4.5 CAPACITANCE OF A SINGLE-CORE CABLE

A single-core cable can be considered to be equivalent to two long co-axial cylinders. The conductor (or core) of the cable is the inner cylinder while the outer cylinder is represented by lead sheath which is at earth potential. Consider a single core cable with conductor diameter d and inner sheath diameter D. Let the charge per metre axial length of the cable be Q coulombs and ε be the permittivity of the insulation material between core and lead sheath.

Obviously *$\varepsilon = \varepsilon_0\, \varepsilon_r$ where εr is the relative permittivity of the insulation.

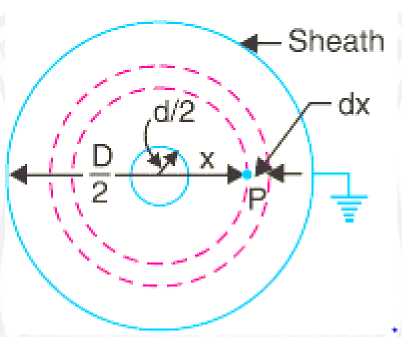

Figure 4.5.1 Single core cable

[Source: "Principles of Power System" by V.K.Mehta Page: 128]

Consider a cylinder of radius x metres and axial length 1 metre. The surface area of this cylinder is = 2 π x × 1 = 2 π x m^2

∴ Electric flux density at any point P on the considered cylinder is

$$D_x = \frac{Q}{2\pi x}\ \text{C/m}^2$$

$$\text{at point } P,\ E_x = \frac{D_x}{\varepsilon} = \frac{Q}{2\pi x \varepsilon} = \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r}\ \text{volts/m}$$

The work done in moving a unit positive charge from point P through a distance dx in the direction of electric field is Ex dx. Hence, the work done in moving a unit positive charge from conductor to sheath, which is the potential difference V between conductor and sheath, is given by :

$$V = \int_{d/2}^{D/2} E_x \, dx = \int_{d/2}^{D/2} \frac{Q}{2\pi x \varepsilon_0 \varepsilon_r} \, dx = \frac{Q}{2\pi \varepsilon_0 \varepsilon_r} \log_e \frac{D}{d}$$

Capacitance of the cable is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi \varepsilon_0 \varepsilon_r} \log_e \frac{D}{d}} \text{ F/m}$$

$$= \frac{2\pi \varepsilon_o \varepsilon_r}{\log_e (D/d)} \text{ F/m}$$

$$= \frac{2\pi \times 8 \cdot 854 \times 10^{-12} \times \varepsilon_r}{2 \cdot 303 \log_{10} (D/d)} \text{ F/m}$$

$$= \frac{\varepsilon_r}{41 \cdot 4 \log_{10} (D/d)} \times 10^{-9} \text{ F/m}$$

If the cable has a length of l metres, then capacitance of the cable is

$$C = \frac{\varepsilon_r \, l}{41 \cdot 4 \log_{10} \frac{D}{d}} \times 10^{-9} \text{ F}$$

Problem 1

A single core cable has a conductor diameter of 1 cm and internal sheath diameter of 1·8 cm. If impregnated paper of relative permittivity 4 is used as the insulation, calculate the capacitance for 1 km length of the cable.

Solution:

$$C = \frac{\varepsilon_r \, l}{41 \cdot 4 \log_{10} (D/d)} \times 10^{-9} \text{ F}$$

$\varepsilon_r = 4$; $l = 1000$ m

$D = 1 \cdot 8$ cm ; $d = 1$ cm

$$C = \frac{4 \times 1000}{41 \cdot 4 \log_{10} (1 \cdot 8 / 1)} \times 10^{-9} \text{ F} = 0 \cdot 378 \times 10^{-6} \text{ F}$$

Problem 2

A 33 kV, 50 Hz, 3-phase underground cable, 4 km long uses three single core cables. Each of the conductor has a diameter of 2·5 cm and the radial thickness of insulation is 0·5 cm. Determine (i) capacitance of the cable/phase (ii) charging current/phase (iii) total charging kVAR. The relative permittivity of insulation is 3.

Solution:

$$C = \frac{\varepsilon_r \; l}{41 \cdot 4 \log_{10}(D/d)} \times 10^{-9} \text{ F}$$

$$\varepsilon_r = 3 \quad ; \quad l = 4 \text{ km} = 4000 \text{ m}$$

$$d = 2 \cdot 5 \text{ cm} \quad ; \quad D = 2.5 + 2 \times 0.5 = 3.5 \text{ cm}$$

$$C = \frac{3 \times 4000 \times 10^{-9}}{41 \cdot 4 \times \log_{10}(3 \cdot 5 / 2 \cdot 5)}$$

$$= 1.984 \times 10^{-6}$$

(ii) Voltage/phase, V_{ph}

$$= \frac{33 \times 10^3}{\sqrt{3}} = 19 \cdot 05 \times 10^3 \text{ V}$$

Charging current/phase, I_C

$$I_C = \frac{V_{ph}}{X_C} = 2\pi f C V_{ph}$$

$$= 2\pi \times 50 \times 1984 \times 10^{-9} \times 19 \cdot 05 \times 10^3$$

$$= 11.87 \text{ A}$$

Total charging kVAR

$$= 3 V_{ph} I_C = 3 \times 19 \cdot 05 \times 10^3 \times 11 \cdot 87$$

$$= 678.4 \times 10^3 \text{ KVAR}$$