

2.2 Loop Antennas

Simple, inexpensive, and very versatile antenna type is the loop antenna. Loop antennas take many different forms such as a rectangle, square, triangle, ellipse, circle, and many other configurations. Because of the simplicity in analysis and construction, the circular loop is the most popular and has received the widest attention. It will be shown that a small loop (circular or square) is equivalent to an infinitesimal magnetic dipole whose axis is perpendicular to the plane of the loop. That is, the fields radiated by an electrically small circular or square loop are of the same mathematical form as those radiated by an infinitesimal magnetic dipole.

Loop antennas are usually classified into two categories, electrically small and electrically large. Electrically small antennas are those whose overall length (circumference) is usually less than about one-tenth of a wavelength ($C < \lambda/10$). However, electrically large loops are those whose circumference is about a free-space wavelength ($C \approx \lambda$). Most of the applications of loop antennas are in the HF (3–30 MHz), VHF (30–300 MHz), and UHF (300–3,000 MHz) bands. When used as field probes, they find applications even in the microwave frequency range.

Loop antennas with electrically small circumferences or perimeters have small radiation resistances that are usually smaller than their loss resistances. Thus they are very poor radiators, and they are seldom employed for transmission in radio communication. When they are used in any such application, it is usually in the receiving mode, such as in portable radios and pagers, where antenna efficiency is not as important as the signal to-noise ratio. They are also used as probes for field measurements and as directional antennas for radiowave navigation. The field pattern of electrically small antennas of any shape (circular, elliptical, rectangular, square, etc.) is similar to that of an infinitesimal dipole with a null perpendicular to the plane of the loop and with its maximum along the plane

of the loop. As the overall length of the loop increases and its circumference approaches one free-space wavelength, the maximum of the pattern shifts from the plane of the loop to the axis of the loop which is perpendicular to its plane.

The radiation resistance of the loop can be increased, and made comparable to the characteristic impedance of practical transmission lines, by increasing (electrically) its perimeter and/or the number of turns. Another way to increase the radiation resistance of the loop is to insert, within its circumference or perimeter, a ferrite core of very

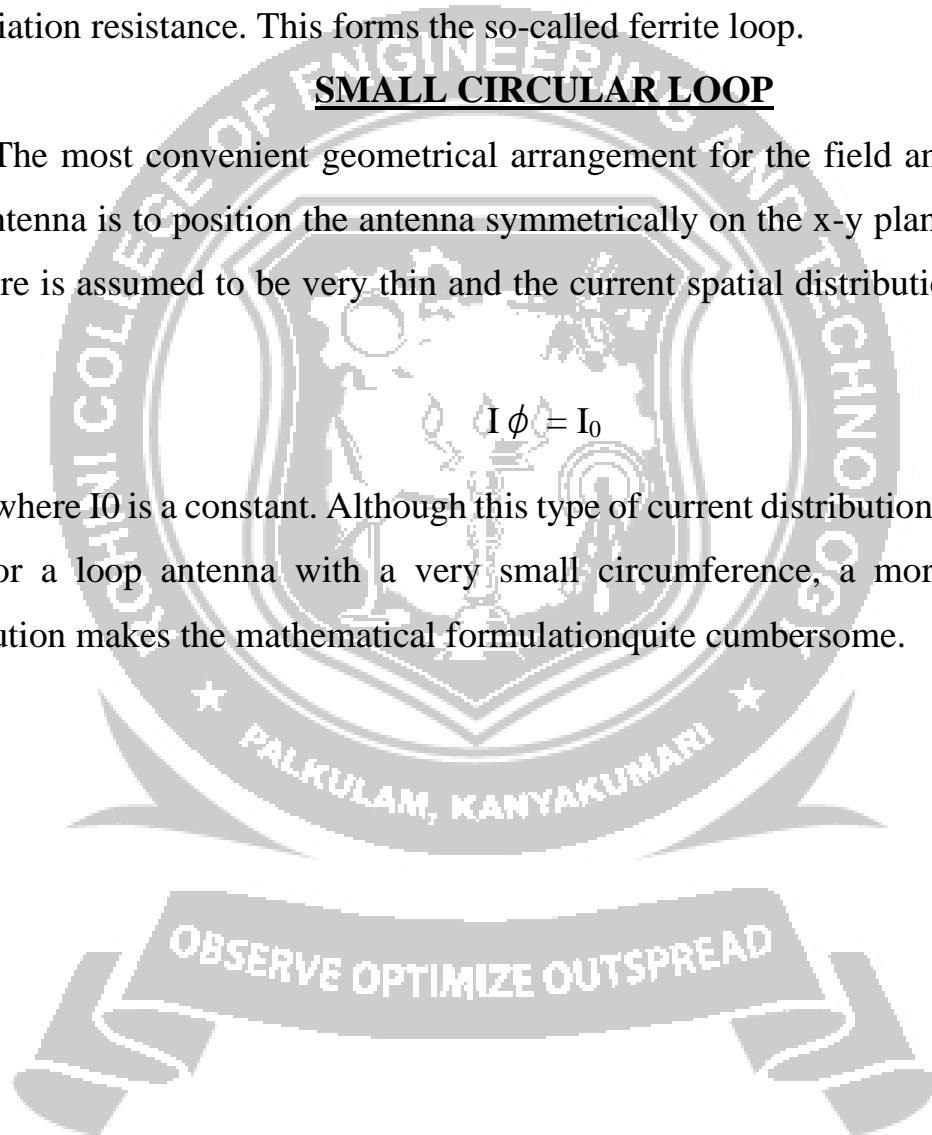
high permeability which will raise the magnetic field intensity and hence the radiation resistance. This forms the so-called ferrite loop.

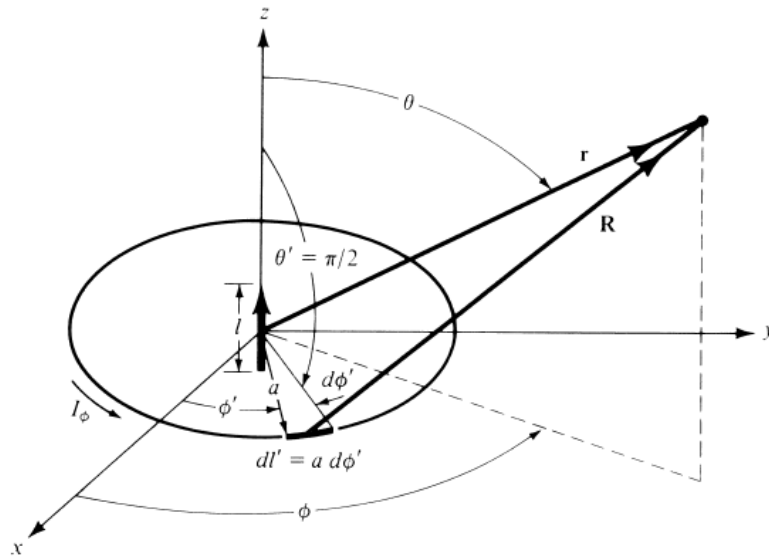
SMALL CIRCULAR LOOP

The most convenient geometrical arrangement for the field analysis of a loop antenna is to position the antenna symmetrically on the x-y plane, at $z = 0$. The wire is assumed to be very thin and the current spatial distribution is given by

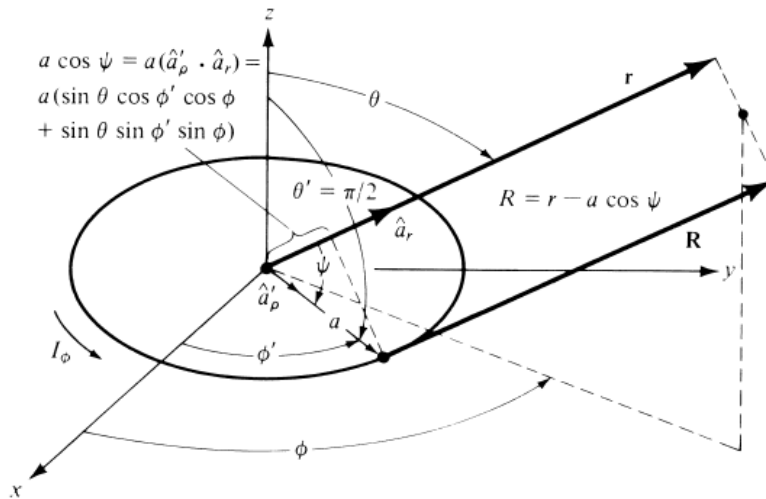
$$I \phi = I_0$$

where I_0 is a constant. Although this type of current distribution is accurate only for a loop antenna with a very small circumference, a more complex distribution makes the mathematical formulation quite cumbersome.





(a) Geometry for circular loop



(b) Geometry for far-field observations

Radiated Fields:

To find the fields radiated by the loop, the same procedure is followed as for the linear dipole. The potential function \mathbf{A} given by,

$$\mathbf{A}(x, y, z) = \frac{\mu}{4\pi} \int_C \mathbf{I}_e(x', y', z') \frac{e^{-jkR}}{R} dl'$$

R is the distance from any point on the loop to the observation point and dl' is an infinitesimal section of the loop antenna. In general, the current spatial distribution $\mathbf{I}_e(x', y', z')$ can be written as

OBSERVE OPTIMIZE OUTSPREAD

$$\mathbf{I}_e(x', y', z') = \hat{\mathbf{a}}_x I_x(x', y', z') + \hat{\mathbf{a}}_y I_y(x', y', z') + \hat{\mathbf{a}}_z I_z(x', y', z')$$

For the circular-loop antenna, whose current is directed along a circular path, it would be more convenient to write the rectangular current components in terms of the cylindrical components using the transformation

$$\begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix} = \begin{bmatrix} \cos \phi' & -\sin \phi' & 0 \\ \sin \phi' & \cos \phi' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_\rho \\ I_\phi \\ I_z \end{bmatrix}$$

which when expanded can be written as

$$\left. \begin{aligned} I_x &= I_\rho \cos \phi' - I_\phi \sin \phi' \\ I_y &= I_\rho \sin \phi' + I_\phi \cos \phi' \\ I_z &= I_z \end{aligned} \right\}$$

$$\left. \begin{aligned} \hat{\mathbf{a}}_x &= \hat{\mathbf{a}}_r \sin \theta \cos \phi + \hat{\mathbf{a}}_\theta \cos \theta \cos \phi - \hat{\mathbf{a}}_\phi \sin \phi \\ \hat{\mathbf{a}}_y &= \hat{\mathbf{a}}_r \sin \theta \sin \phi + \hat{\mathbf{a}}_\theta \cos \theta \sin \phi + \hat{\mathbf{a}}_\phi \cos \phi \\ \hat{\mathbf{a}}_z &= \hat{\mathbf{a}}_r \cos \theta - \hat{\mathbf{a}}_\theta \sin \theta \end{aligned} \right\}$$

$$\begin{aligned} \mathbf{I}_e &= \hat{\mathbf{a}}_r [I_\rho \sin \theta \cos(\phi - \phi') + I_\phi \sin \theta \sin(\phi - \phi') + I_z \cos \theta] \\ &+ \hat{\mathbf{a}}_\theta [I_\rho \cos \theta \cos(\phi - \phi') + I_\phi \cos \theta \sin(\phi - \phi') - I_z \sin \theta] \\ &+ \hat{\mathbf{a}}_\phi [-I_\rho \sin(\phi - \phi') + I_\phi \cos(\phi - \phi')] \end{aligned}$$

For the circular loop, the current is flowing in the ϕ direction (I_ϕ)

$$\mathbf{I}_e = \hat{\mathbf{a}}_r I_\phi \sin \theta \sin(\phi - \phi') + \hat{\mathbf{a}}_\theta I_\phi \cos \theta \sin(\phi - \phi') + \hat{\mathbf{a}}_\phi I_\phi \cos(\phi - \phi')$$

The distance R , from any point on the loop to the observation point, can be written as

$$R = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$x^2 + y^2 + z^2 = r^2$$

$$x' = a \cos \phi'$$

$$y' = a \sin \phi'$$

$$z' = 0$$

$$x'^2 + y'^2 + z'^2 = a^2$$

$$R = \sqrt{r^2 + a^2 - 2ar \sin \theta \cos(\phi - \phi')}$$

the differential element length is given by,

$$dl' = a d\phi'$$

$$A_\phi = \frac{a\mu}{4\pi} \int_0^{2\pi} I_\phi \cos(\phi - \phi') \frac{e^{-jk\sqrt{r^2+a^2-2ar \sin \theta \cos(\phi-\phi')}}}{\sqrt{r^2+a^2-2ar \sin \theta \cos(\phi-\phi')}} d\phi'$$

For small loops, the function

$$f = \frac{e^{-jk\sqrt{r^2+a^2-2ar \sin \theta \cos \phi'}}}{\sqrt{r^2+a^2-2ar \sin \theta \cos \phi'}}$$

$$f = f(0) + f'(0)a + \frac{1}{2!} f''(0)a^2 + \dots + \frac{1}{(n-1)!} f^{(n-1)}(0)a^{n-1} + \dots$$

$$f(0) = \frac{e^{-jkr}}{r}$$

$$f'(0) = \left(\frac{jk}{r} + \frac{1}{r^2} \right) e^{-jkr} \sin \theta \cos \phi'$$

$$f \simeq \left[\frac{1}{r} + a \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta \cos \phi' \right] e^{-jkr}$$

$$A_\phi \simeq \frac{a\mu I_0}{4\pi} \int_0^{2\pi} \cos \phi' \left[\frac{1}{r} + a \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta \cos \phi' \right] e^{-jkr} d\phi'$$

$$A_\phi \simeq \frac{a^2 \mu I_0}{4} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta$$

$$A_r \simeq \frac{a\mu I_0}{4\pi} \sin \theta \int_0^{2\pi} \sin \phi' \left[\frac{1}{r} + a \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta \cos \phi' \right] e^{-jkr} d\phi'$$

$$A_\theta \simeq -\frac{a\mu I_0}{4\pi} \cos \theta \int_0^{2\pi} \sin \phi' \left[\frac{1}{r} + a \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin \theta \cos \phi' \right] e^{-jkr} d\phi'$$

which when integrated reduce to zero. Thus

$$\begin{aligned} \mathbf{A} &\simeq \hat{\mathbf{a}}_\phi A_\phi = \hat{\mathbf{a}}_\phi \frac{a^2 \mu I_0}{4} e^{-jkr} \left[\frac{jk}{r} + \frac{1}{r^2} \right] \sin \theta \\ &= \hat{\mathbf{a}}_\phi j \frac{k \mu a^2 I_0 \sin \theta}{4r} \left[1 + \frac{1}{jkr} \right] e^{-jkr} \end{aligned}$$

the magnetic field components are,

$$H_r = j \frac{ka^2 I_0 \cos \theta}{2r^2} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_0 \sin \theta}{4r} \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2} \right] e^{-jkr}$$

$$H_\phi = 0$$

the corresponding electric-field components can be written as

$$E_r = E_\theta = 0$$

$$E_\phi = \eta \frac{(ka)^2 I_0 \sin \theta}{4r} \left[1 + \frac{1}{jkr} \right] e^{-jkr}$$

Problem:

Find the radiation resistance of a single-turn and an eight-turn small circular loop. The radius of the loop is $\lambda/25$ and the medium is free-space.

Solution:

$$S = \pi a^2 = \pi \left(\frac{\lambda}{25} \right)^2 = \frac{\pi \lambda^2}{625}$$

$$R_r \text{ (single turn)} = 120\pi \left(\frac{2\pi}{3} \right) \left(\frac{2\pi^2}{625} \right)^2 = 0.788 \text{ ohms}$$

$$R_r \text{ (8 turns)} = 0.788(8)^2 = 50.43 \text{ ohms}$$