Inverse Filtering:

The simplest approach to restoration is direct inverse filtering where we complete an estimate $\hat{F}(u,v)$ of the transform of the original image simply by dividing the transform of the degraded image G(u,v) by degradation functionH(u,v)

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

We know that

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Therefore

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

From the above equation we observe that we cannot recover the undegraded image exactly because N(u,v) is a random function whose Fourier transform is not known.

One approach to get around the zero or small-value problem is to limit the filter frequencies to values near the origin.

We know that H(0,0) is equal to the average values of h(x,y).

By Limiting the analysis to frequencies near the origin we reduce the probability of encountering zero values.

MINIMUM MEAN SQUARE ERROR (WIENER) FILTERING:

The inverse filtering approach has poor performance. The wiener filtering approach uses the degradation function and statistical characteristics of noise into the restoration process.

The objective is to find an estimate $\hat{f}$ of the uncorrupted image *f* such that the mean square error between them is minimized.

The error measure is given by

$$e^2 = E\{[f(x) - \hat{f}(x)]^2$$

Where E{.} is the expected value of the argument.

We assume that the noise and the image are uncorrelated one or the other has zero mean.

The gray levels in the estimate are a linear function of the levels in the degraded image.

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

Where H(u,v)= Degradation function

H*(u,v)=complex conjugate of H(u,v)

$| H(u,v)|^2$=H* (u,v) H(u,v)

$S_n(u,v)=|N(u,v)|^2$= power spectrum of the noise

$S_f(u,v)=|F(u,v)|^2$= power spectrum of the underrated image

The power spectrum of the under graded image is rarely known. An approach used frequently when these quantities are not known or cannot be estimated then the expression used is

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

Where K is a specified constant

CONSTRAINED LEAST SQUARES FILTERING:

The wiener filter has a disadvantage that we need to know the power spectra of the under graded image and noise. The constrained least square filtering requires only the knowledge of only the mean and variance of the noise. These parameters usually can be calculated from a given degraded image this is the advantage with this method. This method produces a optimal result. This method require the optimal criteria which is important we express the

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$\mathbf{g} = \mathbf{Hf} + \boldsymbol{\eta}$$

The optimality criteria for restoration are based on a measure of smoothness, such as the second derivative of an image (Laplacian).

The minimum of a criterion function C defined as

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\nabla^2 f(x,y)]^2$$

Subject to the constraint

$$\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\|^2 = \|\boldsymbol{\eta}\|^2$$

Where $\|\mathbf{w}\|^2 \triangleq \mathbf{w}^T\mathbf{w}$ is a Euclidean vector norm $\hat{\mathbf{f}}$ is estimate of the under graded image. ∇^2 is laplacian operator.

The frequency domain solution to this optimization problem is given by

$$\hat{F}(u, v) = \left[\frac{H^*(u, v)}{|H(u, v)|^2 + \gamma|P(u, v)|^2}\right]G(u, v)$$

Where γ is a parameter that must be adjusted so that the constraint is satisfied. P(u,v) is the Fourier transform of the laplacian operator

$$p(x, y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

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