1.2 Baye's Theorem

If $B_1, B_2, \dots, B_n$ be a set of exhaustive and mutually exclusive events associated with a random experiment and D is another event associated with B_i, then

$P(D/B_i) = \frac{P(B_i) \cdot P(D/B_i)}{\sum_{i=1}^{n} P(B_i)\, P(D/B_i)}$

State and Prove Bayes Theorem

(OR)

State and Prove Theorem of Probability of Causes.

Soln :

Statement

If $B_1, B_2, \dots$ B_n be a set of exhaustive and mutually exclusive events associated with random experiment and D is another event associated with (or caused)by Bi. Then

$$P(D \mid B_i) = \frac{P(B_i) \cdot P(D/B_i)}{\sum_{i=1}^{n} P(B_i) P(D/B_i)}$$

Proof :

$$P(B_i \cap D) = P(B_i) \cdot P\left(\frac{D}{B_i}\right)$$

$$P(D \cap B_i) = P(D) \cdot P\left(\frac{B_i}{D}\right)$$

$$P(B_i \mid D) = \frac{P(B_i) \cdot P(D/B_i)}{P(D)} \dots \dots \dots (1)$$

The inner circle reprosents the events D.D can occur along with $B_1, B_2, \ldots B_n$ that are exhaustive and mutually exclusive

$\therefore DB_1, DB_1, \ldots . DB_n$ are also mutually exclusive such that

$D = DB_1 + DB_2 + \cdots + DB_n$

$$\therefore D = \sum DB_i$$

$$P[D] = P[\Sigma DB_i]$$

$$= \sum P[DB_i]$$

$$= \sum P[D \cap B_{\mathrm{i}}]$$

$$P[D] = \sum_{i=1}^{n} P(B_i) \cdot P(D/B_i)$$

Substitute $P[D]$ in eqn (1)

$$(1) \Rightarrow P[B_i/D] = \frac{P(B_i) \cdot P(D/B_i)}{\sum_{i=1}^{n} P(B_{\mathrm{i}}) P(D/B_i)}$$

Hence the proof ,

Problem based on Baye's Theorem

1. **Four boxes A,B,C,D contain fuses. The boxes contain 5000, 3000, 2000 and 1000 fuses respectively. The percentages of fuses in boxes which are defective are 3%, 2%, 1% and 5% respectively.one fuse in selected at random arbitrarily from one of the boxes. It is found to be defective fuse. Find the probability that it has come from box D.**

(OR)

Four boxes A,B,C,D contain fuses. Box A contain 5000 fuses , box B contain 3000 fuses, box C contain 2000 fuses and box D contain 1000 fuses. The percentage of fuses in boxes which are defective are 3%, 2%, 1% and 0.5% respectively. One fuse is select at random from one of the boxes. It is found to be defective fuse . What is the probability that it has come from box D.

Soln:

Since selection ratio is not given

Assume selection ratio is 1 : 1 : 1 : 1

Total = 1+1+1+1 = 4

$P(A) = \frac{1}{4}$

$P(B) = \frac{1}{4}$

$P(C) = \frac{1}{4}$

$P(D) = 1/4$

Let E be the event selecting a defective fuse from any one of the machine

$P(E/A) = 3\% = 0.03$

$P(E/B) = 2\% = 0.02$

$P(E/C) = 1\% = 0.01$

$P(E/D) = 5\% = 0.05$

$P(E) \ = P(A)P(E/A) + P(B)P(E/B) + P(C)P(F/\text{C}) + P(D)(F/\text{D})$

$= \frac{1}{4} \times 0.03 + 1/4 \times 0.02 + 1/4 \times 0.01 + 1/4 \times 0.05$

$= 0.0275$

$$P(D/E) \ = \frac{P(D)P(E/D)}{P(E)}$$

$$= \frac{\frac{1}{4}\times 0.05}{0.0275} = 0.4545$$

$= 0.4545$

2. **In a bolt Factory, Machines A,B and C manufacture respectively 25%, 35% and 40% of total output . also out of these output of A,B,C are 5,4,2 percent respectively are defective. A bolt is drawn at random from the total output and it is found to be defective. What is the probability that it was manufactured by the machine B?**

(OR)

In a company machine A, B and C manufactured bolts, 25%, 35% and 40% of total output . also out of these output of A,B,C are 5,4,2 percent respectively are defective. A bolt is taken random from the total output and it is found to be defective. Find the probability that it was manufactured by the machine B?

Soln:

Given ,P(E_1) =P(A) =25% = 0.25

P(E_2) =P(B) =35% = 0.35

P(E_3) =P(C) =40% = 0.40

Let D be the event of drawing defective bolt

$P(D/E_1) = 5\% = \frac{5}{100} = 0.05$

$P(D/E_2) = 4\% = 0.04$

$P(D/E_3) = 2\%) = 0 \cdot 02$

To find $P(\, E_2/D)$

By Bayes theorem

$$P(\, E_2/D) = \frac{P(E_2)P(D/E_2)}{P(E_1)\, P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)}$$

$$= \frac{(0.35)(0.04)}{(0.25)(0.05)+(0.35)(0.04)+(0.4)(0.02)}$$

$$= \frac{0.014}{0.0345}$$

= 0.406

3. **A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the Probability that it was drawn from bag B**

(OR)

A box A contains 2 white and 3 red balls and a box B contains 4 white and 5 red balls at random one ball is taking and is found to be red. What is the probability that it was drawn from bag B?

Soln:

Let B_1 be the event that the ball is drawn from the bag A.

Let B_2 be the event that the ball is drawn from the bag B.

Lat A be the event that the drawn ball is red

$P(B_1) = P(B_2) = \frac{1}{2}$

$P(A/B_1) = \frac{3C_1}{5C_1} = \frac{3}{5}$

$P(A/B_2) = \frac{5C_1}{9C_1} = \frac{5}{9}$

$$P(B_2/A) = \frac{P(B_2)P(A/B_2)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{(\frac{1}{2})(\frac{5}{9})}{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)+(\frac{1}{2})(\frac{5}{9})}$$

$$= \frac{\frac{5}{18}}{\frac{52}{90}}$$

$P(B_2/A)= \frac{25}{52}$