Shifting theorem

$$L[e^{-at}f(t)] = L[f(t)]_{s\to s+a}$$

$$L[e^{at}f(t)] = L[f(t)]_{s\to s-a}$$

Example: 5.7 Find the Laplace transform for the following:

i. te^{-3t}	vii. $t^2 2^t$
ii. t^3e^{2t}	viii. $t^3 2^{-t}$
iii. $e^{4t}sin2t$	ix. $e^{-2t}sin3tcos2t$
iv. $e^{-5t}cos3t$	x. $e^{-3t}cos4tcos2t$
v. $sinh2tcos3t$	xi. $e^{4t}cos3tsin2t$
vi. cosh3tsin2t	

(i) te^{-3t}

$$L[te^{-3t}] = L[t]_{s\to s+3}$$

$$= \left(\frac{1}{s^2}\right)_{s\to s+3} \qquad \because L(t) = \frac{1}{s^2}$$

$$\therefore L[te^{-3t}] = \frac{1}{(s+3)^2}$$

(ii) t^3e^{2t}

$$L[t^3e^{2t}] = L[t^3]_{s\to s-2}$$

$$= \left(\frac{3!}{s^4}\right)_{s\to s-2} \qquad \because L(t) = \frac{3!}{s^{3+1}}$$

$$\therefore L[t^3e^{2t}] = \frac{6}{(s-2)^4}$$

(iii) $e^{4t}sin2t$

$$L[e^{4t}sin2t] = L[sin2t]_{s\to s-4}$$

$$= \left(\frac{2}{s^2+2^2}\right)_{s\to s-4}$$

$$= \frac{2}{(s-4)^2+4}$$

$$= \frac{2}{s^2-8s+16+4}$$

$$\therefore L[e^{4t}sin2t] = \frac{2}{s^2-8s+20}$$

(iv) $L\left[e^{-5t}cos3t\right]$

$$L[e^{-5t}cos3t] = L[cos3t]_{s\to s+5}$$

$$= \left(\frac{s}{s^2+3^2}\right)_{s\to s+5}$$

$$= \frac{s+5}{(s+5)^2+9}$$

$$= \frac{s+5}{s^2+10s+25+9}$$

$$\therefore\ L[e^{-5t}cos3t] = \frac{s+5}{s^2+10s+34}$$

(v) $L[sinh2tcos3t]$

$$L[sinh2tcos3t] = L\left[\left(\frac{e^{2t}-e^{-2t}}{2}\right)cos3t\right]$$

$$= \frac{1}{2}[L(e^{2t}cos3t) - L(e^{-2t}cos3t)]$$

$$= \frac{1}{2}[L(cos3t)_{s\to s-2} - L(cos3t)_{s\to s+2}]$$

$$= \frac{1}{2}\left[\left(\frac{s}{s^2+3^2}\right)_{s\to s-2} - \left(\frac{s}{s^2+3^2}\right)_{s\to s+2}\right]$$

$$\therefore L[sinh2tcos3t] = \frac{1}{2}\left[\frac{s-2}{(s-2)^2+9} - \frac{s+2}{(s+2)^2+9}\right]$$

(vi) $L[cosh3tsin2t]$

$$L[cosh3tsin2t] = L\left[\left(\frac{e^{3t}+e^{-3t}}{2}\right)sin2t\right]$$

$$= \frac{1}{2}[L(e^{3t}sin2t) + L(e^{-3t}sin2t)]$$

$$= \frac{1}{2}[L(sin2t)_{s\to s-3} + L(sin2t)_{s\to s+3}]$$

$$= \frac{1}{2}\left[\left(\frac{2}{s^2+2^2}\right)_{s\to s-3} + \left(\frac{2}{s^2+2^2}\right)_{s\to s+3}\right]$$

$$\therefore\ L[cosh3tsin2t] = \frac{1}{2}\left[\frac{2}{(s-3)^2+4} + \frac{2}{(s+3)^2+4}\right]$$

(vii) $t^2 2^t$

$$L[t^2 2^t] = L\left[t^2 e^{log2^t}\right]$$

$$= L[t^2 e^{tlog2}] = L[t^2]_{s\to s-log2}$$

$$= \left(\frac{2!}{s^3}\right)_{s\to s-log2}$$

$$= \frac{2}{(s-log2)^3}$$

$$\therefore L[t^2 2^t] = \frac{2}{(s-log2)^3}$$

(viii) $t^3 2^{-t}$

$$L[t^3 2^{-t}] = L\left[t^3 e^{log2^{-t}}\right]$$

$$= L[t^3 e^{-tlog2}] = L[t^3]_{s\to s+log2}$$

$$= \left(\frac{3!}{s^4}\right)_{s\to s+log2}$$

$$= \frac{6}{(s+log2)^4}$$

$$\therefore L[t^3 2^{-t}] = \frac{6}{(s+log2)^4}$$

(ix) $L[e^{-2t}sin3tcos2t]$

$$L[e^{-2t}sin3tcos2t] = L[sin3tcos2t]_{s\to s+2}$$

$$= \frac{1}{2}L[\sin(3t+2t)+\sin(3t-2t)]_{s\to s+2}$$

$$= \frac{1}{2}L[\sin 5t + \sin t]_{s\to s+2}$$

$$= \frac{1}{2}[\mathrm{L}(\sin 5t) + L(\sin t)]_{s\to s+2}$$

$$= \frac{1}{2}\left[\frac{5}{s^2+5^2} + \frac{1}{s^2+1^2}\right]_{s\to s+2}$$

$$= \frac{1}{2}\left[\frac{5}{(s+2)^2+25} + \frac{1}{(s+2)^2+1}\right]$$

$$\therefore L[e^{-2t}sin3tcos2t] == \frac{1}{2}\left[\frac{5}{(s+2)^2+25} + \frac{1}{(s+2)^2+1}\right]$$

(x) $L[e^{-3t}cos4tcos2t]$

$$L[e^{-3t}cos4tcos2t] = L[cos4tcos2t]_{s\to s+3}$$

$$= \frac{1}{2}L[\cos(4t+2t)+\cos(4t-2t)]_{s\to s+3}$$

$$= \frac{1}{2}L[\cos 6\mathrm{t} + \cos 2t]_{s\to s+3}$$

$$= \frac{1}{2}[\mathrm{L}(\cos 6\, t) + L(\cos 2\, t)]_{s\to s+3}$$

$$= \frac{1}{2}\left[\frac{s}{s^2+6^2} + \frac{s}{s^2+2^2}\right]_{s\to s+3}$$

$$= \frac{1}{2}\left[\frac{s+3}{(s+3)^2+36} + \frac{s+3}{(s+3)^2+4}\right]$$

$$\therefore L[e^{-3t}cos4tcos2t] = \frac{1}{2}\left[\frac{s+3}{(s+3)^2+36} + \frac{s+3}{(s+3)^2+4}\right]$$

(xi) $L[e^{4t}cos3tsin2t]$

$$L[e^{4t}cos3tsin2t] = L[cos3tsin2t]_{s\to s-4}$$

$$= \frac{1}{2}L[\sin(3t+2t)-\sin(3t-2t)]_{s\to s-4}$$

$$= \frac{1}{2}L[\sin 5t - \sin t]_{s\to s-4}$$

$$= \frac{1}{2}[\mathrm{L}(\sin 5t) - L(\sin t)]_{s\to s-4}$$

$$= \frac{1}{2}\left[\frac{5}{s^2+5^2} - \frac{1}{s^2+1^2}\right]_{s\to s-4}$$

$$= \frac{1}{2}\left[\frac{5}{(s-4)^2+25} + \frac{1}{(s-4)^2+1}\right]$$

$$\therefore L[e^{4t}cos3tsin2t] = \frac{1}{2}\left[\frac{5}{(s-4)^2+25} + \frac{1}{(s-4)^2+1}\right]$$