Equivalent Circuit of Induction Motor

We have already seen that the induction motor can be treated as generalized transformer. Transformer works on the principle of electromagnetic induction. The induction motor also works on the same principle. The energy transfer from stator to rotor of the induction motor takes place entirely with the help of a flux mutually linking the two. Thus stator acts as a primary while the rotor acts as a rotating secondary when induction motor is treated as a tranformer.

If E_1 = Induced voltage in stator per phase
E_2 = Rotor induced e.m.f. per phase on standstill k = Rotor turns / Stator turns

then $k = E_2/ E_1$

Thus if V_1 is the supply voltage per phase to stator, it produces the flux which links with both stator and rotor. Due to self induction E_1, is the induced e.m.f. in stator per phase while E_2 is the induced e.m.f. in rotor due to mutual induction, at standstill. In running condition the induced e.m.f. in rotor becomes E_{2r} which is s E_2.

Now E_{2r} = Rotor induced e.m.f. in running condition per phase R_2 = Rotor resistance per phase
X_{2r} = Rotor reactance per phase in running condition R_1 = Stator resistance per phase
X_1 = Stator reactance per phase

So induction motor can be represented as a transformer as shown in the Fig. 3.25.

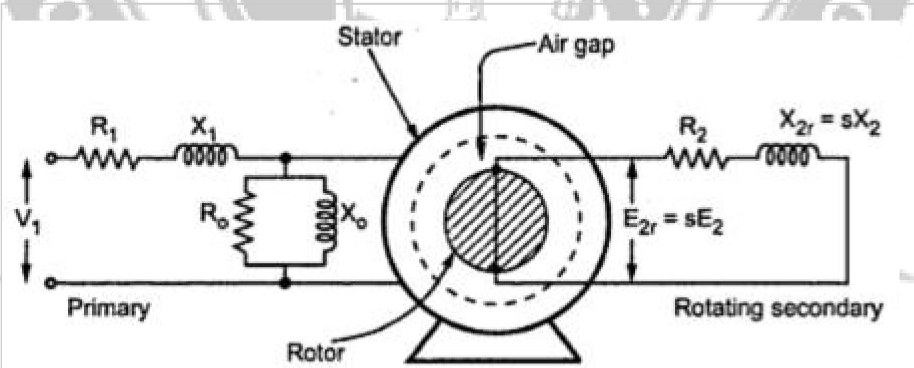

Fig. 3.25 Induction motor as a transformer

When induction motor is on no load, it draws a current from the supply to produce the flux in air gap and to supply iron losses.

1. I_c = Active component which supplies no load losses
2. I_m = Magnetizing component which sets up flux in core and air gap These two currents give us the elements of an exciting branch as,

R_o = Representing no load losses = V_1 / I_c and
X_o = Representing flux set up = V_1/I_m Thus,
$\bar{I}_o = \bar{I}_c + \bar{I}_m$

The equivalent circuit of induction motor thus can be represented as shown in the Fig. 3.26.

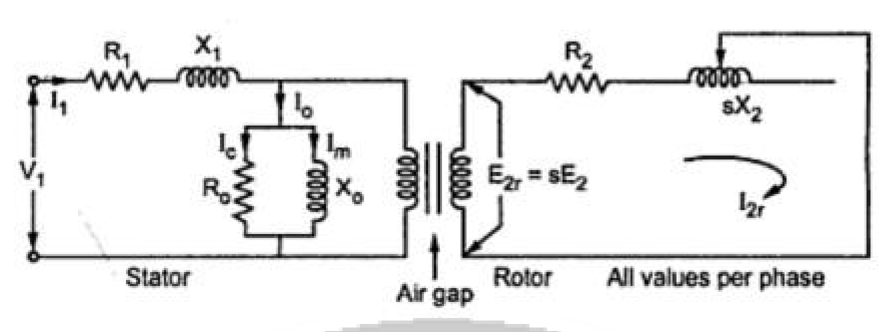

Fig. 3.26 Basic equivalent circuit

The stator and rotor sides are shown separated by an air gap.

I_{2r}= Rotor current in running condition

$= E_{2r}/Z_{2r} = (s E_2)/\sqrt{R_2^2 + (s X_2)^2}$

It is important to note that as load on the motor changes, the motor speed changes. Thus slip changes. As slip changes the reactance X_{2r} changes. Hence $X_{2r} = sX_2$ is shown variable.

Representing of rotor impedance :

It is shown that, $I_{2r} = (sE_2)/\sqrt{R_2^2 + (s X_2)^2} = E_2/\sqrt{(R_2/s)^2 + X_2^2}$

So it can be assumed that equivalent rotor circuit in the running condition has fixed reactance X_2, fixed voltage E_2 but a variable resistance R_2/s, as indicated in the above equation.

Now $R_2/s = R_2 + (R_2/s) - R_2$

$$\therefore \quad \frac{R_2}{s} = R_2 + R_2\left(\frac{1}{s} - 1\right) = R_2 + R_2\left(\frac{1-s}{s}\right)$$

So the variable rotor resistance R_2/s has two parts.

1. Rotor resistance R_2 itself which represents copper loss.
2. $R_2(1 - s)/s$ which represents load resistance R_L. So it is electrical equivalent of mechanical load on the motor.

Key Point : Thus the mechanical load on the motor is represented by the pure resistance of value $R_2(1 - s)/s$.

So rotor equivalent circuit can be shown as,

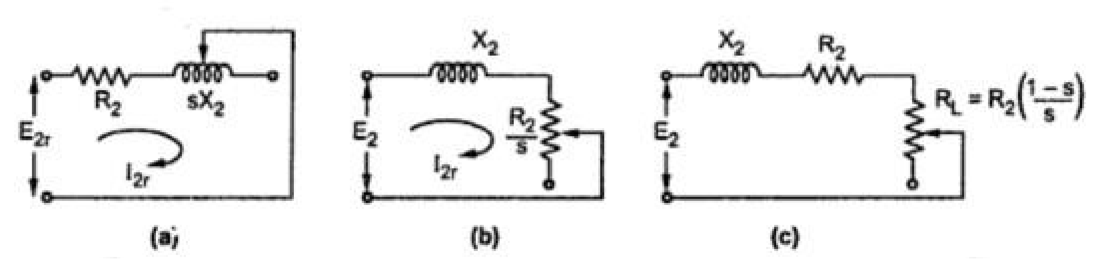

Fig. 3.27 Rotor equivalent circuit

Now let us obtain equivalent circuit referred to stator side.

Equivalent circuit referred to stator :

Transfer all the rotor parameters to stator, $k = E_2/E_1$ = Transformation ratio

$E_2' = E_2/k$

The rotor current has its reflected component on the stator side which is I_{2r}'.

$I_{2r}' = k\ I_{2r} = (k\ s\ E_2)/\sqrt{(R_2^2 + (s\ X_2)^2)}$
$X_2' = X_2/K^2$ = Reflected rotor reactance R_2' = R_2/K^2 = Reflected rotor resistance R_L' = $R_L/K^2 = (R_2/K^2)(1\text{-}s / s)$
$= R_2'\ (1\text{-}s / s)$

Thus R_L' is reflected mechanical load on stator.

So equivalent circuit referred to stator can be shown as in the Fig. 4

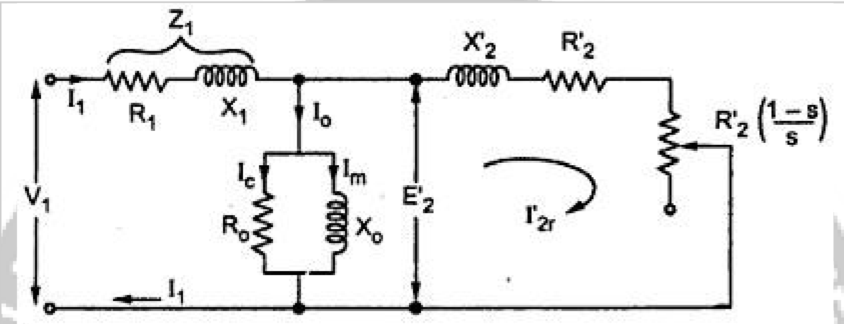

Fig. 3.28 Rotor equivalent circuit

The resistance R_2' (1 -s)/ s = R_L' is fictitious resistance representing the mechanical load on the motor.

Approximate Equivalent Circuit

Similar to the transformer the equivalent circuit can be modified by shifting the exciting current (R_o and X_o) purely across the supply, to the left of R_1 and X_1. Due to this, we are neglecting the drop across R_1 and X_1 due to I_o, which is very small. Hence the circuit is called approximate equivalent circuit. The circuit is shown in the Fig.3.29.

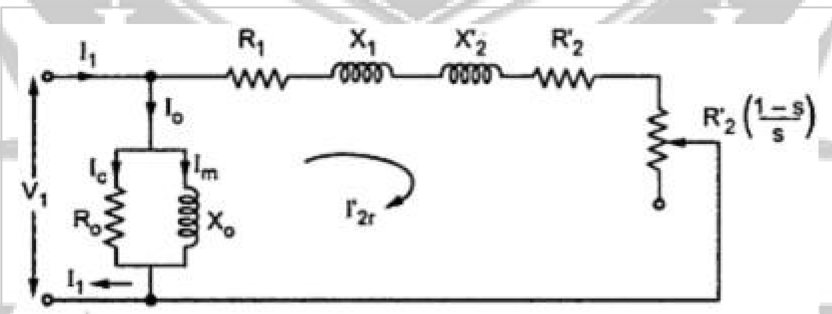

Fig. 3.29 Approximate equivalent circuit

Now the resistance R_1 and R_2' while reactance X_1 and X_2' can be combined.

So we get,

R_{1e} = Equivalent resistance referred to stator = $R_1 + R_2'$ X_{1e} = Equivalent reactance referred to stator = $X_1 + X_2'$ $R_{1e} = R_1 + (R_2/K^2)$

and $X_{1e} = X_1 + (X_2/K^2)$

While $\bar{I}_1 = \bar{I}_o + \bar{I}_{2r}'$.............................. phasor diagram

and $\bar{I}_o = \bar{I}_c + \bar{I}_m$

Thus the equivalent circuit can be shown in the Fig.3.30.

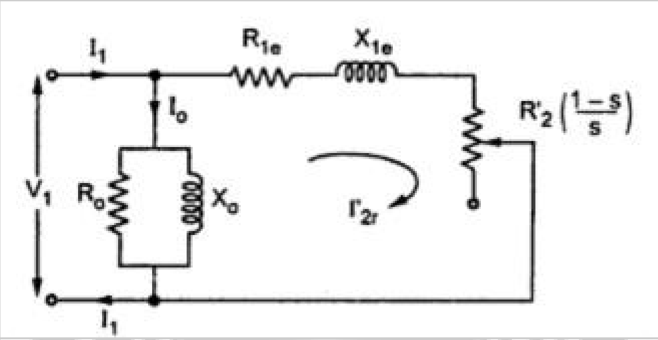

Fig. 3.30 equivalent circuit

Power Equations from Equivalent Circuit

With reference to approximate equivalent circuit shown in the Fig. 3.30, we can write various power equations as,

P_{in} = input power = 3 $V_1 I_1 \cos \Phi$

where V_1 = Stator voltage per phase

I_1 = Current drawn by stator per phase $\cos \Phi$

= Power factor of stator

Stator core loss = $I_m^2 R_o$

Stator copper loss = 3 $I_1^2 R_o$

where R_1 = Stator resistance per phase

P_2 = Rotor input = $(3 I_{2r}'^2 R_2')/s$

P_c = Rotor copper loss = 3 $I_{2r}'^2 R_2'$

Thus $P_c = s\, P_2$

P_m = Gross mechanical power developed

$$\therefore \quad P_m = P_2 - P_c = \frac{3 (I'_{2r})^2 R'_2}{s} - 3 (I'_{2r})^2 R'_2 = 3 (I'_{2r})^2 R'_2 \left(\frac{1-s}{s}\right)$$

T = Torque developed

$$\therefore \quad T = \frac{P_m}{\omega} = \frac{3 (I'_{2r})^2 R'_2 \left(\frac{1-s}{s}\right)}{\frac{2\pi N}{60}}$$

where N = Speed of motor

But $N = N_s (1-s)$ =, so substituting in above

$$T = \frac{\frac{3(I'_{2r})^2 R'_2}{s}}{\frac{2\pi N_s}{60}} = 9.55 \times \frac{\frac{3(I'_{2r})^2 R'_2}{s}}{N_s} \quad \text{N-m}$$

and $\qquad I_{2r}' = V_1 /((R_{1e} + R_L') + j X_{1e})$

where $\qquad R_L' = R_2' (1-s)/s$

$$I_{2r}' = V_1/\sqrt{(R_{1e} + R_L')^2 + X^2}$$

Key Point : Remember that in all the above formula all the values per phase values.

Maximum Power Output

Consider the approximate equivalent circuit as shown in the Fig.7

In this circuit, the exciting current I_o is neglected hence the exciting no load branch is not shown.

$\therefore \qquad I_1 = I_{2r}'$

The total impedance is given by,

$Z_T = (R_{1e} + R_L')+ \qquad$ where $R_L' = R_2' (1-s)/s$ I_1

$= V_1 /\sqrt{(R_{1e} + R_L')^2 + (X_{1e})^2}$

The power supplied to the load i.e. P_{out} per phase is, Per phase

$P_{out} = I_1^2 R_L'$ watts per phase

$\therefore \quad$ Total $= 3 I_1^2 R_L'$

$$\therefore \qquad P_{out} = 3 \frac{V_1^2}{[(R_{1e} + R'_L)^2 + (X_{1e})^2]} (R'_L)$$

To obtain maximum output power, differentiate the equation of total P_{out} with respect to variable R_L' and equal to zero.

$$\therefore \qquad \frac{d}{dR'_L}\left[\frac{3 V_1^2 (R'_L)}{[(R_{1e} + R'_L)^2 + (X_{1e})^2]}\right] = 0$$

$$\therefore \qquad [(R_{1e} + R'_L)^2 + (X_{1e})^2] [3 V_1^2] - 3 V_1^2 (R'_L) [2 (R_{1e} + R'_L)] = 0$$

$$\therefore \qquad (R_{1e} + R'_L)^2 + (X_{1e})^2 - 2 (R'_L)(R_{1e} + R'_L) = 0 \qquad \text{... Taking } 3 V_1^2 \text{ common}$$

$$\therefore \qquad R_{1e}^2 + (R'_L)^2 + 2 R_{1e} R'_L + X_{1e}^2 - 2 R_{1e} R'_L - 2(R'_L)^2 = 0$$

$$\therefore \qquad R_{1e}^2 + X_{1e}^2 = (R'_L)^2$$

But $\qquad Z_{1e} = \sqrt{R_{1e}^2 + X_{1e}^2}$ = Leakage impedance referred to stator

$\therefore \qquad Z_{1e}^2 = R_L'^2$

Thus the mechanical load on the induction motor should be such that the equivalent load resistance referred to stator is equal to the total leakage impedance of motor referred to stator.

Slip at maximum P_{out} : This can be obtained as,

$R_L' = Z_{1e} = R_2'(1-s)/s$ where $R_L' = R_2/K^2$

$\therefore \quad s\,Z_{1e} = R_2' - sR_2'$

$\therefore \quad s(Z_{1e} + R_2') = R_2'$

$$s = \frac{R_2'}{(R_2' + Z_{1e})}$$

This is slip at maximum output.

Expression for maximum P_{out} : Using the condition obtained in expression of total P_{out} , we can get maximum P_{out}.

$\therefore \quad (P_{out})_{max} = 3\, I_L^2\, Z_{1e}$

$$= 3\,\frac{V_1^2}{(R_{1e} + Z_{1e})^2 + (X_{1e})^2} \cdot Z_{1e} \quad \text{as} \quad R_L' = Z_{1e}$$

$$= 3\,\frac{V_1^2}{(R_{1e}^2 + 2R_{1e}Z_{1e} + Z_{1e}^2 + X_{1e}^2)} \cdot Z_{1e}$$

But $R^2 + X^2 = Z$

$$\therefore \quad (P_{out})_{max} = 3\,\frac{V_1^2}{2Z_{1e}^2 + 2R_{1e}Z_{1e}} \cdot Z_{1e} = 3\,\frac{V_1^2}{2Z_{1e} + (R_{1e} + Z_{1e})} \cdot Z_{1e}$$

$$\therefore \quad (P_{out})_{max} = \frac{3V_1^2}{2(R_{1e} + Z_{1e})} \text{ watts}$$

The condition for maximum torque can be obtained from maximum power transfer theorem. When $I_{2r}'^2 R_2'/s$ is maximum consider the approximate equivalent circuit of induction motor as shown in The Fig. 8.

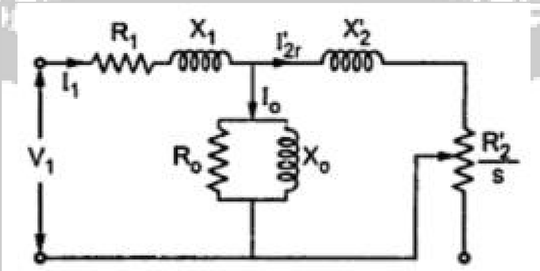

Fig. 8

The value of R_o is assumed to be negligible. Hence the circuit will be reduced as shown below.

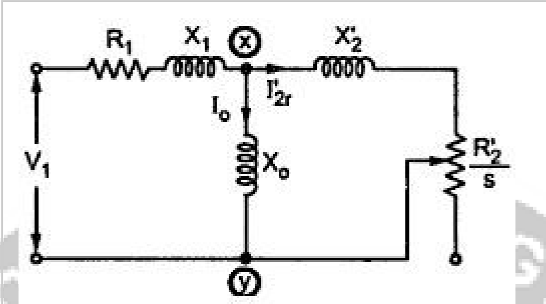

Fig. 3.31 Thevenin's Equivalent Circuit

The thevenin's equivalent circuit for the above network is shown in the Fig.10 across the terminals x and y.

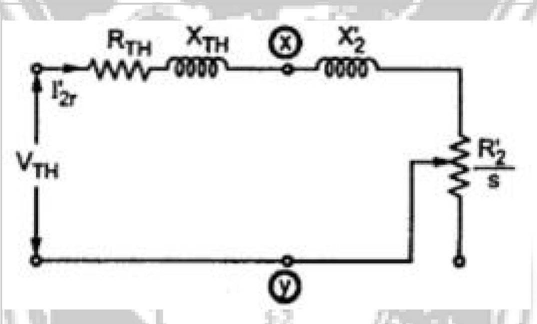

Fig. 3.32 Equivalent Circuit

Now, $$Z_{TH} = R_{TH} + jX_{TH} = (R_1 + j X_1) \| j X_o$$

$$V_{TH} = \frac{V_1(jX_o)}{R_1 + j(X_{TH} + X_o)}$$

The mechanical torque developed by rotor is maximum if there is maximum power transfer to the resistor R_2'/s. This takes place when R_2'/s equals to impedance looking back into the supply source.

$$\frac{R_2'}{s} = R_{TH} + j(X_{TH} + X_2')$$

$$\frac{R_2'}{s} = \sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}$$

$$\therefore \quad s = s_m = \frac{R_2'}{\sqrt{R_{TH}^2 + (X_{TH} + X_2')^2}}$$

This is the slip corresponding to the maximum torque. The maximum torque is given by,

$$T_m = \frac{3}{\omega_s} \cdot \frac{R'_2}{s_m} \cdot (I'_{2r})^2$$

$$\frac{R'_2}{s_m} = \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}$$

$$\therefore \quad I'_{2r} = \frac{V_{TH}}{\sqrt{\left(R_{TH} + \frac{R'_2}{s_m}\right)^2 + (X_{TH} + X'_2)^2}}$$

$$I'^2_{2r} = \frac{V_{TH}^2}{\left(R_{TH} + \frac{R'_2}{s_m}\right)^2 + (X_{TH} + X'_2)^2}$$

Substituting,

$$T_m = \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{V_{TH}^2}{\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\right]^2 + (X_{TH} + X'_2)^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{V_{TH}^2}{2R_{TH}^2 + 2R_{TH}\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} + 2(X_{TH} + X'_2)^2}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{R_{TH}^2}{2\left[R_{TH}^2 + (X_{TH} + X'_2)^2 + R_{TH}\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\right]}$$

$$= \frac{3}{\omega_s} \cdot \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2} \cdot \frac{0.5\, V_{TH}^2}{\sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}\right]}$$

$$\therefore \quad T_m = \frac{3}{\omega_s} \cdot \frac{0.5\, V_{TH}^2}{R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X'_2)^2}}$$

From the above expression, it can be seen that the maximum torque is independent of rotor resistance.

Synchronous Watt

The torque produced in the induction motor is given by,

$$T = \frac{\frac{3(I'_{2r})^2\, R'_2}{s}}{\frac{2\pi N_s}{60}} = \frac{P_2}{\frac{2\pi N_s}{60}} \text{ N-m}$$

Thus torque is directly proportional to the rotor input. By defining new unit of torque which is synchronous watt we can write,

$T = P_2$ synchronous-watts

If torque is given in synchronous-watts then it can be obtained in N-m as,

$$1 \text{ syn-watt} = \frac{60}{2\pi N_s} \text{ N-m}$$

i.e. $$1 \text{ N-m} = \frac{2\pi N_s}{60} \text{ syn-watt}$$

Key Point : Unit synchronous watt can be defined as the torque developed by the motor such that the power input to the rotor across the air gap is 1 W while running at synchronous speed.