UNIT III

TORSION AND SPRINGS

3.1 INTRODUCTION:

A shaft is said to be in torsion, when equal and opposite forces are applied at the two ends of the shaft. The torque is equal to the product of the force applied and radius of the shaft. Due to the application of the force at the ends the shaft is subjected to a twisting moment. This causes the shear stress and shear strains in the material of the shaft.

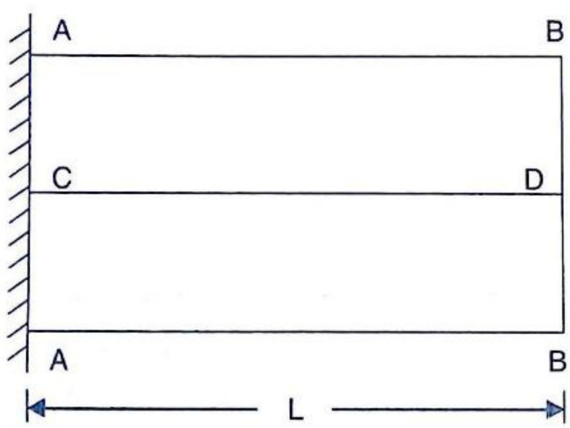
3.2 DERIVATION OF SHEAR STRESS PRODUCED IN A CIRCULAR SHAFT SUBJECTED TO TORSION:

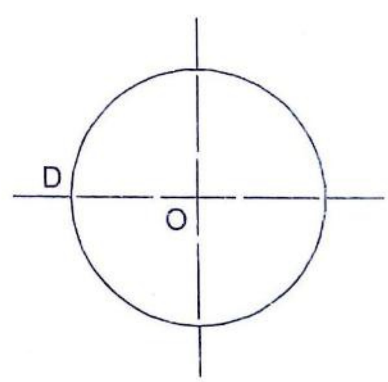
Before the derivation of shear stress produced in a circular shaft the following assumption are to be made as:

Assumption made in the Derivation of Shear Stress Produced in a Circular Shaft Subjected to Torsion:

1. The material of the shaft is uniform throughout.
2. The twist along the shaft is uniform.
3. The shaft is uniform circular section throughout.
4. Cross section of the shaft, which are plane before and after twist.
5. All radii which are straight before and after twist.

Consider a shaft of length *l*, radius R fixed at one end and free other end is free is subjected to a torque T as shown in figure.

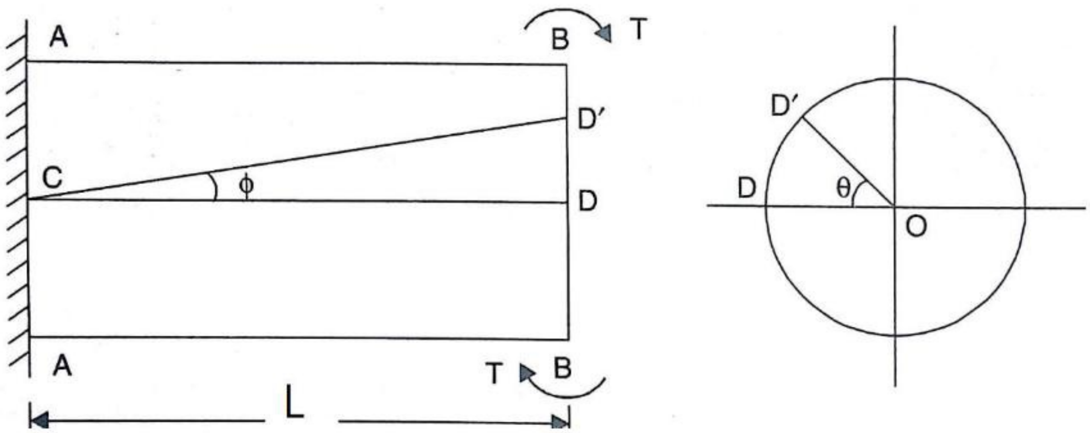




Let C = Modulus of rigidity of the material

τ = Shear stress induced at the surface of the shaft due to torque T

Let 'O' be the centre of the shaft D a point on surface and AB be the line on the shaft parallel to the axis of the shaft.



When the shaft is subjected to torque T then D is moved to D^l . If 'ϕ' be the shear strain and 'θ' be the angle of twist in length l then

Shear strain ϕ = Distortion per unit length = $\frac{\text{Distortion at the outer surface}}{\text{Length of the shaft}}$

$$\phi = \frac{DD^1}{l}$$

Then $DD^l = l\,\phi$

But, Arc length $DD^l = R\,\theta = l\,\phi$ …………….. (3.1)

Shear strain $\phi = \frac{\text{Shear stress}}{\text{Modulus of rigidity}} = \frac{\tau}{C}$ ……………... (3.2)

Substitute ϕ value in eqn 3.1

$$= R\,\theta = l\,\frac{\tau}{C}$$

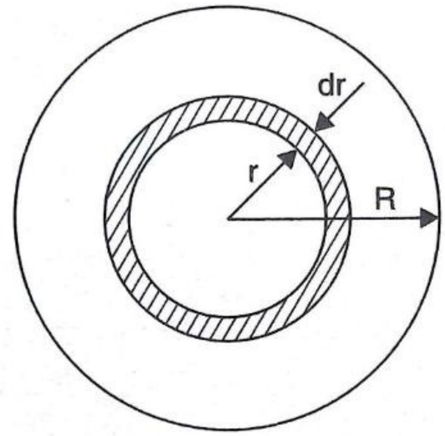
$$= \frac{C\theta}{l} = \frac{\tau}{R}$$ ……………… (3.3)

3.3 STRENGTH (OR) MAXIMUM TORQUE TRANSMITTED BY A CIRCULAR SOLID SHAFT

The strength of a shaft means the maximum torque or maximum power the shaft can transmit.

The maximum torque transmitted by a circular shaft is obtained from the maximum shear stress induced at the outer surface of the solid shaft. ie., $\tau \propto R$

Consider a shaft subjected to a torque T. Also consider a small elementary circular ring of thickness dr at a distance r from the center as shown in figure.



Let τ = Shear stress induced at the surface of the shaft due to torque T

R = Radius of the shaft

q = shear stress at the radius' r' from the centre.

The area of the ring $dA = 2\pi r dr$

If q is the shear stress induced at a radius r from the centre of the shaft then

$$\frac{q}{r} = \frac{\tau}{R}$$

∴ Shear stress at the radius r,

$$q = \frac{\tau}{R} r$$

Tuning force on the elementary circular ring

= Shear stress acting on the ring x area of ring

= q x dA

$= \frac{\tau}{R} \times r \times 2\pi r dr$

$= \frac{\tau}{R} \times 2\pi r^2 dr$

Now tuning moment due to tuning force on the elementary circular ring

dT = Tuning force x distance of the ring from axis

$= \frac{\tau}{R} \times 2\pi r^2 dr \times r$

$= \frac{\tau}{R} \times 2\pi r^3 dr$

Now the total turning moment or torque on the shaft is obtained by integrating the above eqn. ( ) between the limit 0 to R

$$T = \int_o^R dT = \int_0^R \frac{\tau}{R} 2\pi r^3 dr$$

$$= \frac{\tau}{R} \times 2\pi \int_0^R r^3 dr$$

$$= \frac{\tau}{R} \times 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{\tau}{R} \times 2\pi \times \frac{R^4}{4}$$

$$= \frac{\tau}{2} \times \pi R^3$$

$$= \frac{\tau}{2} \times \pi \times \left(\frac{D}{2}\right)^3 \qquad \left(R = \frac{D}{2}\right)$$

$$= \frac{\tau}{2} \times \pi \times \frac{D^3}{8}$$

$$T = \frac{\pi}{16} \tau D^3 \qquad \text{.................(3.6)}$$

3.4 DERIVE THE TORSIONAL EQUATION:

From the eqn (3.3) we know that

$$\frac{C\theta}{l} = \frac{\tau}{R}$$

But from torque transmission on a shaft of eqn. (3.6)

$$\tau = \frac{T \times 16}{\pi \times D^3}$$

Substitute the τ value in the eqn (3.3)

$$\frac{C\theta}{l} = \frac{T \times 16}{\frac{D}{2} \times \pi \times D^3}$$

$$\frac{C\theta}{l} = \frac{T}{\frac{\pi}{32} D^4}$$

Where $\frac{\pi}{32}$ D^4 is the **polar moment of inertia (J)** of the solid shaft. Then the above equation become $\frac{C\theta}{l} = \frac{T}{J}$ 3.10

Similarly polar moment of inertia of hollow circular shaft = $\frac{\pi}{32}\ (D^4 - d^4)$

Where D = outer diameter and d = inner diameter of hollow shaft

From eqn. (3.3) and eqn. (3.10)

$$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$$

3.5 TORQUE TRANSMITTED BY A HOLLOW CIRCULAR SHAFT:

Consider a hollow circular shaft of outer and inner radius are Ro and Ri is subjected to a torque T. Take an elementary circular ring of thickness 'dr' at a distance r from the centre as shown in figure

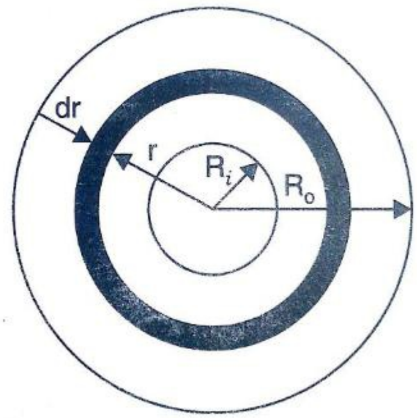
Let q = shear stress induced on the elementary ring.

dA = 2πrdr area of the elementary circular ring

shear stress at the elementary ring is obtained from shear stress ratio

$$\frac{q}{r} = \frac{\tau}{R_o}$$

$$q = \frac{\tau}{R_o} r$$



Tuning force on the elementary circular ring

= Shear stress acting on the ring x area of ring

= q x dA

$$= \frac{\tau}{R_o} \text{ x r x } 2\pi r dr$$

$$= \frac{\tau}{R_o} \text{x } 2\pi r^2 dr$$

Now tuning moment due to tuning force on the elementary circular ring

dT = Tuning force x distance of the ring from axis

$$= \frac{\tau}{R_o} \text{x } 2\pi r^2 dr \text{ x r}$$

$$= \frac{\tau}{R_o} \text{x } 2\pi r^3 dr$$

Now the total turning moment or torque on the shaft is obtained by integrating the above eqn. between the limit Ri to Ro

$$T = \int_{R_i}^{R_o} dT = \int_{R_i}^{R_o} \frac{\tau}{R_o} \times 2\pi r^3 dr$$

$$= \frac{\tau}{R_o} \times 2\pi \int_{R_i}^{R_o} r^3 dr$$

$$= \frac{\tau}{R_o} \times 2\pi \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{\tau}{R_o} \times 2\pi \text{ x } \left[\frac{R_o{}^4 - R_i{}^4}{4}\right]$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{R_o{}^4 - R_i{}^4}{R_o}\right]$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{\left(\frac{D_o}{2}\right)^4 - \left(\frac{D_i}{2}\right)^4}{\frac{D_o}{2}}\right] \qquad \left(R_o = \frac{D_o}{2} \; ; R_i = \frac{D_i}{2}\right)$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{D_o{}^4 - D_i{}^4}{16} \text{ x } \frac{2}{D_o}\right]$$

$$T \quad = \frac{\pi}{16} \, \tau \left[\frac{D_o{}^4 - D_i{}^4}{D_o}\right]$$

3.6 TORSIONAL RIGIDITY:

The product of modulus of rigidity and polar moment of inertia of a circular shaft is known as torsional rigidity. It is denoted by (K).

Torsional rigidity (K) = C x J

Torsional rigidity is also defined as the torque required to produce a twist of radian per unit length of the shaft.

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$ gives $CJ = \frac{Tl}{\theta}$

If l = 1metre and θ = 1radian then Torsional rigidity = Torque

Since C, J and l are constant for a given shaft, the angle of twist θ is directly proportional to the torque (T). The term CJ is known as torsional rigidity.

3.7 POLAR MODULUS:

It is the ratio between the polar moment of inertia and the radius of the shaft. It is denoted by (Z).Its unit is mm^3. It is also called torsional section modulus.

Polar Modulus (Z) = $\frac{\text{Polar moment of inertia}}{\text{Radius of shaft}} = \frac{J}{R}$

For solid shaft (J) = $\frac{\pi}{32} D^4$ then Z = $\frac{\frac{\pi}{32} D^4}{\frac{D}{2}}$ $= \frac{\pi}{16} D^3$

For hollow shaft (J) = $\frac{\pi}{32}\left(D^4 - d^4\right)$ then Z = $\frac{\frac{\pi}{32}\left(D^4 - d^4\right)}{\frac{D}{2}}$ $= \frac{\pi}{16}\left[\frac{D^4-d^4}{D}\right]$

3.8 POWER TRANSMITTED BY SHAFT:

Once the torque (T) for a solid or a hollow shaft is obtained, power transmitted by the shaft can be determined,

Let N = Speed of shaft in rpm

T = mean torque transmitted in Nm

Power P = $\frac{2\pi N T_{mean}}{60}$ Watts or P = T x ω where $\omega = \frac{2\pi N}{60}$

3.9 SAVING OF MATERIAL AND WEIGHT OF SOLID AND HOLLOW SHAFT:

(i) Percentage of saving in Material:

$$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \times 100$$

Let D = diameter of solid shaft

l_S = length of solid shaft

ρ_S = density of solid shaft

l_H = length of hollow shaft

D_H = Outer diameter of hollow shaft

d_H = inner diameter of hollow shaft

ρ_H = density of Hollow shaft

$$= \frac{\frac{\pi \times D^2}{4} - \frac{\pi \times \left({D_H}^2 - {d_H}^2\right)}{4}}{\frac{\pi \times D^2}{4}} \times 100 \qquad = \frac{D^2 - \left({D_H}^2 - {d_H}^2\right)}{D^2} \times 100$$

(ii) Percentage of saving in weight:

$$= \frac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{Weight of solid shaft}} \times 100$$

Weight of solid shaft = density x volume = density x area x length

$$= \rho_S \times \frac{\pi \times D^2}{4} \times l_S$$

Weight of hollow shaft = $\rho_H \times \frac{\pi \times (D_H{}^2 - d_H{}^2)}{4} \times l_H$

Then Percentage of saving in material

$$= \frac{\rho_S \times \frac{\pi \times D^2}{4} \times l_S - \rho_H \times \frac{\pi \times (D_H{}^2 - d_H{}^2)}{4} \times l_H}{\rho_S \times \frac{\pi \times D^2}{4} \times l_S} \text{ x } 100$$

For same material and same length $\rho_S = \rho_H$ and $l_S = l_H$ then

Percentage of saving in material = $\frac{D^2 - (D_H{}^2 - d_H{}^2)}{D^2}$ x 100

Problem 3.1. A solid shaft of is to transmit a torque of 25kNm. If the shearing stress is not to exceed 60 Mpa. Find the minimum diameter of the shaft.

Given Data:

Torque transmitted T = 25kNm = 25×10^6 Nmm

Shear stress τ = 60 Mpa = 60 N/mm^2

To find:

Diameter of the shaft D = ?

Solution:

Wkt Torque transmitted T = $\frac{\pi}{16} D^3$ then D = $\sqrt[3]{\frac{T \times 16}{\pi}}$

$$= \sqrt[3]{\frac{25 \times 10^6 \, x \, 16}{\pi}} \quad = \mathbf{128.5\ mm}$$

Result:

Diameter of the shaft D = **128.5 mm**

Problem 3.2. A hollow circular shaft of external diameter 50mm and internal diameter 40mm transmit a torque of 10 kNm. Find the maximum shear induced in the shaft.

Given Data:

Torque transmitted T = 10kNm = 10×10^6 Nmm

External diameter D = 50mm

Internal diameter d = 40mm

To find:

Shear stress τ =?

Solution:

Wkt Torque transmitted T $= \frac{\pi}{16}\ \tau \left[\frac{D^4-d^4}{D}\right]$

$10 \times 10^6 = \frac{\pi}{16}\ \tau \left[\frac{50^4-40^4}{50}\right]$

τ **= 690.1 N/mm²**

Result:

Shear stress τ = **690.1 N/mm²**

Problem 3.3. Find the power that can be transmitted by a shaft of 50mm diameter at a speed of 120 rpm. If the shear stress is 60 N/mm²

Given Data:

Diameter D = 50mm

Speed N = 120 rpm

Shear stress τ = 60 N/mm²

To find:

Power P =?

Solution:

Wkt Power transmitted P $= \frac{2\pi N T_{mean}}{60}$

But $T_{mean} = \frac{\pi}{16}\ \tau\ D^3 \quad = \frac{\pi}{16}\ 60\ \times 50^3$

T_{mean} = 1472621.5 Nmm = 1.472×10^3 Nm

Then P $= \frac{2 \times\pi\ \times 120\ \times 1.472\times 10^3}{60}$ = 18476.5 W

= 18.476 kW

Result:

Power P = **18.476 kW**

Problem 3.4. A solid circular shaft transmits 85kW power at 200 rpm. Find the shaft diameter if the shear stress is 50 MN/ m^2.

Given Data:

Power $P = 85kW = 85 \times 10^3 W$

Speed $N = 200$ rpm

Shear stress $\tau = 50$ MN/ m^2 = 50 N/mm^2

To find:

Shaft diameter D =?

Solution:

Wkt Power transmitted $P = \frac{2\pi NT}{60}$

Then $85 \times 10^3 = \frac{2 \times \pi \times 200 \times T}{60}$

$T = 4.05 \times 10^3 Nm = 4.05 \times 10^6 Nmm$

But Wkt, $T = \frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \times 50 \times D^3$

D = **74.4 mm**

Result:

Shaft diameter D = **74.4 mm**

Problem 3.5. A hollow shaft is to transmit 200kW at 80 rpm. If the shear stress is not to exceed 70 MN/m^2. and internal diameter is 0.5 of the external diameter. Find the external and internal diameters assuming that maximum torque is 1.6 times the mean.

Given Data:

Power $P = 200kW = 200 \times 10^3 W$

Internal Diameter $d = 0.5 D$

Speed $N = 80$ rpm

Shear stress $\tau = 70$ MN/m^2 = 70×10^6 N/m^2 = 70 N/mm^2

Max. Torque $T_{max} = 1.6\ T_{mean}$

To find:

External and internal diameter D, d =?

Solution:

Wkt Power transmitted P $= \frac{2\pi NT}{60}$

$$200 \times 10^3 = \frac{2\times\pi\times 80\times T}{60}$$

$$T = T_{mean} = 23.87 \times 10^3 Nm = 23.87 \times 10^6\ Nmm$$

In given data $T_{max} = 1.6\ T_{mean}$

Then $T_{max} = 1.6\times 23.87 \times 10^6 = 38.19 \times 10^6\ Nmm$

But $T_{max} = \frac{\pi}{16}\ \tau \left[\frac{D^4 - d^4}{D}\right]$

$$T_{max} = \frac{\pi}{16}\ \tau \left[\frac{D^4 - (0.5\ D)^4}{D}\right] \qquad (\because d = 0.5\ D)$$

$$38.19 \times 10^6 = \frac{\pi}{16} \times 70 \times D^4 \left[\frac{1 - 0.5^4}{D}\right]$$

$$D = 143.6\ mm$$

$\Rightarrow$ $d = 0.5 \times 143.6 = 71.82\ mm$

Result:

Outer diameter D = **143.6 mm**

Inner diameter d = **71.82 mm**

Problem 3.6. Find the maximum torque that can be safely applied to a shaft of 120 mm diameter. If the allowable twist is 3° in a length of 1.5m. Take $C = 1 \times 10^5\ N/mm^2$

Given Data:

Diameter D = 120 mm

Angle of twist $\theta = 3^\circ = 3 \times \frac{\pi}{180} = 0.05$ rad

Length $l = 1.5m = 1.5\times 10^3$ mm

Modulus of rigidity $C = 1 \times 10^5\ N/mm^2$

To find:

Maximum torque transmitted T =?

Solution:

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia = $\frac{\pi}{32} \times D^4$

$$J = \frac{\pi}{32} \times 120^4 \quad = 20.3 \times 10^6 \ \text{mm}^4$$

Substitute J value in the torsion equation, then

$$T = \frac{1 \times 10^5 \times 0.05}{1.5 \times 10^3} \times 20.3 \times 10^6$$

$$T = 67.6 \times 10^6 \ \text{Nmm}$$

Result:

Torque transmitted T = $\mathbf{67.6 \times 10^6}$ **Nmm**

Problem 3.7. A solid shaft of diameter 100mm is require d to transmit 150kW at 120 rpm. If the length of the shaft is 4m and modulus of rigidity for shaft is 75 Gpa, find the angle of twist.

Given Data:

Diameter D = 100 mm

Power P = 150 kW = 150×10^3 W

Speed N = 120 rpm

Length $l = 4$ m = 4×10^3 mm

Modulus of rigidity C = 75 Gpa = 75×10^9 pa = 75×10^9 N/ m^2

= 75×10^3 N/mm^2

To find:

Angle of twist θ = ?

Solution:

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia = $\frac{\pi}{32} \times D^4$

$$J = \frac{\pi}{32} \times 100^4 \quad = 9.81 \times 10^6 \ \text{mm}^4$$

Wkt, Power transmitted P $= \frac{2\pi NT}{60}$

$$150 \times 10^3 = \frac{2\times\pi\times100\times T}{60}$$

$\gg$ $T = 11.93\times 10^3$ Nm $= 11.93\times 10^6$Nmm

Substitute J and T value in the torsion equation, then

$$\frac{11.93\times10^6}{9.81\times10^6} = \frac{75 \times10^6\times\theta}{4\times10^3}$$

$\gg$ $\theta = 0.06$ rad

$$\theta = 0.06 \times \frac{180}{\pi} = 3.7^o$$

Result:

Angle of twist $\theta =$ **3.7º**

Problem 3.8. A hollow shaft of 120mm external diameter and 80mm internal diameter is required to transmit 200kW at 120 rpm. If the angle of twist is not to exceed 3º find the length of the shaft. Take modulus of rigidity for shaft is 80 Gpa.

Given Data:

External Diameter $D = 120$ mm

Internal diameter $d = 80$ mm

Power $P = 200$ kW $= 200 \times 10^3$ W

Speed $N = 120$ rpm

Angle of twist $\theta = 3^o = 3 \times \frac{\pi}{180} = 0.05$ rad

Modulus of rigidity $C = 80$ Gpa $= 80 \times 10^9$ pa $= 80 \times 10^9$ N/ m^2

$= 80 \times 10^3$ N/mm^2

To find:

Length of shaft $l = ?$

Solution:

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia $= \frac{\pi}{32} \times [D^4 - d^4]$

$$J = \frac{\pi}{32} \times [120^4 - 80^4]$$

$$= 16.3 \times 10^6 \quad mm^4$$

Wkt, Power transmitted P $= \frac{2\pi NT}{60}$

$$200 \times 10^3 = \frac{2\times\pi\times120\times T}{60}$$

≫ T = 15.93× 10^3 Nm = 11.93× 10^6Nmm

Substitute J and T value in the torsion equation, then

$$\frac{15.93\times10^6}{16.3\times10^6} = \frac{80\times10^3\times0.05}{l}$$

≫ l = 4264.6 mm

Result:

Length of shaft l = **4264.6 mm**

Problem 3.9: Find the maximum torque that can be safely applied to a shaft of 120mm diameter. The permissible shear stress and allowable twist are 200 N/mm^2 and 3^o respectively. Take C = 75Gpa and length of shaft = 4m.

Given data:

Diameter D = 120 mm

Shear stress τ = 200 N/mm^2

Angle of twist $\theta = 3^o = 3 \times \frac{\pi}{180} = 0.052$ rad

Modulus of rigidity C = 75 Gpa = 75 × 10^9 pa = 75 × 10^9 N/ m^2

= 75 × 10^3 N/mm^2

Length of shaft l = 4m = 4 × 10^3 mm

To find:

Maximum torque T_{max} = ?

Solution:

Consider based on shear stress

Torque $T = \frac{\pi}{16} \tau D^3$

$$= \frac{\pi}{16} \times 200 \times 120^3 = 67.8 \times 10^6 \text{ Nmm}$$

Considering angle of twist

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia = $\frac{\pi}{32} \times D^4 = \frac{\pi}{32} \times 120^4 = 20.35 \times 10^6$ mm^4

Then torsion equation become

$$\frac{T}{20.35 \times 10^6} = \frac{75 \times 10^3 \times 0.052}{4 \times 10^3}$$

Then $T = 19.8 \times 10^6$ Nmm

From the above two torque value we have to find the maximum value that can be safely applied on the shaft is take the minimum value as 19.8×10^6 Nmm.

Result:

Maximum torque T_{max} = $\mathbf{19.8 \times 10^6}$ **Nmm**

Problem 3.10: A solid circular shaft transmit 70kW power at 175 rpm. Calculate the shaft diameter if the twist in the shaft is not to exceed 2° in 2 meter length of shaft and shear stress is limited to 50×10^3 kN/m^2 . Take C = 100×10^3 MN/m^2.

Given data:

Power $P = 70$ kW $= 70 \times 10^3$ W

Speed $N = 175$ rpm

Angle of twist $\theta = 2^\circ = 2 \times \frac{\pi}{180} = 0.034$ rad

Length of shaft $l = 2\text{m} = 2 \times 10^3$ mm

Shear stress $\tau = 50 \times 10^3$ kN/m^2 = 50 N/mm^2

Modulus of rigidity C = 100×10^3 MN/m^2 = 100×10^9 N/m^2

= 100×10^3 N/ mm^2

To find:

Diameter of shaft D = ?

Solution:

Wkt, Power $P = \frac{2\pi NT}{60}$

$$70 \times 10^3 = \frac{2 \times \pi \times 175 \times T}{60}$$

≫ Torque $T = 3.81\times 10^3$ Nm $= 3.81\times 10^6$ Nmm

Consider based on shear stress

Torque $T = \frac{\pi}{16} \tau D^3$

$$3.81\times 10^6 = \frac{\pi}{16} \times 200 \times D^3$$

≫ D = 72.9 mm

Considering angle of twist

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia $= \frac{\pi}{32} \times D^4 = 0.098D^4$

Then torsion equation become

$$\frac{3.81\times 10^6}{0.098D^4} = \frac{100 \times 10^3 \times 0.034}{2 \times 10^3}$$

Then D = 69.12 mm

From the above two cases, we have to find the suitable diameter for the shaft is take the greatest value as 72.9 = 73 mm.

Result:

Shaft diameter **D = 73 mm**

Problem 3.11: A hollow shaft is to transmit 300kW power at 80 rpm. If the shear stress is not to exceed 50 MN/m^2 and diameter ratio is 3/7. Find the external and internal diameter if the twist of shaft is 1.2^o in 2 meter length. Assuming maximum torque is 20% greater than mean. Take C = 80 GN/m^2.

Given data:

Power P = 300 kW = 300×10^3 W

Speed N = 80 rpm

Shear stress τ = 50 MN/m^2 = 50 N/mm^2

Diameter ratio d/D = 3/7 ≫ d = 0.428D

Angle of twist $\theta = 1.2^{o} = 1.2 \times \frac{\pi}{180} = 0.020$ rad

Length of shaft $l = 2\text{m} = 2 \times 10^3$ mm

Maximum torque $T_{\max}$ = 20% greater than T_{mean} = (100% +20%)T_{mean}

$= 1.2\, T_{mean}$

Modulus of rigidity C = Modulus of rigidity C = 80 GN/m^2 = 80×10^9 N/m^2

$= 80 \times 10^3$ N/ mm^2

To find:

External and internal diameter of shaft D,d =?

Solution:

Wkt, Power $P = \frac{2\pi NT}{60}$

$$300 \times 10^3 = \frac{2 \times \pi \times 80 \times T}{60}$$

≫ Torque T= $T_{mean} = 35.8\times 10^3$ Nm $= 35.8\times 10^6$ Nmm

In our data, $T_{\max} = 1.2\, T_{mean} = 1.2\times 35.8\times 10^6 = 42.96\times 10^6$Nmm

Consider based on shear stress

Torque $T_{\max} = \frac{\pi}{16}\, \tau \left[\frac{D^4 - d^4}{D}\right]$

$$42.96\times 10^6 = \frac{\pi}{16} \times 50 \times \left[\frac{D^4 - (0.428D)^4}{D}\right]$$

$$42.96\times 10^6 = \frac{\pi}{16} \times 50 \times D^3[1 - (0.428)^4]$$

≫ D = 165.4 mm, then d = 0.428×165.4 = 70.8 mm

Considering angle of twist

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia $= \frac{\pi}{32} \times [D^4 - d^4] = \frac{\pi}{32} \times [D^4 - (0.428D)^4]$

$= \frac{\pi}{32} \times D^4[1 - (0.428)^4] = 0.095D^4$

Then torsion equation become

$$\frac{42.96\times10^6}{0.095D^4} = \frac{80 \times10^3 \times0.02}{2 \times10^3}$$

» $D = 154.21mm$, then $d = 0.428 \times 154.21 = 66mm$

From the above two cases, we have to find the suitable diameter for the shaft is take the greatest value as

External diameter D = 165.4mm and Internal diameter d = 70.8mm

Result:

External diameter of shaft D = **165.4mm**

Internal diameter of shaft d = **70.8mm**

3.10 REPLACING SHAFT PROBLEMS:

Problem 3.12: A solid shaft of 50mm diameter is to be replaced by a hollow steel shaft whose internal diameter is 0.5 times outer diameter. Find the diameter of the hollow shaft and percentage of saving in weight and material, the maximum shearing stress being the same.

Given data:

Solid shaft diameter D = 50mm

Hollow shaft internal diameter $d = 0.5D_H$

Shear stress τ = same for solid and hollow shaft

To find:

External and internal diameter of shaft D_H, d =?

Solution:

Wkt, the torque transmitted by the solid shaft should be equal to that hollow shaft when solid shaft is replaced by hollow shaft.

Torque transmitted by the solid shaft

$$T = \frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \times \tau \times 50^3 = 24543.6\, \tau \text{ Nmm}$$

Torque transmitted by the solid shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_H^4 - d^4}{D_H}\right]$$

$$= \frac{\pi}{16} \times \tau \times \left[\frac{D_H^4 - (0.5D_H)^4}{D_H}\right]$$

$$= \frac{\pi}{16} \times \tau \times D_H^3 [1 - (0.5)^4]$$

≫ $T = 0.184\,\tau D_H{}^3$ Nmm,

Toque transmitted in both shafts are same,

≫ $24543.6\,\tau = 0.184\,\tau D_H{}^3$

Based on given data shear stress are same (not given assume same), then

≫ $$D_H = \sqrt[3]{\frac{24543.6\,\tau}{0.184\,\tau}} = 51.09 \text{ mm}$$

Now internal diameter of hollow shaft $d = 0.5 D_H = 0.5 \times 51.09 = 25.54$mm

Percentage of saving in weight =

$$= \frac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{Weight of solid shaft}} \times 100$$

Weight of solid shaft = density ×Area×length

$$= \rho_S \times \frac{\pi \times D^2}{4} \times l_S = \rho_S \times \frac{\pi \times 50^2}{4} \times l_S$$

$$= 1963.4\,\rho_S \,.\, l_S$$

Weight of hollow shaft = $\rho_H \times \frac{\pi \times (D_H{}^2 - d^2)}{4} \times l_H$

$$= \rho_H \times \frac{\pi \times (51.09^2 - 25.54^2)}{4} \times l_H = 1537.5\,\rho_H .\, l_H$$

≫ % of saving in weight $= \frac{1963.4\,\rho_S .l_S - 1537.5\,\rho_H .l_H}{1963.4\,\rho_S .l_S} \times 100$

For same material and same length $\rho_S = \rho_H .\ l_H = l_S$ then

≫ % of saving in weight $= \frac{1963.4 - 1537.5}{1963.4} \times 100 = 21.7\%$

Percentage of saving in material =

$$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \times 100$$

Area of solid shaft $= \frac{\pi \times D^2}{4} = \frac{\pi \times 50^2}{4} = 1963.4$ mm^2

Area of hollow shaft $= \frac{\pi \times (D_H{}^2 - d^2)}{4} = \frac{\pi \times (51.09^2 - 25.54^2)}{4} = 1537.5$ mm^2

≫ % of saving in material $= \frac{1963.4 - 1537.5}{1963.4} \times 100 = 21.7\%$

Result:

External diameter of hollow shaft D_H = **51.09mm**

Internal diameter of hollow shaft d = **25.54mm**

% of saving in weight = **21.7%**

% of saving in material = **21.7%**

3.10.1 MODULUS OF RUPTURE:

The maximum shear stress calculated by the torsion formula by using the experimentally found maximum torque required to rupture a shaft.

Modulus of rupture in torsion $\tau_r = \frac{T_u \times R}{J}$

Where T_u = Ultimate torque

R = Radius of shaft

J = polar moment of inertia

3.11 FLANGED COUPLING:

A flanged coupling is used to connect two shaft are in coaxial. The following fig. shows the flange coupling.

The flanges of the two shafts are jointed together by bolts and nuts and torque is then transferred from one shaft to another through the bolts. Connection between each shaft and coupling is provided by the key. The bolts are arranged along a circle called the pitch circle. The bolts are subjected to shear stress when torque is transmitted from one shaft to another.

Let τ = shear stress in the shaft

q = shear stress in the bolt

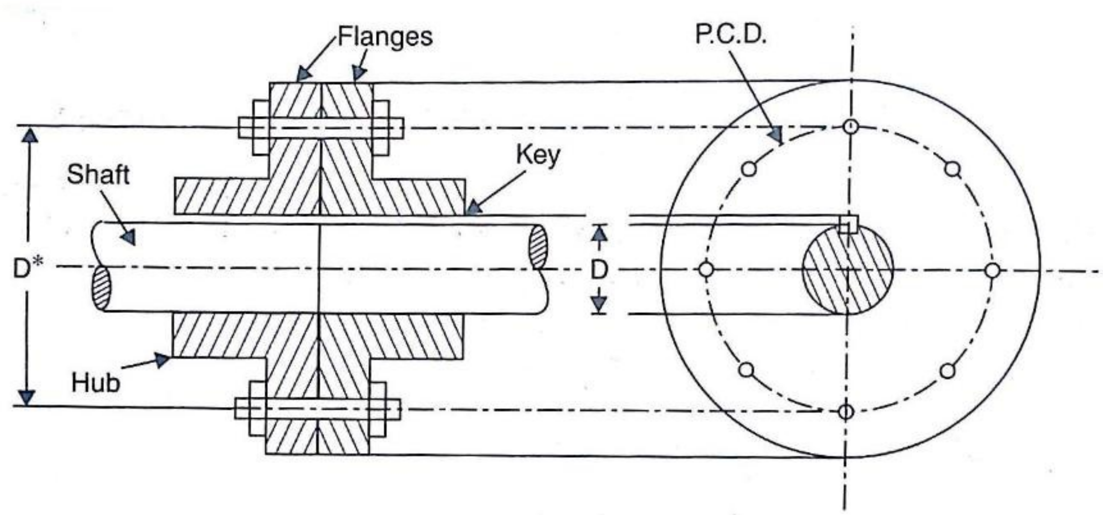
D = diameter of shaft

d = diameter of bolt

D_p = diameter of pitch circle

n = number of bolt

Maximum load that can be resisted by one bolt



$= \text{Stress in bolt x Area of one bolt} = q \times \frac{\pi \times d^2}{4}$

Torque resisted by one bolt

= load resisted by one bolt x radius of pitch circle

$= q \times \frac{\pi \times d^2}{4} \times \frac{D_p}{2}$

∴ Total torque resisted by n bolt

$= n \times q \times \frac{\pi \times d^2}{4} \times \frac{D_p}{2}$

But torque transmitted by the shaft,

$T = \frac{\pi}{16} \tau D^3$

Since the torque resisted by the bolts should be equal to the torque transmitted by the shafts, therefore equating (3.15) and (3.16), we get

$$n \times q \times \frac{\pi \times d^2}{4} \times \frac{D_p}{2} = \frac{\pi}{16} \tau D^3$$

Based on the above equation we have to calculate the unknown parameter.

3.12 STRENGTH OF A SHAFT OF VARYING SECTION

When a shaft is made up of different lengths and of different diameters, the torque transmitted by individual section should be calculated first. The strength of such a shaft is the minimum value of these torques. This varying cross section of shaft arranged in series is called ***series shaft.***

3.12.1SHAFT IN SERIES

In this case, each shaft transmits the same torque but the angle of twist is the sum of the angle of twist of the two shaft connected in series.

Torque transmitted, $T = T_1 = T_2$

Angle of twist $\theta = \theta_1 + \theta_2$ (but $\theta_1 = \theta_2$)

From the torsion equation, $\theta = \frac{T \times l}{C \times J}$

Then angle of twist, $\theta = \frac{T_1 \times l_1}{C_1 \times J_1} + \frac{T_2 \times l_2}{C_2 \times J_2}$

$$\theta = T\left(\frac{l_1}{C_1 \times J_1} + \frac{l_2}{C_2 \times J_2}\right) \quad (\therefore T = T_1 = T_2)$$

Where shafts are made of same material, then $C_1 = C_2$

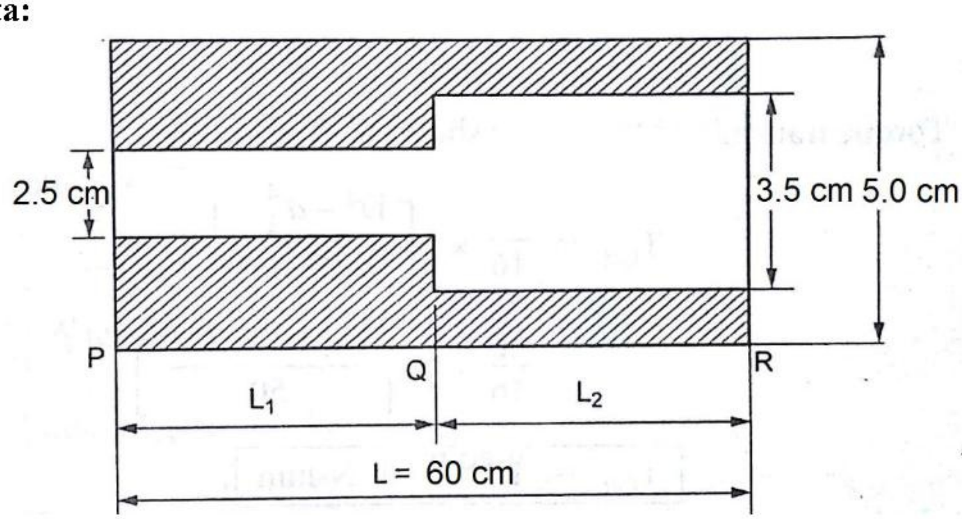
$$\therefore \quad \theta = \frac{T}{C}\left(\frac{l_1}{J_1} + \frac{l_2}{J_2}\right)$$

Here the driving toque is applied at one end and the resisting toque at the other end.

Problem 3.13: A shaft ABC of 60cm length and 5cm external diameter is bored for a part of its length AB to a 2.5cm diameter and for the remaining length BC to a 3.5 cm diameter bore. If the shear stress is not to exceed 70 N/mm^2, find the maximum power that the shaft can transmit at the speed of 150 rpm.

If the angle of twist in the length of 2.5cm diameter bore is equal to that in the 3.5cm diameter bore, find the length of the shaft that has been bored to 2.5cm and 3.5cm diameter.

Given data:



Let Shaft a AB = shaft 1 and shaft BC = shaft 2

Total length of shaft L = 60 cm = 600mm

Outer Diameter of shaft 1 D_1 = 5cm = 50mm

Inner diameter of shaft 1 d_1 = 2.5cm = 25mm

Outer Diameter of shaft 2 D_2 = 5cm = 50mm

Inner diameter of shaft 1 d_2 = 3.5cm = 35mm

Shear stress $\tau = 70$ N/mm^2

Speed N = 150 rpm

Also given as $\theta_1 = \theta_2$

To find out:

Length of shaft 1 l_1 =? Power P =?

Length of shaft 2 l_2 =?

Solution:

Wkt, Condition for series shaft T = T_1 = T_2 $\theta = \theta_1 + \theta_2$

Torque transmitted by the hollow shaft 1

$$T_1 = \frac{\pi}{16}\,\tau\left[\frac{{D_1}^4 - {d_1}^4}{D_1}\right] = \frac{\pi}{16} \times 70 \times \left[\frac{50^4 - 25^4}{50}\right]$$

$$= 1.61 \times 10^6 \text{ Nmm}$$

Torque transmitted by the hollow shaft 2

$$T_1 = \frac{\pi}{16}\,\tau\left[\frac{{D_2}^4 - {d_2}^4}{D_2}\right] = \frac{\pi}{16} \times 70 \times \left[\frac{50^4 - 35^4}{50}\right]$$

$$= 1.30 \times 10^6 \text{ Nmm}$$

From the above two torque value we have to find the maximum value that can be safely applied on the shaft is take the minimum value as 1.30×10^6 Nmm.

Hence torque transmitted T = 1.30×10^6 Nmm = 1.30×10^3Nm

Power transmitted $$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 150 \times 1.30 \times 10^3}{60} = 20.41 \times 10^3 \text{ W}$$

$$= 20.41 \text{ kW}$$

To find the length of shaft

Using the another condition $\theta_1 = \theta_2$

From the torsional equation $\theta_1 = \frac{T_1 \times l_1}{C_1 \times J_1}$ and $\theta_2 = \frac{T_2 \times l_2}{C_2 \times J_2}$

Based on the above condition

$$\frac{T_1 \times l_1}{C_1 \times J_1} = \frac{T_2 \times l_2}{C_2 \times J_2}$$

For same material and same torque transmission $C_1 = C_2$ and $T_1 = T_2$, then the above relation becomes

$$\frac{l_1}{J_1} = \frac{l_2}{J_2} \qquad \gg \ l_1 = \frac{l_2}{J_2} \times J_1$$

Where

$$J_1 = \frac{\pi}{32} \times \left[{D_1}^4 - {d_1}^4\right] = J_1 = \frac{\pi}{32} \times [50^4 - 25^4] = 5.75 \times 10^5 \text{ mm}^4$$

$$J_2 = \frac{\pi}{32} \times \left[{D_2}^4 - {d_2}^4\right] = J_2 = \frac{\pi}{32} \times [50^4 - 35^4] = 4.66 \times 10^5 \text{ mm}^4$$

$\gg$ $$l_1 = \frac{l_2}{4.66 \times 10^5} \times 5.75 \times 10^5$$

$\gg$ $$l_1 = 1.23\ l_2$$

But $l_2 = \text{L} - l_1$ $\quad \gg$ $l_2 = 600 - l_1$, then

$$l_1 = 1.23\ (600 - l_1) \quad \gg \quad l_1 = 330.94 \text{ mm}$$

Then $$l_2 = 600 - 330.94 = 296.06 \text{ mm}$$

Result:

Power transmitted P = **20.41 kW**

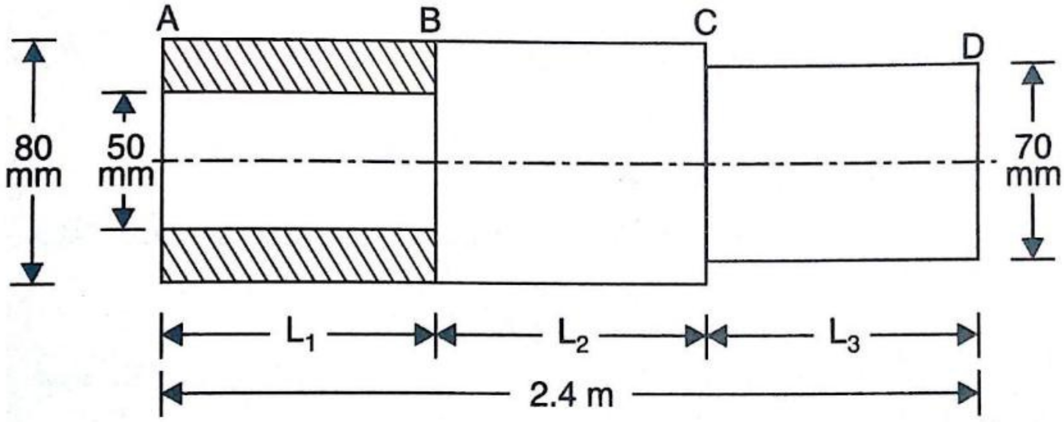
Length of shaft 1 l_1 = **330.94 mm**

Length of shaft 2 l_2 = **296.06 mm**

Problem 3.14: A steel shaft ABCD having a total length of 2.4 m consist of three lengths having different sections as follows;

AB is hollow having outside and inside diameter of 80mm and 50mm respectively and BC and CD are solid, BC having a diameter of 80mm and CD a diameter of 70mm. If the angle of twist is the same for each section, determine the length of each section and the total angle of twist if the maximum shear stress in the hollow potion is 50 N/mm^2. Take C = 8.2×10^4 N/mm^2.

Given data:



Let Shaft a AB = shaft 1, shaft BC = shaft 2 and shaft CD = shaft 3

Total length of shaft L = 2.4m = 2400mm

Outer Diameter of shaft 1 D_1 = 80 mm

Inner diameter of shaft 1 d_1 = 50 mm

Outer Diameter of shaft 2 D_2 = 80mm

Outer Diameter of shaft 3 D_3 = 70mm

Also given as $\theta_1 = \theta_2 = \theta_3$

Shear stress τ = 50 N/mm^2

Modulus of rigidity C = 8.2×10^4 N/mm^2

To find out:

Length of shaft 1	l_1 =?	Length of shaft 3	l_3 =?
Length of shaft 2	l_2 =?	Total angle of twist	θ =?

Solution:

Wkt, Condition for series shaft T = T_1 = T_2 $\theta_1 = \theta_2 = \theta_3$

Using the condition $\theta_1 = \theta_2 = \theta_3$

From the torsional equation $\theta_1 = \frac{T_1 \times l_1}{C_1 \times J_1}$; $\theta_2 = \frac{T_2 \times l_2}{C_2 \times J_2}$ and $\theta_2 = \frac{T_3 \times l_3}{C_3 \times J_3}$

Based on the above condition

$$\frac{T_1 \times l_1}{C_1 \times J_1} = \frac{T_2 \times l_2}{C_2 \times J_2} = \frac{T_3 \times l_3}{C_3 \times J_3}$$

For same material and same torque transmission $C_1 = C_2 = C_3$ and $T_1 = T_2 = T_3$ then the above relation becomes

$$\frac{l_1}{J_1} = \frac{l_2}{J_2} = \frac{l_3}{J_3}$$

≫ $l_1 = \frac{l_2}{J_2} \times J_1$ and $l_2 = \frac{l_3}{J_3} \times J_2$

Where

$J_1 = \frac{\pi}{32} \times [{D_1}^4 - {d_1}^4] = J_1 = \frac{\pi}{32} \times [80^4 - 50^4] = 34.09 \times 10^5$ mm^4

$J_2 = \frac{\pi}{32} \times {D_2}^4 = J_2 = \frac{\pi}{32} \times 80^4 = 40.24 \times 10^5$ mm^4

$J_3 = \frac{\pi}{32} \times {D_3}^4 = J_2 = \frac{\pi}{32} \times 70^4 = 23.58 \times 10^5$ mm^4

≫ $$\frac{l_1}{34.09 \times 10^5} = \frac{l_2}{40.24 \times 10^5} = \frac{l_3}{23.58 \times 10^5}$$

≫ $l_1 = \frac{l_3}{23.58 \times 10^5} \times 34.09 \times 10^5$ and $l_2 = \frac{l_3}{23.58 \times 10^5} \times 40.24 \times 10^5$

≫ $l_1 = 1.44\, l_3$ and $l_2 = 1.71\, l_3$

But L = $l_1 + l_2 + l_3$

Substitute l_1 and l_2 value in the above equation

≫ $2400 = 1.44\, l_3 + 1.71\, l_3 + l_3$

≫ $4.15 l_3 = 2400$ ≫ $l_3 = 578.35$ mm

≫ $l_1 = 1.44 \times 578.35 = 832.75$ mm

≫ $l_2 = 1.71 \times 578.35 = 988.80$ mm

As the shear stress is given in shaft AB. The angle of twist of shaft AB can be obtained by using equation

$$\frac{\tau}{R} = \frac{C \times \theta}{l}$$ where $R = \frac{D_1}{2} = \frac{80}{2} = 40$mm

Then $$\theta_1 = \frac{\tau_1 \times l_1}{C_1 \times R_1} = \frac{50 \times 832.75}{8.2 \times 10^4 \times 40} = 0.01269 \text{ rad}$$

$$= 0.01269 \times \frac{180}{\pi} = 0.7273^{\circ}$$

∵ Total angle of twist of the whole shaft

$$\theta = \theta_1 + \theta_2 + \theta_3 = 3\ \theta_1 = 3 \times 0.7273 = 2.1819^{\circ}$$

Result:

Length of shaft 1 l_1 = **832.75 mm**

Length of shaft 2 l_2 = **988.80 mm**

Length of shaft 3 l_3 = **578.35 mm**

Total angle of twist θ = **2.1819°**

3.13 COMPOSITE SHAFT

A shaft made up of two or more different materials and behaving as a single shaft is known as composite shaft. Hence in a composite shaft one type of is rigidly sleeved over another type of shaft. The total torque transmitted by a composite shaft is the sum of the torques transmitted by each individual shaft. But the angle of twist in each shaft will be equal.

3.13.1 PARALLEL SHAFT

It is also same as the composite shaft, here the shaft are arranged in parallel to one thick cylinder covered with another sleeve shaft and covered with number of sleeve of different material is named as parallel shaft.

The shaft are said to be in parallel when the driving torque is applied at the junction of the shaft and the resisting torque is at the other ends of the shafts. Here the angle of twist is same for each shaft, but the applied torque is divided between the two shafts.

Angle of twist are same $\theta = \theta_1 = \theta_2$

From the torsion equation, $\theta = \frac{T \times l}{C \times J}$

Then $$\frac{T_1 \times l_1}{C_1 \times J_1} = \frac{T_2 \times l_2}{C_2 \times J_2}$$

And $T = T_1 + T_2$

If the shaft are made of same material then $C_1 = C_2$

Then $\frac{T_1 \times l_1}{J_1} = \frac{T_2 \times l_2}{J_2}$ or $\frac{T_1}{T_2} = \frac{J_1 \times l_2}{J_2 \times l_1}$

When torque is shared equally by both the shafts then

$$T_1 = T_2 \text{ then } J_1 \times l_2 = J_2 \times l_1$$

Problem: 3.15: A composite shaft consists of a steel rod 60mm diameter surrounded by a closely fitting tube of brass. Find the outer diameter of the tube so that when a torque of 1000Nm is applied to the composite shaft, it will be shared equally by the two materials. Take C for steel 8.4×10^4 N/mm^2 and C for brass 4.2×10^4 N/mm^2. Find also the maximum shear stress in each material and common angle of twist in a length of 4 m.

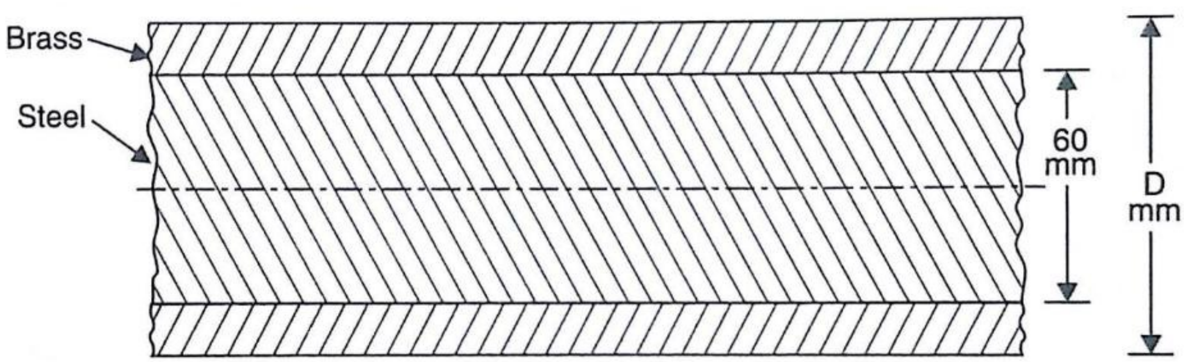
Given data:

Diameter of steel rod d = 80mm

Total torque T = 1000 Nm = 1000×10^3 Nmm

C for steel $C_s = 8.4 \times 10^4$ N/mm^2

C for brass $C_b = 4.2 \times 10^4$ N/mm^2



To find out:

Outside diameter, maximum shear stress on steel and brass, twist angle.

Solution:

Based on the condition of parallel shaft T = $T_S + T_b$ and $\theta_1 = \theta_2$

Based on given data $T_S = T_b$

∴ Total torque T = $T_S + T_b = T_S + T_s = 2T_S$

Then $T_S = \frac{1000 \times 10^3}{2} = 500 \times 10^3$ Nmm

∴ $T_S = T_b = 500 \times 10^3$ Nmm

From the torsion equation $\frac{T}{J} = \frac{C\theta}{l}$, which gives $T = \frac{C \times \theta \times J}{l}$

For steel rod $T_S = \frac{C_S \times \theta_S \times J_S}{l_S}$ and for brass rod $T_b = \frac{C_b \times \theta_b \times J_b}{l_b}$

But $T_S = T_b$ then

$$\frac{C_S \times \theta_S \times J_S}{l_S} = \frac{C_b \times \theta_b \times J_b}{l_b}$$ for compound shaft $l_s = l_s$ and $\theta_s = \theta_b$

Now the above equation become

$$C_s \times J_s = C_b \times J_b$$

Wkt polar moment of inertia for steel $J_s = \frac{\pi \times d^4}{32} = \frac{\pi \times 60^4}{32} = 1.27 \times 10^6$ mm^4

And polar moment of inertia for brass $J_b = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}(D^4 - 60^4)$

Substitute the value in the above equation

$$8.4 \times 10^4 \times 1.27 \times 10^6 = 4.2 \times 10^4 \times \frac{\pi}{32}(D^4 - 60^4)$$

D = 78.98mm

To find the Shear stress on each section

Wkt $\frac{T}{J} = \frac{\tau}{R}$ then $\tau = \frac{T \times R}{J}$

For steel rod $\tau_S = \frac{T_S \times R_S}{J_S} = \frac{500 \times 10^3 \times 30}{1.27 \times 10^6} =$ **11.79 N/mm²**

For brass sleeve $\tau_b = \frac{T_b \times R_b}{J_b} = \frac{500 \times 10^3 \times 78.98}{\frac{\pi}{32}(78.98^4 - 60^4)} =$ **7.76 N/mm²**

Common angle of twist

From the equation $T_S = \frac{C_S \times \theta_S \times J_S}{l_S}$ $\theta_S = \frac{T_S \times l_S}{C_S \times J_S}$

$$= \frac{500 \times 10^3 \times 4000}{8.4 \times 10^4 \times 1.27 \times 10^6} = \mathbf{0.0181\ rad}$$

$$= 0.0181 \times \frac{180}{\pi} = \mathbf{1.072^o}$$

Result:

Outside diameter = **78.98mm**

Maximum shear stress on steel = **11.79 N/mm²**

Maximum shear stress on brass = **7.76 N/mm^2**

Common twist angle = **1.072°**

3.14 COMBINED BENDING AND TORSION

When a shaft is transmitting torque, it is due to shear stresses. At the same time the shaft is also subjected to bending moment due to the inertia loads. Due to bending moment, bending stresses are also set up in the shaft. Hence each particle in a shaft is subjected to shear stress and bending moment.

Consider any point on the cross-section of a shaft.

Let T = Torque at the section

D = Diameter of the shaft

M = B.M. at the section

The torque T will produce shear stress at the point whereas the B.M. will produce bending stress.

Let q = shear stress at the point produced by torque Tand

σ = Bending stress at the point produced by B.M.

The shear stress at any point due to torque (T) is given by

$$\frac{q}{r} = \frac{T}{J}$$

or $$q = \frac{T}{J} r$$

The bending stress at any point due to bending moment (M) is given by

$$\frac{M}{I} = \frac{\sigma}{y} \qquad \text{or} \qquad \sigma = \frac{M \times y}{I} =$$

When two mutual perpendicular force and shear force act on a shaft, we know that the angle θ made by the plane of maximum shear with the normal cross-section is given by,

$$\tan 2\theta = \frac{2\,\tau}{\sigma}$$

The bending stress and shear stress is maximum at a point on the surface of the shaft. where $r = R = \frac{D}{2}$ and $y = \frac{D}{2}$

Let σ = Maximum bending stress i.e., on the surface of the shaft.

$$= \frac{M \times (\frac{D}{2})}{\frac{\pi}{64} D^4} = \frac{32\ M}{\pi D^3}$$

$$\tau = \frac{T}{J} \times R = \frac{T}{\frac{\pi}{32} D^4} \times \frac{D}{2} = \frac{16\,T}{\pi D^3}$$

$\therefore$ $$\tan 2\theta = \frac{2\,\tau}{\sigma} = \frac{2 \times \frac{16\,T}{\pi D^3}}{\frac{32\ M}{\pi D^3}} = \frac{T}{M}$$

Major Principal stress

$$= \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \frac{32\ M}{2 \times \pi D^3} + \sqrt{\left(\frac{32\ M}{2 \times \pi D^3}\right)^2 + \left(\frac{16\,T}{\pi D^3}\right)^2}$$

$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right) \quad \text{and}$$

Minor Principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress $= \frac{\text{Major Principal Stress} - \text{Minor Principal Stress}}{2}$

$$= \frac{16}{\pi D^3}\left(\sqrt{M^2 + T^2}\right)$$

For hollow shaft

Major Principal stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(M + \sqrt{M^2 + T^2}\right)$

Minor Principal stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(M - \sqrt{M^2 + T^2}\right)$

Maximum shear stress $= \frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(\sqrt{M^2 + T^2}\right)$

Problem: 3.16: A solid shaft of diameter 80mm is subjected to bending and twisting moment of 5MNmm and 8MNmm respectively at a point. Determine the Principal stresses, Maximum shear stress and position of plane on which they act.

Given data:

Diameter of shaft D = 80mm

Bending moment $M = 5$ MNmm $= 5 \times 10^6$ Nmm

Twisting moment $T = 8$ MNmm $= 8 \times 10^6$ Nmm

To find out:

Principal stresses, Maximum shear stress and position of plane on which they act.

Solution:

Major Principal stress $= \frac{16}{\pi D^3}\left(M + \sqrt{M^2+T^2}\right)$

$= \frac{16}{\pi D^3}\left(5 \times 10^6 + \sqrt{(5 \times 10^6)^2+(8 \times 10^6)^2}\right)$

= **143.57 N/mm^2**

Minor Principal stress $= \frac{16}{\pi D^3}\left(M - \sqrt{M^2+T^2}\right)$

$= \frac{16}{\pi D^3}\left(5 \times 10^6 - \sqrt{(5 \times 10^6)^2+(8 \times 10^6)^2}\right)$

= - 44.1 = **44.1 N/mm^2**

Maximum shear stress $= \frac{16}{\pi D^3}\left(\sqrt{M^2+T^2}\right)$

$= \frac{16}{\pi D^3}\left(\sqrt{(5 \times 10^6)^2+(8 \times 10^6)^2}\right)$

=

position of plane $= \tan 2\theta$ $= \frac{T}{M} = \frac{8 \times 10^6}{5 \times 10^6} = 1.6$

$2\theta = \tan^{-1} 1.6$ = 57°59’ or 237°59’

∴ θ = 28°59’ or **118°59’**

Result:

Major Principal stress = **143.57 N/mm^2**

Minor Principal stress = **44.1 N/mm^2**

Max. shear stress =

Position of plane = **118°59’**

Problem: 3.17: The maximum allowable shear stress in a hollow shaft of external diameter equal to twice the internal diameter is 80 N/mm^2. Determine the diameter of the shaft if it is subjected to a torque of 4×10^6 Nmm and a bending moment of 3×10^6 Nmm.

Given data:

Maximum shear stress = 80 N/mm^2

Diameter of shaft $D_o = 2\ D_i$

Bending moment M = 3×10^6 Nmm

Twisting moment T = 4×10^6 Nmm

To find out:

Diameter of the shaft

Solution:

Wkt, Maximum shear stress = $\frac{16 D_o}{\pi[{D_o}^4 - {D_i}^4]}\left(\sqrt{M^2 + T^2}\right)$

$$80 = \frac{16 D_o}{\pi[{D_o}^4 - (0.5\,D_o)^4]}\left(\sqrt{(3 \times 10^6)^2 + (4 \times 10^6)^2}\right)$$

$$80 = \frac{16 D_o}{\pi {D_o}^4[1 - (0.5)^4]}(5 \times 10^6)$$

$${D_o}^3 = \frac{16}{\pi \times 80[1 - (0.5)^4]}(5 \times 10^6) = 0.3395 \times 10^6$$

$$D_o = \sqrt[3]{0.3395 \times 10^6} \qquad = \mathbf{69.78\ mm}$$

Then $$D_i = \frac{69.78}{2} \qquad = \mathbf{34.89\ mm}$$

Result:

Outer diameter D_o = **69.78 mm**

Inner diameter D_i = **34.89 mm**

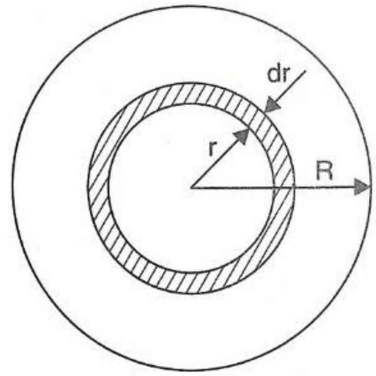
3.15 STRAIN ENERGY STORED IN A SHAFT DUE TO TORSION:

Consider a solid circular shaft of length l and radius R, subjected to a torque T producing a twist θ in the length of the shaft.

Strain energy stored U = Average torque x angle of twist

$$U = \frac{1}{2} \text{ x T x } \theta$$

Wkt Torsion equation $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$



Where T = Torque

J = polar moment of inertia

C = Modulus of rigidity

τ = Maximum shear stress

From the torsion equation $\frac{T}{J} = \frac{J\tau}{R}$ $and\ \theta = \frac{l\tau}{CR}$

Substitute the T and θ value n the energy stored equation

Then, Strain energy stored U = $\frac{1}{2}$ x $\frac{J\tau}{R}$ x $\frac{l\tau}{CR}$

Where polar moment of inertia J = $\frac{\pi}{32}$ D^4 = $\frac{\pi}{2}$ R^4 (∵ D = 2R)

∴ Strain energy stored U = $\frac{1}{2}$ x $\frac{\frac{\pi}{2}R^4\,\tau}{R}$ x $\frac{l\tau}{CR}$

$$= \frac{\pi R^2 \tau^2 l}{4\,C}$$

$$= \left(\frac{\tau^2}{4C}\right) \text{x } \pi R^2 l$$

∴ Strain energy stored (U) = $\left(\frac{\tau^2}{4C}\right)$ x V (∵ $\pi R^2 x\, l = Volume = V$)

Strain Energy Stored in hollow shaft due to torsion

Strain energy stored (U) = $\left(\frac{\tau^2}{4CD^2}\right)$ x $(D^2 + d^2)$V

Where D = Outer diameter of hollow shaft

d = Inner diameter of hollow shaft

Problem3.18. Determine the maximum strain energy stored in a solid shaft of diameter 10cm and of length 1.25 m, if the maximum allowable shear stress is 50 N/mm^2. Take C 8×10^4 N/mm^2.

Given data:

Length of shaft l = 1.25 m = 1250 mm

Diameter of shaft D = 10 cm = 100mm

Maximum shear stress τ = 50 N/mm^2

Modulus of rigidity C = 8 × 10^4 N/mm^2

To find out:

Maximum strain energy stored

Solution:

Wkt, Strain energy stored for solid shaft (U) =$\left(\frac{\tau^2}{4C}\right)$ x V

Where Volume V = $\pi\frac{D^2}{4}$ x l = $\pi\frac{100^2}{4}$ x 1250 = 9.8175 × 10^6 mm^3

Now, Strain energy stored (U) =$\left(\frac{50^2}{4\times 8\times 10^4}\right)$x 9.8175 × 10^6 = **7.669× 10^4 Nmm**

Result:

Strain energy stored (U) = **7.669× 10^4 Nmm**

Problem3.19. A solid circular shaft of 4m length and 10cm diameter is to transmit 112.5 kW power at 150 rpm. Determine the maximum shear stress and Strain energy stored in the shaft. Take C = 8 × 10^4 N/mm^2.

Given data:

Length of shaft l = 4 m = 4× 10^3 mm

Diameter of shaft D = 10 cm = 100 mm

Power transmit P = 112.5 kW = 112.5 × 10^3 W

Speed N = 150 rpm

Modulus of rigidity C = 8 × 10^4 N/mm^2

To find out:

Maximum shear stress, strain energy stored

Solution:

Wkt, power transmitted P = $\frac{2\pi NT}{60}$

$$112.5 \times 10^3 = \frac{2\times\pi\times 150\times T}{60}$$

$$T = \frac{60\times 112.5\times 10^3}{2\times\pi\times 150} = 7159 \text{ Nm} = 7.159\times 10^6 \text{ Nmm}$$

But mean torque T = $\frac{\pi}{16}\,\tau\, D^3$

Then, shear stress $\tau = \frac{T \times 16}{\pi\, D^3} = \frac{7.159 \times 10^6 \times 16}{\pi\, 100^3} = 36.5\ \text{N}/mm^2$

Strain energy stored for solid shaft (U) $= \left(\frac{\tau^2}{4C}\right) \times V$

Where Volume $V = \pi \frac{D^2}{4} \times l = \pi \frac{100^2}{4} \times 4 \times 10^3 = 2.27 \times 10^8\ \text{mm}^3$

Now, Strain energy stored (U) $= \left(\frac{36.5^2}{4 \times 8 \times 10^4}\right) \times 2.27 \times 10^8 =$ **1.308×10^5 Nmm**

Result:

Strain energy stored (U) = **1.308×10^5 Nmm**

Problem3.20. A hollow shaft of internal diameter 10cm is subjected to pure torque and attains a maximum shear stress q on the outer surface of the shaft. If the Strain energy stored in the hollow shaft is given by $\frac{\tau^2}{3C} V$, determine the external diameter of the shaft.

Given data:

Internal diameter $d = 10$ cm = 100 mm 4 m = 4×10^3 mm

Strain energy stored U = $\frac{\tau^2}{3C} V$

To find out:

External diameter of hollow shaft D

Solution:

Wkt, Strain energy stored in hollow shaft (U) $= \left(\frac{\tau^2}{4CD^2}\right) \text{x} \ (D^2 + d^2)\text{V}$

Equating the two values of strain energy, we get

$$\left(\frac{\tau^2}{4CD^2}\right) \text{x} \ (D^2 + d^2)\text{V} = \frac{\tau^2}{3C} V$$

$$\frac{D^2 + d^2}{4D^2} = \frac{1}{3} \qquad \left(\text{Cancelling } \frac{\tau^2}{3C} \text{ V on both sides}\right)$$

$$3D^2 + 3d^2 = 4D^2$$

$$3d^2 = 4D^2 - 3D^2$$

$$\frac{D^2}{d^2} = 3 \text{ then } \frac{D}{d} = \sqrt{3} = 1.732$$

Then D = 1.732 d D = 1.732 x 10 = **17.32 cm**

Result: External Diameter (D) = **17.32 cm**

3.16 DEFINITION FOR SPRING:

Springs are the elastic bodies which absorb energy due to resilience. The absorbed energy may be released as and when required. A spring which is cable of absorbing the greatest amount energy for the given stress, without getting permanently distorted, is known as the best spring. The two important types of springs are:

1. Laminated or leaf spring and
2. Helical spring

3.17 CLASSIFICATION OF SPRINGS:

Based on the shape behavior obtained by some applied force, springs are classified into the following ways:

SPRINGS

HELICAL SPRINGS

LEAF SPRINGS

1. SPIRAL SPRINGS
2. TORSION SPRING

TENSION HELICAL SPRING

COMPRESSION HELICAL SPRING

I. HELICAL SPRINGS:

DEFINITON:

It is made of wire coiled in the form of helix.

CROSS-SECTION:

Circular, square or rectangular

CLASSIFICATION:

1) Open coil springs (or) Compression helical springs

2) Closed coil springs (or) Tension helical springs

1) HELICAL TENSION SPRINGS:

CHARACTERISTICS:

- Figure1 shows a helical tension spring. It has some means of transferring the load from the support to the body by means of some arrangement.

- It stretches apart to create load.
- The gap between the successive coils is small.
- The wire is coiled in a sequence that the turn is at right angles to the axis of the spring.
- The spring is loaded along the axis.
- By applying load the spring elongates in action as it mainly depends upon the end hooks

APPLICATIONS:

1) Garage door assemblies
2) Vise-grip pilers
3) carburetors

2) HELICAL COMPRESSION SPRINGS:

CHARACTERISTICS:

- The gap between the successive coils is larger.
- It is made of round wire and wrapped in cylindrical shape with a constant pitch between the coils.
- By applying the load the spring contracts in action.
- There are mainly four forms of compression springs as shown in figure..

They are as follows:

1) Plain end
2) Plain and ground end
3) Squared end
4) Squared and ground end

Among the four types, the plain end type is less expensive to manufacture. It tends to bow sideways when applying a compressive load.

APPLICATIONS:

1) Ball point pens
2) Pogo sticks
3) Valve assemblies in engines

3) TORSION SPRINGS:

CHARACTERISTICS:

- It is also a form of helical spring, but it rotates about an axis to create load.
- It releases the load in an arc around the axis as shown in figure4.
- Mainly used for torque transmission
- The ends of the spring are attached to other application objects, so that if the object rotates around the center of the spring, it tends to push the spring to retrieve its normal position.

APPLICATIONS:

- Mouse tracks
- Rocker switches
- Door hinges
- Clipboards
- Automobile starters

4) SPIRAL SPRINGS:

CHARACTERISTICS:

- It is made of a band of steel wrapped around itself a number of times to create a geometric shape as shown in figure5.
- Its inner end is attached to an arbor and outer end is attached to a retaining drum.
- It has a few rotations and also contains a thicker band of steel.
- It releases power when it unwinds.

APPLICATIONS:

- Alarm timepiece
- Watch
- Automotive seat recliners

II. LEAF SPRING:

DEFINITION:

A Leaf spring is a simple form of spring commonly used in the suspension vehicles.

CHARACTERISTICS:

- Figure shows a leaf spring. Sometimes it is also called as a semi-elliptical spring, as it takes the form of a slender arc shaped length of spring steel of rectangular cross section.
- The center of the arc provides the location for the axle, while the tie holes are provided at either end for attaching to the vehicle body.
- Heavy vehicles, leaves are stacked one upon the other to ensure rigidity and strenth.
- It provides dampness and springing function.
- It can be attached directly to the frame at the both ends or attached directly to one end, usually at the front, with the other end attched through a shackle a short swinging arm.

- The shackle takes up the tendency of the leaf spring to elongate when it gets compressed and by which the spring becomes softer.

- Thus depending upon the load bearing capacity of the vehicle the leaf spring is designed with graduated and Ungraduated leaves as shown in figure
- Because of the difference in the leaf length, different stress will be there at each leaf. To compensate the stress level, prestressing is to be done. Prestressing is achieved by bending the leaves to different radius of curvature before they are assembled with the center clip.
- The radius of curvature decreases with shorter leaves.
- The extra intail gap found between the extra full length leaf and graduated length leaf is called as nip. Such prestressing achieved by a difference in the radius of curvature is known as nipping.

APPLICATIONS:

Mainly in automobiles suspension systems.

ADVANTAGES:

- It can carry lateral loads.
- It provides braking torque.
- It takes driving torque and withstand the shocks provided by the vehicles.

SPRING MATERIALS:

The mainly used material for manufacturing the springs are as follows:

1. Hard drawn high carbon steel.
2. Oil tempered high carbon steel.
3. Stainless steel
4. Copper or nickel based alloys.
5. Phosphor bronze.
6. Iconel.
7. Monel
8. Titanium.
9. Chrome vanadium.
10. Chrome silicon.

Depending upon the strength of the material, the material is Selected for the design of the spring.

3.18 NOMENCLATURE OF SPRING:

The below figure shows the momenclature of the spring under loading conditions.

Active Coils: Those coils which are free to deflect under load.

Angular relationship of ends: The relative position of the plane of the hooks or loops of extension spring to each other.

Buckling: Bowing or lateral deflection of compression springs when compressed, related to the slenderness ration (L/D).

Closed ends: Ends of compression springs where the pitch of the end coils is reduced so that the end coils touch.

Closed and ground ends: As with closed ends, except that the end is ground to provide a flat plane.

Close-wound: Coiled with adjacent coils touching.

Deflection: Motion of the spring ends or arms under the application or removal of an external load.

Elastic limit: Maximum stress to which a material may be subjected without permanent set.

Endurance limit: Maximum stress at which any given material may operate indefinitely without failure for a given minimum stress.

Free angle: Angle between the arms of a torsion spring when the spring is not loaded.

Free length: The overall length of a spring in the unloaded position.

Frequency (natural): The lowest inherent rate of free vibration of a spring itself (usually in cycles per second) with ends restrained.

Hysteresis: The mechanical energy loss that always occurs under cyclical loading and unloading of a spring, proportional to the are between the loading and unloading load-deflection curves within the elastic range of a spring.

Initial tension: The force that tends to keep the coils of an extension spring closed and which must be overcome before the coil starts to open.

Loops: Coil-like wire shapes at the ends of extension springs that provide for attachment and force application.

Mean coil diameter: Outside wire diameter minus one wire diameter.

Modulus in shear or torsion: Coefficient of stiffness for extension and compression springs.

Modulus in tension or bending: Coefficient of stiffness used for torsion and flat springs. (Young's modulus).

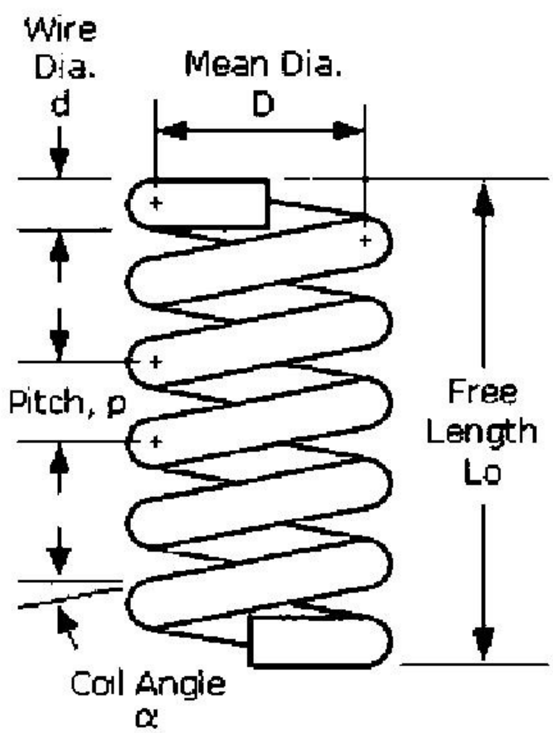
Open ends, not ground: End of a compression spring with a constant pitch for each coil.

Open ends ground: "Opens ends, not ground" followed by an end grinding operation.

Permanent set: A material that is deflected so far that its elastic properties have been exceeded and it does not return to its original condition upon release of load is said to have taken a "permanent set".

Pitch: The distance from center to center of the wire in adjacent active coils.

Spring Rate (or) Stiffness (or) spring constant: Changes in load per unit of deflection, generally given in Kilo Newton per meter. (KN/m).



Remove set: The process of closing to a solid height a compression spring which has been coiled longer than the desired finished length, so as to increase the elastic limit.

Set: Permanent distortion which occurs when a spring is stressed beyond the elastic limit of the material.

Slenderness ratio: Ratio of spring length to mean coil diameter.

Solid height: Length of a compression spring when under sufficient load to bring all coils into contact with adjacent coils.

Spring index: Ratio of mean coil diameter to wire diameter.

Stress range: The difference in operating stresses at minimum and maximum loads.

Squareness of ends: Angular deviation between the axis of a compression spring and a normal to the plane of the other ends.

Squareness under load: As in *squareness of ends*, except with the spring under load.

Torque: A twisting action in torsion springs which tends to produce rotation, equal to the load multiplied by the distance (or moment arm) from the load to the axis of the spring body. Usually expressed in inch-oz, inch-pounds or in foot-pounds.

Total number of coils: Number of active coils plus the coils forming the ends.

Spring index: The ratio between Mean diameter of coil to the diameter of the wire.

Solid length: It is the product of total number of coils and the diameter of the wire when the spring is in the compressed state. It is otherwise called as Solid height also.

3.19 SPRINGS IN PARALLEL AND SERIES:

In many situations, the combination of two or more springs either may be connected in series or parallel are required.

3.19.1SPRINGS IN SERIES:

Two springs of stiffness K_1 and K_2 are connected in series and loaded with W as shown in figure.

In this case, each spring is subjected to the same load applied at the end of one spring. Therefore the load deflection of the assembly is equal to the algebraic sum of the deflection of the two springs.

Total deflection $\frac{W}{K} = \frac{W}{K_1} + \frac{W}{K_2}$

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$$

Combined stiffness, $K = \frac{K_1 K_2}{K_1 + K_2}$

3.19.2 SPRING IN PARALLEL:

Two springs of stiffness K_1 and K_2 are connected in parallel and loaded with W as shown in figure. Let the load shared by the two springs be W_1 and W_2 therefore the deflection of each spring is same

Total load $W = W_1 + W_2$

Common deflection $\delta = \frac{W}{K} = \frac{W_1}{K_1} = \frac{W_2}{K_2}$

From that $W_1 = W\frac{K_1}{K}$ and $W_2 = W\frac{K_2}{K}$

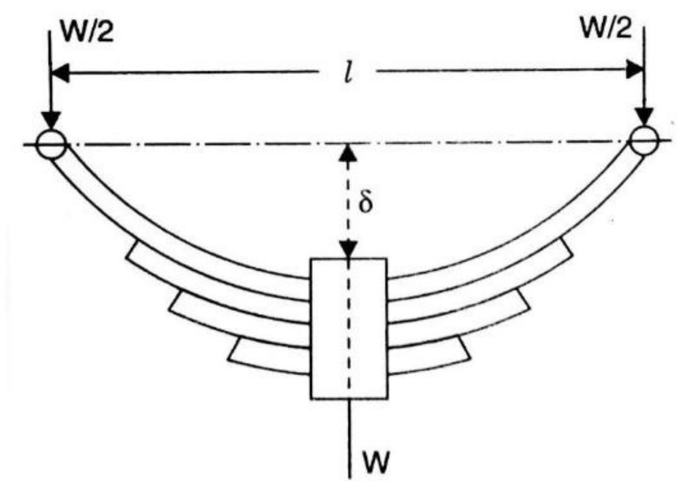
Then total load $W = W\frac{K_1}{K} + W\frac{K_2}{K}$

$$W = \frac{W}{K}(K_1 + K_2)$$

$$K = K_1 + K_2$$

3.20 LAMINATED OR LEAF SPRINGS:

The laminated are used to absorb shocks in railway wagons, coaches and road vehicles (such as cars, lorries etc..).



The above shows a laminated spring which consists of a number of parallel strip of a metal having different lengths and same width, placed one over the other. Initially all the plates are bent to the same radius and are free to slide one over the other. Fig. 16.11 shows the initial position of the spring, which is having some central deflection $\boldsymbol{\delta}$.The spring rests on the axis of the vehicle and its top plate is pinned at the ends to the chassis of the vehicle.

When the springs are loaded to the designed load W, all the plates become flat and the central deflection ($\boldsymbol{\delta}$)disappears.

Let b = Width of each plate

n = Number of plates

l = Span of spring

σ= Maximum bending stress developed in the plates

t = Thickness of each plate

W = Point load acting at the centre of the spring and

δ = Original deflection of the spring.

Expression for maximum bending stress developed in the plates. The load W acting at the centre of the lowermost plate, will be shared equally on the two ends of the top plate as shown in Fig.

∴ B.M. at the centre = Load at one end $\times \frac{l}{2}$

$$M = \frac{W}{2} \times \frac{l}{2} = \frac{W.l}{4} \qquad \ldots. (1)$$

or The moment of inertia of each plate, $I = \frac{bt^3}{12}$

But the relation among bending stress (σ), bending moment (M) and moment of inertia (I) is given by

$$\frac{M}{I} = \frac{\sigma}{y} \qquad (Here\ y = \frac{t}{2})$$

or

$$M = \frac{\sigma}{y} \times I = \frac{\sigma \times \frac{bt^3}{12}}{\frac{t}{2}} = \frac{\sigma.bt^3}{6} \qquad \ldots. (2)$$

∴ Total resisting moment by n plates

$$= n \times M = \frac{n \times \sigma . bt^2}{6}$$

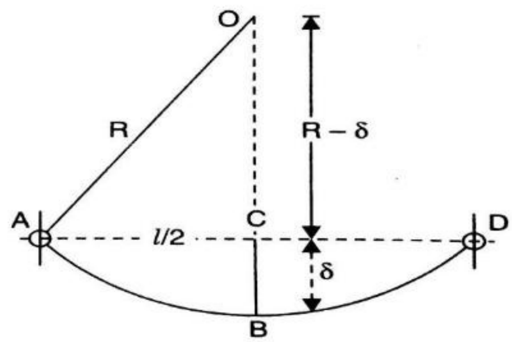
As the maximum B.M due to load is equal to the total resisting moment, therefore equating (1) and (2),

$$\frac{W . l}{4} = \frac{n\sigma . bt^2}{6}$$

$$\sigma = \frac{6W . l}{4 . n . b . t^2} = \frac{3Wl}{2nbt^2} \quad .... (3)$$

Equation (3) gives the maximum stress developed in the plate of the spring.

Expression For Central Deflection of The Leaf Spring



Now R = Radius of the plate to which they are bent.

From triangle ACO of Fig. 16.12, we have

$$AO^2 = AC^2 + CO^2$$

or $$R^2 = \left(\frac{l}{2}\right)^2 + (R - \boldsymbol{\delta})^2$$

$$= \frac{l^2}{4} + R^2 + \boldsymbol{\delta}^2 - 2R\,\boldsymbol{\delta}$$

$$= \frac{l^2}{4} + R^2 - 2R\boldsymbol{\delta}$$

∴ $$2R\boldsymbol{\delta} = \frac{l^2}{4}$$

∴ $$\boldsymbol{\delta} = \frac{l^2}{4 \times 2R} = \frac{l^2}{8R} \quad .... (3)$$

But the relation between bending stress, modulus of elasticity and radius of curvature (R) is given by $\frac{\sigma}{y} = \frac{E}{R}$

$\therefore \qquad R = \frac{E \times y}{\sigma} = \frac{E \times t}{2\sigma} \left(Here\ y = \frac{t}{2} \right)$

Substituting this value of R in equation (3), we get

$$\delta = \frac{l^2 \times 2\sigma}{8 \times E \times t} = \frac{\sigma \cdot l^2}{4Et} \qquad \text{.... (4)}$$

Equation (4) gives the central deflection of the spring.

Problem 3.21. A leaf spring carries a central load of 3000 N. The leaf spring is to be made of 10 steel plates 5 cm wide and 6 mm thick. If the bending stress is limited to 150 N/mm^2 determine :

i. Length of the Spring and
ii. Deflection at the centre of the spring. Take $E = 2 \times 10^5$ N/mm^2

Sol. Given

Central load, $W = 3000$ N

No. of plates, $n = 10$

Width of each plate, $b = 5$ cm $= 50$ mm

Thickness, $t = 6$mm

Bending stress, $\sigma = 150$ N/mm^2

Modulus of elasticity, $E = 2 \times 10^5$ N/mm^2.

Let l = Length of spring

δ = Deflection at the centre of spring.

Wkt, $\sigma = \frac{3Wl}{2nbt^2}$

$$150 = \frac{3 \times 3000 \times l}{2 \times 10 \times 50 \times 6^2}$$

$$l = \frac{150 \times 2 \times 10 \times 50 \times 6^2}{3 \times 3000} = \mathbf{60\ mm.}$$

Using equation (4) for deflection,

$$\delta = \frac{\sigma \cdot l^2}{4Et} = \frac{150 \times 600^2}{4 \times 2 \times 10^5 \times 6} = \mathbf{11.25\ mm.}$$

Problem 3.22. A laminated spring 1m long is made up of plates each 5 cm wide and 1cm thick.If the bending stress in the plate is limited to 100 N/mm^2, how many plates would be required to enable the spring to carry a central point load of 2 kN . If E = 2.1 × 10^5 N/mm^2, What is the deflection under the load .

Sol. Given :

Length of Spring, l = 1 m = 1000 mm

Width of each plate, b = 5 cm = 50 mm

Thickness of each plate, t = 1 cm = 10 mm

Bending stress, σ = 100 N/mm^2

Central load on spring, W = 2 kN = 2000 N

Young's modulus, E = 2.1 × 10^5 N/mm^2

Let n = Number of plates and

δ = Deflection under the load.

Using the equation (3),

$$\sigma = \frac{3Wl}{2nbt^2} \quad \text{or} \quad 100 = \frac{3 \times 2000 \times 1000}{2 \times n \times 50 \times 10^2}$$

or

$$n = \frac{3 \times 2000 \times 1000}{100 \times 2 \times 50 \times 100} = 6. \ \textbf{Ans.}$$

Deflection under load Using equation (4),

$$\delta = \frac{\sigma \cdot l^2}{4Et} = \frac{100 \times 1000^2}{4 \times 2.1 \times 10^5 \times 10} = 11.9 \text{ mm. } \textbf{Ans.}$$

3.21 HELICAL SPRING.

Helical springs are the thick spring wires coiled into a helix.

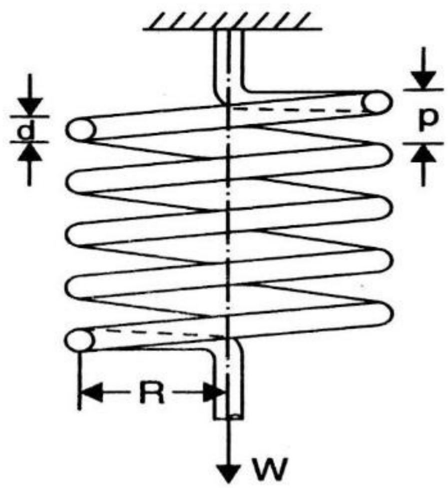
They are of two types :

1. Close-coiled helical springs and
2. Open-coiled helical springs.

Close-coiled helical springs. Close- coiled helical springs are the springs in which helix angle is very small or in other words the pitch between two adjacent turns is small. A close-coiled helical spring carrying an axial load is shown in Fig. As the helix angle in case of Close-coiled helical springs are small, hence the bending effect on the spring is ignored and we assume that the coils of a close-coiled helical springs are to stand purely torsional stresses.

Expression For Max. Shear Stress Induced In Wire.

The below shows a close-coiled helical springs subjected to an axial load.



Let d = Diameter of spring wire

P = Pitch of the helical spring

n = Number of coils

R = Mean radius of spring coil

W = Axial load on spring

C = Modulus of rigidity

τ = Max. shear stress induced in the wire

θ = Angle of twist in spring wire, and

δ = Deflection of spring due to axial load

l = Length of wire

Now twisting moment on the wire,

$$T = W \times R \quad \text{.... (1)}$$

But twisting moment is also given by

$$T = \frac{\pi}{16}\tau d^3 \quad \text{.... (2)}$$

Equating equations (1) and (2), we get

$$W \times R = \frac{\pi}{16}\tau d^3 \text{or } \tau = \frac{16\, W \times R}{\pi d^3} \quad \text{.... (3)}$$

Equation (3) gives the max. shear stress induced in the wire

Expression for deflection of spring

Now length of one coil $= \pi D$ or $2\pi R$

$\therefore$ Total Length of the wire = Length of one coil ×No. of coils

or $l = 2\pi R \times n.$

As the every section of the wire is subjected to torsion, hence the strain energy stored by the spring due to torsion is given by equation(16.20).

$\therefore$ Strain energy stored by the spring,

$$U = \frac{\tau^2}{4C}.\text{ Volume}$$

$$= \left(\frac{16W\,.\,R}{\pi d^3}\right)^2 \times \frac{1}{4C} \times \left(\frac{\pi}{4}d^2 \times 2\pi R.n\right)$$

$$\left(\because \tau = \frac{16WR}{\pi d^3}\ and\ Volume = \frac{\pi}{4d^2} \times Total\ Length\ of\ wire\right)$$

$$= \frac{32W^2R^2}{Cd^4}.\text{ R . n } = \frac{32W^2R^3\,.\,n}{Cd^4} \quad \text{.... (5)}$$

Work done on the spring = Average load × Deflection

$$= \frac{1}{2}W \times \delta$$

Equating the work done on spring to the energy stored, we get

$$\frac{1}{2}W\,.\,\delta = \frac{32W^2R^3\,.\,n}{Cd^4}$$

$$\therefore \delta = \frac{64WR^3\,.\,n}{Cd^4} \quad \text{.... (6)}$$

Expression for stiffness of spring

The stiffness of spring,

s = Load per unit deflection

$$= \frac{W}{\delta} = \frac{W}{\frac{64.WR^3 . n}{Cd^4}} = \frac{Cd^4}{64 . R^3 . n} \quad .... (7)$$

Note. The solid length of the spring means the distance between the coils when the coils are touching each other. There is no gap between the coils. The solid length is given by Solid length = Number of coils × Dia. Of wire = n × d

Problem 3.23. A closely coiled helical spring is to carry a load of 500 N. Its means coil diameter is to be 10 times that of the wire diameter. Calculate these diameters if the maximum shear stress in the material of the spring is to be 80 N/mm^2.

Given :

Load on spring, W = 500 N

Max. shear stress, $\tau = 80$ N/mm2

Let d = Diameter of wire

D = Mean diameter of coil

∴ D = 10 d.

Using equation (4), $\tau = \frac{16\,W\,R}{\pi d^3}$

or $80 = \frac{16 \times 500 \times \left(\frac{D}{2}\right)}{\pi d^3} \left(R = \frac{D}{2}\right)$

$= \frac{8000 \times \left(\frac{10d}{2}\right)}{\pi d^3}$

or $80 \times \pi d^3 = 8000 \times 5d$

$d^2 = \frac{8000 \times 5}{80 \times \pi} = 159.25$

∴ $d = \sqrt{159.25} = 12.6$ mm = 1.26 cm. **Ans.**

∴ $D = 10 \times d = 10 \times 1.26 = 12.6\ cm$. **Ans.**

Problem 3.24. In above problem if the stiffness of the spring is 20 N per mm deflection and modulus of rigidity = 8.4×10^4 N/mm^2, find the number of coils in the closely coiled helical spring .

Given :

Stiffness, s = 20 N/mm

Modulus of rigidity, C = 8.4×10^4 N/mm^2

From problem 5, W = 500 N, τ= 80 N/mm^2

d = 12.6 mm and D = 126 mm

∴ R = D/2 = 126/2 = 63 mm

Let n = Number of coils in the spring

We know, stiffness $= \frac{Load}{\delta}$

$$20 = \frac{500}{\delta}$$

∴ $\delta = \frac{500}{20} = 25$ mm

Using equation (6),

$$\delta = \frac{64WR^3 \cdot n}{Cd^4}$$

$$25 = \frac{64 \times 500 \times 63^3 \times n}{8.4 \times 10^4 \times 12.6^4}$$

$$n = \frac{25 \times 8.4 \times 10^4 \times 12.6^4}{64 \times 500 \times 63^3} = 6.6 \text{ say } 7.0$$

∴ n = 7. **Ans.**

Problem 3.25: A closely coiled helical spring of round steel wire 10 mm in diameter having 10 complete turns with a mean diameter of 12 cm is subjected to an axial load of 200 N. Determine :

1) The deflection of the spring
2) Maximum shear stress in the wire,
3) Stiffness of the spring. Take C = 8×10^4 N/mm^2.

Given :

Dia. of wire, d = 10 mm

No. of turns, n = 10

Mean dia. of coil, D = 12 cm = 120 mm

∴ Radius of coil, R = D/2 = 60 mm

Axial load, W = 200 N

Modulus of rigidity, C = 8×10^4 N/mm^2

Let δ = Deflection of the spring

τ = Max. shear stress in the wire

s = Stiffness of the spring.

1) Using equation (6),

$$\delta = \frac{64WR^3 . n}{Cd^4} = \frac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4} = 34.5 \text{ mm. } \textbf{Ans.}$$

2) Using equation (4),

$$\tau = \frac{16\,W\,R}{\pi d^3} = \frac{16 \times 200 \times 60}{\pi \times 10^3} = 61.1 \text{ N/mm}^2\text{. } \textbf{Ans.}$$

3) Stiffness of the spring,

$$s = \frac{W}{\delta} = \frac{200}{34.5} = 5.8 \text{ N/mm. } \textbf{Ans.}$$

Problem 3.26. A close coiled helical spring of 10 cm mean diameter is made up of 1 cm diameter rod and has 20 turns. The spring carries an axial load of 200 N. Determine the shearing stress. Taking the value of modulus of rigidity = 8.4×10^4 N/mm^2, determine the deflection when carrying this load. Also calculate the stiffness of the spring and the frequency of free vibration for a mass hanging from it.

Given :

Mean diameter of coil, D = 10 cm = 10 mm

Mean radius of coil, R = 5 cm = 50 mm

Diameter of rod, d = 1cm = 10 mm

Number of turns, n = 20

Axial load, W = 200 N

Modulus of rigidity, $C = 8.4 \times 10^4$ N/mm^2

Let τ = Shear stress in the material of the spring

δ = Deflection of the spring due to axial load

s = Stiffness of spring

τ = Frequency of free vibration.

Using equation (16.24),

$$\tau = \frac{16\,W\,R}{\pi d^3} = \frac{16\ \times 200 \times 50}{\pi \times 10^3} = 50.93\ \text{N/mm}^2.\ \textbf{Ans.}$$

Deflection of the spring

Using equation (16.26),

$$\delta\ = \frac{64WR^3 .\ n}{Cd^4} = \frac{64 \times 200 \times 50^3\ \times 20}{8.4 \times 10^4 \times 10^4} = 38.095\ \text{mm.}\ \textbf{Ans.}$$

Stiffness of the spring

$$\text{Stiffness} = \frac{Load\ on\ spring}{Deflection\ of\ spring} = \frac{200}{38.095} = 5.25\ \text{N/mm.}\ \textbf{Ans.}$$

Frequency of free vibration

$$\delta = 38.095\ \text{mm} = 3.8095\ \text{cm}$$

Using the relation, $\tau = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{981}{3.8095}} = 2.55$ cycles/sec. **Ans.**

Problem 3.27: A close coiled helical spring of mean diameter 20 cm is made of 3 cm diameter rod and has 16 turns. A weight of 3 kN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18 cm. Take $C = 8 \times 10^4$ N/mm^2.

Sol. Given :

Mean dia of coil, D =20 cm = 200 mm

Mean radius of coil, R = 200/2 = 100 mm

Dia. of spring rod, d = 3 cm = 30 mm

Number of turns, n =16

Weight dropped, W = 3 kN = 3000 N

Compression of the spring, δ = 18 cm = 180 mm

Modulus of rigidity , C =8 × 10^4 N/mm^2

Let h = Height through which the weight W is dropped

W = Gradually applied load which produces the compression of spring equal to 180 mm.

Now using deflection equation,

$$\delta = \frac{64WR^3 . n}{Cd^4}$$

or $$180 = \frac{64 \times W \times 100^3 \times 16}{8 \times 10^4 \times 30^4}$$

or $$W = \frac{180 \times 8 \times 10^4 \times 30^4}{64 \times 100^3 \times 16} = 11390 \text{ N}$$

Work done by the falling weight on spring

= Weight falling (h + δ) = 3000 (h + 180) N-mm

Energy stored in the spring = $\frac{1}{2}$ × 11390 × 180 = 1025100 N-mm.

Equating the work done by the falling weight on the spring to the energy stored in the spring, we

get

$$3000 (h + 180) = 1025100$$

$$h + 180 = \frac{1025100}{3000} = 341.7 \text{ mm}$$

∴ h = 341.7 – 180 = 161.7 mm. **Ans.**

Problem 3.28. The stiffness of a close coiled helical spring is 1.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm^2. The solid length of the spring (when the coils are touching) is given as 5 cm . Find : 1) Diameter of wire,(2) mean diameter of the coils and (3) number of coils required. Take C = 4.5 × 10^4 N/mm^2.

Sol. Given :

Stiffness of spring, s = 1.5 N/mm

Load on spring, W = 60 N

Maximum shear stress, $\tau = 125\ N/mm^2$

Solid length of spring, $= 5\ cm = 50\ mm$

Modulus of rigidity, $C = 4.5 \times 10^4\ N/mm^2$

Let d = Diameter of wire,

D = Mean dia, of coil, and

R = Mean radius of coil = $\frac{D}{2}$

n = Number of coils.

Using stiffness equation

$$s = \frac{Cd^4}{64\ .\ R^3\ .\ n} \text{ or } 1.5 = \frac{4.5 \times 10^4 \times d^4}{64 \times R^3 \times n}$$

$$\therefore \quad d^4 = \frac{1.5 \times 64 \times R^3 \times n}{4.5 \times 10^4} = 0.002133 R^3 \times n \quad \text{.... (1)}$$

Using shear stress equation,

$$\tau = \frac{16\ W\ R}{\pi d^3} \text{ or } 125 = \frac{16 \times 60 \times R}{\pi d^3}$$

$$\therefore \quad R = \frac{125 \times \pi d^3}{16 \times 60} = 0.40906\ d^3 \text{.... (2)}$$

Substituting the value of R in equation (1) , we get

$$d^4 = 0.002123 \times (\ 0.40906 d^3)^3 \times n$$

$$= 0.002123 \times (0.40906^3) \times d^9 \times n = 0.00014599 \times d^9 \times n$$

or

$$\frac{d^9\ .\ n}{d^4} = \frac{1}{0.00014599} \text{ or } d^5\ .\ n = \frac{1}{0.00014599} \quad \text{.... (3)}$$

Now

Solid length $= n \times d$ or $50 = n \times d$

$$n = \frac{50}{d} \quad \text{.... (4)}$$

Substituting this value of n in equation (3), we get

$$d^5 \times \frac{50}{d} = \frac{1}{0.00014599}$$

$$d^4 = \frac{1}{0.00014599} \times \frac{1}{50} = 136.99$$

$d = (136.99)^{1/4} = 3.42$ mm. **Ans.**

Substituting this value in equation (4)

$$n = \frac{50}{d} = \frac{50}{3.42} = 14.62 \text{ say } 15. \textbf{ Ans.}$$

Also from equation (2),

$$R = 0.40906\, d^3 = 0.40906 \times (3.42)^3 = 16.36 \text{ mm}$$

i.e., Mean dia. of coil, $D = 2R = 2 \times 16.36 = 32.72$ mm. Ans.

Problem 3.29: A close coiled helical spring has a stiffness of 10 N/mm. Its length when fully compressed, with adjacent coils touching each other is 40 cm. The modulus of rigidity of the material of the spring is 0.8×10^5 N/mm^2.

1) Determine the wire diameter and mean coil diameter if their ratio is $\frac{1}{10}$.
2) If the gap between any two adjacent coil is 0.2 cm, what maximum load can be applied before the spring becomes solid, i.e., adjacent coils touch.
3) What is the corresponding maximum shear stress in the spring

Sol. Given :

Stiffness of spring, $s = 10$ N/mm

Length of spring when fully compressed i.e., solid length

= 40 cm = 400 mm

Modulus of rigidity, $C = 0.8 \times 10^5$ N/mm^2

Let d = Diameter of wire of spring

D = Mean coil diameter

n = Number of turns

W = Maximum load applied when spring becomes solid

τ = Maximum shear stress induced in the wire.

Now $\frac{d}{D} = \frac{1}{10}$

Gap between any two adjacent coil = 0.2 cm = 2.0 mm

∴ Total gap in coils = Gap between two adjacent coil × Number of turns

$= 2 \times n$ mm.

When spring is fully compressed, there is no gap in the coils and hence maximum compression of the coil will be equal to the total gap in the coil.

∴ Maximum compression, $\delta = 2\times n$ mm

Now using equation (16.27),

$$s = \frac{Cd^4}{64\,.\,R^3\,.\,n} \text{ or } 10 = \frac{0.8\times10^5\times d^4}{64\,.\,R^3\,.\,n}$$

$$d^4 = \frac{10\times64\times R^3\times n}{0.8\times10^5} = \left(\frac{8}{10^3}\right)R^3\times n \qquad \text{.... (1)}$$

But from equation (16 .28),

$$\text{Solid length} = n\times d \text{ or } 400 = n\times d$$

$$n = \frac{400}{d} \qquad \text{.... (2)}$$

Substituting the value of n in equation (1),

$$d^4 = \left(\frac{8}{10^3}\right)\times R^3\times\frac{400}{d} = 3.2\times\frac{R^3}{d}$$

But mean coil radius,

$$R = \frac{D}{2}$$

∴ $$d^5 = 3.2\times\left(\frac{D}{2}\right)^3 = \frac{3.2\times D^3}{8} = 0.4\,D^3$$

$$\frac{d^5}{D^3} = 0.4 \text{ or } \frac{d^3}{D^3}\,.\,d^2 = 0.4$$

$$\left(\frac{1}{10}\right)^3.\,d^2 = 0.4 \qquad \left(\frac{d}{D} = \frac{1}{10}\right)$$

∴ $$d^2 = 0.4\times10^3 = 400$$

∴ $$d = \sqrt{400} = 20 \text{ mm} = 2\text{cm}\ .\ \textbf{Ans.}$$

But $$\frac{d}{D} = \frac{1}{10}$$

$$D = 10\times d = 10\times2 = 20.0 \text{ cm. } \textbf{Ans.}$$

Let us find first number of turns.

From equation (2), we have

$$n = \frac{400}{d} = \frac{400}{20} = 20 \qquad (d=20)$$

$$\delta = 2\times n = 2\times20 = 40\ mm$$

We know, stiffness of spring is given by

$$s = \frac{W}{\delta} \text{ or } 10 = \frac{W}{40}$$

$$W = 10 \times 40 = 400\ N. \textbf{ Ans.}$$

Using eqution (16.24), we have

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 400 \times 100}{\pi \times 20^3} \qquad \left(R = \frac{D}{2} = \frac{20}{2} = 10 \text{ cm} = 100 \text{ mm} \right)$$

$$= 25.465 \text{ N/mm}^2. \textbf{ Ans.}$$

Problem 3.30. Two close- colied concentric helical springs of the same length, are wound out of the same wire, circular in cross- section and supports a compressive load 'P'. The inner spring consists of 20 turns of mean diameter 16 cm and the outer spring has 18 turns of mean diameter 20 cm. Calculate the maximum stress produced in each spring if the diameter of wire = 1cm and P = 1000 N.

Sol. Given :

Total load supported, P = 1000 N

Both the springs are of the same length of the same material and having same dia . of wire.

Hence values of L,C and 'd' will be same.

For inner spring

No. of turns, $n_i = 20$

Mean dia.., $D_i = 16\ cm = 160$ mm $\therefore R_i = \frac{160}{2} = 80\ mm$

Dia of wire, $d_i = 1$ cm = 10 mm

For outer spring

No. of turns, $n_o = 18$

Mean dia.., $D_o = 20$ cm = 200 mm $\therefore R_o = \frac{200}{2} = 100$ mm

Dia. of wire, $d_o = 1$ cm = 10 mm

Let W_i = Load carried by inner spring

W_o = Load carried by outer spring

τ_i = Max. shear stress produced in inner spring

τ_o = Max. shear stress produced in outer spring.

Now $\quad W_i + W_o$ = Total load carried = 1000 …. (1)

Since there are two close-coiled concentric helical springs, hence deflection of both the springs will be same.

$\delta_o = \delta_i$ where δ_o = Deflection of outer spring

δ_i = Deflection of inner spring.

The deflection of close-coiled spring is given by equation (16.26) as

$$\delta = \frac{64WR^3 \,.\, n}{Cd^4}$$

Hence for outer spring , we have

$$\delta_o = \frac{64W_o \times R_o \times n_o}{C \times d_o^4} = \frac{64W_o \times 100^3 \times 18}{C \times 10^4} \qquad (\ R_o = 100,\ d_o = 10)$$

Similarly for inner spring , we have

$$\delta_i = \frac{64W_i \times R_i \times n_i}{C \times d_i^4} = \frac{64W_i \times 80^3 \times 20}{C \times 10^4}$$

(Material of wires is same. Hence value of C will be same.)

But $\quad \delta_o = \delta_i$

$$\therefore \quad \frac{64W_o \times 100^3 \times 18}{C \times 10^4} = \frac{64W_i \times 80^3 \times 20}{C \times 10^4}$$

$$W_o \times 100^3 \times 18 = W_i \times 80^3 \times 20$$

$$W_o = \frac{W_i \times 80^3 \times 20}{100^3 \times 18} = 0.569\ W_i$$

Substituting the value of W_o in equation (1), we get

$W_i + 0.569\ W_i = 1000$ or $1.569\ W_i = 1000$

$$\therefore \quad W_i = \frac{1000}{1.596} = 637.3 \text{ N.}$$

But from equation (1), $W_i + W_o = 1000$

$$\therefore \quad W_o = 1000 - W_i = 1000 - 637.3 = 362.7 \text{ N.}$$

The maximum shear stress produced is given by equation (16.24) as

$$\tau = \frac{16WR}{\pi d^3}$$

For outer spring, the maximum shear stress produced is given by,

$$\tau_0 = \frac{16W_o \times R_o}{\pi \times d_0^3} = \frac{16 \times 362.7 \times 100}{\pi \times 10^3}$$

$= 184.72$ N/mm^2. **Ans.**

Similarly for inner spring, the maximum shear stress produced is given by,

$$\tau_i = \frac{16W_i \times R_i}{\pi \times d_i^3} = \frac{16 \times 637.3 \times 80}{\pi \times 10^3}$$

$= 259.66$ N/mm^2. **Ans.**

Problem 3.31. A closely coiled helical spring made of 10 mm diameter steel wire has 15 coils of 100 mm mean diameter. The spring is subjected to an axial load of 100 N. Calculate:

i. The maximum shear stress induced,
ii. The deflection, and
iii. Stiffness of the spring n Take modulus of rigidity, C = 8.16×10^4 N/mm^2.

Sol. Given :

Dia. of wire, d = 10 mm

Number of coils, n =15

Mean dia. of coil, D = 100 mm

Mean radius of coil, R = $\frac{100}{2}$ = 50 mm

Axial load, W = 100 N

Modulus of rigidity, C = 8.16×10^4 N/mm^2.

i. Maximum shear stress induced

Wkt $$\tau = \frac{16WR}{\pi d^3} = \frac{16 \times 100 \times 50}{\pi \times 10^3} = 24.46 \text{ N/mm}^2.$$ **Ans.**

The deflection (δ)

Wkt

$$\delta = \frac{64WR^3 \cdot n}{Cd^4} = \frac{64 \times 100 \times 50^3 \times 15}{8.16 \times 10^4 \times 10^4} = 14.7 \text{ mm.}$$ **Ans.**

Stiffness of the spring

$$\text{Stiffness} = \frac{Load\ on\ spring}{Deflection\ of\ spring}$$

$$= \frac{100}{14.7} = 6.802 \text{ N/mm. } \textbf{Ans.}$$

IMPORTANT TERMS

TORSION OF SHAFT		
Torsion Equation	$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$	T = Toque (T_{max}) l = length of shaft θ = Angle of twist in radian J = Polar moment of Inertia $= \frac{\pi}{32}D^4$ For solid shaft $\frac{\pi}{32}(D^4 - d^4)$ for hollow shaft
Toque on shaft(T)	$T = \frac{\pi}{16}\tau D^3$	D = Outer diameter of shaft d = inner diameter of hollow shaft
	$T = \frac{\pi}{16}\tau\left(\frac{D^4 - d^4}{D}\right)$	τ = shear stress C = Modulus of rigidity of shaft
Torsional rigidity	= C X J	
Power transmission	$P = \frac{2\pi NT}{60}$	N = Speed in rpm T = Torque (T_{mean})
% of saving material	$= \frac{W_s - W_h}{W_s}\ x\ 100$	$W_s = Weight\ of\ solid\ shaft$ $W_h = Weight\ of\ hollow\ shaft$
% of saving material for same material and same length	$= \frac{A_s - A_h}{A_s}\ x\ 100$	$A_s = Area\ of\ solid\ shaft$ $A_h = Area\ of\ hollow\ shaft$
Find suitable diameter of shaft	Find diameter of shaft in two method 1. Based on shear stress $T = \frac{\pi}{16}\tau D^3$ 2. Based on angle of twist $\frac{T}{J} = \frac{C\theta}{l}$	Largest diameter is the suitable diameter of shaft Same procedure followed for hollow shaft
SERIES SHAFT (Shaft with varying cross section)	1. Angle of twist $\theta = \theta_1 + \theta_2$ 2. $Length\ L = l_1 + l_2$ 3. $Torque\ T = T_1 = T_2$	
PARALLEL SHAFT (Composite shaft)	1. Angle of twist same $\theta = \theta_1 = \theta_2$ 2. $Length\ L = l_1 = l_2$ 3. $Torque\ T = T_1 + T_2$	

COMBINED BENDING AND TORSION ACT ON A SHAFT		
Major Principal Stress	$=\frac{16}{\pi D^3}(M+\sqrt{M^2+T^2}\ )$ $=\frac{16D}{\pi(D^4-d^4)}(M+\sqrt{M^2+T^2})$	M = Bending moment T = Torque V = Volume = Area x length $=\frac{\pi D^3}{4}l$ for solid shaft $=\frac{\pi}{4}(D^2-d^2)l$ for hollow shaft
Minor Principal Stress	$=\frac{16}{\pi D^3}(M-\sqrt{M^2+T^2}\ )$ $=\frac{16D}{\pi(D^4-d^4)}(M-\sqrt{M^2+T^2})$	
Maximum shear stress	$=\frac{16}{\pi D^3}(\sqrt{M^2+T^2}\ )$ $=\frac{16D}{\pi(D^4-d^4)}(\sqrt{M^2+T^2})$	
Strain energy stored in a body due to torsion	$U=\frac{\tau^2}{4C}V$	
SPRING (CLOSED COIL HELICAL SPRING)		
Deflection (δ)	$\delta=\frac{64WR^3n}{Cd^4}$	W = axial load on the spring R = mean radius of coil spring n = number of turn in coil C = modulus of rigidity of spring d = diameter of spring wire δ = deflection of spring
Energy Stored in spring (U)	= Average load x deflection = W x δ/2 $=\frac{32W^2R^3n}{Cd^4}$	
Spring stiffness (s)	= W /δ $=\frac{Cd^4}{64\ R^3n}$	
Shear stress induced in spring (τ)	$\tau=\frac{16\ WR}{\pi d^3}$	
Solid length of spring	= n x d	
OPEN COIL HELICAL SPRING		
Deflection (δ) Due to axial load	$\delta=\frac{64WR^3nsec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C}+\frac{2\sin^2\alpha}{E}\right]$	W = axial load on the spring R = mean radius of coil spring n = number of turn in coil C = modulus of rigidity of spring d = diameter of spring wire δ = deflection of spring $\alpha = helix\ angle\ of\ spring$ E = young's modulus
Deflection (δ) Due to axial twisting couple	$\delta=\frac{64M_0R^2nsin\alpha}{d^4}\left[\frac{1}{C}-\frac{2}{E}\right]$	
Angle of rotation (β) Due to axial load	$\beta=\frac{64WR^2nsin\alpha}{d^4}\left[\frac{1}{C}-\frac{2}{E}\right]$	

Angle of rotation (β) Due to axial twisting couple	$\beta = \frac{64M_0 Rnsec\alpha}{d^4}\left[\frac{\sin^2\alpha}{C} + \frac{2\cos^2\alpha}{E}\right]$	
Bending Stress	$f = \frac{32\,WR\,sin\alpha}{\pi d^3}$	
Shear stress	$\tau = \frac{16\,WRcos\alpha}{\pi d^3}$	

THEORETICAL PROBLEMS

1. Define the term torsion, torsional rigidity and polar moment of inertia.
2. Derive an expression for the shear stress produced in a circular shaft which is subjected to torsion. what are the assumptions made in the derivation
3. Prove that the torque transmitted by a solid shaft when subjected to torsion is given by T = $\pi/16\tau D^3$ where D =dia of shaft and τ = max shear stress
4. When a circular shaft is subjected to torsion show that the shear stress varies linearly from the axis to the surface
5. Derive the relation for circular shaft when subjected to torsion as shown

   T/J = τ/R = Cθ/L

   Where T = torque transmitted

   J = polar moment of inertia

   τ = max shear stress

   R = radius of shaft

   C = modulus of rigidity

   θ = angle of twist

   L = length of shaft

6. Find an expression for the torque transmitted by a hollow circular shaft of ext dia D_O and int dia D_i
7. Define the term polar modulus. Find the expression for a solid shaft and hollow shaft
8. What do you mean by strength of shaft
9. Define torsional rigidity ofshaft. Prove that the torsional rigidity is the torque required to produce a twist of one radian in a unit length of shaft
10. Prove that the strain energy stored in a body due to shear stress is given by

$$U = \tau^2/2C * V$$

Where τ = shear stress

C = modulus of rigidity

V = volume of body

11. Find an expression for strain energy stored in a body due to torsion
12. A hollow shaftof ext dia D and int dia d is subjected to torsion, prove that the strain energy stored is given by

    $U = \tau^2/4CD^2\ (D^2 + d^2)$ where V = volume of hollow shaft. τ = shear stress
13. What is a spring. Name the two important types of spring
14. Prove that the central deflection of the laminated spring is given by

    $$\delta = \sigma l^2/4Et$$

    Where σ = max stress

    E = modulus of elasticity

    L = length of leaf spring

    T = thickness of each plate
15. Define helical springs. Name the two important types of helical springs.
16. Prove that the max shear stress induced in wire of close coiled helical spring is given by

    $$\tau = 16WR/\pi d^3$$

    Where τ = max shear stress

    W = axial load

    R = mean radius of spring coil
17. Find an expression for the strain energy stored by the close coiled helical spring when subjected to axial load W
18. Prove that the deflection of a close coiled helical spring at the centres due to axial load W is given by $\delta = 64WnR^3/Cd^4$

    Where R = mean radius of spring coil

N = number of coil

C = modulus of rigidity

D = dia of wire

NUMERICAL PROBLEMS

1. The shearing stress in a solid shaft is not exceed 45 N/mm^2 when the torque transmitted is 4000 Nm. Determine the min dia of the shaft. Ans = 16.49 mm
2. Find the max torque transmitted by a hollow circular shaft of ext dia 30cm and int dia 15cm if the shear stress is not to exceed 40N/mm^2. Ans = 198.8Kn
3. Two shafts of the same material and of same length are subjected to same torque if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose int dia is 0.7 times the outside dia and the max shear stress developed in each shaft is the same, compare the weight of the shaft. Ans =(1.633/1)
4. Find the max shear stress induced in a solid circular shaft of dia 20cm when the shaft transmit 187.5kW at 200rpm. Ans = 5.7N/mm^2
5. A solid circular shaft is to transmit 375kW at 150 rpm. Find the dia of shaft if the shear stress is not to exceed 65N/mm^2. What percent saving in weight would be obtained if this is replaced by a hollow shaft whose int dia equal to 2/3 of ext dia, the length, the material and max shear stress being the same. Ans = 12.29 cm and 35.71%
6. A solid shaft has to transmit 112.5 kW at 250 rpm. taking allowable shear stress as 70 N/mm^2. Find suitable dia for the shaft if the max torque transmitted at each revolution exceeds the mean by 20%. Ans = 7.20 cm
7. A hollow shaft is to transmit 337.5 kW at 100 rpm. If the shear stress is not to exceed 65N/mm^2 and the int dia is 0.6 of the ext dia, find the ext and int dia assuming the max torque is 1.3 times the mean. Ans = 15.52 cm and 9.312 cm
8. Determine the dia of solid steel shaft which will transmit 112.5 kW at 200 rpm. Also determine the length of shaft if the twist must not exceed 1.5 over the entire length. The max shear stress is limited to 55N/mm^2. Take the value of modulus of rigidity = 8*$10^4 N/mm^2$. Ans = 7.9 cm and 150.4 cm

9. Determine the dia of solid shaft which will transmit 337.5 kW at 300 rpm. The max shear stress should not exceed 35N/mm^2 and twist should not be more than 1 in a shaft region of 2.5m. take modulus of rigidity = 9*10^4 N/mm^2. Ans = 11.57 cm
10. and the coupling are equally strong in torsion. Ans = 11.6 cm and 0.848 cm
11. A hollow shaft of 1.5 m long has ext dia 60mm. It has 30mm int dia for a part of length and 40mm int dia for rest of length. If the max shear stress in it is not to exceed 85 N/mm^2 determine the max horse power transmitted at 350 rpm. If the twist produced in two portions of the shafts are equal find the length. Ans = 141.37, 808.23mm, 691.77mm
12. A leaf spring carries a central load of 2.5 kN. The leaf spring is to be made of 10 steel plates 6 cm wide and 5mm thick. If the bending stress is limited to 100 N/mm^2. Determine the length of spring and deflection at the centre of the spring. Ans = 40 cm, 0.4cm
13. A laminated leaf spring 0.9m long is made up of plates each 5cm wide and 1cm thick. If the bending stress in the plate is limited to 120 N/mm^2, how many plates would be required to enable the spring to carry a central pont load of 2.65 kN. What is the deflection under the load. Ans = 6 plates, 1.215 cm
14. A closely coiled helical spring is to carry a load of 1kN. Its mean coil dia is to be 10 times the wire dia. Calculate these dia if te max shear stress in the materiall of the spring is to be 90 N/mm^2. Ans = 16.82 cm and 1.68cm
15. In ques 16 if stiffness of spring is 20N/mm deflection and modulus of rigidity = 8.4*10^4 N/mm^2 find the number of coils in the closely coiled helical spring. Ans = 9
16. A closely coiled helical spring of round steel wire 8mm in dia having 10 complete turns with a mean dia of 10 cm is subjected to an axial load of 250 N. Determine the deflection of spring, max shear stress in wire, stiffness of spring. Ans = 6.1cm, 124.34 N/mm^2, 4.1 N/mm^2