their vertical and horizontal components

Inclined load at C is having horizontal component = 300 x cos 30°= 259.8N

Vertical component = 300 x sin30°= 150N

Inclined load at D is having horizontal component = 200 x cos 45°= 141.4N

Vertical component = 200 x sin45°= 141.4 N

Inclined load at E is having horizontal component = 100 x cos 60°= 50N

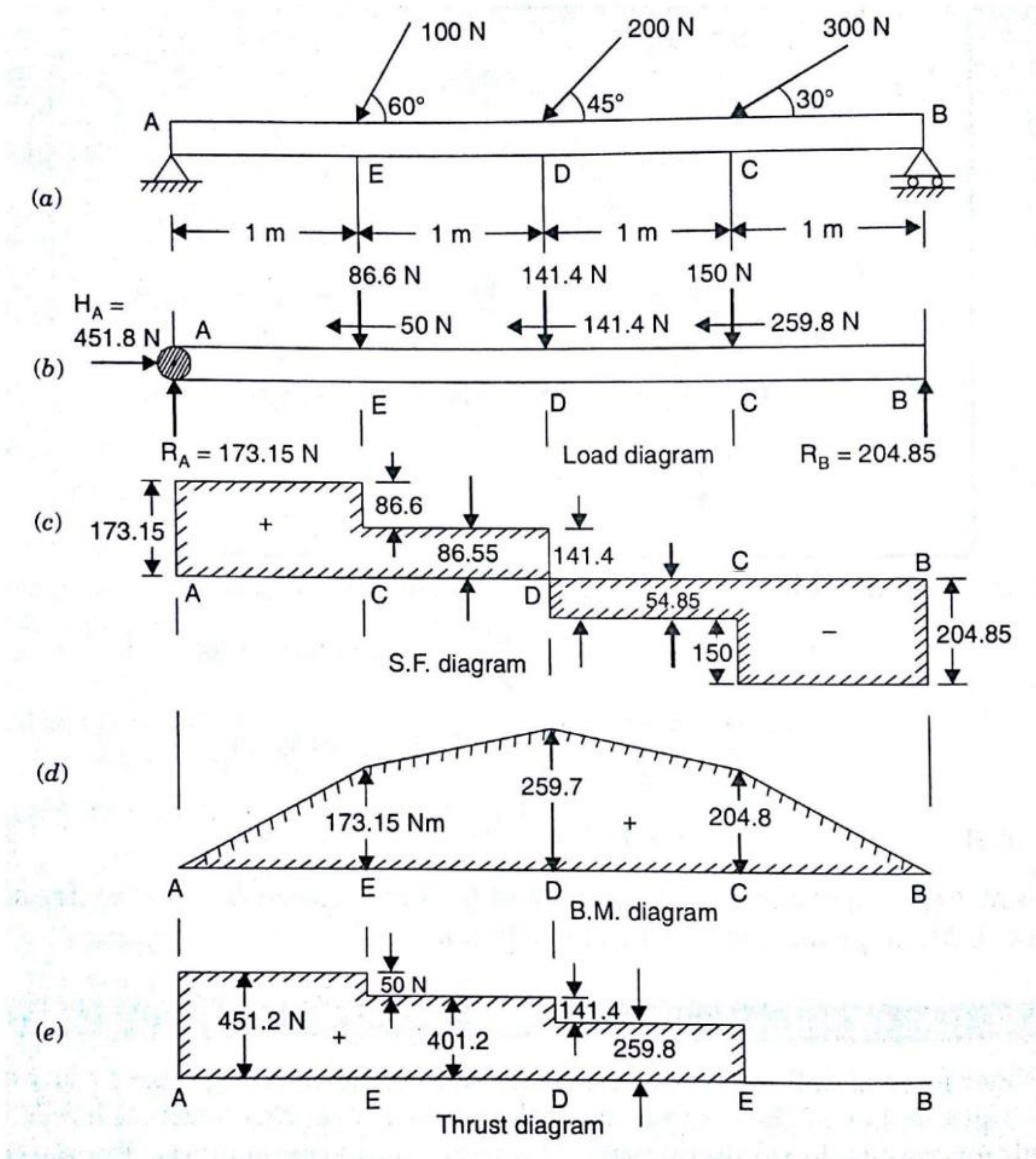
Vertical component = 100 x sin60°= 86.6N

As the beam is supported on roller at B, hence have vertical reaction as R_B. But at A having hinged support, hence Horizontal and vertical reaction are there as R_A and H_B

Horizontal Reaction at B = H_B = Sum all horizontal component of inclined loads

= 50 + 141.4 + 259.8 = 451.20 N

(Based on horizontal force direction put + or – sign)



Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

$+ R_B$ x4 - (150 x 3) - (141.4 x 2) – (86.6 x 1)= 0

R_B x4 = (150 x 3) + (141.4 x 2)+(86.6 x 1) ≫ R_B = 204.85 N

Step2: Sum of upward force = Sum of downward force

$R_A + R_B$ = 150 + 141.4 + 86.6≫ R_A = 378 - R_B

≫ R_A = 378 – 204.85 = 173.15 N

Shear Force Calculation: (Sum of vertical forces)

SF at B = -R_B = - 204.85N

SF at C = - 204.85 +150 = - 54.85 N

SF at D =- 204.85 +150 + 141.4= +86.55 N

SF at E = - 204.85 +150 + 141.4 + 86.6 = +173.15 N

SF at A = - 204.85 +150 + 141.4 + 86.6 = +173.15N

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign.

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = -+ (R_B x 0) = 0Nm

BM at C = +(204.85 x 1) – (150 x 0) = + 204.85Nm

BM at D = + (204.85 x 2) –(150 x 1) – (141.4 x 0) = + 259.7Nm

BM at E = +(204.85 x 3) – (150 x 2) – (141.4 x 1) –(86.6 x 0) = +173.15Nm

BM at A = +(204.85 x 4) – (150 x 3) – (141.4 x 2) – (86.6 x 1) = 0Nm

Bending moment Diagram:

PL are drawn as inclined line based on their sign.

Thrust or Axial force calculation :(Sum of horizontal component of forces)

TF at B = -0 kN = 0N

TF at C = 0 + 259.8= +259.8 N

TF at D =0+ 259.8+ 141.4 = + 401.2 N

TF at E = 0+ 259.8 + 141.4+ 50 = +451.2 N

TF at A = 0+ 259.8 + 141.4+ 50 = +451.2 N

Thrust or Axial force Diagram:

PL are drawn as straight line based on their sign.

Result:

The SFD,BMD and TFD are drawn as shown in fig.

2.16. S.F AND B.M. DIAGRAM FOR BEAMS CARRYING INCLINED LOAD, SUPPORTS AND SUBJECTED TO COUPLE

When a beam subjected to couple at a section, only the B.M. at the section of the couple changes suddenly in magnitude equal to that of the couple. But the S.F. does not changes at the section of the couple as there is no change in load due to couple at the section. But while calculating the reactions, the magnitude of the couple is taken into account. Hence the B.M. calculate at the couple acting section with or without consider the couple.

Problem 2.11: Draw the SF and BM diagram for a beam shown in fig.

Given Data: shown in figure.

To find: SFD and BMD

Solution:

The horizontal force 5 kN acting on the top of a 1m lever at D causes a clockwise moment of 5 kNm at D and a horizontal thrust of 5kN in the beam. Since the horizontal thrust n beam effects neither the S.F in the beam nor the B.M. in it, for purpose of analyzing the shear and the bending moment in the beam the given force of 5kN can be replaced by a clockwise moment of 5kNm

Find the reaction at A and B as R_A and R_B

Step1: Take moment about A is equal to zero

$-(10 \text{ x } 8)+ (R_C \text{ x } 6) - 5 - (2 \text{ x } 4 \text{ x } \frac{4}{2})= 0$

$R_C \text{ x}6 = (10 \text{ x } 8) + 5+(2 \text{ x } 4) \quad \gg R_C = 101/6 = 16.83\text{kN}$

Step2: Sum of upward force = Sum of downward force

$R_A + R_C = 10 + (2 \text{ x } 4) \gg R_A = 18 - R_C$

$\gg R_A = 18 - 16.83 = 1.17\text{kN}$

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 10 kN

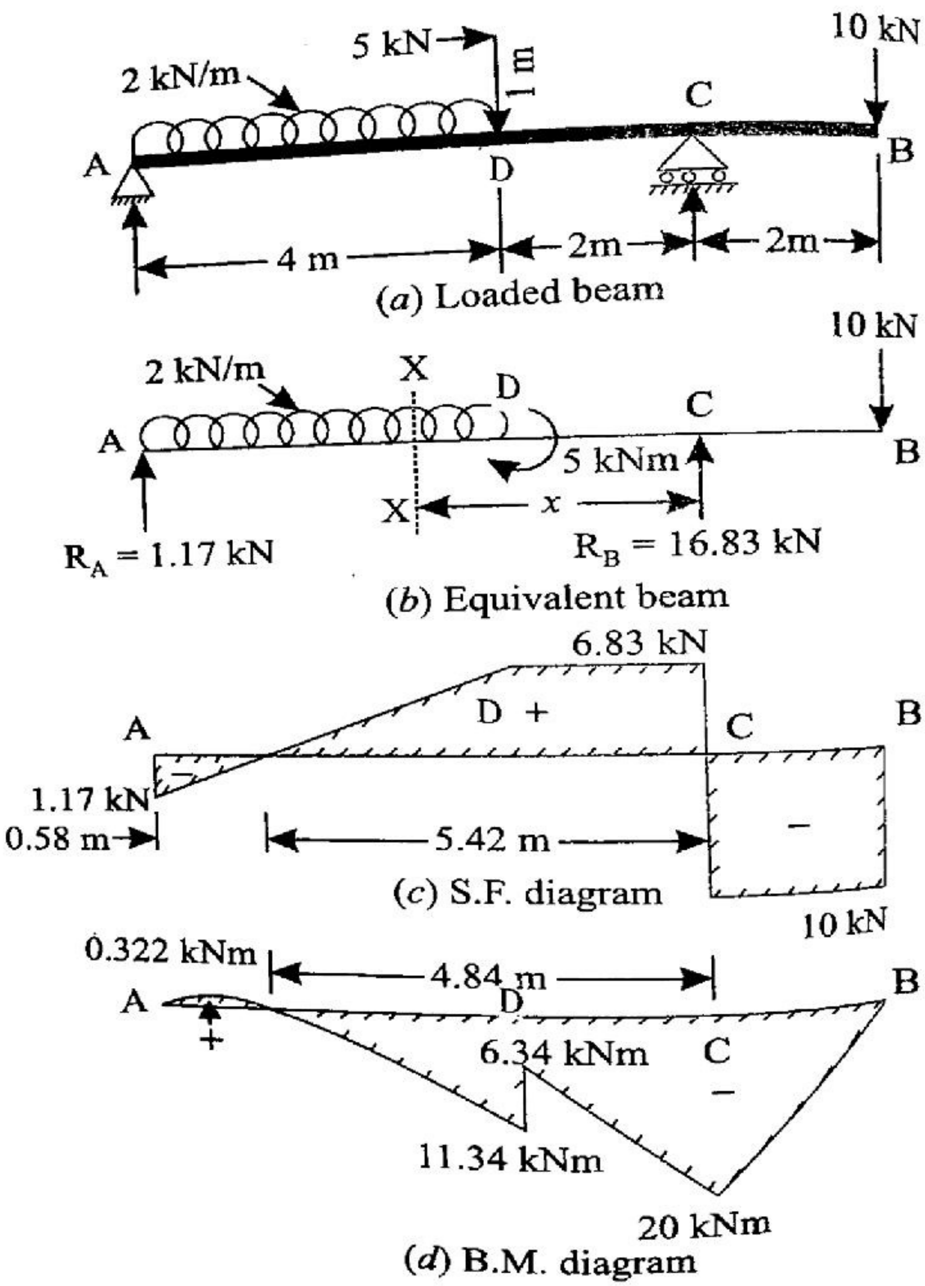
SF at C = + 10 – 16.83 = - 6.83 N

SF at D =+ 10 – 16.83 + 0 = - 6.83 N

SF at A = + 10 – 16.83 + (2 x 4) = 1.17 N

Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign. UDL are drawn as inclined line based on their sign.



Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = - (10 x 0)= 0 Nm

BM at C = - (10 x 2) + (16.83 x 0) = - 6.83 N = + 204.85 Nm

BM at D = - (10 x 4) + (16.83 x 2) - 5 = + 259.7 Nm

BM at A = - (10 x 4) + (16.83 x 2) - 5 – (2 x 4 x $\frac{4}{2}$) = 0 Nm

Bending moment Diagram:

PL are drawn as inclined line and UDL drawn as parabolic curved line based on their sign.

Calculate Maximum Bending Moment:

The maximum BM will occur where the shear force becomes zero. Let us take a point X at a distance of *x* from D where SF become zero is shown in fig.

SF at X =+ 10 – 16.83 + (2 x *x*)= 0 ≫x = (16.83 – 10)/2 = 3.42 m

Then BM at X = - [10 x (4+3.42)] + [16.83 x (2 + 3.42)] - 5 – [2 x 3.42 x $\frac{3.42}{2}$]]

= 0.322kNm

Calculate the position of point of contraflexure or point of inflexion:

The point of contraflexure or point of inflexion lie at a point where BM become zero. In the fig. the BM act at Y at a distance y m from B.

BM at Y = - (10 x y) + [16.83 x (y - 2)] - 5 – [2 x (y-4) x $\frac{(y-4)}{2}$] = 0

≫ = - 10y+ 16.83y - 37.66 – 5 - y^2 +8y– 16 = 0

≫ y^2 – 14.83y– 58.66 = 0

Then y = $\frac{-(-14.83) \pm \sqrt{(-14.83)^2 - 4 \text{ x } 1 \text{ x } (-58.66)}}{2\text{x } 1}$ = 6.84m or 8.45m

Now x = 6.84m and other value 8.45m being inadmissible.

Result:

The SFD, BMD are drawn as shown in fig.

Point of contraflexure act at a distance of **6.84m** from B

Problem 2.12: Calculate the reaction at A and C and position of point of contraflexure of the given beam with load shown in fig. Also Draw the SF and BM diagram for the beam.

Given Data: shown in figure.

To find: SFD, BMD and distance of point of contraflexure.

Solution:

In this figure roller support has one reaction R_A and hinged support has horizontal and vertical reaction R_{C_H} and R_{C_v}. A force applied 4kN through a bracket 0.5m away from the point D. Now apply equal and opposite load of 4kN at D. This will be equivalent to a anticlockwise couple of value (4 x 0.5) = 2kNm acting at D together with a vertical downward load of 4 kN at D.

The inclined load 4kN resolved into vertical and horizontal component of forces.

Horizontal component of force = B_H = 4 x cos30° = $2\sqrt{3}$kN

Vertical component of force = B_V = 4 x sin30° = 2 kN

The horizontal force balanced by a force at the hinged support. Which gives

Find the reaction at A and C as R_A and R_C

Step1: Take moment about A is equal to zero

$$-(2\text{x } 7) + (R_{C_v}\text{ x } 6) - (4 \text{ x } 4) + 2 - (1 \text{ x } 2\times \frac{2}{2}) = 0$$

$$R_{C_v}\text{x}6 = (2 \text{ x } 7) + (4 \text{ x } 4) - 2 + (1 \text{ x } 2 \times \frac{2}{2}) \gg R_{C_v} = 30/6 = 5\text{kN}$$

Step2: Sum of upward force = Sum of downward force

$$R_A + R_{C_v} = 2 + 4 + (1 \text{ x } 2) \quad \gg R_A = 8 - R_{C_v}$$

$$\gg R_A = 8 - 5 = 3\text{kN}$$

Total reaction at C (R_C) = $\sqrt{{R_{C_H}}^2 + {R_{C_v}}^2} = \sqrt{(2\sqrt{3})^2 + 5^2}$ = 6.08 kN

Location with horizontal θ = $\tan^{-1}\frac{R_{C_v}}{R_{C_H}} = \tan^{-1}\frac{5}{2\sqrt{3}}$ = 55.3°

Shear Force Calculation: (Sum of vertical forces)

SF at B = + 2kN

SF at C = + 2 – 5 = - 3 kN

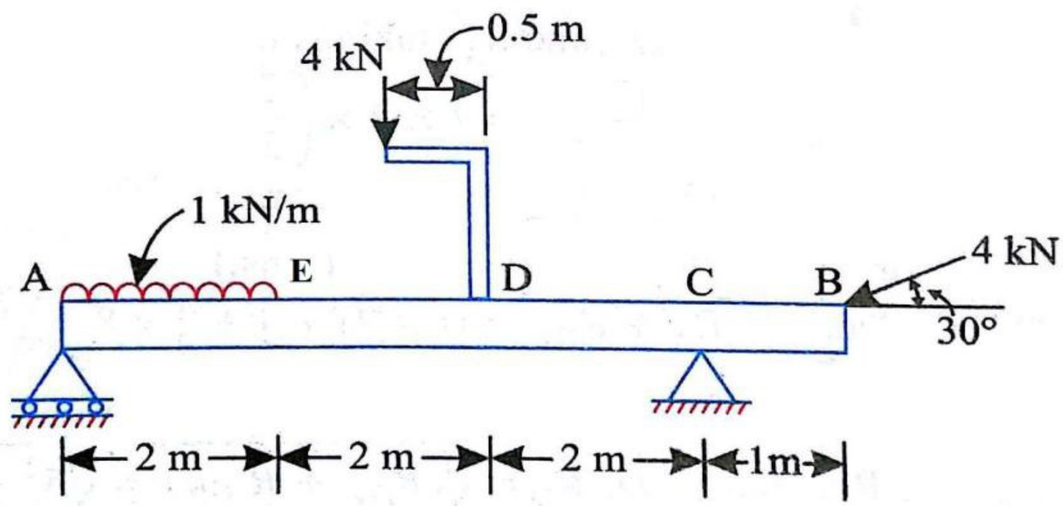
SF at D = + 2 – 5 + 4 = + 1kN

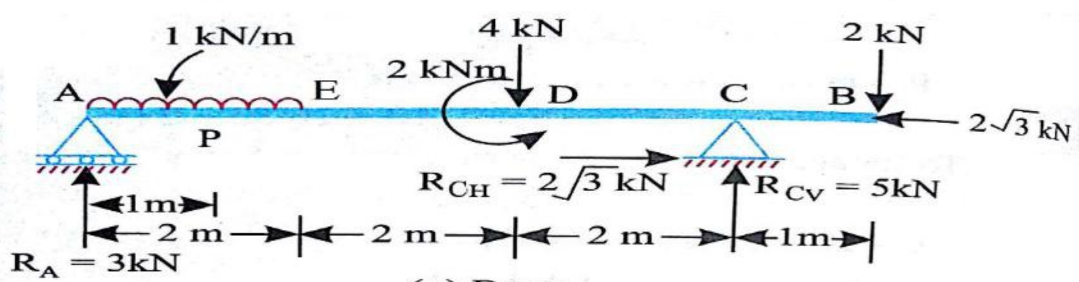
SF at E = + 2 – 5 + 4 = + 1kN

SF at A = + 2 – 5 + 4 + (1 x 2) = + 3 kN

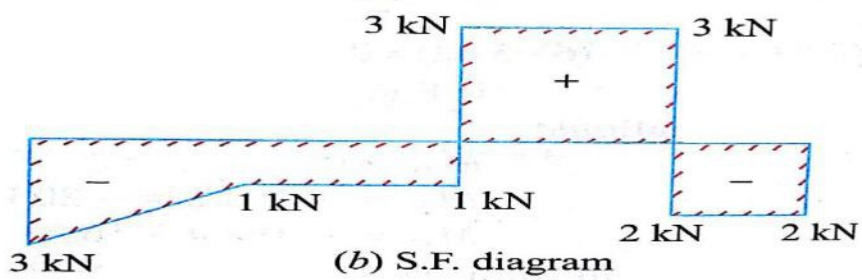
Shear Force Diagram:

Vertical upward reaction and downward load are drawn as vertical line based on sign. UDL are drawn as inclined line based on their sign.

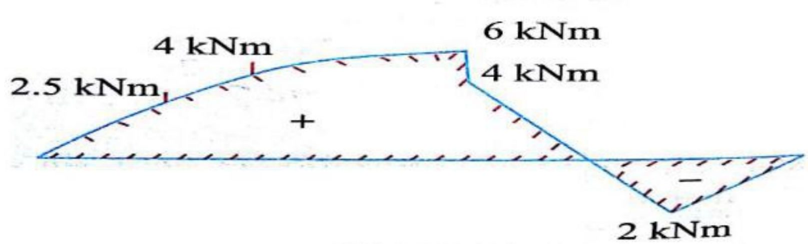




(a) Beam



(b) S.F. diagram



(c) B.M. diagram

Bending moment Calculation: [Sum of (Vertical force x Distance of load acting from required section)]

BM at B = - (2 x 0) = 0kNm

BM at C = - (2 x 1) + (5 x 0) = - 2kNm

BM at D = - (2 x 3) + (5 x 2) -(4 x 0) +2 = + 6kNm

BM at E = - (2 x 5) + (5 x 4) - (4 x 2)+ 2 = + 4 kNm

BM at E = - (2 x 7) + (5 x 6) - (4 x 4) + 2 – (1 x 2 x $\frac{2}{2}$) = 0 kNm

Bending moment Diagram:

PL are drawn as inclined line and UDL drawn as parabolic curved line based on their sign. Couples are drawn as vertical line

Calculate the position of point of contraflexure or point of inflexion:

The point of contraflexure or point of inflexion lie at a point where BM become zero. In the fig. the BM act at X at a distance x m from B.

BM at X = - (2 x x) + [5 x (x-1)] = 0 kNm

= - 2.x +5.x -5 = 0 ≫ 3x = 5 ≫ x = 5/3 = 1.67m

Result:

The SFD, BMD are drawn as shown in fig.

Reaction forces $R_A = 3$ kN

R_C = 6.08 kN which makes an angle with horizontal θ= 55.3°

Point of contraflexure act at a distance of **1.67m** from B

THEORETICAL QUESTIONS

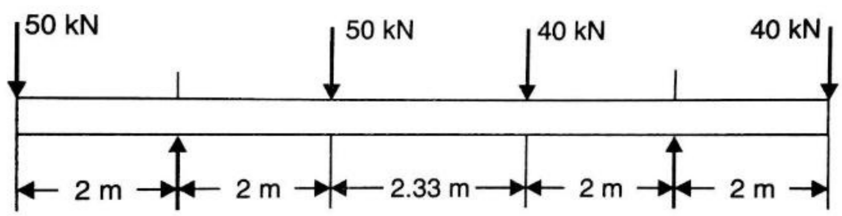
1. Define and explain the following terms :
   Shear force, bending moment, shear force diagram and bending moment diagram.
2. What are the different types of beams? Differentiate between a cantilever and a simply supported beam.
3. What are the different types of loads acting on a beam? Differentiate between a point load and a uniformly distributed load.
4. What are the sign conventions for shear force and bending moment in general?
5. Draw the S.F. and B.M. diagrams for a cantilever of length L carrying a point load W at the free end.

6. Draw the S.F and B.M. diagrams for a cantilever of length L carrying a uniformly distributed load of w per m length over its entire length.
7. Draw the S.F and B.M. diagrams for a cantilever of length L carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.
8. Draw the S.F and B.M. diagrams for a simply supported beam of length L carrying a point load W at its middle point.
9. Draw the S.F and B.M. diagrams for a simply supported beam carrying a uniformly distributed load of w per unit length over the entire span. Also calculate the maximum B.M.
10. Draw the S.F and B.M. diagrams for a simply supported beam carrying a uniformly varying load from zero at each end to w per unit length at the centre.
11. What do you mean by point of contraflexure ? Is the point of contraflexure and the point of inflexion different?
12. How many points of contraflexure you will have for simply supported beam overhanging at one end only ?
13. How will you draw the S.F. and B.M. diagrams for a beam which is subjected to inclined loads?
14. What do you mean by thrust diagram ?
15. Draw the S.F and B.M. diagrams for a simply supported beam of length L which is subjected to a clockwise couple μ at the centre of the beam.

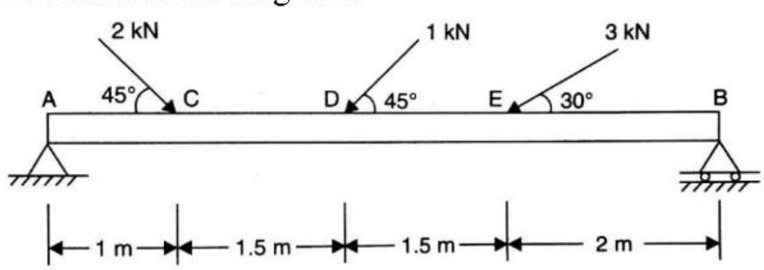
NUMERICAL PROBLEMS

1. A cantilever beam of length 2 m carries a point load of 1 kN at its free end, and another load of 2 kN at a distance of 1 m from the free end. Draw the S.F. and B.M. diagrams for the cantilever.
2. A cantilever beam of length 2 m carries a uniformly distributed load of 3 kN/m run over a length of 1 m from the fixed end. Draw the S.F. and B.M. diagrams.
3. A cantilever of length 5 m carries a uniformly distributed load of 2 kN/m length over the whole length and a point load of 4 kN at the free end. Draw the S.F. and B.M. diagrams for the cantilever.
4. A cantilever 2 m long is loaded with a uniformly distributed load of 2 kN/m run over a length of 1 m from the free end. It also carries a point load of 4 kN at a distance of 0.5 m from the free end. Draw the S.F. and B.M. diagrams.
5. A simply supported beam of length 8 m carries point loads of 4 kN and 6 kN at the distance of 2 m and 4 m from the left end. Draw the S.F. and B.M. diagrams for the beam.
6. A simply supported beam of length 10 m carries point loads of 30 kN and 50 kN at the distance of 3 m and 7 m from the left end. Draw the S.F. and B.M. diagrams for the beam.

7. A simply supported beam of length 8 m carries point loads of 4 kN, 10 kN, and 7 kN at a distance of 1.5 m, 2.5 m and 2 m respectively from left end A. Draw the S.F. and B.M. diagrams for the simply supported beam.
8. A simply supported beam is carrying a uniformly distributed load of 2 kN/m over a length of 3 m from the right end. The length of the beam is 6 m. Draw the S.F. and B.M. diagrams for the beam and also calculate the maximum B.M. on the section.
9. A simply supported beam of length 8 m rests on supports 5 m apart, the right hand end is overhanging y 2 m and the left hand end is overhanging by 1 m. The beam carries a uniformly distributed load of 5 kN/m over the entire length. It also carries two point loads of 4kN and
   6 kN at each end of the beam. The load of 4 kN is at the extreme left of the beams, whereas the load of 6 kN is at the extreme right of the beam. Draw S.F. and B.M. diagrams for the beam and find the points of contraflexure.
10. A beam is loaded as shown in Fig Draw the S.F. and B.M. diagrams and find:
    i. maximum S.F. , maximum B.M., point of inflexion



11. A beam is loaded as shown in fig. Find the reactions at A and B. Also draw the S.F., B.M. and thrust diagrams.



12. A simply supported beam of length 5 m, carries a uniformly distributed load of 100 N/m extending from the left end to a point 2 m away. There is also a clockwise couple of 1500 Nm applied at the centre of the beam. Draw the S.F. and B.M. diagrams for the beam and find the maximum bending moment.
13. A simply supported beam of length 8 m carries point loads of 4 kN, 10 kN, and 7 kN at a distance of 1.5 m, 2.5 m and 2 m respectively from left end A. Draw the S.F. and B.M. diagrams for the simply supported beam.

14. A simply supported beam is carrying a uniformly distributed load of 2 kN/m over a length of 3 m from the right end. The length of the beam is 6 m. Draw the S.F. and B.M. diagrams for the beam and also calculate the maximum B.M. on the section.
15. A simply supported beam of length 8 m rests on supports 5 m apart, the right hand end is overhanging y 2 m and the left hand end is overhanging by 1 m. The beam carries a uniformly distributed load of 5 kN/m over the entire length. It also carries two point loads of 4kN and
6 kN at each end of the beam. The load of 4 kN is at the extreme left of the beams, whereas the load of 6 kN is at the extreme right of the beam. Draw S.F. and B.M. diagrams for the beam and find the points of contraflexure.

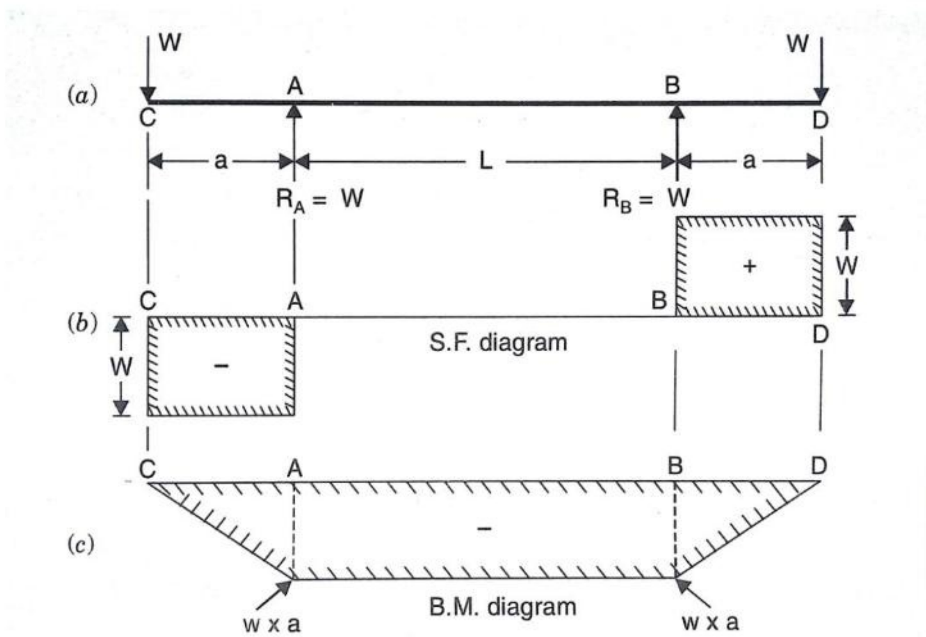
BENDING STRESS IN BEAM

2.17. BENDING STRESS IN BEAM

When some external load acts on a beam, the shear force and bending moments are set up at all sections of the beam. Due to the shear force and bending moment, the beam undergoes certain deformation. The stresses introduced by bending moment is known as bending stresses.

2.18. PURE BENDING OR SIMPLE BENDING

If a length of a beam is subjected to a constant bending moment and no shear force (i .e, zero shear force), then the stresses will be set up in that length of the beam due to B.M. only and that length of beam is known as pure bending or simple bending. The stresses set up in that length of beam are known as bending stresses.



A beam simply supported at A and B and overhanging by same length at each support. A point load W is applied at each end of the overhanging portion. It is clear that there is no shear force between A and B but the B.M. between A and B is known as pure bending or simple bending.

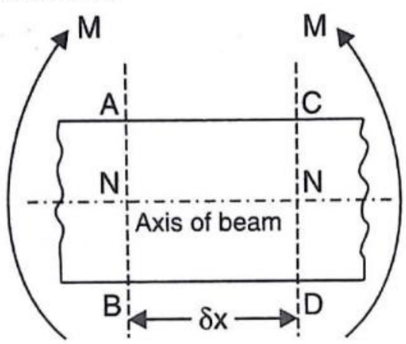
2.19. THEORY OF SIMPLE BENDING WITH ASSUMPTIONS MADE

Assumptions made in the theory of simple bending. The following are the important assumptions:

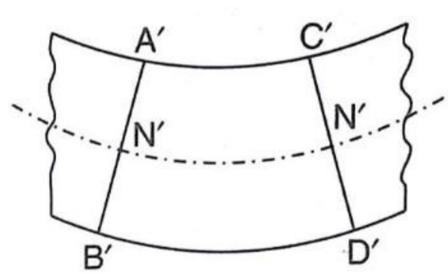
1. The material of the beam is homogeneous and isotropic.
2. The value of young's modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending , remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular areas with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of the cross section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

Theory of Simple Bending

The fig. shows A part of a beam is subjected to simple bending. Consider a small length d x of this part of beam. Consider two sections AB and CD which are normal to the axis of the beam N-N. Due to the action of the bending moment, the part of length d x will be deformed.



(a) Before bending



(b) After bending

The top layer such as AC has deformed to the shape A'C'. This layer has been elongated. It is clear that some of the layers have been shortened while some of them are elongated. At a level between the top and bottom of the beam there will be a layer which is neither shortened nor elongated. This layer is known as neutral layer or neutral surface.

This layer is N'-N'. The line of intersection of the neutral layer on a cross section of the beam is known as the neutral axis.

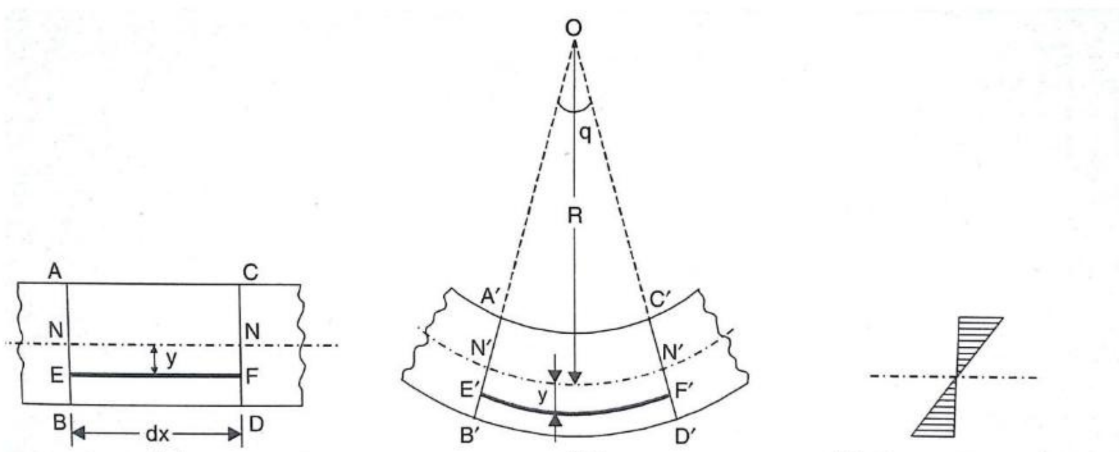
Hence the amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N-N. This theory of bending is known as theory of simple bending.

2.20. EXPRESSION FOR BENDING STRESS

A small length d x of a beam subjected to a simple bending. Due to the action of bending, the part of length d x will be deformed. Let A'B' and C'D' meet at O.

Let R= Radius of neutral layer N'N'

θ=Angle subtended at O by A'B' and C''D' produced.



(a) (b) (c) Stress Diagram

2.20.1. Strain Variation along the Depth of Beam

Consider a layer EF at a distance y below the neutral layer NN. After bending this layer will be elongated to E'F'.

Original length of layer EF=d x.

Also length of neutral layer NN=d x.

After bending, the length of neutral layer N'N' will remain unchanged. But the length of layer E'F' will increase.

N'N'=NN=d x.

N'N'=R × θ

E'F'= (R + y)× θ

But N'N'=NN=d x

Hence d x=R× θ

∴ Increase in length of layer EF=E'F'-EF=(R+ y)θ -R× θ

= y × θ

∴ Strain in layer EF $= \frac{Increase\ in\ length}{original\ length}$

$= y \times \frac{\theta}{EF} \quad = \frac{y \times \theta}{R \times \theta} \quad = \frac{y}{R}$

2.20.2. Stress Variation

Let σ = stress in layer EF

E = young's modulus of beam

E = stress in layer EF/strain in layer EF

= σ/(y/R)

$\sigma = E \times \frac{y}{R} = \frac{E}{R} \times y$

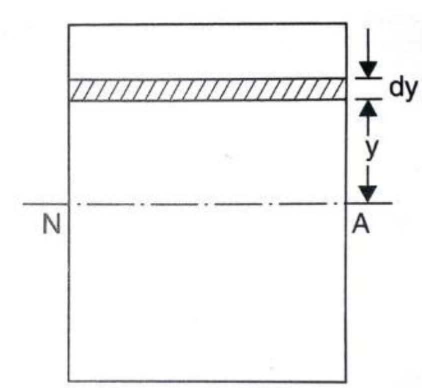
It can also be written as $\frac{\sigma}{y} = \frac{E}{R}$

2.21. NEUTRAL AXIS AND MOMENT OF RESISTANCE

The neutral axis of any transverse section of beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as NA.

The stress at a distance y from the neutral axis is given by

$\sigma = \frac{E}{R} \times y$



Let d A = Area of layer

Force on layer = stress on layer ×area of layer

$= \sigma \times dA$

$= \frac{E}{R} \times y \times d A$

Total force on beam section $= \int \frac{E}{R} \times y \times d A$

$= \frac{E}{R} \int y \times d A$

But for pure bending there is no force on the section of beam

$\therefore$ $$\frac{E}{R}\int y \times dA = 0$$

$$\int y \times dA = 0$$

Hence $\int y \times dA$ represents the moment of entire area of the section about neutral axis.

2.21.1. Moment of Resistance

Due to pure bending the layers above the NA are subjected to compressive stresses whereas the layers below the NA are subjected to tensile stresses. Due to these stresses forces acts on the layers.

The force on the layer at a distance y from neutral axis.

$$\text{Force on layer} = \frac{E}{R} \times y \times dA$$

Moment of this force about NA

$$= \text{force on layer} \times y$$

$$= \frac{E}{R} \times y \times dA \times y$$

$$= \frac{E}{R} \times y^2 \times dA$$

Total moment of the forces on the section of beam $= \int \frac{E}{R} \times y^2 \times dA$

$$= \frac{E}{R}\int y^2 \times dA$$

Let M = External moment applied on the beam section.

$$M = \frac{E}{R}\int y^2 \times dA$$

But the expression $\int y^2 \times dA$ represent the moment of inertia of the area of the section about the neutral axis I

Then $$M = \frac{E}{R} \times I$$

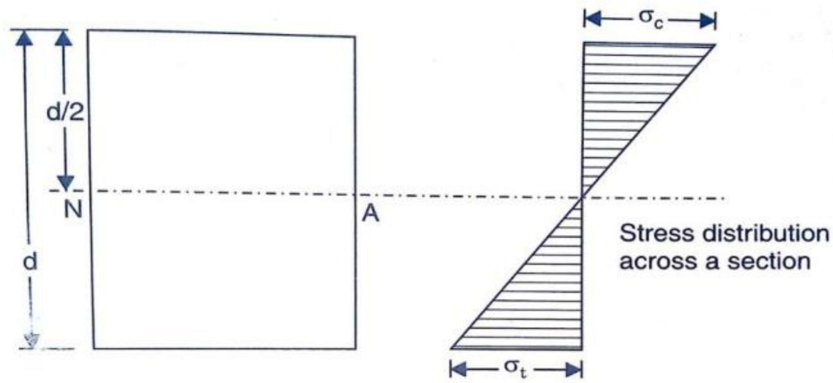
We know that $$\frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

It is known as the bending equation.

2.22. BENDING STRESSES IN SYMMETRICAL SECTIONS

The neutral axis of a symmetrical section lies at a distance of $d/2$ from the outermost layer of the section where d is the diameter or depth. There is no stress at the neutral axis. But the stress at a point is directly proportional to its distance from the neutral axis. The maximum shear stress takes place at the outermost layer. For a simply supported beam, there is a compressive stress above the neutral axis and a tensile stress below it. If we plot these stresses, we will get a figure shown below.



Problem 2.13. A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will provide the maximum stress. Take E = $2 \times 10^5 \text{N}/mm^2$.

Given:

Width b= 120mm

Thickness t=20mm

Moment of inertia I = $\frac{bt^3}{12} = 120 \times \frac{20^3}{12} = 8 \times 10^4 mm^4$

Radius of curvature R =10m = 10×10^3mm

Young's modulus E = 2×10^5 N/mm^2

Solution:

Wkt $$\frac{\sigma}{y} = \frac{E}{R}$$

$$y_{max} = \frac{t}{2} = \frac{20}{2} = 10\text{mm}$$

$$\sigma_{max} = \frac{E}{R} \times y_{max}$$

$$= \frac{2 \times 10^5}{10 \times 10^3} \times 10$$

$$= 200 \text{ N}/mm^2$$

Wkt $\frac{M}{I} = \frac{E}{R}$

$$\text{M} = \frac{E}{R} \times \text{I}$$

$$= \frac{2 \times 10^5}{10 \times 10^3} \times\times 8 \times 10^4$$

$= 16 \times 10^5$Nmm = **1.6KNm**

Problem 2.13. Calculate the maximum stress induced in a cast iron pipe of external diameter 40mm of internal diameter 20mm and of length 4m when the pipe is supported on its ends and carries a point load of 80N at its centre.

Given

Ext dia. D = 40mm

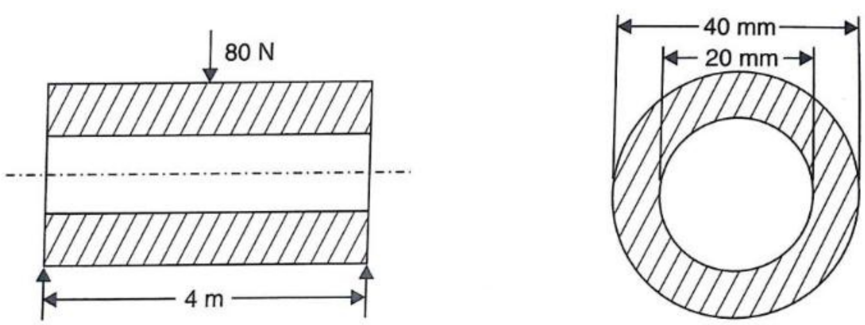
Int dia. d = 20mm

Length L= 4m = 4×1000 = 4000mm

Point load W = 80 N

Solution:

In this case of simply supported beam carrying a point load at the centre, the maximum bending moment is at the centre of the beam.



(a) (b) Area of cross-section

Max bending moment BM $= \frac{W \times L}{4}$

$= 80 \times 4000/4 = 8 \times 10^4$Nmm

Moment of inertia of hollow pipe I = $\frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(40^4 - 20^{4)}$

$$= 117809.7\ mm^4$$

Now $$\frac{M}{I} = \frac{\sigma}{y}$$

$$y_{max} = D/2 = 40/2 = 20\text{mm}$$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$= 8 \times 10^4 \times \frac{20}{117809.7}$$

$$= \mathbf{13.58\ N/mm^2}$$

2.23. SECTION MODULUS

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$Z = \frac{1}{y_{max}}$$

Where I = moment of inertia about neutral axis

y_{max} = distance of the outermost layer from the neutral axis

Wkt $$\frac{M}{I} = \frac{\sigma}{y}$$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

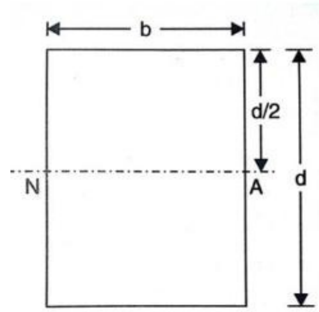
$$M = \sigma_{Max} \times \frac{I}{y_{max}}$$

But $$\frac{I}{y_{max}} = Z$$

∴ $$M = \sigma_{max} \times Z$$

2.24. SECTION MODULUS FOR VARIOUS SHAPES OR BEAM SECTIONS

1. Rectangular Section



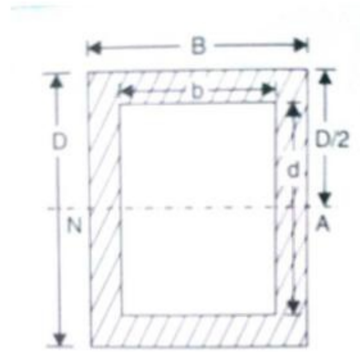
Moment of inertia I = $bd^3/12$

Distance of outermost layer from NA

$$y_{max} = d/2$$

∴ Section modulus $$Z = \frac{I}{y_{max}} = \frac{b\,d^3/12}{d/2} = \frac{bd^2}{6}$$

2. Hollow Rectangular Section



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}(BD^3 - bd^3)$$

$$y_{max} = D/2$$

$$\therefore \quad Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{1}{12}(BD^3 - bd^3)}{D/2}$$

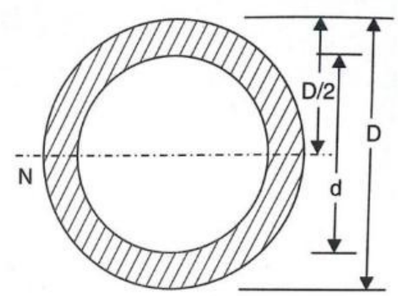
$$= \frac{1}{6D}(BD^3 - bd^3)$$

3. Circular Section

$$I = \frac{\pi}{64}d^4 \quad \text{and} \quad y_{max} = d/2$$

$$\therefore \quad Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}d^4}{d/2} = \frac{\pi}{32}d^3$$

4. Hollow Circular Section



$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$y_{max} = D/2$$

$$\therefore \quad Z = I/y_{max}$$

$$= \frac{\pi}{32\,D}(D^4 - d^4)$$

Problem 2.14. A cantilever of length 2m fails when a load of 2kN is applied at a free end. If the section of the beam is 40mm×60mm. Find the stress at the failure.

Given

Length	L = 2m
Load	W = 2Kn
Width	b = 40mm
Depth	d = 60mm

Solution:

Section modulus of rectangular section $Z = bd^2/6$

$=40\times 60^2/6$

$= 24000mm^3$

Max bending moment for cantilever at fixed end M = W×L

$=2000\times 2\times 10^3$

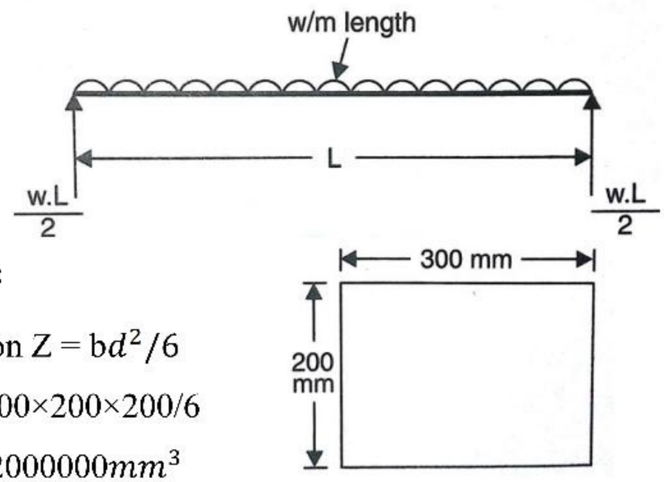
$= 4\times 10^6 Nm$

$M = Z \times \sigma_{max}$

σ_{max} = M/Z = **166.67 N/mm^2**

Problem 2.15. A rectangular beam 200mm deep and 300 mm wide is simply supported over a span of 8m. What uniformly distributed load per metre the beam is carrying if the bending stress is not to exceed 120N/mm^2

Given



Depth d = 200mm

Width b = 300mm

Length L = 8m

Max bending stress σ_{max} = 120N/mm^2

Section modulus for rectangular section Z = b$d^2/6$

= 300×200×200/6

= 2000000mm^3

Max bending moment for simply supported beam carrying uniform load M

$= w\times L^2/8$

= 8w Nm = 8w×1000Nmm

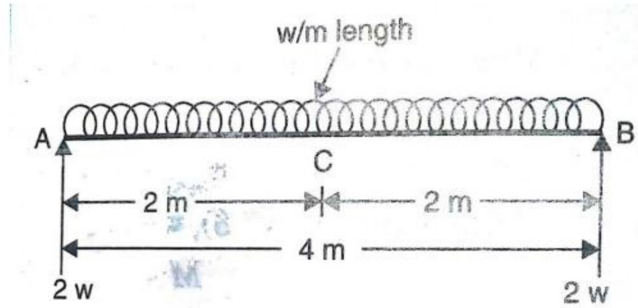
= 8000w Nmm

$M = Z\times \sigma_{max}$

$8000w = 120\times 2000000$

w = **30KN/m**

Problem 2.16: A rectangular beam 300 mm deep is simply supported over a span of 4m. Determine the uniformly distributed load per metre which the beam may carry if the bending stress should not exceed 120N/mm^2. Take I = 8×$10^6 mm^4$



Given

Depth d = 300mm

Span L = 4m

Max bending stress M = 120N/mm^2

Moment of inertia I = $8\times 10^6 mm^4$

Max BM = 2w×2-2w×1

= 4w-2w

= 2wNm

= $2w\times1000 = 2000w$ Nmm

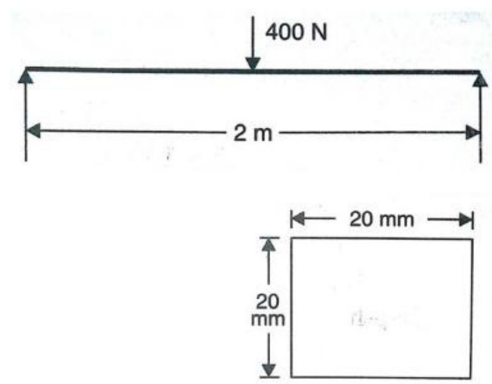
M = 2000w Nmm

M = $\sigma_{max}\times Z$

Z = $I/y_{max} = 8\times 10^6/150$

∴ $2000w = 120\times Z$ **w= 3200 N/m**

Problem 2.17: A square beam 20mm×20mm in section and 2m long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per metre length will break a cantilever of the same material 40mm wide 60mm deep and 3m long.



Given

Depth d = 20mm

Width b = 20mm

Length L =2m

Point load W = 400N

Solution:

Section modulus Z = $bd^3/6$

= 4000/3 mm^3

Max bending moment M = w×L/4 = 400×2/4

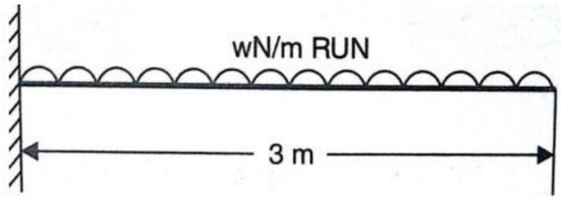
= 200Nm

$= 200\times1000 = 200000$ Nmm

$M = Z\times \sigma_{max}$

$20000 = \sigma_{max} \times (4000/3)$

$\sigma_{max} = 150$ N/mm^2

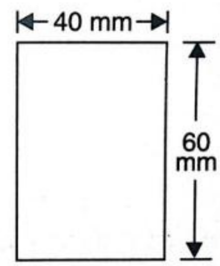


Let w = uniformly distributed load per m run

Let b = 40 mm

d = 60 mm

L = 3m



Section modulus $Z = bd^2/6 = 40\times 60^2/6 = 24000\ mm^3$

Max bending moment $= wL^2/2$

$M = 4.5\times1000w$ Nmm

Wkt $M = \sigma_{max} \times Z$

$4.5\times1000w = 150\times24000$

w = **800N/m**

Problem 2.18: A beam is simply supported and carries a uniformly distributed load of 40KN/m run over the whole span. The section of beam is rectangular having depth as 500mm. If the max stress in the material of beam is 120N/mm^2 and a moment of inertia of the section is $7\times 10^8 mm^4$. Find the span of beam

Given

U.D.L w = 40KN/m

Depth d = 500mm

Max stress $\sigma_{max} = 120$ N/mm^2

M.O.I $I = 7\times 10^8 mm^4$

Section modulus $Z = I/Y_{max}$

$y_{max} = d/2 = 500/2 = 250$ mm

$Z = 7\times 10^8/250 = 28\times 10^5 mm^3$

Max B.M for a simply supported beam carrying U.D.L = $w\times L^2/8$

$M = 40000\times L^2/8$

$= 5000L^2 \times 1000$Nmm

Wkt $M = \sigma_{max} \times Z$

$5000 \times 1000 \times L^2 = 120 \times 28 \times 10^5$

$L = \sqrt{2.4 * 28} =$ **8.197m**

Problem 2.19: A timber beam of rectangular section is to support a load of 20 kN uniformly distributed over a span of 3.6 m when beam is simply supported. If the depth of the section is to be twice the breadth and the stress in the timber is not to exceed 7N/mm^2. Find the dimensions of how would you modify the cross section of the beam if it carries a concentrated load of 20 k N placed at the centrevwith the same ratio of breadth to depth.

Given

Total load W = 20 k N

Span L = 3.6m

Max stress σ_{max} = 7N/mm^2

Depth d = 2b mm

Section modulus Z = b$d^2/6$

$Z = b \times (2b) \times (2b)/6 = \frac{2b^3}{6} mm^3$

Max B.M $= wL^2/8$ or WL/8

M = WL/8 = 9000 Nm = 9000×1000Nmm

W k t $M = Z \times \sigma_{max}$

$9000 \times 1000 = 7 \times \frac{2b^3}{3}$

$b^3 = 1.92857 \times 10^6$

b = 124.5mm

d = 2b = 2×124.5 = 249 mm

B.M is maximum at the centre and is equal to WL/4

M = WL/4 = 20000×3.6/4 = 18000 N m

= 18000×1000 Nmm

$\sigma_{max} = 7$ N/mm^2

$Z = 2b^3/3$

We get $M = Z \times \sigma_{max}$

$18000 \times 1000 = 7 \times 2b^3/3$

b = **156.82mm**

d = 2×156.82 = **313.64mm**

Problem 2.20: A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L of 12kN/m run over the entire length and a point load of 10k N at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8N/mm^2. Find the suitable dimensions of the section.

Given

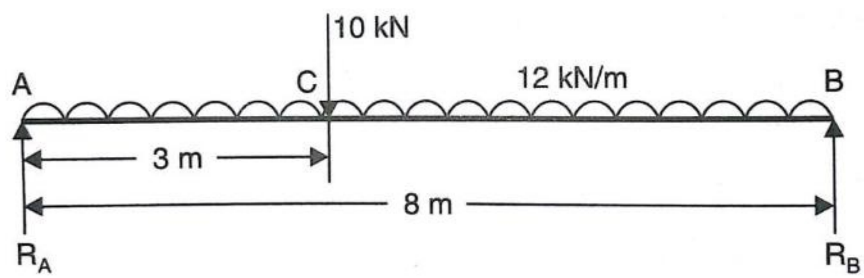
Length L = 8m

UDL w = 12000N/m

Point load W = 10000N

Depth of beam d = 2b

σ_{max} = 8N/mm^2



Solution:

Taking moment about A

We get, $R_B \times 8 = [(12000 \times 8) \times \frac{8}{2}] + (10000 \times 3)$ scoop

R_B = 51750 N

R_A = total load- R_B

= (12000×8)+10000 -5175 = 54250N

S.F at B = -R_B = -51750N

S.F at C = -51750+ (12000×5)= +8250N (without PL)

S.F at C = -51750+ (12000×5) +10000= - 18250N (with PL)

S.F at A = $+R_A$ = +54250N

Let SF is zero at x metre from B

Equating $(12000 \times x) - R_B = 0$

$(120000\times x) - 51750 = 0$

∴ $x = 4.3125$m

∴ Max BM will occur at 4.3125m from B

∴ Max BM $M = R_B \times 4.3125 - 12000 \times 4.3125 \times \frac{4.3125}{2}$

$=111585.9375\times10^3$Nmm

Section modulus for rectangular beam $Z = \frac{bd^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2b^3}{3}$

Wkt $M = Z\times \sigma_{max}$

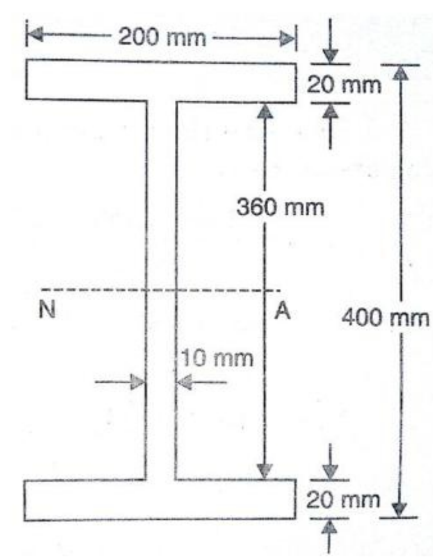
$111586.9375\times10^3 = 8\times \frac{2b^3}{3}$

$b^3 = 20.9223\times 10^6$

b = **275.5mm**

d = 2×275.5 = **551mm**

Problem 2.21: A rolled steel joist of I section has the dimensions as shown. This beam of I section carries a UDL of 40 kN/m run on a span of 10m. Calculate the max stress produced due to bending



Given

UDl w= 40 K N/m = 40000N/m

Span L = 10m

Solution:

Moment of inertia about the neutral axis

$= 200\times 400^3/12 - (200\text{-}10)\times 360^3/12$

$= 327946666mm^4$

Max BM. $M = w\times L^2/8 = \frac{40000 \times 10^2}{8} = 500000$ Nm

$= 5\times 10^8$ N mm

Now using the relation $\frac{M}{I} = \frac{\sigma}{y}$

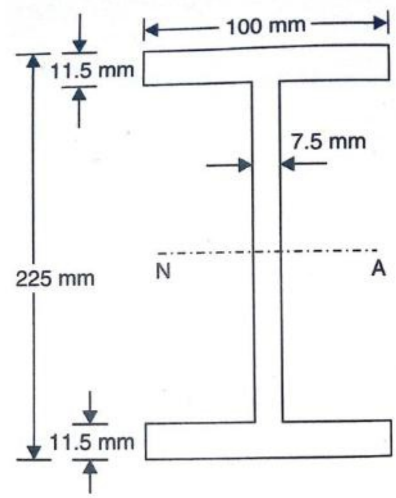
$\therefore \quad \sigma = \frac{M}{I} \times y$

$\sigma_{max} = \frac{M}{I} \times y_{max}$

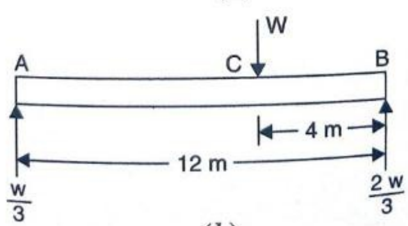
$= 5\times \frac{10^8}{327946666} \times 200$

$= \mathbf{304.92\ N/mm^2}$

Problem 2.22: An I section shown in figure is simply supported over a span of 12m. If the max permissible bending stress is 80 N/mm^2. What concentrated load can be carried at a distance of 4m from one support?



(a)



Given

Bending stress $\sigma_{max} = 80$ N/mm^2

Load W = 4m from support B in N

Solution:

Taking moments about A, we get

$R_B \times 12 = W\times 8$

$\therefore \quad R_B = \frac{8W}{12} = \frac{2}{3}$ W

and $\quad R_A = W - R_B = W - \frac{2}{3} W = \frac{W}{3}$

B.M at C = $R_A \times 8$ = 8/3 W Nm

But B.M at C is maximum

$\therefore$ Max B.M $M_{max} = \frac{8}{3}$ W Nm

$= \frac{8}{3}$ W×1000 N mm

$= \frac{8000}{3}$ W Nmm

Now find the MOI of the given I section about the N.A

$\therefore \quad I = \frac{100\times 225^3}{12} - \frac{(100-7.5)\times(225-2\times 11.5)^3}{12}$

$= 31386647.45 mm^4$

Now using the relation, $\frac{M}{I} = \frac{\sigma}{y}$

$$y_{max} = \frac{225}{2} = 112.5 \text{ mm}$$

Now substituting the values, we get

$$= \frac{\left(\frac{8000}{3}W\right)}{31386647.45} = \frac{80}{112.5}$$

W = **8369.77 N**

Problem 2.23: Two circular beams where one is solid of dia. D and other is a hollow 0f outer dia. D_0 and the inner dia. D_i are of same length, same material and of same weight. Find the ratio of section modulus of these circular beams.

Given

Dia. Of solid beam = D

Dia. Of hollow beam = D_0 and D_i

Let L be the length W be the weight and ρ be the density

Solution:

Weight of solid beam = ρ ×g ×area of section × L

$$= \rho \times g \times L \times \pi/4D^2$$

Weight of hollow beam = ρ ×g ×area of section × L

$$= \rho \times g \times L \times \frac{\pi}{4}(D_0^2 - D_i^2)$$

But the weights are same $\rho \times g \times L \times \pi/4D^2 = \rho \times g \times L \times \frac{\pi}{4}(D_0^2 - D_i^2)$

$$D^2 = (D_0^2 - D_i^2) \qquad \text{……………..(1)}$$

Now the section modulus of solid section $Z = \frac{\pi}{32}D^3$

Section modulus of hollow section

$$Z_1 = \frac{\pi}{32D_0}(D_0^4 - D_i^4)$$

$$= \frac{\pi}{32D_0}(D_0^2 - D_i^2)(D_0^2 + D_i^2)$$

$$\therefore \frac{\text{Section modulus of solid section}}{\text{Section Modulus of hollow section}}$$

$$= \frac{\frac{\pi}{32}D^3}{\frac{\pi}{32D_0}(D_0^2-D_i^2)(D_0^2+D_i^2)}$$

$$= \frac{D^3 \times D_0}{(D_0^2-D_i^2)(D_0^2+D_i^2)} \quad = \frac{D \times D_0 \times D^2}{(D_0^2-D_i^2)(D_0^2+D_i^2)}$$

Substitute eqn.1 in the above calculation, then

$$= \frac{D \times D_0 \times (D_0^2-D_i^2)}{(D_0^2-D_i^2)(D_0^2+D_i^2)}$$

$$= \frac{D \times D_0}{D_0^2+D_i^2} \qquad \text{................(2)}$$

Also from eqn.1

$$D^2 = D_0^2 - D_i^2 \qquad \text{or} \qquad D_i^2 = D_0^2 - D^2$$

Substituting the value D_i^2 in the section modules ratio

$$\frac{\text{Section modulus of solid section}}{\text{Section Modulus of hollow section}} = \frac{D \times D_0}{D_0^2+D_0^2-D^2} = \frac{D \times D_0}{2D_0^2-D^2}$$

or

$$\frac{\text{Section modulus of hollow section}}{\text{Section Modulus of solid section}} = \frac{2D_0^2-D^2}{D \times D_0} = \frac{2D_0^2}{D \times D_0} - \frac{D^2}{D \times D_0}$$

$$= \frac{2D_0}{D} - \frac{D}{D_0}$$

Problem 2.24: A water main of 500mm internal dia and 20mm thick is running full. The water main is of cast iron and is supported at two points 10m apart. Find the max stress in the metal. The cast iron and water weigh 72000 N/m^3 respectively.

Given:

Internal diameter D_i = 500mm = 0.5m

Thickness of pipe t = 20mm 0.02

Outer diameter = $D_0 = D_i$ + 2×t = 0.5+2×0.02 = 0.54m

Length of pipe L = 10m

Weight density of cast iron = 72000 N/m^3

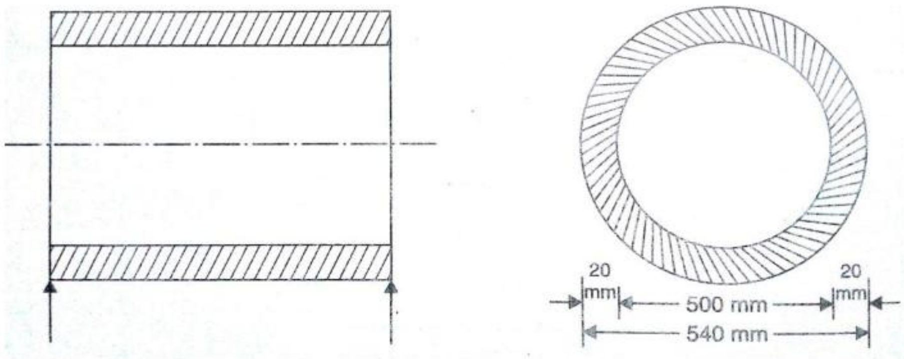
Weight density of water = 10000 N/m^3

Solution:

Internal area of pipe $= \frac{\pi}{4} D_i^2 = \frac{\pi}{4} \times 0.5^2 = 0.1960\ m^2$

This is equal to the Area of water section

∴ Area of water section = $0.1960\ m^2$



Outer area of pipe $= \frac{\pi}{4} D_0^2 = \frac{\pi}{4} \times 0.54^2\ m^2$

Area of pipe section $= \frac{\pi}{4} D_0^2 - \frac{\pi}{4} D_i^2$

$= \frac{\pi}{4}\left(D_0^2 - D_i^2\right) = \frac{\pi}{4}\left(0.54^2 - 0.5^2\right) = 0.0327 m^2$

Moment of Inertia of pipe section about the neutral axis

$$I = \frac{\pi}{64}(D_0^4 - D_i^4) = \frac{\pi}{64}(0.54^4 - 0.5^4) = 1.105 \times 10^9\ m^4$$

Let us now find the weight of pipe and weight of water for one meter length,

Weight of pipe for one meter length

= weight density of cast iron × volume of pipe

= 72000×area of pipe section ×length

= 72000×0.0327×1 (∵ length = 1m)

= 2354 N

Weight of water for one meter run

= wt. density of water × volume of water

= 10000× area of water section × length

= 10000×0.196×1 = 1960N

∴ Total weight on pipe for one meter run = 2354+1960 =4314N

Hence the above weight is the UDL on the pipe. the maximum bending moment due to UDL is $\frac{w \times L^2}{8}$, where w = rate of UDL = 4314N per meter length of beam.

∴ Max bending moment due to UDL M = $\frac{w \times L^2}{8}$

$$= \frac{4314 \times 10^2}{8} = 53925 \text{ Nm} = 5.392 \times 10^7 \text{Nmm}$$

Now using $\frac{M}{I} = \frac{\sigma}{y}$

∴ $\sigma = \frac{M}{I} \times y$

to find the maximum maxim um stress it will acts at y is maximum

$$y = \frac{D_0}{2} = \frac{540}{2} = 270\text{mm}$$

∴ $y_{max} = 270$ mm

∴ $\sigma_{max} = \frac{5.392 \times 10^7}{1.105 \times 10^9} \times 270 =$ **13.18N/mm^2**

2.25. BENDING STRESS IN UNSYMMETRICAL SECTION

In case of symmetrical section, the neutral axis passes through the geometrical centre of the section. but in case of unsymmetrical section such as L, T section, the neutral axis does not passes through the geometrical centre of the section. Hence the value of y for the topmost layer or bottom layer of the section from neutral axis will not be same. For finding the bending stress in the beam, the bigger value of y is used. As the neutral axis passes through the centre of gravity of the section, hence in unsymmetrical section, first the centre of gravity is calculated and followed to find other values.

Problem 2.25: A cast iron bracket subject to bending has the cross section of I form with unequal flanges. The dimensions of the sections are shown. Find the position of the neutral axis and M.O.I. of the section about the neutral axis. If the max bending moment on the section is 40 MN mm. Determine the max bending stress. What is the nature of the stress.

Given

Max. BM M = 40MN mm

Wkt, centroid about x axis $\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$

Where

A_1 = Area of bottom flange = $b_1 \times d_1$ = 130×50 = $6500mm^2$

y_1 = distance of C.G of A_1 from bottom f ace = $\frac{50}{2}$ = 25mm

A_2 = area of web = $b_2 \times d_2$ = 50×200 = 10000 mm^2

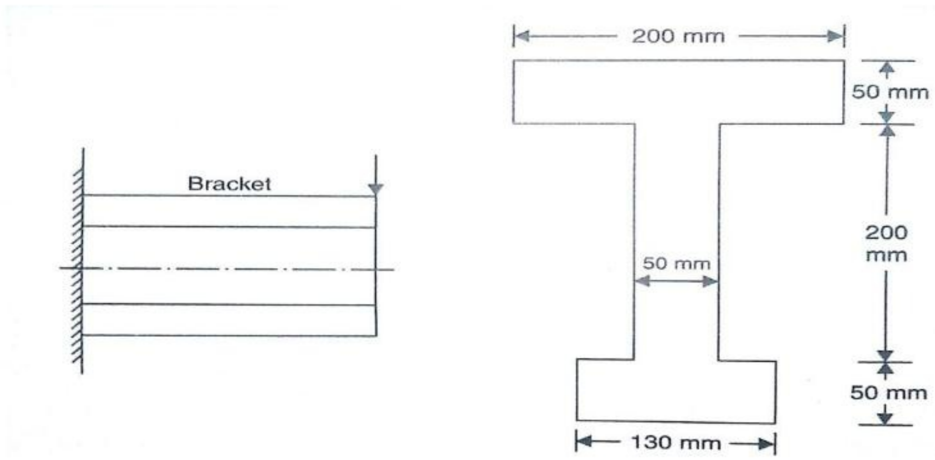
y_2 = distance of C.G of A_2 from bottom face = 50 + $\frac{200}{2}$ = 150 mm

A_3 = area of top flange = $b_3 \times d_3$ = 200×50 = $10000mm^2$

y_3 = dist of C.G of A_3 from bottom face = 50+200+$\frac{50}{2}$ = 275mm

Then,

$$\bar{y} = \frac{6500\times25+10000\times150+10000\times275}{6500+10000+10000} = 166.51\text{mm}$$



Hence the neutral axis is at a distance of 166.51mm from the bottom face

M.O.I of the section about the neutral axis I = $I_1 + I_2 + I_3$

Where I_1 = M.O.I of bottom flange about N.A

$= I_{C.G} + A_1 \times (distance\ of\ its\ C.G. from\ N.A.)^2$

$= \frac{b_1 \times {d_1}^3}{12} + A_1(\bar{y} - y_1)^2$

$= \frac{130\times50^3}{12} + 6500 \times (166.51 - 25)^2 = 131517186.6mm^4$

Similarly $I_2 = \frac{b_2 \times {d_2}^3}{12} + A_2(\bar{y} - y_2)^2$

$= \frac{50\times200^3}{12} + 10000 \times (166.51 - 150)^2$

= 3360913.43 mm^4.

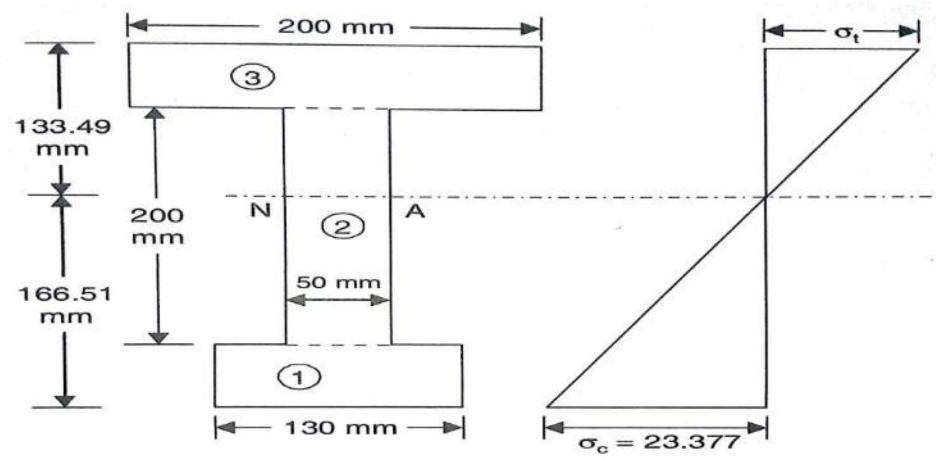
$$I_3 = \frac{b_3 \times {d_3}^3}{12} + A_3(\bar{y} - y_3)^2$$

$$= \frac{200 \times 50^3}{12} + 10000 \times (166.51 - 275)^2$$

$$= 119784134.3 mm^4.$$

∴ $\quad I = I_1 + I_2 + I_3 = 131517186.6 + 3360913.43 + 119784134.3$

$$= \mathbf{284907234.9 mm^4}$$



Now distance of C.G. from the upper top fibre

$$= 300 - \bar{y} = 300 - 166.51 = 133.49mm$$

Distance of C.G. from the bottom fibre = 166.51mm

Hence we shall take the value of y = 166.51mm for maximum bending stress.

Now using bending equation

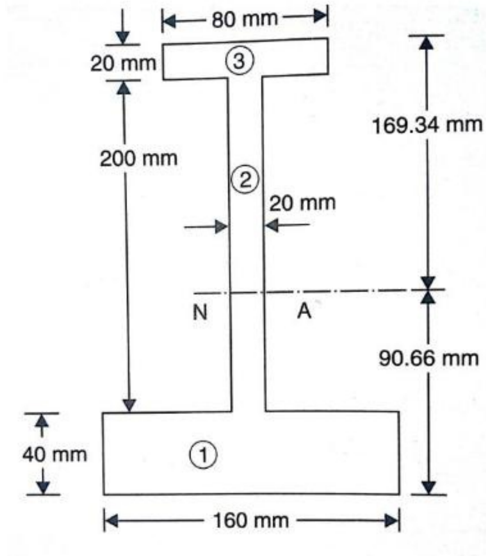
$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y = \frac{40 \times 10^6}{284907234.9} \times 166.51 = 23.377 \text{ N}/mm^2$$

∴Max bending stress = **23.377N/mm^2**

Problem 2.26: A cast iron beam is of I section. The beam is simply supported on a span of 5m. If the tensile stress is not to exceed 20N/mm^2. Find the safe uniformly load which the beam can carry. Find also the max compressive stress.

Given



Length $L = 5m$

Max tensile stress $\sigma_t = 20N/mm^2$

Solution:

Wkt, centroid about x axis $\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$

Where

A_1 = Area of bottom flange

$= b_1 \times d_1 = 160 \times 40$

$= 6400mm^2$

y_1 = distance of C.G of A_1 from bottom f ace

$= \frac{40}{2} = 20mm$

A_2 = area of web $= b_2 \times d_2 = 20 \times 200 = 4000\ mm^2$

y_2 = distance of C.G of A_2 from bottom face $= 40 + \frac{200}{2} = 140$ mm

A_3 = area of top flange $= b_3 \times d_3 = 80 \times 20 = 1600mm^2$

y_3 = dist of C.G of A_3 from bottom face $= 40 + 200 + \frac{20}{2} = 250mm$

Then,

$$\bar{y} = \frac{6400 \times 20 + 4000 \times 140 + 1600 \times 250}{6400 + 4000 + 1600} = 90.66mm$$

Hence the neutral axis is at a distance of 166.51mm from the bottom face or 260-90.66 = 169.34 mm from the top face.

M.O.I of the section about the neutral axis $I = I_1 + I_2 + I_3$

Where I_1 = M.O.I of bottom flange about N.A

$= I_{C.G} + A_1 \times (distance\ of\ its\ C.G.from\ N.A.)^2$

$= \frac{b_1 \times {d_1}^3}{12} + A_1(\bar{y} - y_1)^2$

$= \frac{160 \times 40^3}{12} + 6400 \times (90.66 - 20)^2 = 32807481.17mm^4$

Similarly $I_2 = \frac{b_2 \times {d_2}^3}{12} + A_2(\bar{y} - y_2)^2$

$= \frac{20 \times 200^3}{12} + 4000 \times (90.66 - 140)^2$

$= 23071075.73mm^4$

$I_3 = \frac{b_3 \times {d_3}^3}{12} + A_3(\bar{y} - y_3)^2$

$= \frac{80 \times 20^3}{12} + 1600 \times (90.66 - 250)^2$

$= 40676110.29mm^4$

∴ I = $I_1 + I_2 + I_3$ = 32807481.17+ 23071075.73+ 40676110.29

$= 96554667.21mm^4$

For a simply supported beam the tensile stress will be at the extreme bottom and the compressive stress will be at the extreme top fibre

Here max. tensile stress = 20N/mm^2

Hence for max tensile stress y = 90.66mm

Using $\frac{M}{I} = \frac{\sigma}{y}$

$M = \frac{\sigma}{y} \times I$

$= \frac{20}{90.66} \times 96554667.21$

= 21300389.85Nmm

Let w = uniformly distributed load in N/m on the simply supported beam

Max bending moment is at the centre and equal to wL^2/8

∴ M = w×5^2/8 Nm = w×25×1000/8 N mm = 3125w N mm

Equating the values of M

3125w = 21300389.85

w = **6816.125N/m**

Max compressive stress

Distance of extreme top fibre from N.A

y_c = 169.34mm

M = 21300389.85

I = 96554667.21

Let σ_c = max compressive stress

Using the relation $\frac{M}{I} = \frac{\sigma}{y}$

∴ $\sigma = \frac{M}{I} \times y = \frac{21300389.85}{96554667.21} \times 169.34$

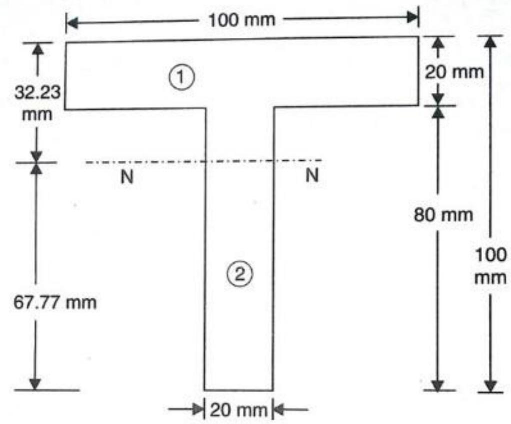
$\sigma_c = \mathbf{37.357\ N/mm^2}$

Problem 2.27: A cast iron beam is of T section as shown. The beam is simply supported on a span of 8m. The beam carries a uniformly distributed load of 1.5kN/m length on the entire span. Determine the max tensile and max compressive stress.

Given

Length L = 8m

U.D.L w = 1.5KN/m



Let y = distance of C.G. of section from the bottom

$Y = (A_{1Y_1} + A_{2Y_2})/(A_1 + A_2)$

$=(100\times20)\times(100\times20)\times(80+(20/2))+80\times20\times80/2)/(100\times2) + (80\times20)$

$= 67.77$mm

N.A lies at the distance of 67.77mm from the bottom face or 100 – 67.77 = 32.23mm from the top face

Moment of inertia $I = I_1 + I_2$

Where I_1 = M.O.I of top flange about N.A

$= \frac{100\times20^3}{12} + (100\times20)\times(32.23 - 10)^2$

$= 1055012.5 mm^4$

I_2 = M.O.I of web

$= 20\times 80^3/12 + (80\times20)\times(67.77 - 40)^2$

$= 2087209.9 mm^4$

$I = I_1 + I_2 = 1055012.5 + 2087209.9 mm^4$

$= 3142222.4 mm^4$

For simply supported beam the tensile stress will be at the extreme bottom and the compressive stress will be at the extreme top

Max B.M M = w× $L^2/8$ = 1500× $8^2/8$ = 12000 Nm

= 12000000 Nmm

Now using relation $\frac{M}{I} = \frac{\sigma}{y}$ ⋙ $\sigma = \frac{M}{I}$×y

1.For max tensile stress y = 67.77mm

$\sigma = \frac{12000000}{3142222.4}$×67.77= **258.81 N/$mm^2$**

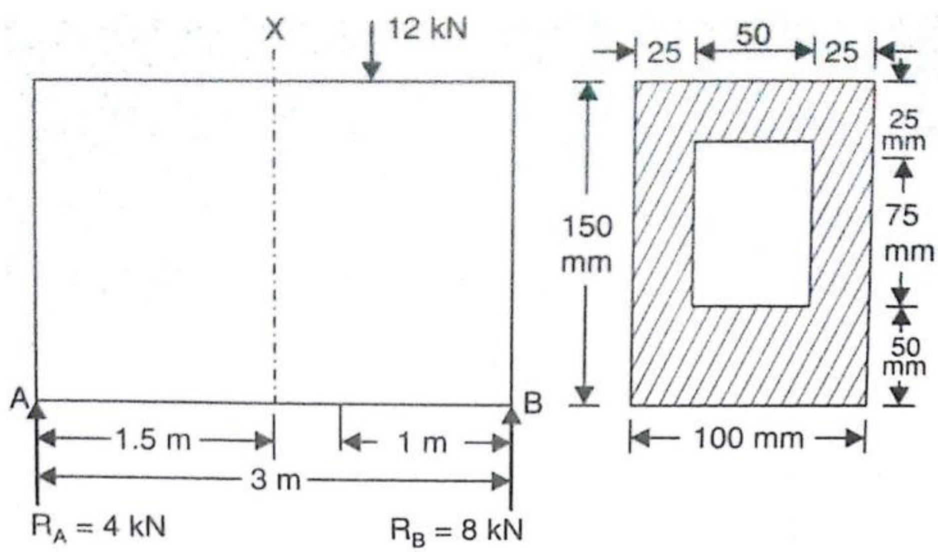
2.for max compressive stress y = 32.33mm

$\sigma = \frac{12000000}{3142222.4}$×32.23 = **123.08 N/$mm^2$**

Problem 2.28: A simply supported beam of length 3m carries a point load of 12k N at a distance of 2m from left support. The cross section of the beam is shown. Determine the max tensile and compressive stress at X-X

Given

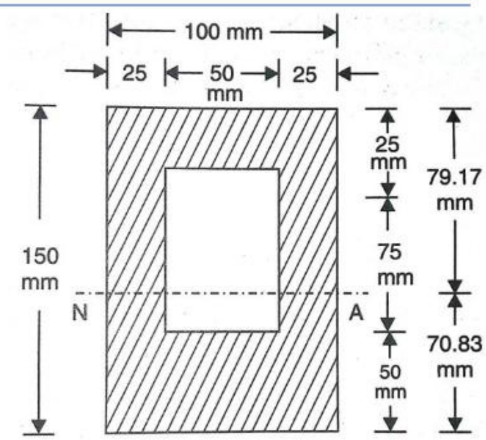
Point load w = 12 K N = 1200N



Solution:

First find the B.M at X-X.

Taking moments about A



R_B ×3 = 12×2

R_B = 8k N

R_A = W- R_B

= 4k N

B.M. at X-X = R_A ×1.5 = 4×1.5 = 6k Nm

= 6000×1000 N mm

M = 6000,000 N mm

Let y = distance of C.G. of the section from the bottom edge

= $(A_{1Y_1}$-$A_{2Y_2})/(A_1 - A_2)$

= (150×100) × 75 -(75×50) ×(50+ (75/2))/(150×100-75×50)

= 70.83mm

Hence N.A will lie at a distance of 70.83mm from the bottom edge or 150 – 70.83 = 79.17mm from the top edge

M.O.I of section I = I_1 - I_2

Where I_1 =M.O.I of outer rectangle about N.A

= M.O.I of rectangle 100×150 about its C.G + A_1

= 100× $150^3/12$ +100×150× $(75 - 70.83)^2$

= 28385833.5mm^4

I_2 = M.O.I of cut out part about N.A

= 50× $75^3/12$ +50×75×(50+75/2 – 70.83)

= 2799895.875mm^4

I = 28385833.5 – 2799895.875 = 25585937.63mm^4

The bottom edge of the section will be subjected to tensile stress whereas the top edge will be subjected to compressive stress. the top edge is at 79.17 mm from N.A whereas bottom edge is 70.83 mm from N.A

Now using relation $\frac{M}{I} = \frac{\sigma}{y}$ ⋙ $\sigma = \frac{M}{I}$×y

For max tensile stress y = 70.83mm

Max tensile stress $\sigma = \frac{6000000}{25585937.63}$×70.83=**16.60 N/$mm^2$**

For max compressive stress y = 79.17mm

$$\sigma = \frac{6000000}{25585937.63} \times 79.17 = \mathbf{18.56\ N/}\ \boldsymbol{mm^2}$$

2.26. STRENGTH OF A SECTION

The strength of a section means the moment of resistance offered by the section and moment of resistance is given by

$$M = Z \times \sigma$$

Where M = moment of resistance

σ = bending stress

Z = section modulus

Problem 2.29: Three beams have the same length same allowable bending stress and the same bending moment. The cross section of the beam are a square rectangle with depth twice the width and a circle. Find the radius of weights of the circular and the rectangular beams with respect to square beams.

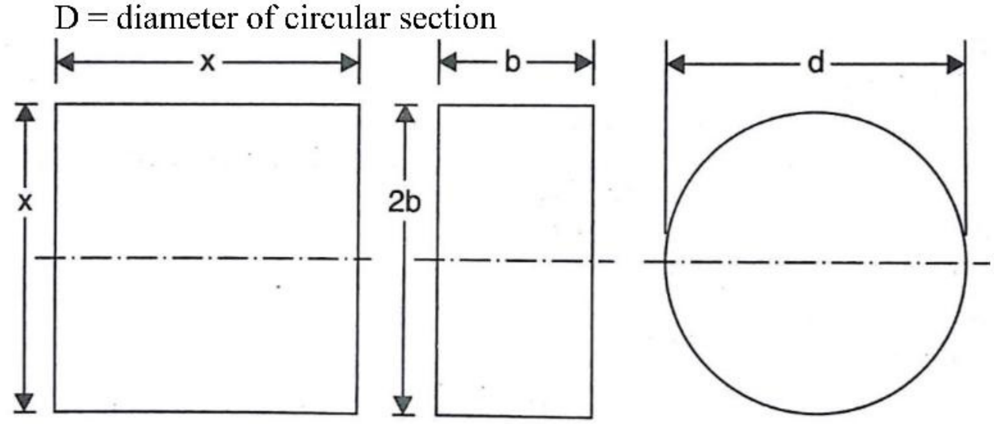
Given

Let x = side of square beam

b = width of rectangular beam

2b = depth of rectangular beam

D = diameter of circular section



Solution:

Moment of resistance of beam M = Z × σ

Where Z = section modulus

Section modulus of square beam = $\frac{I}{y} = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$

$$= \frac{x \times x^3}{12} \times \frac{2}{x} \qquad (\because \quad b = d = x)$$

$$= \frac{x^3}{6}$$

Section modulus of rectangular beam $= \frac{\frac{b\times(2b)^3}{12}}{\frac{2b}{2}}$

$$= \frac{2}{3} b^3$$

Section modulus of a circular beam $= \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^3}{32}$

Equating the section modulus of square beam with that of rectangular beam

$$\frac{x^3}{6} = \frac{2}{3} b^3$$

$$b^3 = 0.25x^3$$

$$b = 0.63x$$

Equating the section modulus of square beam with that of a circular beam

$$\frac{x^3}{6} = \frac{\pi d^3}{32}$$

$$d^3 = \frac{32x^3}{6\pi}$$

$$d = 1.1927x$$

Weight of beams are proportional to their cross sectional areas. Hence

$$\frac{Weight\ of\ rectangular\ beam}{weight\ of\ square\ beam} = \frac{Area\ of\ rectangular\ beam}{Area\ of\ square\ beam}$$

$$\frac{b \times 2b}{x \times x} = \frac{0.63x \times 2 \times 0.63x}{x \times x} = \mathbf{0.7938}$$

$$\frac{Weight\ of\ rectangular\ beam}{weight\ of\ square\ beam} = \frac{Area\ of\ rectangular\ beam}{Area\ of\ square\ beam}$$

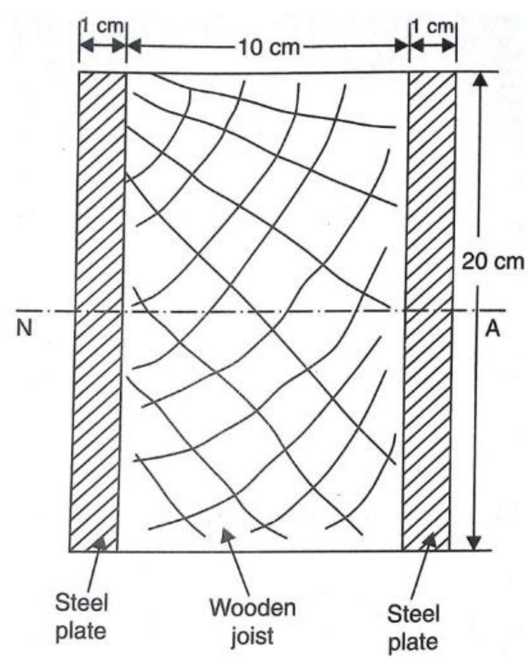
$$= \frac{\frac{\pi d^2}{4}}{x^2} = \frac{\pi d^2}{4\,x^2} \qquad (\because d = 1.1927\,x)$$

$$= \frac{\pi\,(1.1927\,x)^2}{4\,x^2} = \mathbf{1.1172}$$

2.27. COMPOSITE BEAMS (FLITCHED BEAMS)

A beam made up of two or more different materials assumed to be rigidly connected together and behaving like a single piece is known as a composite beam or a wooden flitched beam. The strain at the common surface will be same for both materials. Also the total moment of resistance will be equal to the sum of the moments of individual sections.

Problem 19. a flitched beam consist of a wooden joist 10cm wide and 20cm deep strengthed by two steel plates 10mm thick and 20cm deep. If the max stress in the wooden joist is 7 N/mm^2. Find the corresponding max stress attained in steel. Find also the moment of resistance of the composite section. Take youngs modulus for steel = 2× 10^5 N/mm^2 and for wood = 1× 10^4 N/mm^2



Given

Let width of wooden joist b_2 = 10cm

Depth of wooden joist d_2 = 20cm

Width of one steel plate b_1 = 1cm

Depth of one steel plate d_1 = 20cm

Number of steel plate = 2

Max stress in wood σ_2 = 7N/mm^2

E for steel E_1 = 2× 10^5 N/mm^2

E for wood E_2 = 1× 10^4 N/mm^2

Solution:

M.O.I. of wooden joist about N.A.

$$I_2 = b_{2\,d_2^2} / 12 = 6666.66 cm^4$$

M.O.I of two steel plates about N.A

$$I_2 = 2\times b_{1d_1^3} = 1333.33\times 10^4\ mm^4$$

Now using $\sigma_{1/E_1} = \sigma_{2/E_2}$

$$\sigma_1 = 20\times7 = \mathbf{140\ N/mm^2}$$

Total moment M = $M_1 + M_2$

Where $M_1 = \frac{\sigma_1}{y}\times I_1$

$= \frac{140}{100}\times1333.33\times 10^4$

$= 18666.620$ Nm

$M_2 = \sigma_{\frac{2}{y}} \times I_2$

$= \frac{7}{100}\times 6666.66\times 10^4$ N mm

$= 4666.662$ Nm

M = $M_1 + M_2$

$= 18666.620 + 4666.662$

= **23333.282 Nm**

IMPORTANT TERMS

Shear force	Adding of vertical forces from right side to the consider point of the beam Symbol: Downward force = + ve Upward force = - ve	**Diagram:** Point load (W) = vertical line (upward force = downward line Downward force = upward line) UVL (w) – Inclined line UVL (w) – parabolic curve Cantilever Beam : +ve side SSB : + ve or – ve OHB : + ve or – ve
Bending moment	Adding of bending moment from right side to the consider point of the beam. Symbol: Clockwise direction = - ve	**Diagram:** Point load (W) – Inclined line (upward force = downward line Down force = upward line

	Anticlockwise direction = +ve CLB : free end = 0 SSB : Both end = 0 OHB : Both end = 0	UVL (w) – parabolic curve UVL (w) – Cubic Curve Cantilever Beam : - ve side SSB : + ve OHB : + ve or – ve
Cantilever Beam	Adding of vertical forces	PL = add only W UDL = Add (Force x distance)
SSB	Step 1: To find reaction forces at two support (R_A, R_B) Take moment about A = 0 to find Reaction R_B Sum of upward force = downward force; to find reaction R_A	UDL acting point = midpoint = *l*/2 UVL = add (w*l*/2) UVL acting point from small end = 2*l*/3 UVL acting point from big end = *l*/3
OHB	Same procedure as SSB SF with Reaction & without reaction calculate	**Maximum bending moment** at shear force become zero (SF = 0) **Point of contrafluxture** act at Bending moment become zero (BM = 0)
BENDING STRESS IN BEAM		
Bending Equation	$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}} = \frac{E}{R}$ Based on type of beam with support to find M which is available in <u>**IV unit table**</u>	$M = Bending\ Moment$ $I = Moment\ of\ Inertia$ $\sigma = Bending\ stress$ $y = distance\ of\ Neutral\ axis$ $E = Youngs\ modulus$ $R = Bending\ radius$
Section Modulus	$Z = \frac{I}{y}$ $= \frac{bd^2}{6}\ for\ Rectangular\ section$ $= \frac{1}{6D}(BD^3 - bd^3)\ hollow\ Rect$	$= \frac{\pi d^3}{32}\ for\ circular\ section$ $= \frac{\pi}{32D}(D^4 - d^4)\ hollow\ circlr$

For Unsymmetrical section	Step1: to find C.G of the section in y direction = $\bar{y}$ but max value of y is used in bending eqn. Step2: to find Moment of inertia of the section = I Step3: from Moment eqn to find unknown value	
Moment of resistance of a section	$M = \sigma \, x \, Z$	
Composite beam(Flitched beams)	Strain remains same $e_1 = e_2 = \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$	
Modular Ratio	$= \frac{E_1}{E_2}$	

THEORETICAL QUESTIONS

1. Define the terms : bending stress in a beam , neutral axis and section modulus.
2. What do you mean by 'simple bending' or 'pure bending' ? What are the assumptions made in the theory of simple bending ?
3. Derive an expression for bending stress at a layer in a beam.
4. What do you understand by neutral axis and moment of resistance ?
5. Prove that relation,

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M = Bending moment, I = M.O.I.

σ = Bending stress, y = Distance from N.A.

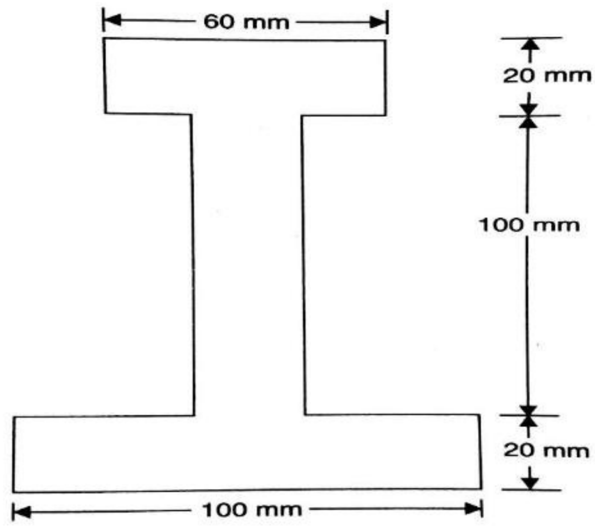
E = Young's modulus, and R = Radius of curvature.

6. What do you mean by section modulus ? Find expression for section modulus for a rectangular, circular and hollow circular sections.
7. How would you find the bending stress in unsymmetrical section ?
8. What is the meaning of 'Strength of a section' ?
9. Define and explain in terms : modular ratio, flitched beams and equivalent section.
10. What is the procedure of finding bending stresses in case of flitched beams when it is of
    i. A symmetrical section and
    ii. An unsymmetrical section ?
11. Explain the terms : Neutral axis, section modulus, and moment of resistance.

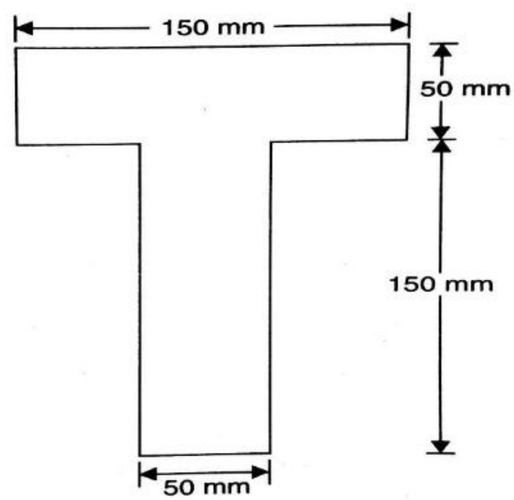
12. Show that for a beam subjected to pure bending , neutral axis coincides with the centroid of the cross- section.
13. Prove that the bending stress in any fibre is proportional to the distance of that fibre from neutral layer in a beam.

NUMERICAL PROBLEMS

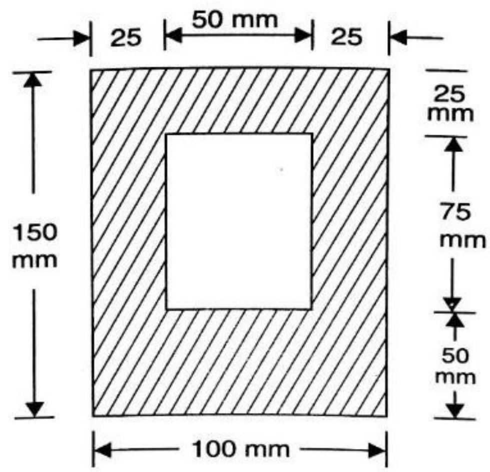
1. A steel plate of width 60 mm and of thickness 10 mm is bent into a circular arc of radius 10 m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2 \times 10^5$ N/mm^2.
2. A cast iron pipe of external diameter 60 mm, internal diameter of 40 mm, and of length 5 m is supported at its ends. Calculate the maximum bending stress induced in the pipe if it carries a point load of 100 N at its centre.
3. A rectangular beam 300 mm deep is simply supported over a span of 4 m. What uniformly distributed load per metre, the beam may carry if the bending stress is not to exceed
   120 N/mm^2 ? Take $I = 8 \times 10^5$ mm^4.
4. A cast iron cantilever of length 1.5 metre fails when a point load W is applied at the free end. If the section of the beam is 40 mm × 60 mm and the stress at the failure is 120 N/mm^2,find the point load applied.
5. A cast iron beam 20 mm × 20 mm in section and 100 cm long is simply supported at the ends. It carries a point load W at the centre. The maximum stress induced is 120 N/mm^2. What uniformly distributed load will break a cantilever of the same material 50 mm wide, 100 mm deep and 2 m long ?
6. A timber beam is 120 mm wide and 200 mm deep and is used on a span of 4 metres. The beam carries a uniformly distributed load of 2.8 kN/m run over the entire length. Find the maximum bending stress induced.
7. A timber cantilever 200 mm wide and 300 mm deep is 3 m long. It is loaded with a U.D.L. of 3 kN/m over the entire length. A point load of 2.7 kN is placed at the free end of the cantilever. Find the maximum bending stress produced.
8. A timber beam is freely supported on supports 6 m apart. It carries a uniformly distributed load of 12 kN/m run and a point load of 9 kN at 3.5 m from the right support. Design a suitable section of the beam making depth twice the width, if the stress in timber is not to exceed
   8 N/mm^2.
9. A beam of an I – section shown in Fig. is simply supported over a span of 4 metres. Determine the load that the beam can carry per metre length, if the allowable stress in the beam is 30.82 N/mm^2.



10. A beam is of T-section as shown in Fig. The beam is simply supported over a span of 4 m and carries a uniformly distributed load of 1.7 kN/m run over the entire span. Determine the maximum tensile and maximum compressive stress.



11. A simply supported beam of length 4 m carries a point load of 16 kN at a distance of 3 m from left support. The cross-section of the beam is shown in Fig. Determine the maximum tensile and compressive stress at a section which is at a distance of 2.25 m from the left support.



12. Prove that the moment of resistance of a beam of square section is equal to $\sigma \times \frac{x^3}{6}$ where 'σ' is the permissible stress in bending, x is the side of the square beam and beam is placed such that its two sides are horizontal.
13. Find the moment of resistance of the above beam, if it is placed such that its one diagonal is vertical , the permissible bending stress is same (i.e., equal to 'σ').
14. A flitched beam consists of a wooden joist 150 mm wide and 300 mm deep strengthened by a steel plate 12 mm thick and 300 mm deep on either side of the joist. If the maximum stress in the wooden joist is 7 N/mm^2 , find the corresponding maximum stress attained in steel. Find also the moment of resistance of the composite section. Take E for steel = 2 × 10^5 N/mm^2 and for wood = 1 × 10^4 N/mm^2.
15. A timber beam 60 mm wide by 80 mm deep is to be reinforced by bolting on two steel flitches, each 60 mm by 5 mm in section. Find the moment of resistance in the following cases: (i) flitches attached symmetrically at top and bottom ; (ii) flitches attached symmetrically at the sides. Allowable timber stress is 8 N/mm^2. What is the maximum stress in the steel in each case ? Take E for steel = 2.1 × 10^5 N/mm^2 and for timber = 1.4 × 10^4 N/mm^2.
16. Two rectangular plates, one of steel and the other of brass each 37.5 mm by 10 are placed to either to from a beam 37.5 mm wide by 20 mm deep, on two supports 75 cm apart, the brass component being on top of the steel component. Determine the maximum central load if the plates are (i) separate and can bend independently , (ii) firmly secured throughout their length. Permissible stresses for brass and steel are 70 N/mm^2 and 100 N/mm^2. Take $E_b = 0.875 \times 10^5$ N/mm^2 and $E_s = 2.1 \times 10^5$ N/mm^2.