2.2 ANALYSIS OF CONTINUOUS BEAMS IN SLOPE DEFLECTION METHOD.

2.2.1 NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.8,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.

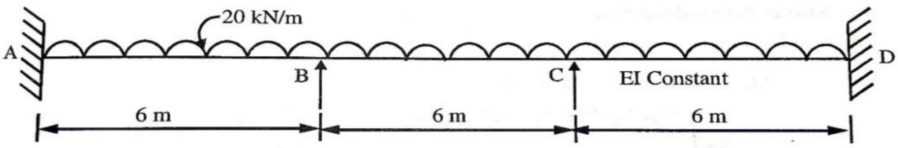


Fig. 2.8

Solutions:

- **Fixed End Moments:**
  MFAB = MFBC = MFCD = $-Wl^2/12$ = $- 20x6^2/12$ = - 60 kNm
  MFBA = MFCB = MFDC = $Wl^2/12 = 20x6^2/12$ = 60 kNm
- **Slope Deflection Equations:**

The structure is symmetrical.So is the loadind.There is no sinking of supports.Hence the following conditions prevail.

- $\theta A = \theta D = 0$
- $\delta = 0$ for all spans
- $\theta B = \theta C$
  Hence there is only one unknown displacement,namely θB.For span AB,the general slope deflection equation is

$$MAB = MFAB + 2EI/6(2\theta A + \theta B + 3\delta/l)$$

$$MAB = - 60 + 2EI/6(\theta B) \quad ----- (2.1)$$

Since $\theta A = 0$ and $\delta = 0$

$$MAB = 60 + 2EI/6(\theta B) \quad ----- (2.2)$$

No other slope deflection equation is needed.

Since θB is the only unknown.

For span BC,

$$MBC = MFBC + 2EI/6(2\theta B + \theta C + 3\delta/l)$$

$$MBC = - 60 + 2EI/6(3\theta B) \quad \text{----- (2.3)}$$

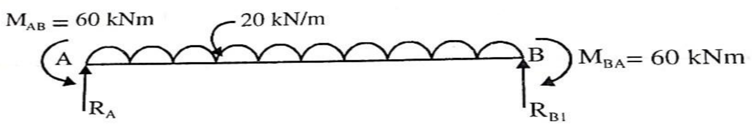
- **Joint Equilibrium Equations:**

MAB + MBC = 0

$60 + 2EI\theta B/3 – 60 + EI\theta B = 0$

Hence, θB = 0

- **Final Moments:**
  MAB = MBC = MCD = - 60 kNm
  MBA = MCB = MDC = 60 kNm
- **Shear Force Diagram:**
  Span AB:



Taking moments about A,on the free body diagram of span AB,

$$- RB1 \times 6 – MAB + MBA + wl^2/2 = 0$$

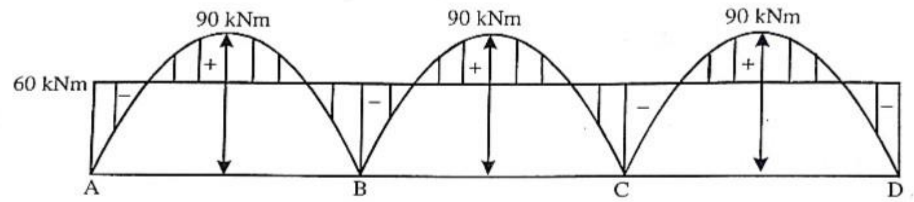
$$- RB1 \times 6 – 60 + 60 + 20 \times 6^2/2 = 0$$

RB1 = 60KN ; RA = 60KN

Similarly in span BC, RB2 = RC1 = 60

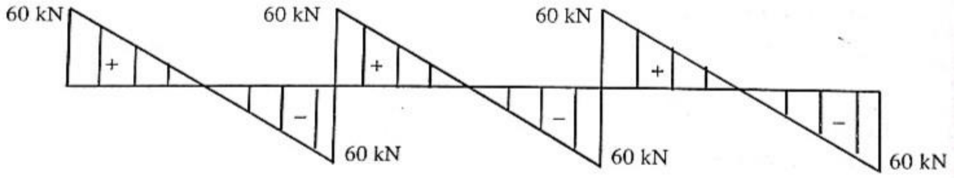
RB = RB1 + RB2 = 120 KN

- **BMD and SFD:**



Bending moment diagram

Simply supported span bending moment $= \frac{wl^2}{8} = \frac{20 \times 6^2}{8} = 90$ kNm



Shear force diagram

PROBLEM NO:02

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using slope deflection method.Support B sinks by 10mm.Take E = 2 x 10^5 N/mm^2,I = 16 x 10^7 mm^4.Sketch the SF and BM diagrams.

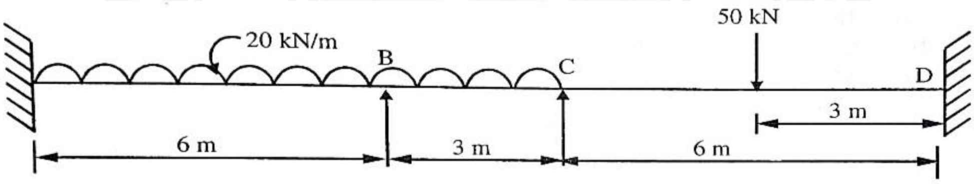


Fig. 2.10

Solution:

- **Fixed End Moments:**

  MFAB = -Wl^2/12 = - 20x6^2/12 = - 60 kNm;

  MFBA = Wl^2/12 = 20x6^2/12 = 60 kNm;

  MFBC = -Wl^2/12 = - 20x3^2/12 = - 15 kNm;

  MFCB = Wl^2/12 = 20x3^2/12 = 15 kNm;

  MFCD = -Wl/8 = - 50x6/8 = - 37.5 kNm;

  MFDC = Wl/8 = 50x6/8 = 37.5 kNm;

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

  = - 60 + EI/3(0 + θB – 1/200) --- (1)

  MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

$= 60 + EI/3(2\theta B – 3 \times 10/6000)$ --- (2)

$MBC = MFBC + 2EI/3(2\theta B + \theta C + 3\delta/l)$

$= - 15 + 2EI/3(2\theta B + \theta C + 1/100)$ --- (3)

$MCB = MFCB + 2EI/3(2\theta C + \theta B + 3\delta/l)$

$= 15 + 2EI/3(2\theta C + \theta B + 1/100)$ --- (4)

$MCD = MFCD + 2EI/6(2\theta C + \theta D + 3\delta/l)$

$= - 37.5 + EI/3(2\theta C)$ --- (5)

$MDC = MFDC + 2EI/6(2\theta D + \theta C + 3\delta/l)$

$= 37.5 + EI/3(\theta C)$ --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

$EI/3(6\theta B+2\theta C+3/200) = - 135$ --- (7)

Joint C:

MCB + MCD = 0

$EI(\theta B+3\theta C+1/100) = 33.75$ --- (8)

Equvating (7 & 8);we get

$\theta C = - 1/464$; $\theta B = - 1/402$

- **Final Moments:**

MAB = - 139.843 kNm;

MBA = - 46.354 kNm;

MBC = 46.3 kNm;

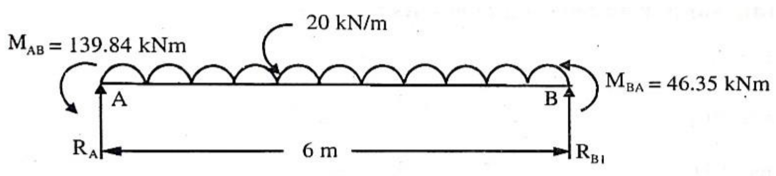
MCB = 83.35 kNm;

MCD = - 83.477 kNm;

MDC = 14.51 kNm;

- **To Draw S.F.D:**

Span AB:

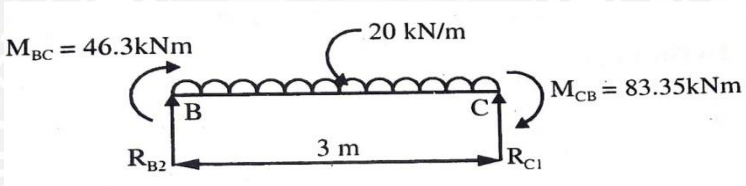


Taking moments about A.

$20 \times 6^2/2 - 46.35 - 139.84 - RB1(6) = 0$; RB1 = 28.97 KN

RA = 20 x 6 – 28.97; RA = 91.03 KN

Span BC:

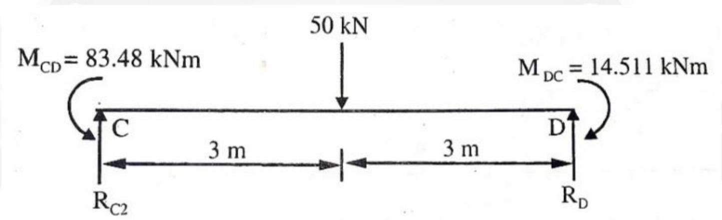


Taking moments about B.

$20 \times 3^2/2 + 83.35 + 46.3 - RC1(3) = 0$; RC1 = 73.22 KN

RB2 = 20 x 3 – 73.22; RB2 = - 13.21 KN

Span CD:



Taking moments about C.

14.511 + 50(3) – 83.48 – RD(6) = 0;

RD = 13.5 KN; RC2 = 36.5 KN

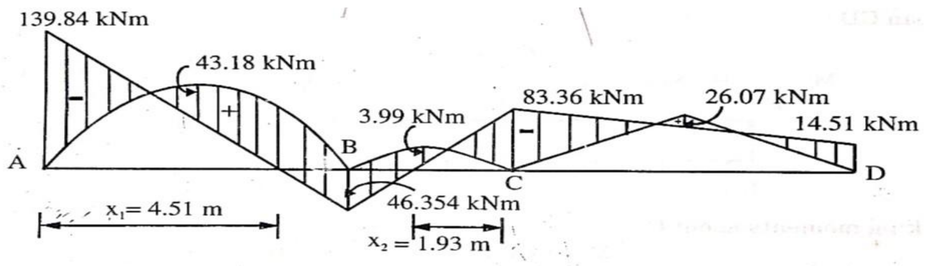
- **Free BMD:**

  MAB = $Wl^2/8 = 20 \times 6^2/8 = 90$ kNm

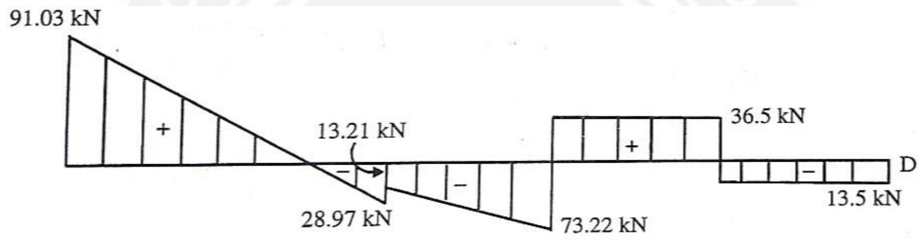
  MBC = $Wl^2/8 = 20 \times 3^2/8 = 22.5$ kNm

  MCD = $Wl/4 = 50 \times 6/4 = 75$ kNm

- **BMD and SFD:**



Bending moment diagram



Shear force diagram

PROBLEM NO:03

Analysis the continuous beam shown in fig.2.3,Calculate the support moments using slope deflection method. Sketch the BM diagrams.

2IAB = IBC = 2ICD = 2I

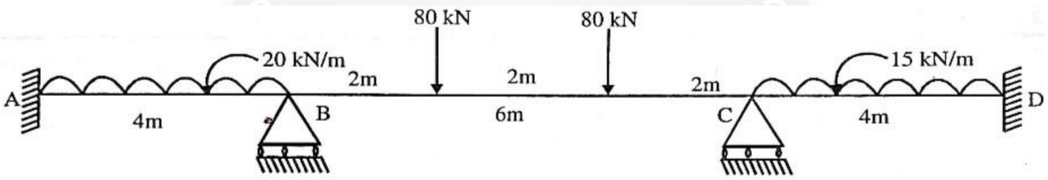


Fig. 2.3

IAB = ICD = I, IBC = 2I, $\theta A = \theta D = 0$ (A and D are fixed)

Solution:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12 = -20 \times 4^2/12 = -26.67$ kNm;

  MFBA = $Wl^2/12 = 20 \times 4^2/12 = 26.67$ kNm;

MFBC = -Wa(a + c)/6 = - 80(4 + 2)/6 = - 106.67 kNm;

MFCB = Wa(a + c)/6 = 80(4 + 2)/6 = 106.67 kNm;

MFCD = $-Wl^2/12$ = - $15x4^2/12$ = - 20 kNm;

MFDC = $Wl^2/12$ = $15x4^2/12$ = 20 kNm;

- **Slope Deflection Equations:**

MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

= - 26.67 + 0.5EIθB --- (1)

MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

= 26.67 + 0.5EIθB --- (2)

MBC = MFBC + 2EI/3(2θB + θC + 3δ/l)

= - 106.67 + EI (1.332θB + 0.666θC) --- (3)

MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

= 106.67 + EI (1.332θC + 0.666θB) --- (4)

MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

= - 20 + EIθC --- (5)

MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

= 20 + EI (0.5θC) --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

2.333θB + 0.666θC = 80/EI --- (7)

Joint C:

MCB + MCD = 0

0.666θB + 2.333θC = 86.67/EI --- (8)

Equvating (7 & 8);we get

θC = - 51.11/EI; θB = 48.88/EI;

- **Final Moments:**

MAB = - 2.23 kNm;

MBA = 75.55 kNm;

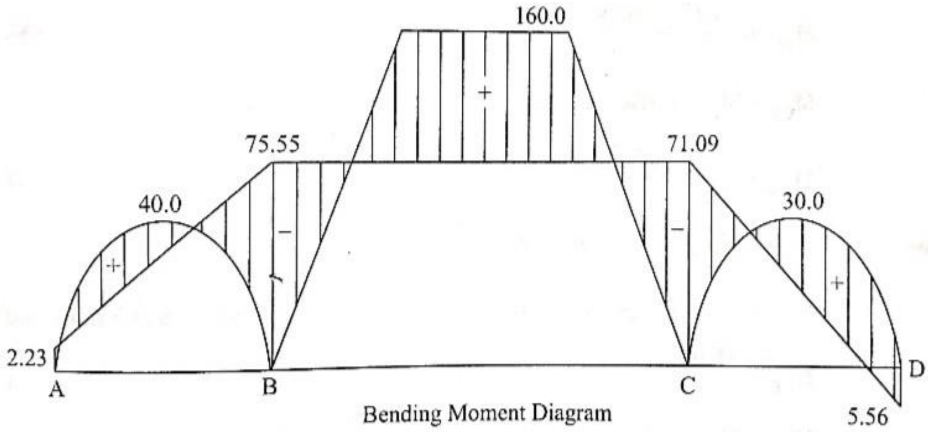
MBC = - 75.55 kNm;

MCB = 71.09 kNm;

MCD = - 71.09 kNm;

MDC = - 5.56 kNm;

- **BMD:**



Bending Moment Diagram