UNIT-III FREE VIBRATION

3.1 INTRODUCTION:

When a system is subjected to an initial disturbance and then left free to vibrate on its own, the resulting vibrations are referred to as free vibrations .**Free vibration** occurs when a mechanical system is set off with an initial input and then allowed to vibrate freely. Examples of this type of vibration are pulling a child back on a swing and then letting go or hitting a tuning fork and letting it ring. The mechanical system will then vibrate at one or more of its "natural frequencies" and damp down to zero.

3.2 BASIC ELEMENTS OF VIBRATION SYSTEM:

- Mass or Inertia
- Springiness or Restoring element
- Dissipative element (often called damper) External excitation
-

3.3 CAUSES OF VIBRATION:

Unbalance: This is basically in reference to the rotating bodies. The uneven distribution of mass in a rotating body contributes to the unbalance. A good example of unbalance related vibration would be the —vibrating alert|| in our mobile phones. Here a small amount of unbalanced weight is rotated by a motor causing the vibration which makes the mobile phone to vibrate. You would have experienced the same sort of vibration occurring in your front loaded washing machines that tend to vibrate during the —spinning|| mode.

Misalignment: This is an other major cause of vibration particularly in machines that are driven by motors or any other prime movers.

Bent Shaft: A rotating shaft that is bent also produces the the vibrating effect since it losses it rotation capability about its center.

Gears in the machine: The gears in the machine always tend to produce vibration, mainly due to their meshing. Though this may be controlled to some extent, any problem in the gearbox tends to get enhanced with ease.

Bearings: Last but not the least, here is a major contributor for vibration. In majority of the cases every initial problem starts in the bearings and propagates to the rest of the members of the machine. A bearing devoid of lubrication tends to wear out fast and fails quickly, but before this is noticed it damages the remaining components in the machine and an initial look would seem as if something had gone wrong with the other components leading to the bearing failure.

3.3.1 Effects of vibration:

(a)Bad Effects:

The presence of vibration in any mechanical system produces unwanted noise, high stresses, poor reliability, wear and premature failure of parts. Vibrations are a great source of human discomfort in the form of physical and mental strains.

(b)Good Effects:

A vibration does useful work in musical instruments, vibrating screens, shakers, relive pain in physiotherapy.

3.4 METHODS OF REDUCTION OF VIBRATION:

- -unbalance is its main cause, so balancing of parts is necessary.
- -using shock absorbers.
- -using dynamic vibration absorbers.
- -providing the screens (if noise is to be reduced)

3.5 TYPES OF VIBRATORY MOTION:

- Free Vibration
- Forced Vibration

3.6 TERMS USED VIBRATORY MOTION:

(a)Time period (or)period of vibration:

It is the time taken by a vibrating body to repeat the motion itself.time period is usually expressed in seconds.

(b) Cycle:

It is the motion completed in one time period.

(c) Periodic motion:

A motion which repeats itself after equal interval of time.

(d)Amplitude (X)

The maximum displacement of a vibrating body from the mean position.it is usually expressed in millimeter.

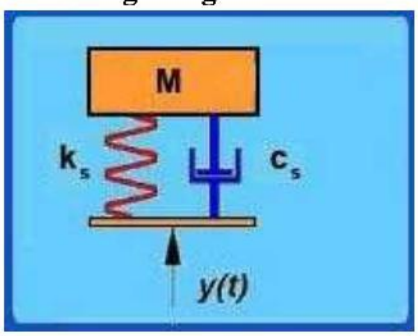
(e) Frequency (f)

The number of cycles completed in one second is called frequency

3.7 DEGREES OF FREEDOM:

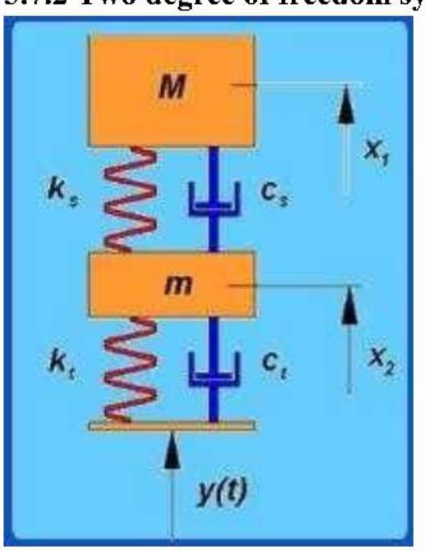
The minimum number of independent coordinates required to specify the motion of a system at any instant is known as D.O.F of the system.

3.7.1 Single degree of freedom system:



The system shown in this figure is what is known as a Single Degree of Freedom system. We use the term degree of freedom to refer to the number of coordinates that are required to specify completely the configuration of the system. Here, if the position of the mass of the system is specified then accordingly the position of the spring and damper are also identified. Thus we need just one coordinate (that of the mass) to specify the system completely and hence it is known as a single degree of freedom system.

3.7.2 Two degree of freedom system:



A two degree of freedom system With reference to automobile applications, this is referred as —quarter car‖ model. The bottom mass refers to mass of axle, wheel etc components which are below the suspension spring and the top mass refers to the mass of the portion of the car and passenger. Since we need to specify both the top and bottom mass positions to completely specify the system, this becomes a two degree of freedom system.

3.8 TYPES OF VIBRATORY MOTION:

The following types of vibratory motion are important from the subject point of view :

1. ***Free or natural vibrations.*** When no external force acts on the body, after giving it an initial displacement, then the body is said to be under ***free or natural vibrations.*** The frequency of the free vibrations is called ***free or natural frequency.***

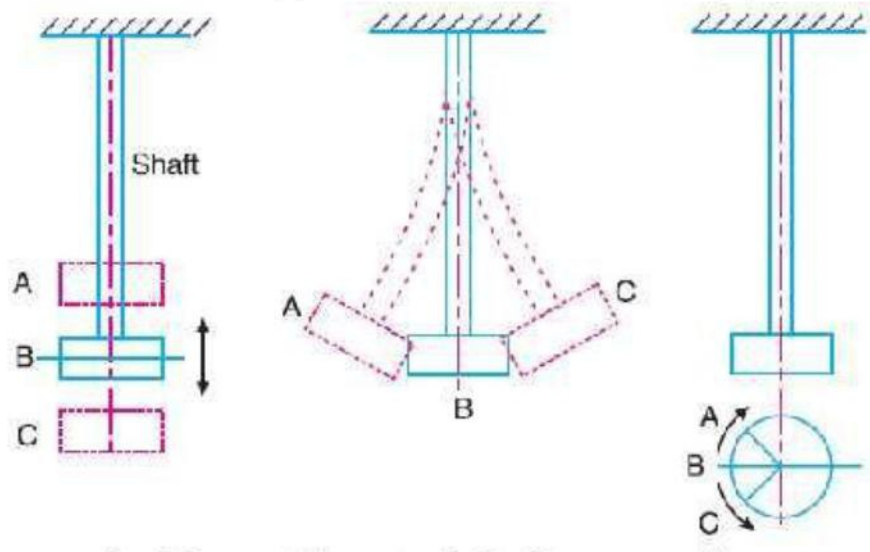
2. ***Forced vibrations.*** When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations.*** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

Note : When the frequency of the external force is same as that of the natural vibrations, resonance takes place.

3. ***Damped vibrations.*** When there is a reduction in amplitude over every cycle of vibration, the motion is said to be ***damped vibration.*** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

Types of Vibration:

(a)Longitudinal vibration (b)Transverse Vibration (c)Torsional Vibration.



B = Mean position ; A and C = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

Longitudinal Vibration:

When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations known as longitudinal vibrations.

Free undamped longitudinal vibrations;

When a body is allowed to vibrate on its own, after giving it an initial displacement, then the ensuring vibrations are known as free or natural vibrations. When the vibrations take place parallel to the axis of constraint and no damping is provided, then it is called free undamped longitudinal vibrations.

3.9 NATURAL FREQUENCY OF FREE UNDAMPED LONGITUDINAL VIBRATION:

3.9.1 Eq u ilib riu m m et h od or Newt on ' s met h od :

Consider a constraint (*i.e.* spring) of negligible mass in an unstrained position.

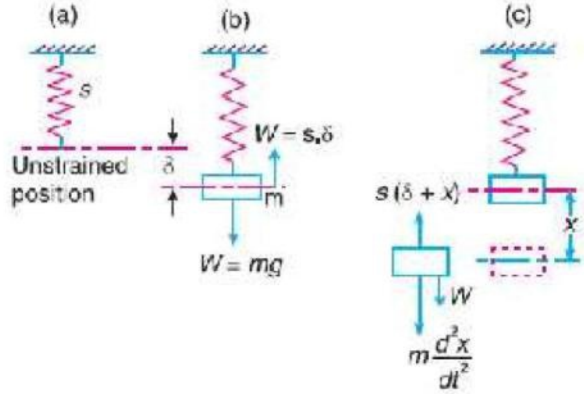
Let s = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/m.

m = Mass of the body suspended from the constraint in kg,

W = Weight of the body in newtons = $m.g$.

δ = Static deflection of the spring in metres due to weight W newtons, and

x = Displacement given to the body by the external force, in metres.



Natural frequency of free longitudinal vibrations.

In the equilibrium position, as shown in Fig. 23.2 (*b*), the gravitational pull $W = m.g$, is balanced by a force of spring, such that $W = s.\delta$.

Since the mass is now displaced from its equilibrium position by a distance x, as shown in Fig. (*c*), and is then released, therefore after time t,

Restoring force $= W - s(\delta + x) = W - s.\delta - s.x$

$= s.\delta - s.\delta - s.x = -s.x$ . . . ($\because W = s.\delta$) . . . (*i*)

and Accelerating force = Mass × Acceleration

$$= m \times \frac{d^2x}{dt^2}$$ . . . (Taking downward force as positive) . . . (*ii*)

Equating equations (*i*) and (*ii*), the equation of motion of the body of mass m after time t is

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \qquad \text{. . . (iii)}$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \qquad \text{. . . (iv)}$$

Comparing equations (*iii*) and (*iv*), we have

$$\omega = \sqrt{\frac{s}{m}}$$

$\therefore$ Time period, $$t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$$

and natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ . . . ($\because$ $m.g = s.\delta$)

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

Note : The value of static deflection δ may be found out from the given conditions of the problem. For longitudinal vibrations, it may be obtained by the relation,

$$\frac{\text{Stress}}{\text{Strain}} = E \quad \text{or} \quad \frac{W}{A} \times \frac{l}{\delta} = E \quad \text{or} \quad \delta = \frac{W.l}{E.A}$$

where δ = Static deflection *i.e.* extension or compression of the constraint,

W = Load attached to the free end of constraint,

l = Length of the constraint,

E = Young's modulus for the constraint, and

A = Cross-sectional area of the constraint.

3.9.2 Energy Method

In free vibrations, no energy is transferred into the system or from the system. Therefore, the total energy (sum of KE and PE)is constant and is same all the times.

We know that the kinetic energy is due to the motion of the body and the potential energy is with respect to a certain datum position which is equal to the amount of work required to move the body from the datum position. In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero.

In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times. In other words,

$$\therefore \quad \frac{d}{dt}(K.E. + P.E.) = 0$$

This industrial compressor uses compressed air to power heavy-duty construction tools. Compressors are used for jobs, such as breaking up concrete or paving, drilling, pile driving, sand-blasting and tunnelling. A compressor works on the same principle as a pump. A piston moves backwards and forwards inside a hollow cylinder, which compresses the air and forces it into a hollow chamber. A pipe or hose connected to the chamber channels the compressed air to the tools.

Note : This picture is given as additional information and is not a direct example of the current chapter.

We know that kinetic energy,

$$K.E. = \frac{1}{2} \times m \left(\frac{dx}{dt} \right)^2$$

and potential energy,

$$P.E. = \left(\frac{0 + s.x}{2} \right) x = \frac{1}{2} \times s.x^2$$

. . . ($\because$ P.E. = Mean force × Displacement)

$$\therefore \quad \frac{d}{dt}\left[\frac{1}{2} \times m \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} \times s.x^2 \right] = 0$$

$$\frac{1}{2} \times m \times 2 \times \frac{dx}{dt} \times \frac{d^2x}{dt^2} + \frac{1}{2} \times s \times 2x \times \frac{dx}{dt} = 0$$

or $$m \times \frac{d^2x}{dt^2} + s.x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$$ . . . (Same as before)

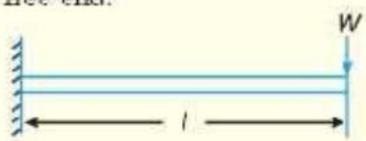
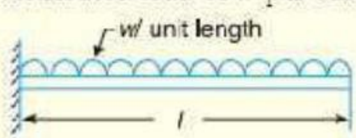
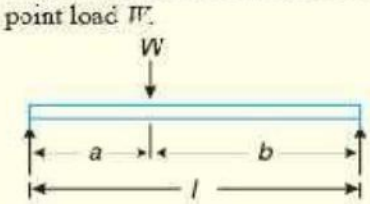
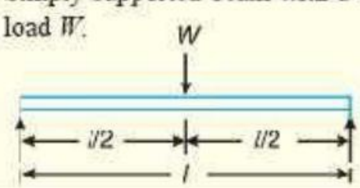
The time period and the natural frequency may be obtained as discussed in the previous method.

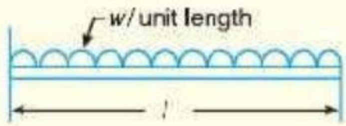
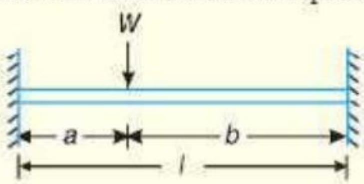
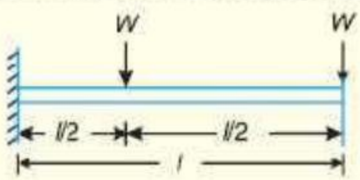
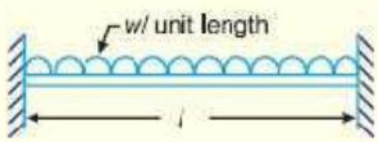
3.9.3 Rayleigh's method

In this method, the maximum kinetic energy at mean position is made equal to the maximum potential energy at the extreme position.

3.10 EQUIVALENT STIFFNESS OF SPRING.

(1) Springs in series
(2) Springs in parallel
(3) Combined springs
(4) Inclined springs

S.No.	*Type of beam*	*Deflection (δ)*
1.	Cantilever beam with a point load W at the free end. 	$\delta = \frac{Wl^3}{3EI}$ (at the free end)
2.	Cantilever beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{wl^4}{8EI}$ (at the free end)
3.	Simply supported beam with an eccentric point load W. 	$\delta = \frac{Wa^2b^2}{3EIl}$ (at the point load)
4.	Simply supported beam with a central point load W. 	$\delta = \frac{Wl^3}{48EI}$ (at the centre)

<table>
<tr><th>S.No.</th><th>Type of beam</th><th>Deflection (δ)</th></tr>
<tr><td>5.</td><td>Simply supported beam with a uniformly distributed load of w per unit length.

</td><td>$\delta = \frac{5}{384} \times \frac{wl^4}{EI}$ (at the centre)</td></tr>
<tr><td>6.</td><td>Fixed beam with an eccentric point load W.

</td><td>$\delta = \frac{Wa^3b^3}{3EIl}$ (at the point load)</td></tr>
<tr><td>7.</td><td>Fixed beam with a central point load W.

</td><td>$\delta = \frac{Wl^3}{192EI}$ (at the centre)</td></tr>
<tr><td>8.</td><td>Fixed beam with a uniformly distributed load of w per unit length.

</td><td>$\delta = \frac{wl^4}{384EI}$ (at the centre)</td></tr>
</table>

3.11 DAMPING:

It is the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibrations.

3.11.1 Types of damping:

(1) Viscous damping
(2) Dry friction or coulomb damping (3) Solid damping or structural damping
(4) Slip or interfacial damping.

3.11.2 Damping Coefficient:

The damping force per unit velocity is known as damping coefficient.

3.11.3 Equivalent damping coefficient:

Dampers may be connected either in series or in parallel to provide required damping.

3.12 DAMPED VIBRATION:

The vibrations associated with this resistance are known as damped vibrations.

3.12.1 Damping factor:

Damping factor can be defined as the ratio of actual damping coefficient to critical damping coefficient.

The ratio of the actual damping coefficient (c) to the critical damping coefficient (c_c) is known as *damping factor* or *damping ratio*. Mathematically,

$$\text{Damping factor} = \frac{c}{c_c} = \frac{c}{2m.\omega_n} \qquad \ldots (\because\ c_c = 2\pi.\omega_n)$$

The damping factor is the measure of the relative amount of damping in the existing system with that necessary for the critical damped system.

Thus mainly three cases arise depending on the value of ξ

$\xi > 1 \Leftrightarrow$ *Overdamped System*

$\xi = 1 \Leftrightarrow$ *Critically damped System*

$\xi < 1 \Leftrightarrow$ *Underdamped System*

When $\xi > 1$ the system undergoes aperiodically decaying motion and hence such systems are said to be **Overdamped Systems**.

An example of such a system is a door damper – when we open a door and enter a room, we want the door to gradually close rather than exhibit $\xi > 1$ oscillatory motion and bang into the person entering the room behind us! So the damper is designed such that

Critically damped motion ($\xi = 1$ a hypothetical borderline case separating oscillatory decay from a periodic decay) I the fastest decaying aperiodic motion.

When — $\xi < 1$ ||, x(t) is a damped sinusoid and the system exhibits a vibratory motion whose amplitude keeps diminishing. This is the most common vibration case and we will spend most of our time studying such systems. These are referred to as **Underdamped systems**.

3.12.2 Logarithmic decrement:

It is defined as the natural logarithm of ratio of any two successive amplitudes of an under damped system. It is a dimensionless quantity.

We define Damping factoras ξ

$$\xi = \frac{c}{2\sqrt{km}}$$

such that $\frac{4km}{c^2} = \frac{1}{\xi^2}$

And $\xi = \frac{c}{2m\omega_n}$

$$\therefore \frac{c}{2m} = \xi\omega_n$$

3.13 TRANSVERSE VIBRATION:

When the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft, then the vibrations known as transverse vibrations.

Consider a shaft of negligible mass, whose one end is fixed and the other end carries a body of weight W, as shown in Fig. 23.3.

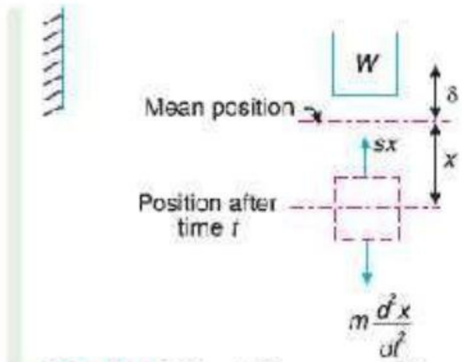


Fig. 23.3. Natural frequency of free transverse vibrations.

Let s = Stiffness of shaft,

δ = Static deflection due to weight of the body,

x = Displacement of body from mean position after time t.

m = Mass of body = W/g

As discussed in the previous article,

Restoring force $= s.x$ . . . (*i*)

and accelerating force $= m \times \frac{d^2x}{dt^2}$ . . . (*ii*)

Equating equations (*i*) and (*ii*), the equation of motion becomes

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$$ . . . (Same as before)

Hence, the time period and the natural frequency of the transverse vibrations are same as that of longitudinal vibrations. Therefore

Time period, $t_p = 2\pi\sqrt{\frac{m}{s}}$

and natural frequency, $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

Note : The shape of the curve, into which the vibrating shaft deflects, is identical with the static deflection curve of a cantilever beam loaded at the end. It has been proved in the text book on Strength of Materials, that the static deflection of a cantilever beam loaded at the free end is

$$\delta = \frac{Wl^3}{3EI} \text{ (in metres)}$$

where W = Load at the free end, in newtons,

l = Length of the shaft or beam in metres,

E = Young's modulus for the material of the shaft or beam in N/m^2, and

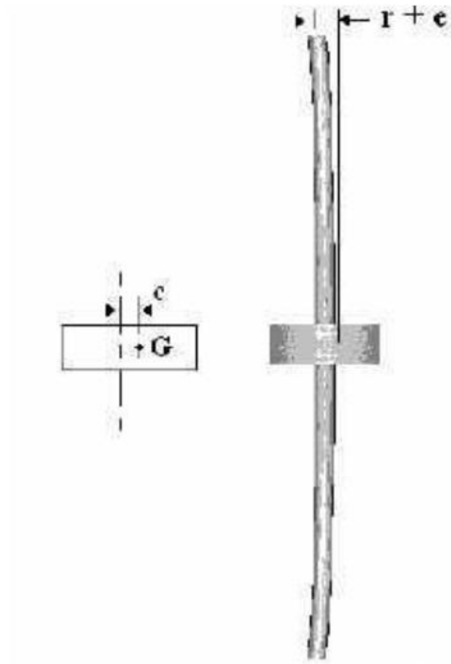
I = Moment of inertia of the shaft or beam in m^4.

3.13.1 Whirling speed of shaft:

The speed, at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.

No shaft can ever be perfectly straight or perfectly balanced. When an element of mass is a distance from the axis of rotation, centrifugal force, will tend to pull the mass outward. The elastic properties of the shaft will act to restore the —straightness‖. If the frequency of rotation is
equal to one of the resonant frequencies of the shaft, whirling will occur. In order to save the machine from failure, operation at such whirling speeds must be avoided.

When a shaft rotates, it may well go into transverse oscillations. If the shaft is out of balance, the resulting centrifugal force will induce the shaft to vibrate. When the shaft rotates at a speed equal to the natural frequency of transverse oscillations, this vibration becomes large and shows up as a whirling of the shaft. It also occurs at multiples of the resonant speed. This can be very damaging to heavy rotary machines such as turbine generator sets and the system must be carefully balanced to reduce this effect and designed to have a natural frequency different to the speed of rotation. When starting or stopping such machinery, the critical speeds must be avoided to prevent damage to the bearings and turbine blades. Consider a weightless shaft as shown with a mass M at the middle. Suppose the centre of the mass is not on the centre line.



The whirling frequency of a symmetric cross section of a given length between two points is given by:

$$N = 94.25\sqrt{\frac{E\ I}{m\ L^3}}\ \text{RPM}$$

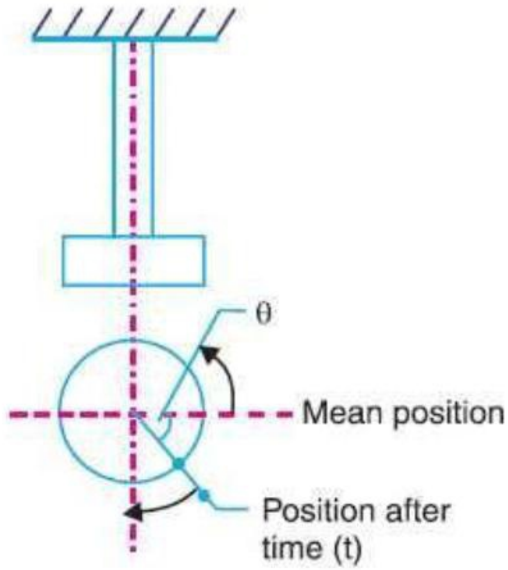
Where E = young's modulus, I = Second moment of area, m = mass of the shaft, L= length of the shaft between points

A shaft with weights added will have an angular velocity of N (rpm) equivalent as follows:

$$\frac{1}{N_N^2} = \frac{1}{N_A^2} + \frac{1}{N_B^2} + \cdots + \frac{1}{N_n^2}$$

3.14 TORSIONAL VIBRATION:

When the particles of the shaft or disc move in a circle about the axis of the shaft, then the vibrations known as tensional vibration



Natural frequency of
free torsional vibrations.

Let θ = Angular displacement of the shaft from mean position after time t in radians,

m = Mass of disc in kg,

I = Mass moment of inertia of disc in kg-m^2 $= m.k^2$,

k = Radius of gyration in metres,

q = Torsional stiffness of the shaft in N-m.

$\therefore$ Restoring force $= q.\theta$ ... (*i*)

and accelerating force $= I \times \frac{d^2\theta}{dt^2}$ ... (*ii*)

Equating equations (*i*) and (*ii*), the equation of motion is

$$I \times \frac{d^2\theta}{dt^2} = -q.\theta$$

or

$$I \times \frac{d^2\theta}{dt^2} + q.\theta = 0$$

$\therefore$

$$\frac{d^2\theta}{dt^2} + \frac{q}{I} \times \theta = 0 \quad \text{. . . } (iii)$$

The fundamental equation of the simple harmonic motion is

$$\frac{d^2\theta}{dt^2} + \omega^2 . x = 0 \quad \text{. . . } (iv)$$

Comparing equations (*iii*) and (*iv*),

$$\omega = \sqrt{\frac{q}{I}}$$

$\therefore$ Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

and natural frequency , $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

The value of the torsional stiffness q may be obtained from the torsion equation,

$$\frac{T}{J}=\frac{C.\theta}{l} \quad \text{or} \quad \frac{T}{\theta}=\frac{C.J}{l}$$

$$q=\frac{C.J}{l} \qquad \ldots \left(\because \frac{T}{\theta}=q\right)$$

where C = Modulus of rigidity for the shaft material,

J = Polar moment of inertia of the shaft cross-section,

$=\frac{\pi}{32}d^4$; d is the diameter of the shaft, and

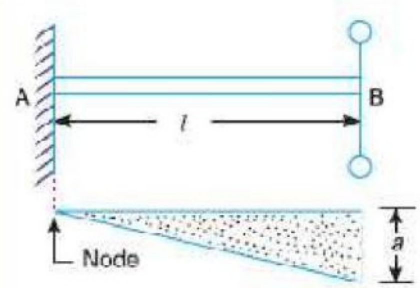
l = Length of the shaft.

3.14.1 Torsional vibration of a single rotor system:

We have already discussed that for a shaft fixed at one end and carrying a rotor at the free end as shown in Fig. the natural frequency of torsional vibration,

$$f_n \quad \frac{1}{2}\sqrt{\frac{q}{I}} \quad \frac{1}{2}\sqrt{\frac{C.J}{l.I}}$$

$$\ldots \quad q \quad \frac{C.J}{l}$$



Free torsional vibrations of a single rotor system.

where C = Modulus of rigidity for shaft material,

J = Polar moment of inertia of shaft

$= \frac{}{32} d^4$

d = Diameter of shaft,

l = Length of shaft,

m = Mass of rotor,

---

k = Radius of gyration of rotor, and

I = Mass moment of inertia of rotor = $m.k^2$

A little consideration will show that the amplitude of vibration is zero at A and maximum at B, as shown in Fig. It may be noted that the point or the section of the shaft whose amplitude of torsional vibration is zero, is known as ***node***. In other words, at the node, the shaft remains unaffected by the vibration.

3.14.2 Torsional vibration of a two rotor system:

Consider a two rotor system as shown in Fig.   It consists of a shaft with two rotors at its ends. In this system, the torsional vibrations occur only when the two rotors A and B move in opposite directions *i.e.* if A moves in anticlockwise direction then B moves in clockwise direction at the same instant and *vice versa*. It may be noted that the two rotors must have the same frequency.

We see from Fig.   that the node lies at point N. This point can be safely assumed as a fixed end and the shaft may be considered as two separate shafts NP and NQ each fixed to one of its ends and carrying rotors at the free ends.

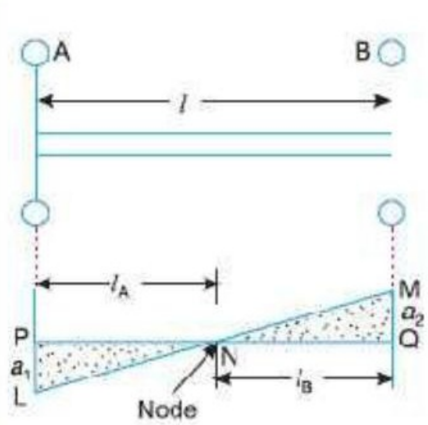


Fig   Free torsional vibrations of a two rotor system.

Let l = Length of shaft,

l_A = Length of part NP *i.e.* distance of node from rotor A,

l_B = Length of part NQ, *i.e.* distance of node from rotor B,

I_A = Mass moment of inertia of rotor A,

I_B = Mass moment of inertia of rotor B,

d = Diameter of shaft,

J = Polar moment of inertia of shaft, and

C = Modulus of rigidity for shaft material.

Natural frequency of torsional vibration for rotor A,

$$f_{nA} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_A.I_A}} \qquad \ldots (i)$$

and natural frequency of torsional vibration for rotor B,

$$f_{nB} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_B.I_B}} \qquad \ldots (ii)$$

Since $f_{nA} = f_{nB}$, therefore

$$\frac{1}{2}\sqrt{\frac{C.J}{l_A.I_A}} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_B.I_B}} \quad \text{or} \quad l_A \cdot I_A = l_B \cdot I_B \qquad \ldots (iii)$$

$$l_A \quad \frac{l_B.I_B}{I_A}$$

We also know that

$$l \quad l_A \quad l_B \qquad \ldots (iv)$$

3.14.3 Torsionally equivalent shaft:

we have assumed that the shaft is of uniform diameter. But in actual practice, the shaft may have variable diameter for different lengths. Such a shaft may, theoretically, be replaced by an equivalent shaft of uniform diameter.

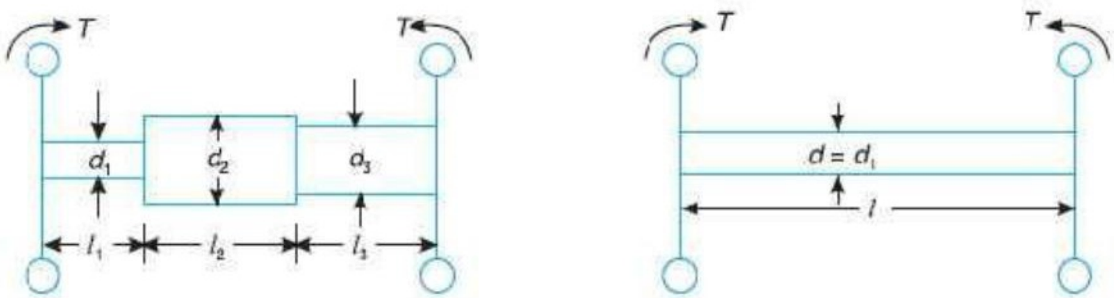
Consider a shaft of varying diameters as shown in Fig. (*a*). Let this shaft is replaced by an equivalent shaft of uniform diameter d and length l as shown in Fig. (*b*). These two shafts must have the same total angle of twist when equal opposing torques T are applied at their opposite ends.

Let d_1, d_2 and d_3 = Diameters for the lengths l_1, l_2 and l_3 respectively,

$_1$, $_2$ and $_3$ = Angle of twist for the lengths l_1, l_2 and l_3 respectively,

= Total angle of twist, and

J_1, J_2 and J_3 = Polar moment of inertia for the shafts of diameters d_1, d_2 and d_3 respectively.



(*a*) Shaft of varying diameters. (*b*) Torsionally equivalent shaft.

Fig 24.8

Since the total angle of twist of the shaft is equal to the sum of the angle of twists of different lengths, therefore

$$_1 \quad _2 \quad _3$$

or
$$\frac{T.l}{C.J} \quad \frac{T.l_1}{C.J_1} \quad \frac{T.l_2}{C.J_2} \quad \frac{T.l_3}{C.J_3}$$

$$\frac{l}{J} \quad \frac{l_1}{J_1} \quad \frac{l_2}{J_2} \quad \frac{l_3}{J_3}$$

3.15 SOLVED ROBLEMS

1.A machine of mass 75 kg is mounted on springs and is fitted with a dashpot to damp out

the amplitude of vibration diminishes from 38.4 mm to 6.4 mm in two oscillations. Assuming that the damping force varies as the velocity, de resistance of the dash-pot at unit velocity ; 2. vibration to the frequen undamped vibration ; and 3. damped vibration.

$$\frac{l}{\frac{}{32} d^4} \quad \frac{l_1}{\frac{}{32}(d_1)^4} \quad \frac{l_2}{\frac{}{32}(d_2)^4} \quad \frac{l_3}{\frac{}{32}(d_3)^4}$$

$$\frac{l}{d^4} \quad \frac{l_1}{(d_1)^4} \quad \frac{l_2}{(d_2)^4} \quad \frac{l_3}{(d_3)^4}$$

In actual calculations, it is assumed that the diameter d of the equivalent shaft is equal to one of the diameter of the actual shaft. Let us assume that $d = d_1$.

$$\frac{l}{(d_1)^4} \quad \frac{l_1}{(d_1)^4} \quad \frac{l_2}{(d_2)^4} \quad \frac{l_3}{(d_3)^4}$$

or
$$l \quad l_1 \quad l_2 \left(\frac{d_1}{d_2}\right)^4 \quad l_3 \left(\frac{d_1}{d_3}\right)^4$$

This expression gives the length l of an equivalent shaft.

vibrations. There are three springs each of stiffness 10 N/mm and it is found that

1. the

the ratio of the frequency of the damped the periodic time of the

Solution. Given : m = 75 kg ; s = 10 N/mm = 10 ×10^3 N/m ; x_1 = 38.4 mm = 0.0384 m ; x_3 = 6.4 mm = 0.0064 m

Since the stiffness of each spring is 10 × 10^3 N/m and there are 3 springs, therefore total stiffness,

$$s = 3\times10\times10^3 = 30\times10^3 \text{ N/m}$$

We know that natural circular frequency of motion,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{30\times10^3}{75}} = 20 \text{ rad/s}$$

1. ***Resistance of the dashpot at unit velocity***

Let c = Resistance of the dashpot in newtons at unit velocity *i.e.* in N/m/s,

x_2 = Amplitude after one complete oscillation in metres, and

x_3 = Amplitude after two complete oscillations in metres.

We know that $\frac{x_1}{x_2} = \frac{x_2}{x_3}$

$$\therefore \quad \left(\frac{x_1}{x_2}\right)^2 = \frac{x_1}{x_3} \quad \ldots \left[\because \frac{x_1}{x_3} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} = \frac{x_1}{x_2} \times \frac{x_1}{x_2} = \left(\frac{x_1}{x_2}\right)^2\right]$$

or

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_3}\right)^{1/2} = \left(\frac{0.0384}{0.0064}\right)^{1/2} = 2.45$$

We also know that

$$\log_e\left(\frac{x_1}{x_2}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

2. ***2. Ratio of the frequency of the damped vibration to the frequency of undamped vibration***

Let f_{n1} = Frequency of damped vibration = $\frac{\omega_d}{2\pi}$

f_{n2} = Frequency of undamped vibration = $\frac{\omega_n}{2\pi}$

$$\therefore \quad \frac{f_{n1}}{f_{n2}} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \frac{\sqrt{(\omega_n)^2 - a^2}}{\omega_n} = \frac{\sqrt{(20)^2 - (2.8)^2}}{20} = 0.99 \text{ Ans.}$$

3. Periodic time of damped vibration

We know that periodic time of damped vibration

$$= \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{2\pi}{\sqrt{(20)^2 - (2.8)^2}} = 0.32 \text{ s Ans.}$$

2. The mass of a single degree damped vibrating system is 7.5 kg and makes 24 free oscillations in 14 seconds when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Determine : 1. stiffness of the spring, 2. logarithmic decrement, and 3. damping factor, i.e. the ratio of the system damping to critical damping.

Solution. Given : m = 7.5 kg

Since 24 oscillations are made in 14 seconds, therefore frequency of free vibrations,

$$f_n = 24/14 = 1.7$$

and

$$\omega_n = 2\pi \times f_n = 2\pi \times 1.7 = 10.7 \text{ rad/s}$$

1. Stiffness of the spring

Let s = Stiffness of the spring in N/m.

We know that $(\omega_n)^2 = s/m$ or $s = (\omega_n)^2 m = (10.7)^2 7.5 = 860$ N/m Ans.

$$\log_e 2.45 = a \times \frac{2\pi}{\sqrt{(20)^2 - a^2}}$$

$$0.8951 = \frac{a \times 2\pi}{\sqrt{400 - a^2}} \quad \text{or} \quad 0.8 = \frac{a^2 \times 39.5}{400 - a^2} \qquad \text{. . . (Squaring both sides)}$$

$\therefore$ $a^2 = 7.94$ or $a = 2.8$

We know that $a = c / 2m$

$\therefore$ $c = a \times 2m = 2.8 \times 2 \times 75 = 420$ N/m/s **Ans.**

3. *Damping factor*

Let c = Damping coefficient for the actual system, and

c_c = Damping coefficient for the critical damped system. .. (Given)

We know that logarithmic decrement (δ),

$$0.28 = \frac{a \times 2\pi}{\sqrt{(\omega_n)^2 - a^2}} = \frac{a \times 2\pi}{\sqrt{(10.7)^2 - a^2}} \qquad \left[\frac{x_4}{x_5} = \frac{x_5}{x_6} \right.$$

or

$$\frac{x_1}{x_2} = \left(\frac{x_1}{x_3}\right)^{1/5} = \left(\frac{x_1}{0.25\,x_1}\right)^{1/5} = (4)^{1/5} = 1.32$$

We know that logarithmic decrement,

$$\delta = \log_e\left(\frac{x_1}{x_2}\right) = \log_e 1.32 = 0.28 \text{ Ans.}$$

$$0.0784 = \frac{a^2 \times 39.5}{114.5 - a^2} \qquad \text{. . . (Squaring both sides)}$$

$8.977 - 0.0784\,a^2 = 39.5\,a^2$ or $a^2 = 0.227$ or $a = 0.476$

We know that $a = c / 2m$ or $c = a \times 2m = 0.476 \times 2 \times 7.5 = 7.2$ N/m/s **Ans.**

and $c_c = 2m\omega_n = 2 \times 7.5 \times 10.7 = 160.5$ N/m/s **Ans.**

$\therefore$ Damping factor $= c/c_c = 7.2 / 160.5 = 0.045$ **Ans.**

3(i) The measurements on a mechanical vibrating system show that it has a mass of 8 kg and that the springs can be combined to give an equivalent spring of stiffness 5 N/mm. If the vibrating system have a dashpot attached which exerts a force of when the mass has a velocity of 1 m/s, find : 1. critical damping coefficient, 2. logarithmic decrement, and 4. ratio of two consecutive amplitudes.

damping factor, 3.

Solution. Given : $m = 8$ kg ; $s = 5.4$ N/mm = 5400 N/m

1. *Critical damping coefficient*

We know that critical damping coefficient,

$$c_c = 2m\omega_n = 2m \times \sqrt{\frac{s}{m}} = 2 \times 8\sqrt{\frac{5400}{8}} = 416 \text{ N/m/s Ans.}$$

Since the force exerted by dashpot is 40 N, and the mass has a velocity of 1 m/s , therefore Damping coefficient (actual),

2. Damping factor

We know that damping factor

$$= \frac{c}{c_c} = \frac{40}{416} = 0.096 \text{ Ans.}$$

c = 40 N/m/s

3. Logarithmic decrement

We know that logarithmic decrement,

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}} = \frac{2\pi \times 40}{\sqrt{(416)^2 - (40)^2}} = 0.6 \text{ Ans.}$$

4. Ratio of two consecutive amplitudes

Let x_n and x_{n-1} = Magnitude of two consecutive amplitudes,

We know that logarithmic decrement,

$$\delta = \log_e \left[\frac{x_n}{x_{n+1}}\right] \text{ or } \frac{x_n}{x_{n+1}} = e^{\delta} = (2.7)^{0.6} = 1.82 \text{ Ans.}$$

3 (ii) An instrument vibrates with a frequency of 1 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 0.9 Hz. Find 1. the damping factor, and 2. logarithmic decrement.

Solution. Given : f_n = 1 Hz ; f_d = 0.9 Hz

1. ***Damping factor***

Let m = Mass of the instrument in kg,

c = Damping coefficient or damping force per unit velocity in N/m/s, and

c_c= Critical damping coefficient in N/m/s.

We know that natural circular frequency of undamped vibrations,

$$\omega_n = 2\pi \times f_n = 2\pi \times 1 = 6.284 \text{ rad/s}$$

and circular frequency of damped vibrations,

$$\omega_d = 2\pi \times f_d = 2\pi \times 0.9 = 5.66 \text{ rad/s}$$

We also know that circular frequency of damped vibrations (ω_d),

$$5.66 = \sqrt{(\omega_n)^2 - a^2} = \sqrt{(6.284)^2 - a^2}$$

Squaring both sides

$$(5.66)^2 = (6.284)^2 - a^2 \text{ or } 32 = 39.5 - a^2$$

$\therefore$ $a^2 = 7.5$ or $a = 2.74$

We know that, $a = c/2m$ or $c = a \times 2m = 2.74 \times 2m = 5.48\, m$ N/m/s

and $c_c = 2m\omega_n = 2m \times 6.284 = 12.568\, m$ N/m/s

$\therefore$ Damping factor,

$$c/c_c = 5.48m/12.568\,m = 0.436 \textbf{ Ans.}$$

4(i) A coil of spring stiffness 4 N/mm supports vertically a mass of 20 kg at the free end. The motion is resisted by the oil dashpot. It is found that the amplitude at the beginning of the fourth cycle is 0.8 times the amplitude of the previous vibration. Determine the damping force per unit velocity. Also find the ratio of the frequency of damped and undamped vibrations.

Solution. Given : s = 4 N/mm = 4000 N/m ; m = 20 kg

Damping force per unit velocity

Let c = Damping force in newtons per unit velocity *i.e.* in N/m/s x_n
= Amplitude at the beginning of the third cycle,

x_{n-1} = Amplitude at the beginning of the fourth cycle = 0.8 x_n

We know that natural circular frequency of motion,

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{4000}{20}} = 14.14 \text{ rad/s}$$

and
$$\log_e\left(\frac{x_n}{x_{n+1}}\right) = a \times \frac{2\pi}{\sqrt{(\omega_n)^2 - a^2}}$$

or
$$\log_e\left(\frac{x_n}{0.8\,x_n}\right) = a \times \frac{2\pi}{\sqrt{(14.14)^2 - a^2}}$$

$$\log_e 1.25 - a \times \frac{2\pi}{\sqrt{200 - a^2}} \quad \text{or} \quad 0.223 = a \times \frac{2\pi}{\sqrt{200 - a^2}}$$

Squaring both sides

$$0.05 = \frac{a^2 \times 4\pi^2}{200 - a^2} = \frac{39.5\,a^2}{200 - a^2}$$

$$0.05 \times 200 - 0.05\,a^2 = 39.5a^2 \quad \text{or} \quad 39.55\,a^2 = 10$$

$\therefore$ $a^2 = 10 / 39.55 = 0.25$ or $a = 0.5$

We know that $a - c / 2m$

$\therefore$ $c - a \times 2m - 0.5 \times 2 \times 20 - 20$ N/m/s Ans.

Ratio of the frequencies

Let f_{n1} = Frequency of damped vibrations = $\frac{\omega_d}{2\pi}$

f_{n2} = Frequency of undamped vibrations = $\frac{\omega_n}{2\pi}$

$\therefore$

$$\frac{f_{n1}}{f_{n2}} = \frac{\omega_d}{2\pi} \times \frac{2\pi}{\omega_n} = \frac{\omega_d}{\omega_n} = \sqrt{\frac{(\omega_n)^2 - a^2}{\omega_n}} = \sqrt{\frac{(14.14)^2 - (0.5)^2}{14.14}}$$

$$\ldots \left(\because\ \omega_d = \sqrt{(\omega_n)^2 - a^2}\right)$$

$= 0.999$ Ans.

4(ii) Derive an expression for the natural frequency of single degrees of freedom system.

We know that the kinetic energy is due to the motion of the body and the potential energy is with respect to a certain datum position which is equal to the amount of work required to move the body from the datum position. In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero.

In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times. In other words,

$$\therefore \quad \frac{d}{dt}(K.E. + P.E.) = 0$$

We know that kinetic energy,

$$K.E. = \frac{1}{2} \times m \left(\frac{dx}{dt}\right)^2$$

and potential energy,

$$P.E. = \left(\frac{0 + s.x}{2}\right) x = \frac{1}{2} \times s.x^2$$

... ($\because$ P.E. = Mean force × Displacement)

$$\therefore \quad \frac{d}{dt}\left[\frac{1}{2} \times m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} \times s.x^2\right] = 0$$

$$\frac{1}{2} \times m \times 2 \times \frac{dx}{dt} \times \frac{d^2x}{dt^2} + \frac{1}{2} \times s \times 2x \times \frac{dx}{dt} = 0$$

or

$$m \times \frac{d^2x}{dt^2} + s.x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2 . x = 0$$

Comparing equations,

$$\omega = \sqrt{\frac{s}{m}}$$

$$\therefore \quad \text{Time period,} \quad t_p = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{s}}$$

and natural frequency,

$$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \quad \text{... } (\because m.g = s.\delta)$$

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

5. A vertical shaft of 5 mm diameter is 200 mm long and is supported in long bearings at its ends. A disc of mass 50 kg is attached to the centre of the shaft. Neglecting any increase in stiffness due to the attachment of the disc to the shaft, find the critical speed of rotation

and the maximum bending stress when the shaft is rotating at 75% of the critical speed. The centre of the disc is 0.25 mm from the geometric axis of the shaft. E = 200 GN/m².

Solution. Given : $d = 5$ mm $= 0.005$ m ; $l = 200$ mm $= 0.2$ m ; $m = 50$ kg ; $e = 0.25$ mm $= 0.25 \times 10^{-3}$ m ; $E = 200$ GN/m² $= 200 \times 10^9$ N/m²

Critical speed of rotation

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.005)^4 = 30.7 \times 10^{-12}\ \text{m}^4$$

Since the shaft is supported in long bearings, it is assumed to be fixed at both ends. We know that the static deflection at the centre of the shaft due to a mass of 50 kg,

$$\delta = \frac{Wl^3}{192\,EI} = \frac{50 \times 9.81 (0.2)^3}{192 \times 200 \times 10^9 \times 30.7 \times 10^{-12}} = 3.33 \times 10^{-3}\ \text{m}$$

$$\ldots (\because W = mg)$$

We know that critical speed of rotation (or natural frequency of transverse vibrations),

$$N_c = \frac{0.4985}{\sqrt{3.33 \times 10^{-3}}} = 8.64 \text{ r.p.s. } \textbf{Ans.}$$

Maximum bending stress

Let σ = Maximum bending stress in N/m², and

N = Speed of the shaft = 75% of critical speed = $0.75\, N_c$ . . . (Given)

When the shaft starts rotating, the additional dynamic load (W_1) to which the shaft is subjected, may be obtained by using the bending equation,

$$\frac{M}{I} = \frac{\sigma}{y_1} \quad \text{or} \quad M = \frac{\sigma . I}{y_1}$$

We know that for a shaft fixed at both ends and carrying a point load (W_1) at the centre, the maximum bending moment

$$M = \frac{W_1 . l}{8}$$

$$\therefore \quad \frac{W_1 . l}{8} = \frac{\sigma \cdot I}{d/2} \qquad \ldots (\because y_1 = d/2)$$

and

$$W_1 = \frac{\sigma . I}{d/2} \times \frac{8}{l} = \frac{\sigma \times 30.7 \times 10^{-12}}{0.005/2} \times \frac{8}{0.2} = 0.49 \times 10^{-6}\sigma\text{N}$$

$\therefore$ Additional deflection due to load W_1,

$$y = \frac{W_1}{W} \times \delta = \frac{0.49 \times 10^{-6}\sigma}{50 \times 9.81} \times 3.33 \times 10^{-3} = 3.327 \times 10^{-12}\sigma$$

We know that

$$y = \frac{\pm e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1} = \frac{\pm e}{\left(\frac{N_c}{N}\right)^2 - 1} \qquad \ldots (\text{Substituting } \omega_c = N_c \text{ and } \omega = N)$$

$$3.327\times10^{-12}\sigma = \frac{\pm0.25\times10^{-3}}{\left(\frac{N_c}{0.75\,N_c}\right)^2 - 1} = \pm0.32\times10^{-3}$$

$$\sigma = 0.32\times10^{-3}/3.327\times10^{-12} = 0.0962\times10^{9}\ \text{N/m}^2 \quad \text{...(Taking + ve sign)}$$

$$= 96.2\times10^{6}\ \text{N/m}^2 = 96.2\ \text{MN/m}^2 \quad \textbf{Ans.}$$

6.(i) A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support. The Young's modulus for shaft material is 200 GN/m². Find the frequency of transverse vibration.

Solution. Given :^{2} = 200 × d = 50 mm = 0.05 m ;10^9 N/m^2l = 3 m, W_1 = 1000 N ; W_2 = 1500 N ; W_3 = 750 N;

E = 200 GN/m

The shaft carrying the loads is shown in Fig. 23.13

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64}\times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307\times10^{-6}\ \text{m}^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3EIl}$$

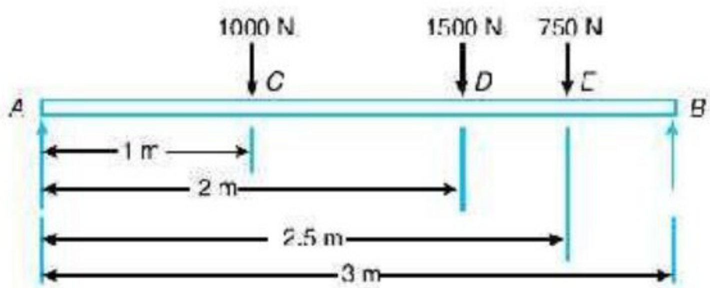


Fig. 23.13

∴ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000\times1^2\times2^2}{3\times200\times10^9\times0.307\times10^{-6}\times3} = 7.24\times10^{-3}\ \text{m}$$

... (Here a = 1 m, and b = 2 m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500\times2^2\times1^2}{3\times200\times10^9\times0.307\times10^{-6}\times3} = 10.86\times10^{-3}\ \text{m}$$

... (Here a = 2 m, and b = 1 m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750\,(2.5)^2\,(0.5)^2}{3\times200\times10^9\times0.307\times10^{-6}\times3} = 2.12\times10^{-3}\ \text{m}$$

... (Here a = 2.5 m, and b = 0.5 m)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1+\delta_2+\delta_3}} = \frac{0.4985}{\sqrt{7.24\times10^{-3}+10.86\times10^{-3}+2.12\times10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz} \text{ Ans.}$$

6.(ii) Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m³, and Young's modulus is 200 GN/m². Assume the shaft to be freely supported.

Solution. Given : d = 20 mm = 0.02 m ; l = 0.6 m ; m_1 = 1 kg ; ρ = 40 Mg/m³ = 40 × 10⁶ g/m³ = 40 × 10³ kg/m³ ; E = 200 GN/m² = 200 × 10⁹ N/m²

The shaft is shown in Fig. 23.15.

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64}\times d^4 = \frac{\pi}{64}(0.02)^4 \text{ m}^4$$

$$= 7.855 \times 10^{-9} \text{ m}^4$$

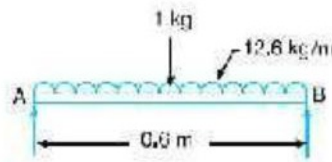


Fig. 23.15

Since the density of shaft material is 40 × 10³ kg/m³, therefore mass of the shaft per metre length,

$$m_S = \text{Area}\times\text{length}\times\text{density} = \frac{\pi}{4}(0.02)^2\times1\times40\times10^3 = 12.6 \text{ kg/m}$$

We know that static deflection due to 1 kg of mass at the centre,

$$\delta = \frac{Wl^3}{48\,EI} = \frac{1\times9.81(0.6)^3}{48\times200\times10^9\times7.855\times10^{-9}} = 28\times10^{-6} \text{ m}$$

and static deflection due to mass of the shaft,

$$\delta_S = \frac{5\,wl^4}{384\,EI} = \frac{5\times12.6\times9.81(0.6)^4}{384\times200\times10^9\times7.855\times10^{-9}} = 0.133\times10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta+\frac{\delta_S}{1.27}}} = \frac{0.4985}{\sqrt{28\times10^{-6}+\frac{0.133\times10^{-3}}{1.27}}}$$

$$= \frac{0.4985}{11.52\times10^{-3}} = 43.3 \text{ Hz}$$

Let N_c = Whirling speed of a shaft.

We know that whirling speed of a shaft in r.p.s. is equal to the frequency of transverse vibration in Hz , therefore

$$N_c = 43.3 \text{ r.p.s.} = 43.3\times60 = 2598 \text{ r.p.m.} \text{ Ans.}$$

3.16 REVIEW QUESTIONS

1. When a body is subjected to transverse vibrations, the stress induced in a body will be ?

2. In under damped vibrating system, if x_1 and x_2 are the successive values of the amplitude on the same side of the mean position, then the logarithmic decrement is equal to?

3. Discuss the effect of inertia of the shaft in longitudinal and transverse vibrations?

4. How the natural frequency of torsional vibrations for a two rotor system is obtained?

5. . At a nodal point in a shaft, the amplitude of torsional vibration is?

3.17 TUTORIAL PROBLEMS

1. A beam of length 10 m carries two loads of mass 200 kg at distances of 3 m from each end together with a central load of mass 1000 kg. Calculate the frequency of transverse vibrations.
Neglect the mass of the beam and take $I = 10^9$ mm^4 and $E = 205\times10^3$ N/mm^2. **[Ans. 13.8 Hz]**

2.A vertical shaft 25 mm diameter and 0.75 m long is mounted in long bearings and carries a pulley of mass 10 kg midway between the bearings. The centre of pulley is 0.5 mm from the axis of the shaft. Find (*a*) the whirling speed, and (*b*) the bending stress in the shaft, when it is rotating at 1700 r.p.m. Neglect the mass of the shaft and E = 200 GN/m^2. **[Ans. 3996 r.p.m ; 12.1 MN/m^2]**

3.A shaft of 100 mm diameter and 1 metre long is fixed at one end and the other end carries a flywheel of mass 1 tonne. The radius of gyration of the flywheel is 0.5 m. Find the frequency of torsional vibrations, if the modulus of rigidity for the shaft material is 80 GN/m^2. [Ans. 8.9 Hz]

4.The flywheel of an engine driving a dynamo has a mass of 180 kg and a radius of gyration of 30 mm. The shaft at the flywheel end has an effective length of 250 mm and is 50 mm diameter. The armature mass 120 kg and its radius of gyration is 22.5 mm. The dynamo shaft is 43 is mm diameter and mm effective length. Calculate the position of node and frequency of 200 torsional oscillation. C = 83 kN/mm^2.[Ans. 205 mm from flywheel, 218 Hz]

5. The two rotors A and B are attached to the end of a shaft 500 mm long. The mass of the rotor A is 300 kg and its radius of gyration is 300 mm. The corresponding values of the rotor B are 500 kg and 450 mm respectively. The shaft is 70 mm in diameter for the first 250 mm ; 120 mm for the next 70 mm and 100 mm diameter for the remaining length. The modulus of rigidity for the shaft material is 80 GN/m^2. Find : 1. The position of the node, and 2. The frequency of torsional vibration. [Ans. 225 mm from A ; 27.3 Hz]