EMF Equation of an Alternator

Let Φ = Flux per pole, in Wb P =
Number of poles
N_s = Synchronous speed in r.p.m.
f = Frequency of induced e.m.f. in Hz Z =
Total number of conductors
Z_{ph} = Conductors per phase connected in series
∴ Z_{ph} = Z/3 as number of phases = 3. Consider a single conductor placed in a slot.
The average value of e.m.f. induced in a conductor
= dΦ/dt
For one revolution of a conductor,
e_{avg} per conductor = (Flux cut in one revolution)/(time taken for one revolution) Total flux cut in one revolution is Φ x P
Time taken for one revolution is 60/N_s seconds.
∴ e_{avg} per conductor = ΦP / (60/N_s)
= Φ (PN_s/60).....................(1)
But f = PN_s/6120 .... PN_s/60= 2f
Substation in (1),
e_{avg} per conductor = 2 f Φ volts
Assume full pitch winding for simplicity i.e. this conductor is connected to a conductor which is 180_o electrical apart. So there two e.m.f.s will try to set up a current in the same direction i.e. the two e.m.f. are helping each other and hence resultant e.m.f. per turn will be twice the e.m.f. induced in a conductor e.m.f. per
turn = 2 x (e.m.f. per conductor)
= 2 x (2 f Φ)
= 4 f Φ volts
Let T_{ph} be the total number of turn per phase connected in series. Assuming concentrated winding, we can say that all are placed in single slot per pole per phase. So induced e.m.f.s in all turns will be in phase as placed in single slot. Hence net e.m.f. per phase will be algebraic sum of the e.m.f.s per turn.
∴ Average E_{ph} = T_{ph} x (Average e.m.f. per turn) Average Eph = T_{ph} x 4 f Φ
But in a.c. circuits R.M.S. value of an alternating quantity is used for the analysis. The form factor is 1.11 of sinusoidal e.m.f.
K_f = (R.M.S.)/Average = 1.11 .... for sinusoidal ∴ R.M.S. value of E_{ph} = K x Average value E = 4.44 x f Φ T_{ph} volts...(2)
Note : This is the basic e.m.f. equation for an induced e.m.f. per phase for full pitch, concentrated type of winding.
Where T_{ph} = Number of turns per phase T_{ph} = Z_{ph} /2 .... as 2 conductors constitute 1 turn
But as mentioned earlier, the winding used for the alternators is distributed and short pitch hence e.m.f. induced slightly gets affected. Let us see now the effect of distributed and short pitch type of winding on the e.m.f. equation

Pitch Factor or Coil Span Factor (Kc)

In practice short pitch coils are preferred. So coil is form by connecting one coil side to another which is less than one pole pitch away. So actual span is less than 180_o . The coil is generally shorted by one or two slots.

Note : The angle by which coil are short pitched is called angle or short pitched is called angle of short pitch denoted as 'α'.

α = Angle by which coils are short pitched. As coils are shorted in terms of number of slots i.e. either by one slot, two slots and so on and slot angle is β then angle of short pitch is always a multiple of the slot angle β.

α = β x Number of slots by which coils are short pitched. or **α** = 180_o -Actual coil span of the coils.

This is shown in the Figure1.14

Now let E be the induced e.m.f. in each coil side. If coil is full pitch coil, the induced e.m.f. in each coil side help each other. Coil connections are such that both will try to set up a current in the same direction in the external circuit. Hence the resultant e.m.f. across a coil will be algebraic sum of the two.

∴ $E_R = E + E = 2E$ ....for full pitch

Now the coil is short pitched by angle 'α', the two e.m.f. in two coil sides no longer remains in phase from external circuit point of view. Hence the resultant e.m.f. is also no longer remains algebraic sum of the two but becomes a phasor sum of the two Obviously E_R in such a case will be less than what it is in case of full pitch coil.

From the geometry of the Figure 1.14 we can write, AC is perpendicular drawn on OB bisecting OB.

∴ l (OC) = l (CB) = E_R /2

and

∟BOA = **α**/2

∴ cos (**α**/2) = OC/OA = $E_R/2E$

∴ $E_R = 2E\cos(\alpha/2)$ .... For short pitch

This is the resultant e.m.f. in case of a short pitch coil which depends on the angle of short pitch 'α'.

Note : Now the factor by which, induced e.m.f. gets reduced due to short pitching is called pitch factor or coil span factor denoted by K_c.

It is defined as the ratio of resultant e.m.f. when coil is short pitch to the resultant

$$K_c = \frac{E_R \text{ when coil is short pitched}}{E_R \text{ when coil is full pitched}} = \frac{2E\cos\left(\frac{\alpha}{2}\right)}{2E}$$

$$\boxed{K_c = \cos\left(\frac{\alpha}{2}\right)}$$

e.m.f. when coil is full pitched. It is always less than one.

where α = Angle of short pitch

Considering full pitch, concentrated winding.

$$\boxed{E_{ph} = 4.44\ K_c\ K_d\ f\ \phi\ T_{ph}\ \text{volts.}}$$

E_{ph} = 4.44 f Φ T_{ph}Volts.

But due to short pitch, distributed winding used in practice, this will reduce by factors and . So generlised expression for e.m.f. equation can be written as For full pitch coil, K_c = 1.

For concentrated winding K_d = 1.

Note : For short pitch and distributed winding K_c and K_d are always less than unity**.**

Parameters of Armature Winding

There are three important parameters of an armature winding of an alternator. These are,

1. Armature resistance R_a
2. Armature leakage reactance X_L
3. Reactance corresponding to armature reaction
4. Armature Resistance

Every armature winding has its own resistance. The effective resistance of an armature winding per phase is denoted as R_{aph} Ω/ph or R_a Ω/ph. Generally the armature resistance is measured by applying the known d.c. voltage and measuring the d.c. current through it. The ratio of applied voltage and measured current is the armature resistance. But due to the skin effect, the effective resistance under a.c. conditions is more than the d.c. resistance. Generally the effective armature resistance under a.c. conditions is taken 1.25 to 1.75 times the d.c. resistance.

While measuring the armature resistance, it is necessary to consider how the armature winding is connected whether in star or delta. Consider a star connected armature winding as shown in the Figure1.14. When the voltage is applied across any two terminals of an armature winding, then the equivalent resistance is the series combination of the two resistance of two different phase windings,

∴ R_{RY} = Resistance between R-Y terminals

= $R_a + R_a = 2R_a$

where R_a = armature resistance per phase

∴ $R_a = R_{RY}/2$ Ω/ph

Thus in star connected alternator, the armature resistance per phase is half of the resistance observed across any two line terminals.

Consider the delta connected alternator as shown in the When voltage is applied across any two terminals, then one phase winding appears in parallel with series combination of other two.

Hence the equivalent resistance across the terminals is parallel combination of the resistance R_a and $2R_a$.

∴ R_{RY} R_a || R_a Ω/ph

Thus in delta connected alternator, the armature resistance per phase is to be calculated from the equivalent resistance observed across any two line terminals.

Armature Leakage Reactance

When armature carries a current, it produces its own flux. Some part of this flux completes its path through the air around the conductors itself. Such a flux is called leakage flux. This is shown in the Figure1.15

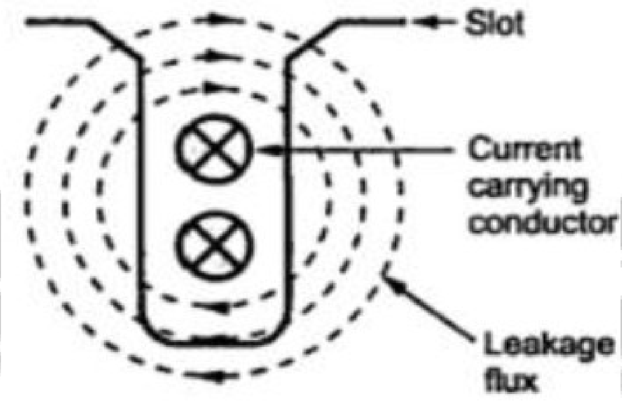

Figure 1.15 Armature leakage reactance

Note : This leakage flux makes the armature winding inductive in nature. So winding possesses a leakage reacatnce, in addition to the resistance. So if 'L' is the leakage inductance of the armature winding per phase, then leakage reactance per phase is given by $X_L = 2 \pi f L \ \Omega/ph$. The value of leakage reactance is much higher than the armature resistance. Similar to the d.c. machines, the value of armature resistance is very very small.

Armature Reaction

When the load is connected to the alternator, the armature winding of the alternator carries a current. Every current carrying conductor produces its own flux so armature of the alternator also produces its own flux, when carrying a current. So there are two fluxes present in the air gap, one due to armature current while second is produced by the filed winding called main flux. The flux produced by the armature is called armature flux.

Note : So effect of the armature flux on the main flux affecting its value and the distribution is called armature reaction.

The effect of the armature flux not only depends on the magnitude of the current flowing through the armature winding but also depends on the nature of the power factor of the load connected to the alternator.

Now we will study the effect of nature of the load power factor on the armature reaction.

Unity Power Factor Load

Consider a purely resistive load connected to the alternator, having unity power factor. As induced e.m.f. E_{ph} drives a current of I_{aph} and load power factor is unity, E_{ph} and I_{ph} are in phase with each other.

If Φ_f is the main flux produced by the field winding responsible for producing E_{ph}then E_{ph} lags Φ_f by 90_o .

Now current through armature I_a, produces the armature flux say Φ_a. So flux $\Phi_a$and I_a are always in the same direction. This relation between Φ_f , Φ_a, E_{ph} and I_{aph} can be shown in the phasor diagram.

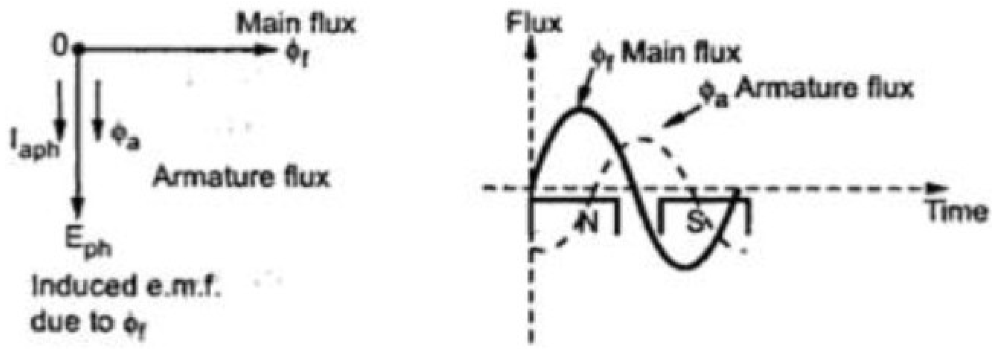

Figure 1.16 Phasor Diagram for Unity Power Factor Load

It can be seen from the phasor diagram that there exists a phase difference of 90_o between the armature flux and the main flux. The waveforms for the two fluxes are also shown in the Figure. 1.16 From the waveforms it can be seen that the two fluxes oppose each other on the left half of each pole while assist each other on the right half of each pole. Hence average flux in the air gap remains constant but its distribution gets distrorted.

Note : Hence such distorting effect of armature reaction under unity p.f. condition of the load is called cross magnetising effect of armature reaction.

Due to such distortion of the flux, there is small drop in the terminal voltage of the alternator.

Zero Lagging Power Factor Load

Consider a purely inductive load connected to the alternator having zero lagging power factor. This indicates that I_{aph} driven by E_{ph} lags E_{ph} by 90_o which is thepower factor angle Φ.

Induced e.m.f. E_{ph} lags main flux Φ_f by 90_o while Φ_a is in the same direction as that of I_a. So the phasor diagram and the waveforms are shown in the Figure1.17.

It can be seen from the phasor diagram that the armature flux and the main flux are exactly in opposite direction to each other.

Note : So armature flux tries to cancel the main flux. Such an effect of armature reaction is called demagnetising effect of the armature reaction.

As this effect causes reduction in the main flux, the terminal voltage drops. This drop in the terminal voltage is more than the drop corresponding to the unity p.f. load

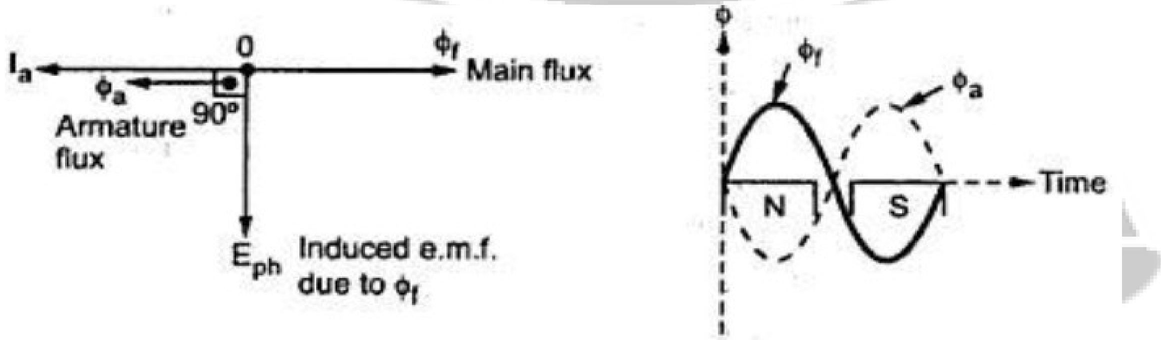

Figure 1.17 Zero Leading Power Factor Load

Zero Leading Power Factor Load Consider a purely capacitive load connected to the alternator having zero leading power factor. This means that armature current Iaph driven by Eph, leads Ephby 90o, which is the power factor angle Φ. Induced e.m.f. Eph lags Φf by 90o while Iaph and Φa are always in the same direction. The phasor diagram and the waveforms are shown in the FiIt can be seen from the phasor diagram and waveforms shown in the Figure 1.17, the armature flux and the main field flux are in the same direction i.e. they are helping each other. This results into the addition in main flux.

Note : Such an effect of armature reaction due to which armature flux assists field flux is called magnetising effect of the armature reaction.

As this effect adds the flux to the main flux, greater e.m.f. gets induced in the armature. Hence there is increase in the terminal voltage for leading power factor loads.

For intermediate power factor loads i.e. between zero lagging and zero leading the armature reaction is partly cross magnetising and partly demagnetising for lagging power factor loads or partly magnetising for leading power factor loads

Armature Reaction Reactance (X_{ar})

In all the conditions of the load power factors, there is change in the terminal voltage due to the armature reaction. Mainly the practical loads are inductive in nature, due to demagnetising effect of armature reaction, there is reduction in the terminal voltage. Now this drop in the voltage due to the interaction of armature and main flux. This drop is not across any physical element.

But to quantify the voltage drop due to the armature reaction, armature winding is assumed to have a fictitious reactance. This fictitious reactance of the armature is called armature reaction reactance denoted as X_{ar} Ω/ph. And the drop due to armature reaction can be accounted as the voltage drop across this reactance as $I_{ar}X_{ar}$.

Concepts of Synchronous Reactance and Impedance

The sum of fictitious armature reaction reactance accounted for considering armature reaction effect and the leakage reactance of the armature called synchronous reactance of the alternator demoted as X_s.

So $\mathbf{X_s = X_L + X_{ar}}$ **Ω/ph**

As both X_L and X_{ar} are ohmic values per phase, synchronous reactance is also specified as ohms per phase.

Now from this, it is possible to define an impedance of the armature winding. Such an impedance obtained by combining per phase values of synchronous reactance and armature resistance is called synchronous impedance of the alternator denoted as Z_s. So $\mathbf{Z_s = R_a + j\,X_s}$ **Ω/ph** And $\mathbf{|\,Z_s\,| = \sqrt{(R_{a2} + j\,(Xs)_2\,)}}$

For getting a standard frequency, alternator is to be driven at synchronous speed. So word synchronous used in specifying the reactance and impedance is referred to the working speed of the alternator. Generally impedance of the winding is constant but in case of alternator, synchronous reactance depends on the load and its power factor condition, hence synchronous impedance also varies with the load and its power factor conditions.